script_cv_1_Euler_method

February 25, 2023

1 Ex. 2 - Motion of point

1.1 Analytical solution

We have calculated the analytical solution using second-order differential equations. We have obtained the results - two functions $v_x = f(t)$ and $x = f(t)$.

$v_x = -\frac{1}{4} \cdot t^2 + 10 \cdot t$

$x = -\frac{1}{12} \cdot t^3 + 5 \cdot t^2$

[ ]:
```
import numpy as np

num_points = 10000 # number of points in the defined time range [0, 20] s

t_ana = np.linspace(0, 20, num_points)

# calculation of the analytical solution
v_ana = -1/4 * t_ana**2 + 10 * t_ana
x_ana = -1/12 * t_ana**3 + 5 * t_ana**2
```

1.2 Numerical solution - Explicit Euler method

We have simply derived the approximate explicit Euler numerical solution from the analytical one. The method is based on the linear approximation of the function using the first-order Taylor polynomial:

$y(x + h) \approx y(x_0) + \dot{y}(x_0) \cdot h$

[ ]:
```
steps = 100 # number of steps in the time range [0, 20] s using for the␣
 ↪numerical solution
# initial conditions
t0 = 0 # [s]
x0 = 0 # [m]
v0 = 0 # [m/s]

t_finish = 20 # [s]
delta_t = t_finish/steps # [s], time step of numerical solution

# initial assignment
```

```python
v_k_1 = v0
x_k_1 = x0

# logs
x_num = []
v_num = []
t_num = []

# do defined steps + 1
for k in range(steps + 1):
    t_k = t0 + k*delta_t
    a_k = -1/2 * t_k + 10

    # calculation of k-step using Taylor polynom (the first derivation of v and␣
 ↪x)
    v_k = v_k_1 + a_k * delta_t
    x_k = x_k_1 + v_k * delta_t

    # assignment for the next step
    v_k_1 = v_k
    x_k_1 = x_k

    # logs
    print(f"Step:{k}, Time:{t_k:.2f} s, x={x_k_1:.2f} m, v={v_k_1:.2f} m/s")

    t_num.append(t_k)
    v_num.append(v_k_1)
    x_num.append(x_k_1)
```

```
Step:0, Time:0.00 s, x=0.40 m, v=2.00 m/s
Step:1, Time:0.20 s, x=1.20 m, v=3.98 m/s
Step:2, Time:0.40 s, x=2.38 m, v=5.94 m/s
Step:3, Time:0.60 s, x=3.96 m, v=7.88 m/s
Step:4, Time:0.80 s, x=5.92 m, v=9.80 m/s
Step:5, Time:1.00 s, x=8.26 m, v=11.70 m/s
Step:6, Time:1.20 s, x=10.98 m, v=13.58 m/s
Step:7, Time:1.40 s, x=14.06 m, v=15.44 m/s
Step:8, Time:1.60 s, x=17.52 m, v=17.28 m/s
Step:9, Time:1.80 s, x=21.34 m, v=19.10 m/s
Step:10, Time:2.00 s, x=25.52 m, v=20.90 m/s
Step:11, Time:2.20 s, x=30.06 m, v=22.68 m/s
Step:12, Time:2.40 s, x=34.94 m, v=24.44 m/s
Step:13, Time:2.60 s, x=40.18 m, v=26.18 m/s
Step:14, Time:2.80 s, x=45.76 m, v=27.90 m/s
Step:15, Time:3.00 s, x=51.68 m, v=29.60 m/s
Step:16, Time:3.20 s, x=57.94 m, v=31.28 m/s
Step:17, Time:3.40 s, x=64.52 m, v=32.94 m/s
Step:18, Time:3.60 s, x=71.44 m, v=34.58 m/s
```

```
Step:19, Time:3.80 s, x=78.68 m, v=36.20 m/s
Step:20, Time:4.00 s, x=86.24 m, v=37.80 m/s
Step:21, Time:4.20 s, x=94.12 m, v=39.38 m/s
Step:22, Time:4.40 s, x=102.30 m, v=40.94 m/s
Step:23, Time:4.60 s, x=110.80 m, v=42.48 m/s
Step:24, Time:4.80 s, x=119.60 m, v=44.00 m/s
Step:25, Time:5.00 s, x=128.70 m, v=45.50 m/s
Step:26, Time:5.20 s, x=138.10 m, v=46.98 m/s
Step:27, Time:5.40 s, x=147.78 m, v=48.44 m/s
Step:28, Time:5.60 s, x=157.76 m, v=49.88 m/s
Step:29, Time:5.80 s, x=168.02 m, v=51.30 m/s
Step:30, Time:6.00 s, x=178.56 m, v=52.70 m/s
Step:31, Time:6.20 s, x=189.38 m, v=54.08 m/s
Step:32, Time:6.40 s, x=200.46 m, v=55.44 m/s
Step:33, Time:6.60 s, x=211.82 m, v=56.78 m/s
Step:34, Time:6.80 s, x=223.44 m, v=58.10 m/s
Step:35, Time:7.00 s, x=235.32 m, v=59.40 m/s
Step:36, Time:7.20 s, x=247.46 m, v=60.68 m/s
Step:37, Time:7.40 s, x=259.84 m, v=61.94 m/s
Step:38, Time:7.60 s, x=272.48 m, v=63.18 m/s
Step:39, Time:7.80 s, x=285.36 m, v=64.40 m/s
Step:40, Time:8.00 s, x=298.48 m, v=65.60 m/s
Step:41, Time:8.20 s, x=311.84 m, v=66.78 m/s
Step:42, Time:8.40 s, x=325.42 m, v=67.94 m/s
Step:43, Time:8.60 s, x=339.24 m, v=69.08 m/s
Step:44, Time:8.80 s, x=353.28 m, v=70.20 m/s
Step:45, Time:9.00 s, x=367.54 m, v=71.30 m/s
Step:46, Time:9.20 s, x=382.02 m, v=72.38 m/s
Step:47, Time:9.40 s, x=396.70 m, v=73.44 m/s
Step:48, Time:9.60 s, x=411.60 m, v=74.48 m/s
Step:49, Time:9.80 s, x=426.70 m, v=75.50 m/s
Step:50, Time:10.00 s, x=442.00 m, v=76.50 m/s
Step:51, Time:10.20 s, x=457.50 m, v=77.48 m/s
Step:52, Time:10.40 s, x=473.18 m, v=78.44 m/s
Step:53, Time:10.60 s, x=489.06 m, v=79.38 m/s
Step:54, Time:10.80 s, x=505.12 m, v=80.30 m/s
Step:55, Time:11.00 s, x=521.36 m, v=81.20 m/s
Step:56, Time:11.20 s, x=537.78 m, v=82.08 m/s
Step:57, Time:11.40 s, x=554.36 m, v=82.94 m/s
Step:58, Time:11.60 s, x=571.12 m, v=83.78 m/s
Step:59, Time:11.80 s, x=588.04 m, v=84.60 m/s
Step:60, Time:12.00 s, x=605.12 m, v=85.40 m/s
Step:61, Time:12.20 s, x=622.36 m, v=86.18 m/s
Step:62, Time:12.40 s, x=639.74 m, v=86.94 m/s
Step:63, Time:12.60 s, x=657.28 m, v=87.68 m/s
Step:64, Time:12.80 s, x=674.96 m, v=88.40 m/s
Step:65, Time:13.00 s, x=692.78 m, v=89.10 m/s
Step:66, Time:13.20 s, x=710.74 m, v=89.78 m/s
```

```
Step:67, Time:13.40 s, x=728.82 m, v=90.44 m/s
Step:68, Time:13.60 s, x=747.04 m, v=91.08 m/s
Step:69, Time:13.80 s, x=765.38 m, v=91.70 m/s
Step:70, Time:14.00 s, x=783.84 m, v=92.30 m/s
Step:71, Time:14.20 s, x=802.42 m, v=92.88 m/s
Step:72, Time:14.40 s, x=821.10 m, v=93.44 m/s
Step:73, Time:14.60 s, x=839.90 m, v=93.98 m/s
Step:74, Time:14.80 s, x=858.80 m, v=94.50 m/s
Step:75, Time:15.00 s, x=877.80 m, v=95.00 m/s
Step:76, Time:15.20 s, x=896.90 m, v=95.48 m/s
Step:77, Time:15.40 s, x=916.08 m, v=95.94 m/s
Step:78, Time:15.60 s, x=935.36 m, v=96.38 m/s
Step:79, Time:15.80 s, x=954.72 m, v=96.80 m/s
Step:80, Time:16.00 s, x=974.16 m, v=97.20 m/s
Step:81, Time:16.20 s, x=993.68 m, v=97.58 m/s
Step:82, Time:16.40 s, x=1013.26 m, v=97.94 m/s
Step:83, Time:16.60 s, x=1032.92 m, v=98.28 m/s
Step:84, Time:16.80 s, x=1052.64 m, v=98.60 m/s
Step:85, Time:17.00 s, x=1072.42 m, v=98.90 m/s
Step:86, Time:17.20 s, x=1092.26 m, v=99.18 m/s
Step:87, Time:17.40 s, x=1112.14 m, v=99.44 m/s
Step:88, Time:17.60 s, x=1132.08 m, v=99.68 m/s
Step:89, Time:17.80 s, x=1152.06 m, v=99.90 m/s
Step:90, Time:18.00 s, x=1172.08 m, v=100.10 m/s
Step:91, Time:18.20 s, x=1192.14 m, v=100.28 m/s
Step:92, Time:18.40 s, x=1212.22 m, v=100.44 m/s
Step:93, Time:18.60 s, x=1232.34 m, v=100.58 m/s
Step:94, Time:18.80 s, x=1252.48 m, v=100.70 m/s
Step:95, Time:19.00 s, x=1272.64 m, v=100.80 m/s
Step:96, Time:19.20 s, x=1292.82 m, v=100.88 m/s
Step:97, Time:19.40 s, x=1313.00 m, v=100.94 m/s
Step:98, Time:19.60 s, x=1333.20 m, v=100.98 m/s
Step:99, Time:19.80 s, x=1353.40 m, v=101.00 m/s
Step:100, Time:20.00 s, x=1373.60 m, v=101.00 m/s
```

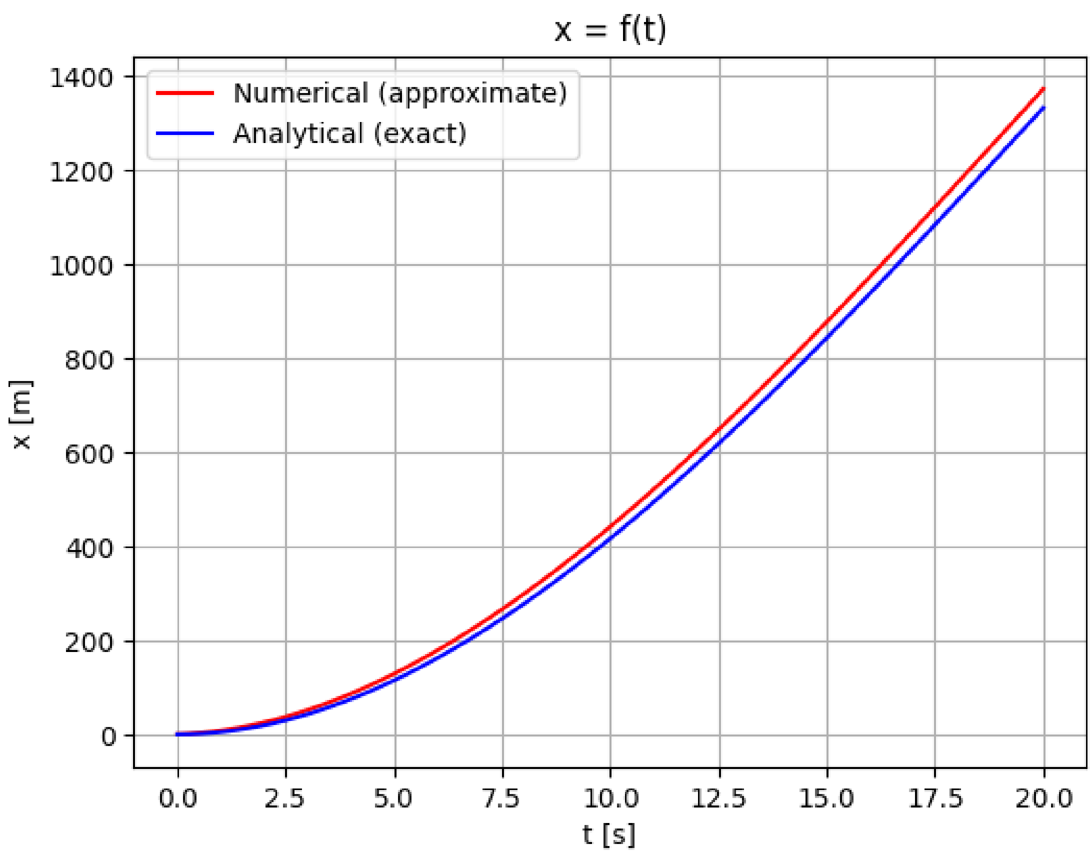
1.3 Plot results

We can compare both obtained results.

[ ]:
```python
import matplotlib.pyplot as plt
# plot x_ana and x_num as f(t)
plt.plot(t_num, x_num, 'r')
plt.plot(t_ana, x_ana, 'b')
plt.legend(['Numerical (approximate)', 'Analytical (exact)'])
plt.xlabel('t [s]')
plt.ylabel('x [m]')
plt.grid(True)
plt.title('x = f(t)')
```

```python
plt.show()

# plot the error of result
print(f"Analytical result: x_ana(t = 20s) = {x_ana[-1]:.2f} m")
print(f"Numerical result: x_num(t = 20s) = {x_num[-1]:.2f} m")
print(f"Error:  |x_num(t = 20s) - x_ana(t = 20s)| = |{abs(x_num[-1] - x_ana[-1]):.2f}| m")
```




```
Analytical result: x_ana(t = 20s) = 1333.33 m
Numerical result: x_num(t = 20s) = 1373.60 m
Error:  |x_num(t = 20s) - x_ana(t = 20s)| = |40.27| m
```