Coherent Quantum States

Podcast Learn & Fun [*]

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In quantum mechanics, coherent states are specific types of quantum states that exhibit properties resembling classical harmonic oscillators. These states have been extensively studied due to their remarkable resemblance to classical physics, and they play a key role in many areas of quantum optics, quantum information, and condensed matter physics.

We will cover the definition of coherent states, their properties, their mathematical construction, and their relevance in physical systems such as the quantum harmonic oscillator. We will also delve into the significance of coherent states in applications like quantum optics and the behavior of quantum fields.

Preliminary Concepts

Before we define and explore coherent states, it's crucial to review the quantum harmonic oscillator (QHO), as it serves as the foundational model for understanding coherent states. The quantum harmonic oscillator is described by the Hamiltonian:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2$$

where $\hat{p}$ is the momentum operator, $\hat{x}$ is the position operator, m is the mass, and ω is the angular frequency. The eigenstates of the harmonic oscillator Hamiltonian, $|n\rangle$, form a discrete set corresponding to energy eigenvalues:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, \quad n = 0, 1, 2, \ldots$$

These eigenstates are called *number states* or *Fock states*. The quantum harmonic oscillator plays an essential role in the study of coherent states

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because coherent states are closely related to the oscillatory behavior of the system.

We define the *raising* (creation) operator $\hat{a}^\dagger$ and the *lowering* (annihilation) operator $\hat{a}$, which can be written in terms of position and momentum as:

$$\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}}\left(m\omega\hat{x} - i\hat{p}\right)$$

$$\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}}\left(m\omega\hat{x} + i\hat{p}\right)$$

These operators satisfy the following commutation relation:

$$[\hat{a}, \hat{a}^\dagger] = 1$$

Definition and Derivation of Coherent States

A coherent state $|\alpha\rangle$ is an eigenstate of the annihilation operator $\hat{a}$:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

where α is a complex number. These states are generally not eigenstates of the number operator $\hat{N} = \hat{a}^\dagger\hat{a}$, but they are superpositions of number states:

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n|n\rangle$$

where c_n are the coefficients of the expansion.

The annihilation operator $\hat{a}$ acts on the number states $|n\rangle$ as follows:

$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$$

This property shows that $\hat{a}$ lowers the quantum number n by 1 and provides a prefactor $\sqrt{n}$.

Now consider the action of $\hat{a}$ on the superposition $|\alpha\rangle$. Since $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, we substitute the expansion for $|\alpha\rangle$:

$$\hat{a}\left(\sum_{n=0}^{\infty} c_n|n\rangle\right) = \alpha\sum_{n=0}^{\infty} c_n|n\rangle$$

Using the action of $\hat{a}$ on the number states, the left-hand side becomes:

$$\sum_{n=0}^{\infty} c_n\hat{a}|n\rangle = \sum_{n=1}^{\infty} c_n\sqrt{n}|n-1\rangle$$

Thus, we have:

$$\sum_{n=1}^{\infty} c_n \sqrt{n} |n-1\rangle = \alpha \sum_{n=0}^{\infty} c_n |n\rangle$$

We shift the summation index on the left-hand side by replacing $n-1$ with n. Now, this can be rewritten as:

$$\sum_{n=0}^{\infty} c_{n+1} \sqrt{n+1} |n\rangle = \alpha \sum_{n=0}^{\infty} c_n |n\rangle$$

Equating the coefficients of each number state $|m\rangle$, we obtain the recurrence relation:

$$c_{n+1}\sqrt{n+1} = \alpha c_n$$

This recurrence relation can be solved iteratively to obtain:

$$c_n = \frac{\alpha^n}{\sqrt{n!}} c_0$$

where c_0 is a normalization constant. The coherent states are now written as:

$$|\alpha\rangle = c_0 \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

To normalize the state $|\alpha\rangle$, we require that:

$$\langle\alpha|\alpha\rangle = 1$$

By using the orthonormal property of the eigenstates $|n\rangle$, we calculate:

$$\langle\alpha|\alpha\rangle = c_0^* c_0 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} \langle n|n\rangle = \|c_0\|^2 \sum_{n=0}^{\infty} \frac{|\alpha|^{2n}}{n!} = 1$$

Recognizing the series as the Taylor expansion of the exponential function, we find:

$$\langle\alpha|\alpha\rangle = \|c_0\|^2 e^{-|\alpha|^2} = 1$$

The c_0 is undetermined up to a phase factor. The convention is that the c_0 be a real number. Therefore, we choose $c_0 = e^{-|\alpha|^2/2}$:

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

This is the final expression for the coherent state $|\alpha\rangle$.

Time Evolution of the Annihilation Operator

We can derive the time evolution of $\hat{a}(t)$ using the Heisenberg equation of motion. For an operator $\hat{O}$ that does not explicitly depend on time, such as the Hamiltonian of a quantum harmonic oscillator, the general Heisenberg equation is:

$$\frac{d}{dt}\hat{O}_H(t) = \frac{i}{\hbar}[\hat{H}, \hat{O}_H(t)].$$

For the annihilation operator $\hat{a}$, this becomes:

$$\frac{d}{dt}\hat{a}_H(t) = \frac{i}{\hbar}[\hat{H}, \hat{a}_H(t)].$$

Substituting the Hamiltonian $\hat{H} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2})$, we calculate the commutator:

$$[\hat{H}, \hat{a}_H(t)] = -i\hbar\omega\hat{a}_H(t).$$

Thus, the Heisenberg equation becomes:

$$\frac{d}{dt}\hat{a}_H(t) = -i\omega\hat{a}_H(t),$$

which is a simple first-order differential equation with the solution:

$$\hat{a}_H(t) = \hat{a}e^{-i\omega t}.$$

We know that the coherent states satisfy:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

Now let us apply $\hat{a}_H(t)$ to the same state:

$$\hat{a}_H(t)|\alpha\rangle = \left(\hat{a}e^{-i\omega t}\right)|\alpha\rangle = e^{-i\omega t}\left(\hat{a}|\alpha\rangle\right) = e^{-i\omega t}\left(\alpha|\alpha\rangle\right) = \left(\alpha e^{-i\omega t}\right)|\alpha\rangle$$

This result indicates that the annihilation operator undergoes a phase rotation at the frequency ω. The eigenvalue associated with a coherent state, $\alpha(t)$, also evolves with this same phase factor. Since a coherent state is an eigenstate of the annihilation operator, this phase evolution of the operator implies that the coherent state itself evolves in time with its eigenvalue $\alpha(t) = \alpha(0)e^{-i\omega t}$. This evolution mirrors the classical motion of a particle in a harmonic potential, where the quantum state oscillates in phase space.

Time Evolution of Coherent States

Coherent states exhibit simple and predictable dynamics, making them a central concept in quantum mechanics and quantum optics. A defining feature of coherent states is that they retain their coherence over time. Unlike general quantum states, which may evolve into superpositions of different eigenstates, a coherent state remains an eigenstate of the annihilation operator throughout its time evolution.

To show this, let us assume that at $t = 0$, the quantum state is at a coherent state, which is expanded in the basis of energy eigenstates $|n\rangle$:

$$|\psi(0)\rangle = |\alpha_0\rangle = e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} |n\rangle$$

For a conservative system where the Hamiltonian is time-independent, the time evolution of the system can be written as:

$$|\psi(t)\rangle = e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} e^{-iE_n t/\hbar} |n\rangle$$

where the exponential phase factor $e^{-iE_n t/\hbar}$ corresponds to each energy eigenvalue of quantum harmonic oscillator:

$$E_n = \hbar\omega \left(n + \frac{1}{2}\right)$$

By using the expression of the energy eigenvalues:

$$\begin{aligned}
|\psi(t)\rangle &= e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{\alpha_0^n}{\sqrt{n!}} e^{-i\hbar\omega(n+\frac{1}{2})t/\hbar} |n\rangle \\
&= e^{-i\omega t/2} e^{-|\alpha_0|^2/2} \sum_{n=0}^{\infty} \frac{(\alpha_0 e^{-i\omega t})^n}{\sqrt{n!}} |n\rangle
\end{aligned}$$

Now let's define α as $\alpha = \alpha_0 e^{-i\omega t}$, which gives $|\alpha|^2 = |\alpha_0|^2$. We can rewrite the time-evolution of the state of the system as:

$$|\psi(t)\rangle = e^{-i\omega t/2} e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle = e^{-i\omega t/2} |\alpha\rangle$$

where the second equation comes from the expression of the coherent states we derived in the last section.

This show that if a state starts with a coherent state, i.e., $|\psi(0)\rangle = |\alpha_0\rangle$, then at a later time, it is in a state:

$$|\psi(t)\rangle = e^{-i\omega t/2}|\alpha_0 e^{-i\omega t}\rangle$$

Since $e^{-i\omega t/2}$ is a global phase factor that does not change the state of the system, this shows that if a quantum state starts with a coherent state, it remains as a coherent state.

Quasi-Classical states

Expectation Value of Position $\hat{x}(t)$ The position operator $\hat{x}$ is related to the annihilation and creation operators $\hat{a}$ and $\hat{a}^\dagger$ as follows:

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger)$$

To calculate the expectation value of $\hat{x}(t)$, we first recall that for a coherent state, $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$. For the time-evolved coherent state $|\alpha(t)\rangle$, we have $\hat{a}(t) = \alpha e^{-i\omega t}$.

The expectation value of position is:

$$\langle \hat{x}(t)\rangle = \langle \alpha(t)|\hat{x}|\alpha(t)\rangle$$

Substitute the expression for $\hat{x}$ in terms of $\hat{a}$ and $\hat{a}^\dagger$:

$$\langle \hat{x}(t)\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\langle \alpha(t)|\hat{a}(t) + \hat{a}^\dagger(t)|\alpha(t)\rangle\right)$$

Since $\hat{a}(t) = \alpha e^{-i\omega t}$, and $\hat{a}^\dagger(t) = \alpha^* e^{i\omega t}$, we compute the expectation value:

$$\langle \hat{x}(t)\rangle = \sqrt{\frac{\hbar}{2m\omega}}\left(\alpha e^{-i\omega t} + \alpha^* e^{i\omega t}\right)$$

Using the definition of the real part of a complex number, $\Re(\alpha) = (\alpha+\alpha^*)/2$, this becomes:

$$\langle \hat{x}(t)\rangle = \sqrt{\frac{2\hbar}{m\omega}}\Re(\alpha e^{-i\omega t})$$

Thus, the expectation value of position evolves in time as a harmonic oscillation, with the same amplitude and frequency as a classical harmonic oscillator.

Expectation Value of Momentum $\hat{p}(t)$ The momentum operator $\hat{p}$ is related to the annihilation and creation operators as follows:

$$\hat{p} = -i\sqrt{\frac{\hbar m\omega}{2}}(\hat{a} - \hat{a}^\dagger)$$

To compute the expectation value of momentum, we proceed in a similar manner as we did for position. The expectation value of $\hat{p}(t)$ is:

$$\langle \hat{p}(t) \rangle = \langle \alpha(t) | \hat{p} | \alpha(t) \rangle$$

Substitute the expression for $\hat{p}$:

$$\langle \hat{p}(t) \rangle = -i\sqrt{\frac{\hbar m\omega}{2}} \left(\langle \alpha(t) | \hat{a}(t) - \hat{a}^\dagger(t) | \alpha(t) \rangle \right)$$

Substitute $\hat{a}(t) = \alpha e^{-i\omega t}$ and $\hat{a}^\dagger(t) = \alpha^* e^{i\omega t}$:

$$\langle \hat{p}(t) \rangle = -i\sqrt{\frac{\hbar m\omega}{2}} \left(\alpha e^{-i\omega t} - \alpha^* e^{i\omega t} \right)$$

Using the definition of the imaginary part of a complex number, $\Im(\alpha) = (\alpha - \alpha^*)/2i$, we obtain:

$$\langle \hat{p}(t) \rangle = \sqrt{2\hbar m\omega}\Im(\alpha e^{-i\omega t})$$

Thus, the expectation value of momentum also evolves in time as a harmonic oscillation, with the same frequency and a phase difference relative to the position.

Classical-Like Behavior of Coherent States The key feature of these results is that the expectation values for both position and momentum exhibit oscillatory motion with constant amplitude and frequency, just like the motion of a classical harmonic oscillator. The position and momentum evolve in time according to classical equations of motion, and the state maintains minimal uncertainty throughout the evolution, as governed by the Heisenberg uncertainty principle. The coherent state follows a classical trajectory in phase space, with the classical amplitude governed by the magnitude of α. The uncertainty in both position and momentum remains minimal, and the state exhibits a well-defined trajectory that closely mimics a classical oscillator.

Minimum Uncertainty States

Coherent states are often referred to as minimum uncertainty states because they saturate the Heisenberg uncertainty principle. The Heisenberg uncertainty principle provides a fundamental limit to the precision with which certain pairs of physical properties (conjugate variables) can be simultaneously known. In particular, for position $\hat{x}$ and momentum $\hat{p}$, the uncertainty relation is given by:

$$\Delta\hat{x}\Delta\hat{p} \geq \frac{\hbar}{2}$$

Here, $\Delta\hat{x}$ and $\Delta\hat{p}$ represent the standard deviations (uncertainties) in position and momentum, respectively. Coherent states are unique in that they saturate this inequality:

$$\Delta\hat{x}\Delta\hat{p} = \frac{\hbar}{2}.$$

Proof: The uncertainties in position and momentum are defined as the standard deviations of their respective quantum mechanical operators:

$$\Delta\hat{x} = \sqrt{\langle\hat{x}^2\rangle - \langle\hat{x}\rangle^2}\,,$$
$$\Delta\hat{p} = \sqrt{\langle\hat{p}^2\rangle - \langle\hat{p}\rangle^2}\,.$$

For a coherent state $|\alpha\rangle$, we know that the expectation values of position and momentum are:

$$\langle\hat{x}\rangle = \sqrt{\frac{2\hbar}{m\omega}}\Re(\alpha e^{-i\omega t})\,,$$
$$\langle\hat{p}\rangle = \sqrt{2\hbar m\omega}\Im(\alpha e^{-i\omega t})\,.$$

However, to calculate the uncertainties, we also need the second moments, $\langle\hat{x}^2\rangle$ and $\langle\hat{p}^2\rangle$.

Now, let's compute the expectation value $\langle\hat{x}^2\rangle$:

$$\langle\hat{x}^2\rangle = \left\langle\alpha\left|\hat{x}^2\right|\alpha\right\rangle = \frac{\hbar}{2m\omega}\left(\langle\hat{a}^2 + (\hat{a}^\dagger)^2 + \hat{a}\hat{a}^\dagger + \hat{a}^\dagger\hat{a}\rangle\right)$$

Using the fact that $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, and applying the properties of coherent states, we find that:

$$\langle\hat{x}^2\rangle = \frac{\hbar}{m\omega}\left(|\alpha|^2 + |\alpha|^2 + 2\right) = \frac{\hbar}{m\omega}\left(\Re(\alpha)^2 + \frac{1}{2}\right).$$

Similarly, for momentum, we compute $\langle\hat{p}^2\rangle$:

$$\langle\hat{p}^2\rangle = \left\langle\alpha\left|\hat{p}^2\right|\alpha\right\rangle = \frac{\hbar m\omega}{2}\left(\langle\hat{a}^2 - (\hat{a}^\dagger)^2 - \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\rangle\right)$$

By similar calculation, we find:

$$\langle \hat{p}^2 \rangle = \frac{\hbar m\omega}{2}\left(|\alpha|^2 + |\alpha|^2 - 2\right) = \frac{\hbar m\omega}{2}\left(\Im(\alpha)^2 + \frac{1}{2}\right).$$

The uncertainties $\Delta\hat{x}$ and $\Delta\hat{p}$ are now computed as:

$$\Delta\hat{x} = \sqrt{\langle \hat{x}^2 \rangle - \langle \hat{x} \rangle^2} = \sqrt{\frac{\hbar}{2m\omega}},$$

$$\Delta\hat{p} = \sqrt{\langle \hat{p}^2 \rangle - \langle \hat{p} \rangle^2} = \sqrt{\frac{\hbar m\omega}{2}}.$$

Thus, the product of the uncertainties is:

$$\Delta\hat{x}\Delta\hat{p} = \sqrt{\frac{\hbar}{2m\omega}}\sqrt{\frac{\hbar m\omega}{2}} = \frac{\hbar}{2}.$$

Therefore, coherent states are indeed minimum uncertainty states, as they saturate the Heisenberg uncertainty relation. This means that coherent states are the closest quantum states to classical particles in terms of their uncertainty properties, with minimal trade-off between uncertainties in position and momentum. This characteristic makes coherent states particularly important in understanding the classical limit of quantum systems.

Overlap Between Coherent States

The overlap between two coherent states $|\alpha\rangle$ and $|\beta\rangle$ is a central concept in quantum mechanics, especially in quantum optics and quantum information theory. Coherent states are typically represented as eigenstates of the annihilation operator $\hat{a}$ with eigenvalues α and β, respectively. The coherent state $|\alpha\rangle$ is defined by:

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

where α is a complex number that characterizes the coherent state. Similarly, for $|\beta\rangle$, we have:

$$\hat{a}|\beta\rangle = \beta|\beta\rangle$$

where β is another complex number.

To compute the overlap $\langle\alpha|\beta\rangle$, we begin by recalling that coherent states are usually represented in terms of the vacuum state $|0\rangle$ (the state with no particles) as follows:

$$|\alpha\rangle = D(\alpha)|0\rangle, \quad |\beta\rangle = D(\beta)|0\rangle$$

where $D(\alpha)$ and $D(\beta)$ are the displacement operators for the coherent states $|\alpha\rangle$ and $|\beta\rangle$, respectively. The displacement operator $D(\alpha)$ is given by:

$$D(\alpha) = \exp\left(\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right)$$

and it acts on the vacuum state $|0\rangle$ to produce a coherent state.

Now, let's compute the overlap $\langle\alpha|\beta\rangle$:

$$\langle\alpha|\beta\rangle = \langle 0|D^\dagger(\alpha)D(\beta)|0\rangle$$

Since $D(\alpha)$ and $D(\beta)$ are unitary, we can use the identity for the inner product of two coherent states:

$$\langle\alpha|\beta\rangle = \langle 0| \exp\left(\alpha \hat{a}^\dagger - \alpha^* \hat{a}\right) \exp\left(\beta \hat{a}^\dagger - \beta^* \hat{a}\right) |0\rangle$$

The Baker-Campbell-Hausdorff formula allows us to combine the two exponential operators:

$$\langle\alpha|\beta\rangle = \exp\left(-\frac{|\alpha|^2 + |\beta|^2}{2} + \alpha^* \beta\right)$$

Thus, the overlap between the two coherent states is:

$$\langle\alpha|\beta\rangle = e^{-\frac{|\alpha|^2+|\beta|^2}{2} + \alpha^* \beta}$$

This result has several important implications:

Non-Orthogonality Coherent states are not orthogonal unless the complex parameters α and β are sufficiently far apart in phase space. Specifically, the overlap is maximized when $\alpha = \beta$, in which case $\langle\alpha|\beta\rangle = 1$. When $\alpha \neq \beta$, the overlap decreases exponentially with the distance between α and β.

Exponential Decay The overlap between coherent states diminishes exponentially as the distance between their corresponding complex parameters increases. This exponential dependence can be understood in terms of the separation between α and β in phase space, where the phase space distance between the two points is given by:

$$|\alpha - \beta|^2 = |\alpha|^2 + |\beta|^2 - 2\Re(\alpha^* \beta)$$

As $|\alpha - \beta|$ increases, the overlap $\langle\alpha|\beta\rangle$ rapidly decays, reflecting the fact that coherent states become increasingly orthogonal as their parameters are more separated in phase space.

Overcomplete Basis The formula for the overlap shows that coherent states form an overcomplete basis in Hilbert space. This means that any quantum state can be expressed as a superposition (or integral) of coherent states. The fact that coherent states are overcomplete is crucial in quantum optics, where they are used to model laser light, as well as in quantum information theory, where they serve as a natural basis for encoding quantum information. Coherent states allow for an efficient representation of quantum states in terms of continuous variables (such as position and momentum or quadratures of the electromagnetic field), which is vital for practical applications in quantum technologies.

Thus, the overlap formula encapsulates the close relationship between coherent states and classical trajectories in phase space, the exponential decay of overlap with increasing separation in phase space, and the overcompleteness of the coherent state basis in Hilbert space. These properties make coherent states highly valuable tools for both theoretical and experimental investigations in quantum mechanics.

Applications

Coherent States in Quantum Optics Coherent states play a central role in quantum optics, particularly in the description of light fields. In this context, the classical electromagnetic field in the coherent regime closely resembles a coherent state, making coherent states a natural description for light with well-defined phase and amplitude.

In quantum optics, the electromagnetic field is typically modeled as a quantum harmonic oscillator. The quantum state of the electromagnetic field can be written as a superposition of number states, or Fock states, which describe the photon occupation number of the field. The general field state is then:

$$|\psi\rangle = \sum_{n=0}^{\infty} c_n |n\rangle,$$

where $|n\rangle$ are the Fock states, and c_n are the expansion coefficients. However, for large average photon numbers (i.e., when the field is highly populated), the quantum state behaves in such a way that it closely approximates the classical behavior of an electromagnetic wave.

In this limit, the quantum state of the field is described by a coherent state $|\alpha\rangle$, which minimizes the quantum mechanical uncertainties in position and momentum, and thus behaves most similarly to a classical field. Coherent states are characterized by a well-defined classical-like amplitude and phase, which closely mimic the oscillatory nature of classical electromagnetic fields.

To describe the field in phase space, the *P-representation* is commonly used. The P-representation is an alternative way of expressing quantum states, particularly useful for coherent states, where the quantum state is expressed in terms of a quasi-probability distribution in phase space. For a coherent state, the Wigner distribution (which represents the quantum state in phase space) is a Gaussian function centered at the classical trajectory. This is a manifestation of the classical limit of quantum mechanics, where the quantum fluctuations become negligible in comparison to the classical behavior of the system.

The P-representation is given by:

$$\langle \hat{O} \rangle = \int d^2\alpha \, P(\alpha) \langle \alpha | \hat{O} | \alpha \rangle,$$

where $\hat{O}$ is an operator, $\langle \alpha | \hat{O} | \alpha \rangle$ is the expectation value of $\hat{O}$ in the coherent state $|\alpha\rangle$, and $P(\alpha)$ is the quasi-probability distribution, which for coherent states is Gaussian in form. This distribution describes the probability amplitude for the system being in a coherent state characterized by the complex number α. The P-representation allows one to compute expectation values of operators in a manner that parallels classical statistical mechanics, bridging the gap between quantum and classical descriptions.

Coherent States in Quantum Field Theory Coherent states are also highly significant in quantum field theory (QFT), where they are used to describe quantized fields, especially in contexts such as quantum electrodynamics (QED) and quantum gravity. In QFT, the vacuum state $|0\rangle$ is typically defined as the ground state of the quantized field, representing the absence of particles. However, coherent states provide a description of fields in quasi-classical configurations, where the field oscillates in a manner similar to classical fields.

In quantum electrodynamics, for example, laser fields can be described as coherent states. A laser, which emits light in a narrow frequency band with a well-defined phase and amplitude, can be modeled by a coherent state. Such a state is an eigenstate of the annihilation operator $\hat{a}$ for the field mode, i.e.,

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle,$$

where α is a complex number that encodes both the amplitude and phase of the electromagnetic field. This means that a laser beam, which behaves classically in many respects (such as its well-defined phase and amplitude), is accurately described by a coherent quantum state.

Furthermore, coherent states in QFT are also useful in the study of quantum gravity and other quantum field systems. In these contexts, coherent states are often used to describe configurations where the field is not in its ground state, but instead in a state that behaves in a nearly classical way. For example, a coherent state can be used to model a gravitational wave with a well-defined classical-like amplitude and phase, or in the study of quantum fluctuations in the early universe.

In summary, coherent states are crucial in QFT as they provide a tool for modeling field configurations that are close to classical, making them particularly important in fields like QED and quantum gravity. They allow for the description of phenomena that bridge the gap between quantum and classical field theories, particularly in high-energy physics, cosmology, and the study of fundamental interactions.

Conclusion

Coherent states provide an elegant bridge between quantum mechanics and classical physics. By minimizing the uncertainty in both position and momentum, they exhibit behavior that is closest to classical harmonic oscillators, making them crucial for understanding the quantum-classical transition. Coherent states are key to many areas of physics, including quantum optics, quantum information, and condensed matter theory. Their study allows for a deeper understanding of the behavior of quantum systems and their interactions with classical systems.