The Dirac Delta Function and the Fourier Transform

Podcast Learn & Fun *

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The Dirac delta function, denoted as $\delta(x)$, is a generalized function (or distribution) used extensively in physics and engineering, especially in signal processing, quantum mechanics, and differential equations. Despite its name, the delta function is not a traditional function but rather a distribution that represents an idealized point source or impulse. In the context of Fourier analysis, the delta function can also be represented using exponential functions.

Properties of the delta function

Sifting Property

$$\int_{-\infty}^{\infty} \delta(x-a) f(x)\, dx = f(a)$$

This property "picks out" the value of the function $f(x)$ at $x = a$.

Normalization

$$\int_{-\infty}^{\infty} \delta(x)\, dx = 1$$

Support The delta function is zero everywhere except at $x = 0$, where it is singular, and its integral over the entire real line is 1.

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Delta Function in Terms of Exponentials

The delta function can be viewed as the inverse Fourier transform of a constant function. Specifically, if we take the Fourier transform of $\delta(x)$, we get:

$$\hat{\delta}(k) = \int_{-\infty}^{\infty} \delta(x) e^{-ikx} \, dx = 1$$

Since the Fourier transform of $\delta(x)$ is 1, the inverse Fourier transform can be written as the integral:

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx} \, dk$$

This shows that the delta function is a superposition of sinusoidal waves with all possible wave numbers, k, each weighted equally.