

Econ201 Course Notes

mzx!

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Chapter 1

Technology

1.1 Inputs and Outputs

Definition 1.1.1

Inputs to production are called Factors of production
Physical Capitals

- Land
- Labor
- Capital
 - Physical Capital
 - * Tractor
 - * Buildings
 - * Computers
 - * Machines of one sort
 - Financial Capital
 - * Money
 - * Stocks
 - * Bonds
- Raw Materials

1.2 Technology Constraints (Single Input)

Definition 1.2.1

Production Set All combinations of inputs and outputs that are technologically feasible

Definition 1.2.2

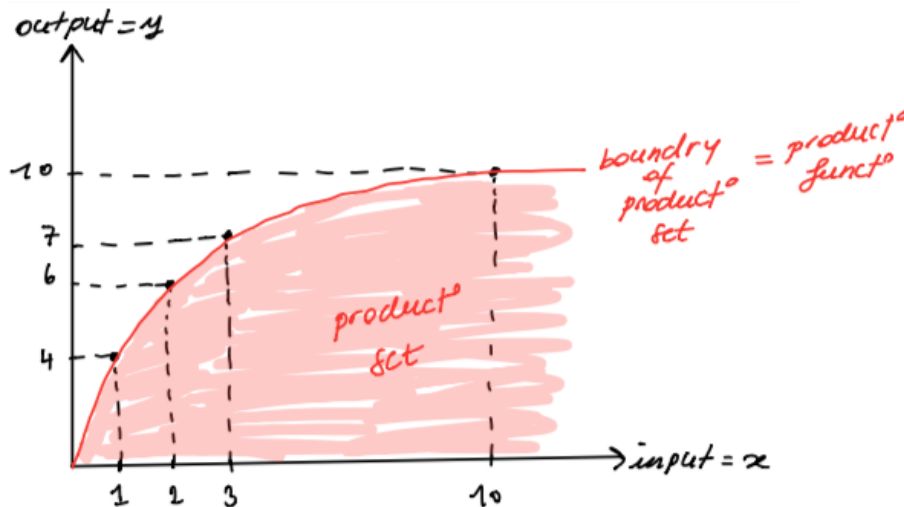
Production Function A function describing the **Boundary of Production Set**

$$y = f(x)$$

x = amount of inputs

y = amount of output

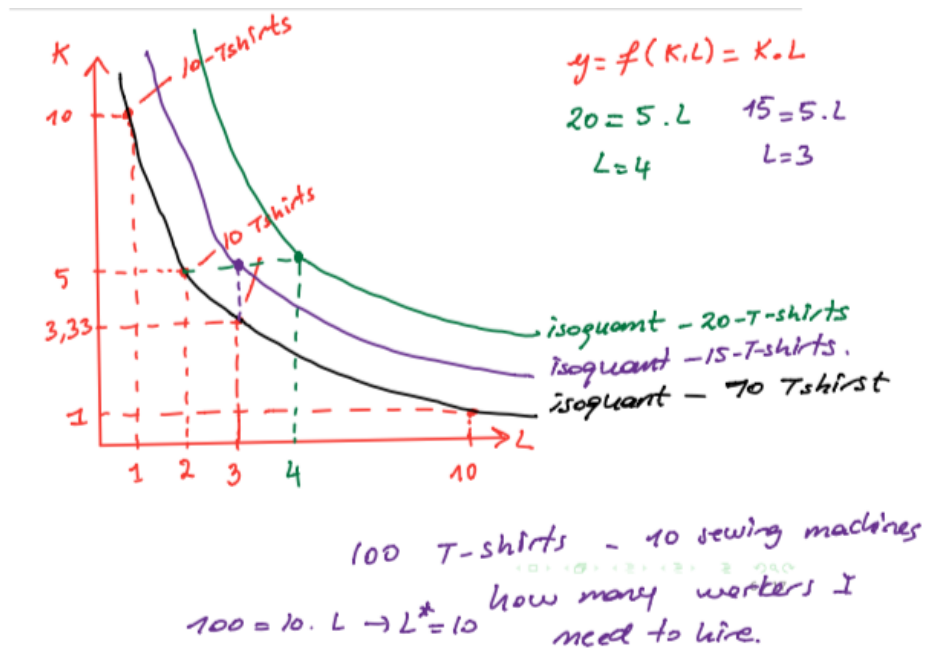
Two production functions do not represent the same technology even if one is a **Monotone Transformation** of the other



1.3 Technology Constraints (Multiple Inputs)

We consider the case of **Two Inputs**, the production function $f(x_1, x_2)$ would measure the maximum amount of output y that we could get if we had x_1 units of input 1. and x_2 units of input 2

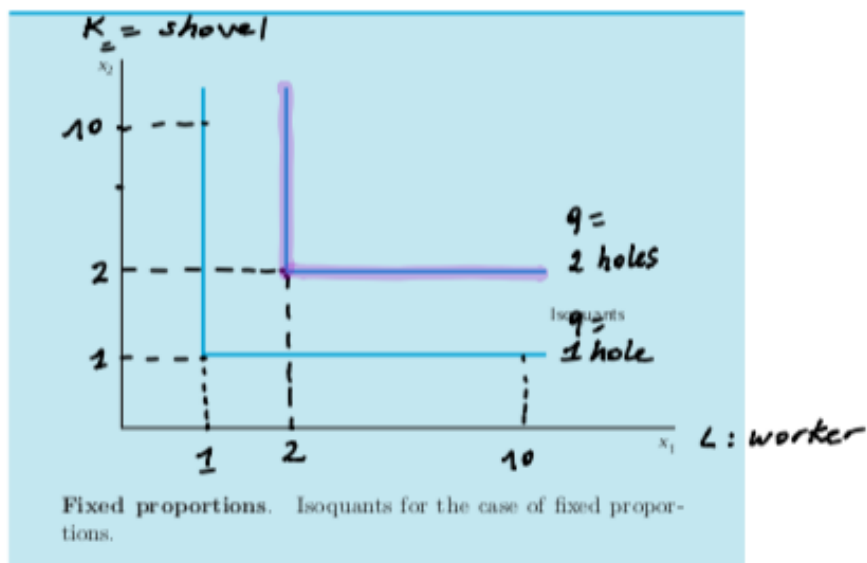
Isoquant In the two-input case, there is a convenient way to depict production relations known as the isoquant



1.4 Example of Technology

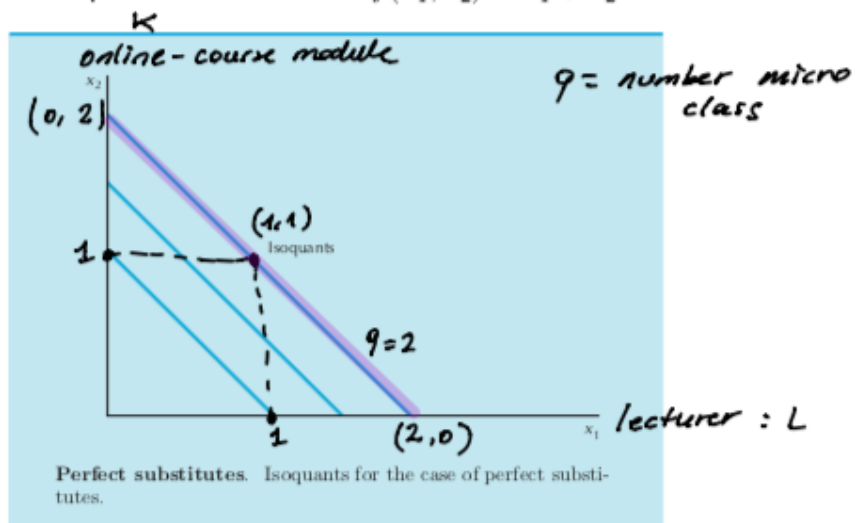
1.4.1 Fixed Proportion

$$f(x_1, x_2) = \min(x_1, x_2)$$



1.4.2 Perfect Substitutes

$$f(x_1, x_2) = x_1 + x_2$$

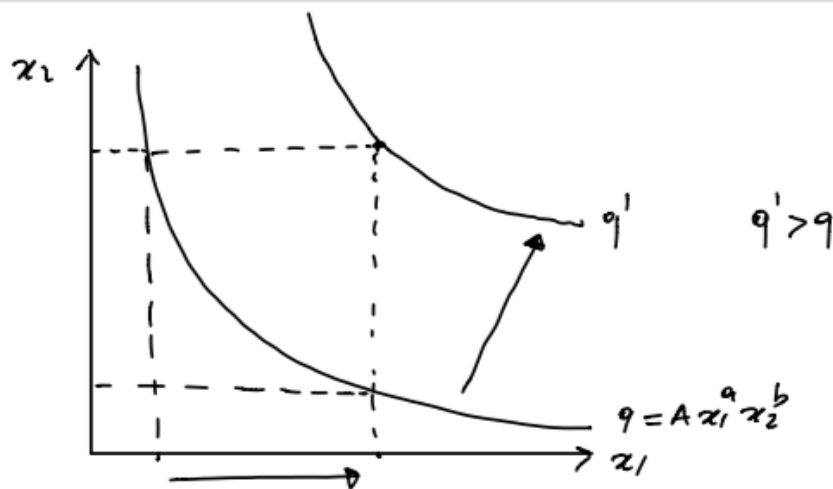


1.4.3 Cobb Douglas

$$q = f(x_1, x_2) = Ax_1^a x_2^b$$

A: Scale of Production (how much output we would get if we used one unit of each input)

a.b: The amount of output responds to changes in the inputs



1.5 Properties of Technology

Definition 1.5.1

- Technology is monotonic
- Technology is convex

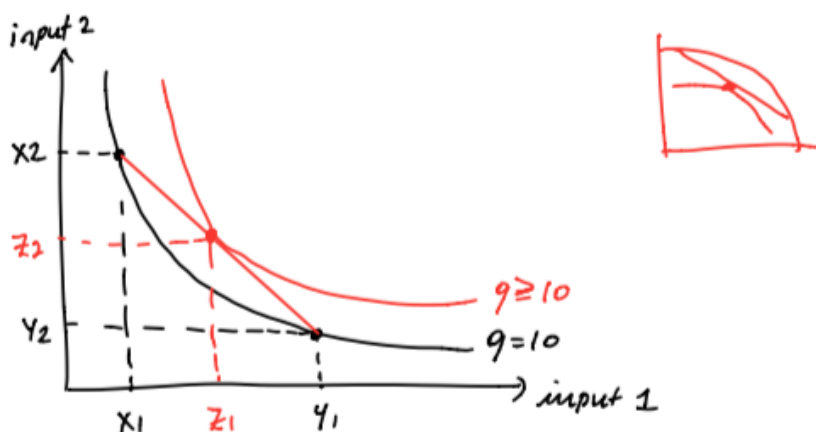
Monotonic If you increase the amount of at least one of the inputs, it should be possible to produce at least as much output as you were producing originally

Convex If you have two ways to produce y units of output, (x_1, x_2) and (z_1, z_2) , then their weighted average will produce at least y units of output

10 T-shirt \rightarrow 5W and 5 machine.



At least 10 T-shirt 8W and 5 machine



1.6 Marginal Product

Definition 1.6.1

Marginal Product of Factor 1

$$MP_1(x_1, x_2) = \frac{\Delta y}{\Delta x_1} = \frac{f(x_1 + \Delta x_1, x_2) - f(x_1, x_2)}{\Delta x_1}$$

or

$$MP_1 = \frac{\partial y(x_1, x_2)}{\partial x_1}$$

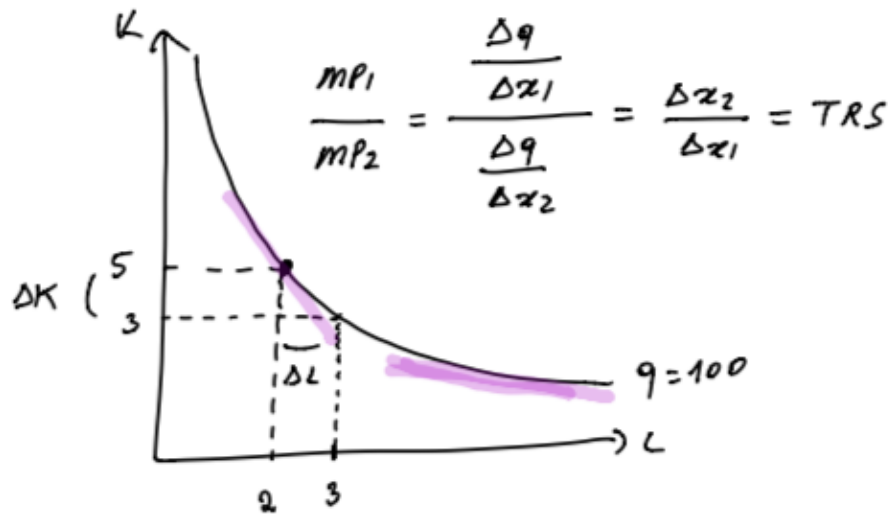
∂ : Partial Derivative. If ∂x_1 , then treat x_2 as constant value
 $f(x_1, x_2) = 2x_1x_2$, then we have $MP_1 = \frac{\partial y(x_1, x_2)}{\partial x_1} = 2x_2$

1.7 Technical Rate of Substitution

Suppose that we are operating at $f(x_1, x_2) = y$, at some input level x_1, x_2 and output level y . We want to adjust x_1, x_2 (ie, add x_1 , less x_2), to get the same output level y .

Definition 1.7.1

$$TRS(x_1, x_2) = \frac{\Delta x_2}{\Delta x_1} = \frac{-MP_1(x_1, x_2)}{MP_2(x_1, x_2)}$$



1.8 Diminishing Marginal Product

Law of Diminishing Marginal Product As long as we have a monotonic technology, we know that the total output will go up As we increase the amount of factor 1

But we expect that it will go up at a **Decreasing Level**

1.9 Long Run and Short Run

Definition 1.9.1

Short Run Some Factors are fixed

Long Run All factors can be varying

1.10 Returns to scale

Scale the amount of all inputs up by some constant factor

This is called the case of **constant returns to scales**

Definition 1.10.1

constant return to scale

$$tf(x_1, x_2) = f(tx_1, tx_2)$$

for some constant t

Definition 1.10.2

Increasing return to scale

$$tf(x_1, x_2) < f(tx_1, tx_2)$$

for some constant t

Definition 1.10.3

Decreasing return to scale

$$tf(x_1, x_2) > f(tx_1, tx_2)$$

for some constant t

Generalization $f(x_1, x_2) = Ax_1^a x_2^b$, then

- $a + b = 1 \rightarrow$ constant R.S
- $a + b > 1 \rightarrow$ Increasing R.S
- $a + b < 1 \rightarrow$ Decreasing R.S

$$y = f(x_1, x_2) = 2 \cdot x_1^{0,3} \cdot x_2^{0,5}$$

$$f(tx_1, tx_2) = 2(tx_1)^{0,3} \cdot (tx_2)^{0,5}$$

$$= 2 \cdot t^{0.13} x_1^{0.13} \cdot t^{0.15} x_2^{0.15}$$

$$= t^{0,8} \cdot \underbrace{2 \cdot x_1^{0,3} \cdot x_2^{0,5}}_{= 4} = t^{0,8} \cdot 4$$

$$t^1 > t^{0,8} \quad f(x_1, x_2) = y$$

If $t=2 \rightarrow 2^1 = 2 > 2^{0.8} = 1.74$

$2 = 2 > 2 = 1.74$
I doubled the input, output increased
by 1.74 times.

$$y = f(x_1, x_2) = \min \{x_1, x_2\}$$

$$f(2x_1, 2x_2) = \min \{2x_1, 2x_2\}$$

$$= 2 \cdot \min \{x_1, x_2\} = 2 \cdot 2.$$

CST Returns to Scale.

$$f(x_1, x_2) = y$$

Chapter 2

Profit Maximization

2.1 Profits

Definition 2.1.1

Profits are defined as **Revenues** minus **Cost**

$$\pi = p \times q - w_1x_1 - w_2x_2$$

- p = price of output
- q = output
- w_1 = price of input 1
- w_2 = price of input 2

$$\pi = \sum_{i=1}^n p_i y_i - \sum_{i=1}^m w_i x_i$$

$$\pi = \underbrace{p_1 \cdot y_1 + p_2 \cdot y_2 + \dots + p_n \cdot y_n}_{\text{Total Revenue}} - \underbrace{(w_1 \cdot x_1 + w_2 \cdot x_2 + \dots + w_m \cdot x_m)}_{\text{Total Cost}}$$

$$\pi = \sum_{i=1}^n p_i \cdot y_i - \sum_{i=1}^m w_i \cdot x_i$$

↳ 2 inputs → 1 output.

x_1	x_2	y	→	$\pi = p \cdot y - (w_1 x_1 + w_2 x_2)$
w_1	w_2	p		$\pi = p y - w_1 x_1 - w_2 x_2$

T-shirt producer produces 100 T-shirt
 Selling price at the market = \$20/per T-shirt.
 Earn: $100 \times 20 = \$2000 = \text{Revenue}$
 Cost of product = \$10/each
 Cost: $100 \times 10 = \$1000 = \text{Cost}$.

Profit = $\pi = \text{Revenue} - \text{Cost}$
 $= 2000 - 1000 = \$1000$

2.2 Short-Run Profit Maximization

Definition 2.2.1

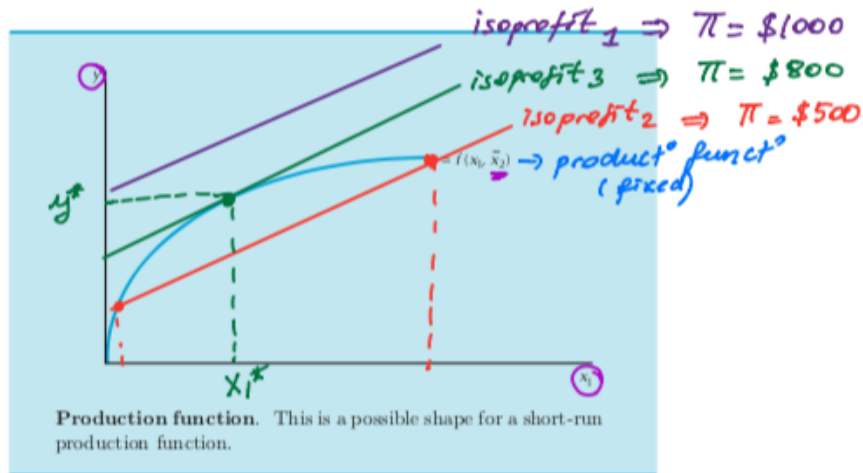
$$pMP_1(x_1^*, x_2) = w_1$$

- p = Output Price
- MP_1 = Marginal Product of factor 1
- x_1^* = the profit-maximizing choice for factor 1
- w_1 = the price of factor 1

Recall

Generalized

- $pMP_1 > w_1 \rightarrow$ Should Increase x_1
- $pMP_1 = w_1 \rightarrow$ Keep x_1
- $pMP_1 < w_1 \rightarrow$ Should Decrease x_1



$$\pi = p \cdot y - w_1 x_1 - w_2 \bar{x}_2$$

$$\pi + w_1 x_1 + w_2 \bar{x}_2 = p \cdot y.$$

$$\underbrace{\frac{\pi}{p}}_{\text{constant} > 0} + \underbrace{\frac{w_2 \cdot \bar{x}_2}{p}}_{\text{constant}} + \underbrace{\frac{w_1}{p}}_{\text{slope}} \cdot \underline{x_1} = y$$

Linear functⁿ
 $ax + b = y$

slope $= \frac{w_1}{p}$
 isoprofit line.
 $\frac{\pi}{p} + \frac{w_2 \bar{x}_2}{p}$

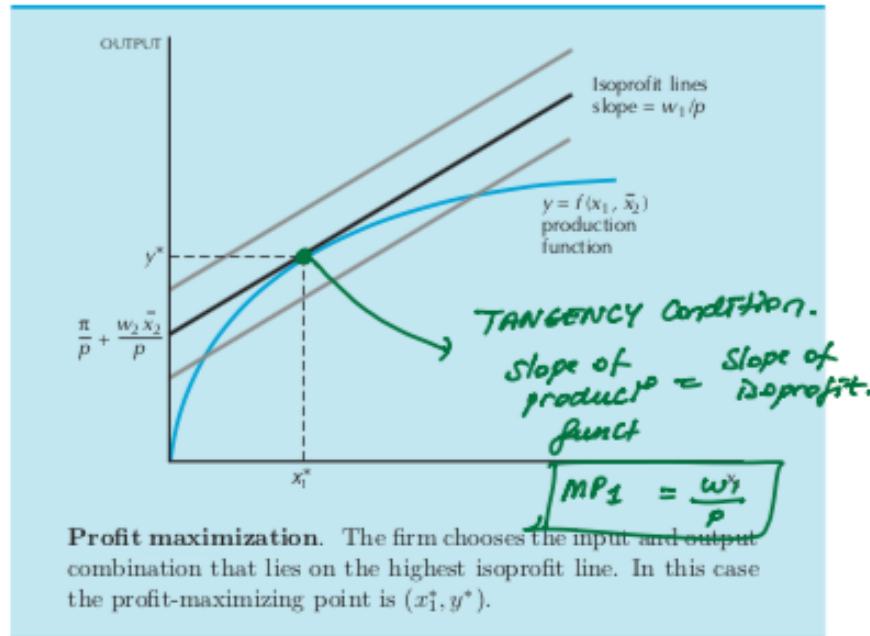
2.2.1 Tangency Condition

The slope of the production function should equal the slope the isoprofit line

$$MP_1 = \frac{w_1}{p}$$

or

$$pMP_1 = w_1$$



2.3 Long-Run Profit Maximization

In the Long-Run, we are free to choose Level of All Inputs

Definition 2.3.1

Long-Run Profit-Maximization

$$\max_{x_1, x_2} pf(x_1, x_2) - w_1x_1 - w_2x_2$$

Generalize

- $pMP_1(x_1^*, x_2^*) = w_1$
- $pMP_2(x_1^*, x_2^*) = w_2$

In Long Run Additional Revenue from x_i = Additional Cost of x_i

2.4 Inverse Factor Demand Curves

Factor Demand Curves measures the relationship between the price of a factor and the profit-maximizing choice of that factor