Image Restoration using 2-D Wiener Filtering

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1. **Introduction**
   Computer assignments aim to study the effects of Wiener Filtering in image processing methods such as De-Blurring, Noise Removal, and Image Restoration. All goals were accomplished using MATLAB code [1].

2. **Theory**

   The Wiener Filtering is the method where the filter input is an additive mixture of valid signal x[n] and noise n[n]. The filter has a recursive structure with an impulse response (Figure 1). The task is to create such a filter so that the MSE is minimal [2].

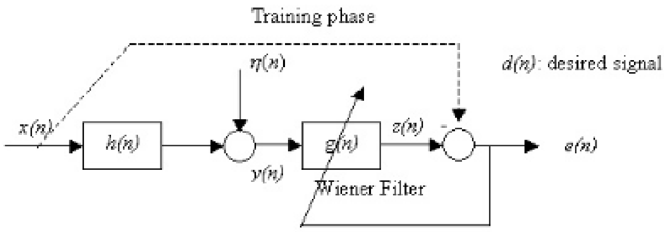

Figure 1 The Wiener Filtering method

This system's DTFT transfer function, which was obtained from MSE criteria, was presented in formula (1).

$$G\left(e^{j\Omega}\right) = \frac{S_{dy}(e^{j\Omega})}{S_{yy}(e^{j\Omega})} \quad (1)$$

The r_{yy} and r_{dy} Signals in this transfer function are fined from Figure 1 (2).

$$r_{yy}(k) = h(k) * h(-k) * r_{xx}(k) + r_{nn}(k) \ (2)$$
$$r_{dy}(k) = \ h(-k) * r_{xx}(k)$$

These signals transform in DTFT space into the following equations (3).

$$S_{dy}(e^{j\Omega}) = \left|H\left(e^{j\Omega}\right)\right|^2 S_{xx}\left(e^{j\Omega}\right) + S_{nn}\left(e^{j\Omega}\right) \ (3)$$
$$S_{yy}(e^{j\Omega}) = \ H^*\left(e^{j\Omega}\right) S_{xx}\left(e^{j\Omega}\right)$$

These equations were used to find the transfer function for the 2D-image signal (4).

$$G(k,l) = \frac{H^*(k,l)S_{xx}(k,l)}{|H(k,l)|^2 S_{xx}(k,l) + S_{nn}(k,l)} \quad (4)$$

3. Results and discussion

a. De-Blurring

The Matlab code provides algorithmic for the blurring and de-blurring image "Lena" by using the Wiener filter. The result of processing the image is presented in Figure 2.

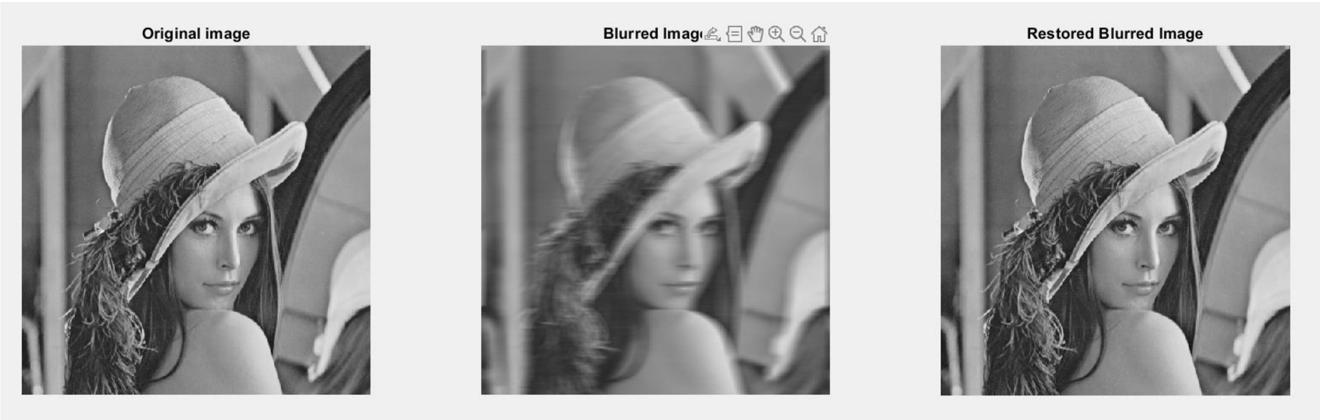

Figure 2 The original, blurred, and de-blurred images.

Wiener deconvolution can be used effectively when the frequency characteristics of the image and additive noise are known to some extent. In the absence of noise, the Wiener filter reduces to an ideal inverse filter. Figure 1 shows the ideal de-blurring, but an assessment is needed to find the transfer function or use a training algorithm to minimize MSE the same as in Machine learning techniques.

b. Noise Removal

The Matlab code provides an algorithm for reducing noise by using a Wiener filter. The result is presented in Figure 3.

Figure 3 The original noise and filtering image.

The noise image has SNR equal to 8.0157 and a filtered image equal to 17.4621, which is almost twice bigger. This is a significant improvement in the image. The Wiener filter considers statistical behaviors of the noise and the signal and theoretically achieves optimum separation. However, in the Wiener filter frequency response, where noise power is greater than the signal, the output has gained the noise, and otherwise, the signal was gained. Thus, to design a Wiener filter, the engineer must know the PDF of the noise and the desired signal. However, the majority of cases do not have this sort of information, but they can be estimated from available data. The result is a departure from the optimum performance.

c. Image Restoration

The Matlab code provides an algorithm for reducing blurring and noise by using a Wiener filter. The result is presented in Figure 4.

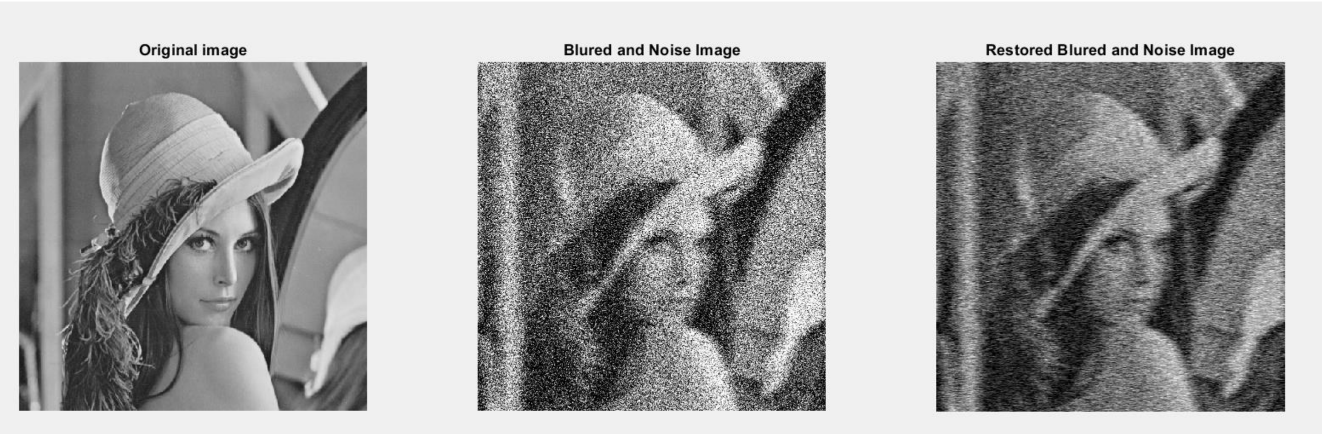

Figure 4. The original, noise and blur, and filtering image

The noise image has SNR equal to 5.0858 and a filtered image equal to 10.5186, almost twice bigger. This is a significant improvement in the image. However, the figure shows that the restored image was not ideal noise and blur reduction from the visual check. Also, the restored image becomes darker than the original. That could be because the prediction of the noise was determined incorrectly.

4. Conclusion

The computer assignments show the application of the Wiener filter. The computer assignments provide a method of decreasing the noise and blur of the image. Also, the project shows that the Wiener filter is the result and optimal if the need to minimize the MSE is as if the noise and image are Gaussian subspaces. However, the MSE problem is not always relevant, and in such cases, the Wiener filter is useless.

Reference

1. The official MATLAB site: https://www.mathworks.com/
2. The professor M.R Azimi lecture 23-24