Study of Factors Affecting the Unemployment Rate in the United States

Multivariate Time Series Analysis

- **Time Series Names and Source:** FRED API
- **Frequency of Time Series:** Quarterly [1Q]
- **Analyzed Period:** Q1 1980 - Q4 2022

Created by: Boglárka Póra and Anna Fischer

The first thing to do is clear the figures and variable values from memory so that we can start the work with a clean slate.

```
rm(list = ls())
graphics.off()
```

We install (where necessary) and load the packages that we will need for our analysis.

```
install.packages('tidyverse')
install.packages('tidyquant')
install.packages('scales')
install.packages('fpp3')
install.packages('timetk')
install.packages('aTSA')
install.packages('gridExtra')
install.packages('plotly')
install.packages('ggpubr')
install.packages('strucchange')
install.packages('urca')
install.packages('vars')
install.packages('MTS')
```

```
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Warning message:
“package ‘tidyverse’ is not available for this version of R

A version of this package for your version of R might be available elsewhere,
see the ideas at
https://cran.r-project.org/doc/manuals/r-patched/R-admin.html#Installing-packages”
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘shape’, ‘future.apply’, ‘numDeriv’, ‘progressr’, ‘SQUAREM’, ‘diagram’, ‘lava’, ‘prodlim’, ‘glob


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘distributional’, ‘ggdist’, ‘fable’, ‘fabletools’, ‘feasts’, ‘tsibble’, ‘tsibbledata’


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘Deriv’, ‘microbenchmark’, ‘doBy’, ‘SparseM’, ‘MatrixModels’, ‘minqa’, ‘nloptr’, ‘RcppEigen’, ‘c


also installing the dependency ‘sandwich’


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)
```

```
Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)

Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)

also installing the dependencies 'rbibutils', 'gbutils', 'Rdpack', 'timeSeries', 'fastICA', 'cvar', 'gss', 'stabledist', 'fGarch'
```

```
library(tidyverse)
library(tidyquant)
library(scales)
library(fpp3)
library(timetk)
library(aTSA)
library(gridExtra)
library(plotly)
library(ggpubr)
library(strucchange)
library(urca)
library(vars)
library(MTS)
```

```
Loading required package: MASS


Attaching package: 'MASS'


The following object is masked from 'package:plotly':

    select


The following object is masked from 'package:dplyr':

    select


Loading required package: lmtest


Attaching package: 'vars'


The following object is masked from 'package:aTSA':

    arch.test


The following object is masked from 'package:fable':

    VAR


The following object is masked from 'package:tidyquant':

    VAR



Attaching package: 'MTS'


The following object is masked from 'package:vars':

    VAR


The following object is masked from 'package:fable':

    VAR


The following object is masked from 'package:tidyquant':

    VAR
```

Loading, Aggregating, Transforming, and Visualizing Time Series Data

The proxy variable for economic growth is the real GDP growth rate. We will calculate this rate and then assign more descriptive names to the variables.

```
rgdp <-  tq_get("GDPC1", get = 'economic.data',
                from = "1980-01-01",
                to = "2022-12-31") %>%
  dplyr::select(date, price)
head(rgdp)
```

A tibble: 6 × 2

date	price
<date>	<dbl>
1980-01-01	7341.557
1980-04-01	7190.289
1980-07-01	7181.743
1980-10-01	7315.677
1981-01-01	7459.022
1981-04-01	7403.745

```
data <- rgdp %>%
  transmute(time = date, rgdp = price)
head(data)
```

A tibble: 6 × 2

time	rgdp
<date>	<dbl>
1980-01-01	7341.557
1980-04-01	7190.289
1980-07-01	7181.743
1980-10-01	7315.677
1981-01-01	7459.022
1981-04-01	7403.745

The next variable is the Consumer Price Index for the United States.

```
inf <- tq_get("USACPIALLMINMEI", get = 'economic.data',
              from = "1980-01-01",
              to = "2022-12-31") %>%
  dplyr::select(date, price)
head(inf)
```

A tibble: 6 × 2

date	price
<date>	<dbl>
1980-01-01	32.82465
1980-02-01	33.28875
1980-03-01	33.79504
1980-04-01	34.17476
1980-05-01	34.51229
1980-06-01	34.89201

```
inf <- inf %>%
  rename(inf = price)
head(inf)
```

A tibble: 6 × 2

date	inf
<date>	<dbl>
1980-01-01	32.82465
1980-02-01	33.28875
1980-03-01	33.79504
1980-04-01	34.17476
1980-05-01	34.51229
1980-06-01	34.89201

```
inf <- inf %>%
  mutate(year = year(date),
         quarter = quarter(date))
head(inf)
```

A tibble: 6 × 4

date	inf	year	quarter
<date>	<dbl>	<dbl>	<int>
1980-01-01	32.82465	1980	1
1980-02-01	33.28875	1980	1
1980-03-01	33.79504	1980	1
1980-04-01	34.17476	1980	2
1980-05-01	34.51229	1980	2
1980-06-01	34.89201	1980	2

```
agg_inf <- inf %>%
  dplyr::select(year, quarter, inf) %>%
  group_by(year, quarter) %>%
  summarise(inf = mean(inf)) %>%
  ungroup()
head(agg_inf)
```

```
`summarise()` has grouped output by 'year'. You can override using the
`.groups` argument.
```

A tibble: 6 × 3

year	quarter	inf
<dbl>	<int>	<dbl>
1980	1	33.30281
1980	2	34.52635
1980	3	35.15922
1980	4	36.08742
1981	1	37.04376
1981	2	37.90164

```
agg_inf <- agg_inf %>%
  mutate(time = yq(paste0(year, '-', quarter))) %>%
  dplyr::select(time, inf)
head(agg_inf)
```

A tibble: 6 × 2

time	inf
<date>	<dbl>
1980-01-01	33.30281
1980-04-01	34.52635
1980-07-01	35.15922
1980-10-01	36.08742
1981-01-01	37.04376
1981-04-01	37.90164

The next variable is the U.S. Federal Reserve's interest rate for overnight lending.

```
int <- tq_get("IRSTFR01USM156N", get = 'economic.data',
              from = "1980-01-01",
              to = "2022-12-31") %>%
  dplyr::select(date, price)
head(int)
```

A tibble: 6 × 2

date	price
<date>	<dbl>
1980-01-01	13.82
1980-02-01	14.13
1980-03-01	17.19
1980-04-01	17.61
1980-05-01	10.98
1980-06-01	9.47

```
int <- int %>%
  rename(int = price)
head(int)
```

A tibble: 6 × 2

date	int
<date>	<dbl>
1980-01-01	13.82
1980-02-01	14.13
1980-03-01	17.19
1980-04-01	17.61
1980-05-01	10.98
1980-06-01	9.47

```
int <- int %>%
  mutate(year = year(date),
         quarter = quarter(date))
head(int)
```

A tibble: 6 × 4

date	int	year	quarter
<date>	<dbl>	<dbl>	<int>
1980-01-01	13.82	1980	1
1980-02-01	14.13	1980	1
1980-03-01	17.19	1980	1
1980-04-01	17.61	1980	2
1980-05-01	10.98	1980	2
1980-06-01	9.47	1980	2

Interest rate aggregation:

```
agg_int <- int %>%
  dplyr::select(year, quarter, int) %>%
  group_by(year, quarter) %>%
  summarise(int = mean(int)) %>%
  ungroup()
head(agg_int)
```

```
`summarise()` has grouped output by 'year'. You can override using the
`.groups` argument.
```

A tibble: 6 × 3

year	quarter	int
<dbl>	<int>	<dbl>
1980	1	15.046667
1980	2	12.686667
1980	3	9.836667
1980	4	15.853333
1981	1	16.570000
1981	2	17.780000

```
agg_int <- agg_int %>%
  mutate(time = yq(paste0(year, '-', quarter))) %>%
  dplyr::select(time, int)
head(agg_int)
```

A tibble: 6 × 2

time	int
<date>	<dbl>
1980-01-01	15.046667
1980-04-01	12.686667
1980-07-01	9.836667
1980-10-01	15.853333
1981-01-01	16.570000
1981-04-01	17.780000

The productivity of the corporate sector in the United States is the fourth dimension.

```
prod <- tq_get("OPHPBS", get = 'economic.data',
               from = "1980-01-01",
               to = "2022-12-31") %>%
  dplyr::select(date, price)
head(prod)
```

A tibble: 6 × 2

date	price
<date>	<dbl>
1980-01-01	48.612
1980-04-01	48.087
1980-07-01	48.147
1980-10-01	48.626
1981-01-01	49.543
1981-04-01	49.096

```
prod <- prod %>%
  rename(prod = price)
head(prod)
```

A tibble: 6 × 2

date	prod
<date>	<dbl>
1980-01-01	48.612
1980-04-01	48.087
1980-07-01	48.147
1980-10-01	48.626
1981-01-01	49.543
1981-04-01	49.096

```
prod <- prod %>%
  mutate(year = year(date),
         quarter = quarter(date))
head(prod)
```

A tibble: 6 × 4

date	prod	year	quarter
<date>	<dbl>	<dbl>	<int>
1980-01-01	48.612	1980	1
1980-04-01	48.087	1980	2
1980-07-01	48.147	1980	3
1980-10-01	48.626	1980	4
1981-01-01	49.543	1981	1
1981-04-01	49.096	1981	2

The next step is to aggregate productivity.

```
agg_prod <- prod %>%
  dplyr::select(year, quarter, prod) %>%
  group_by(year, quarter) %>%
  summarise(prod = mean(prod)) %>%
  ungroup()
head(agg_prod)
```

```
`summarise()` has grouped output by 'year'. You can override using the
`.groups` argument.
```

A tibble: 6 × 3

year	quarter	prod
<dbl>	<int>	<dbl>
1980	1	48.612
1980	2	48.087
1980	3	48.147
1980	4	48.626
1981	1	49.543
1981	2	49.096

```
agg_prod <- agg_prod %>%
  mutate(time = yq(paste0(year, '-', quarter))) %>%
  dplyr::select(time, prod)
head(agg_prod)
```

A tibble: 6 × 2

time	prod
<date>	<dbl>
1980-01-01	48.612
1980-04-01	48.087
1980-07-01	48.147
1980-10-01	48.626
1981-01-01	49.543
1981-04-01	49.096

The next dimension is the number of unemployed as a percentage of the employed.

```
unemp <- tq_get("UNRATE", get = 'economic.data',
                from = "1980-01-01",
                to = "2022-12-31") %>%
  dplyr::select(date, price)
head(unemp)
```

A tibble: 6 × 2

date	price
<date>	<dbl>
1980-01-01	6.3
1980-02-01	6.3
1980-03-01	6.3
1980-04-01	6.9
1980-05-01	7.5
1980-06-01	7.6

```
unemp <- unemp %>%
  rename(unemp = price)
head(unemp)
```

A tibble: 6 × 2

date	unemp
<date>	<dbl>
1980-01-01	6.3
1980-02-01	6.3
1980-03-01	6.3
1980-04-01	6.9
1980-05-01	7.5
1980-06-01	7.6

```
unemp <- unemp %>%
  mutate(year = year(date),
         quarter = quarter(date))
head(unemp)
```

A tibble: 6 × 4

date	unemp	year	quarter
<date>	<dbl>	<dbl>	<int>
1980-01-01	6.3	1980	1
1980-02-01	6.3	1980	1
1980-03-01	6.3	1980	1
1980-04-01	6.9	1980	2
1980-05-01	7.5	1980	2
1980-06-01	7.6	1980	2

Unemployment rate aggregation:

```
agg_unemp <- unemp %>%
  dplyr::select(year, quarter, unemp) %>%
  group_by(year, quarter) %>%
  summarise(unemp = mean(unemp)) %>%
  ungroup()
head(agg_unemp)
```

```
`summarise()` has grouped output by 'year'. You can override using the
`.groups` argument.
```

A tibble: 6 × 3

year	quarter	unemp
<dbl>	<int>	<dbl>
1980	1	6.300000
1980	2	7.333333
1980	3	7.666667
1980	4	7.400000
1981	1	7.433333
1981	2	7.400000

```
agg_unemp <- agg_unemp %>%
  mutate(time = yq(paste0(year, '-', quarter))) %>%
  dplyr::select(time, unemp)
head(agg_unemp)
```

A tibble: 6 × 2

time	unemp
<date>	<dbl>
1980-01-01	6.300000
1980-04-01	7.333333
1980-07-01	7.666667
1980-10-01	7.400000
1981-01-01	7.433333
1981-04-01	7.400000

```
rm(inf, int, prod, rgdp, unemp)
```

Next, we sequentially append the sub-datasets containing each variable to the final dataset.

```
data <- left_join(data, agg_inf, by = "time")
data <- left_join(data, agg_int, by = "time")
data <- left_join(data, agg_prod, by = "time")
data <- left_join(data, agg_unemp, by = "time")
rm(agg_inf, agg_int, agg_prod, agg_unemp)
head(data)
```

A tibble: 6 × 6

time	rgdp	inf	int	prod	unemp
<date>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
1980-01-01	7341.557	33.30281	15.046667	48.612	6.300000
1980-04-01	7190.289	34.52635	12.686667	48.087	7.333333
1980-07-01	7181.743	35.15922	9.836667	48.147	7.666667
1980-10-01	7315.677	36.08742	15.853333	48.626	7.400000
1981-01-01	7459.022	37.04376	16.570000	49.543	7.433333
1981-04-01	7403.745	37.90164	17.780000	49.096	7.400000

Next, we need to convert our data frame into a tsibble object so that we can handle the different dimensions as time series.

```
data <- data %>%
  mutate(date = yearquarter(time)) %>%
  as_tsibble(index = date) %>%
  dplyr::select(-time)
head(data)
```

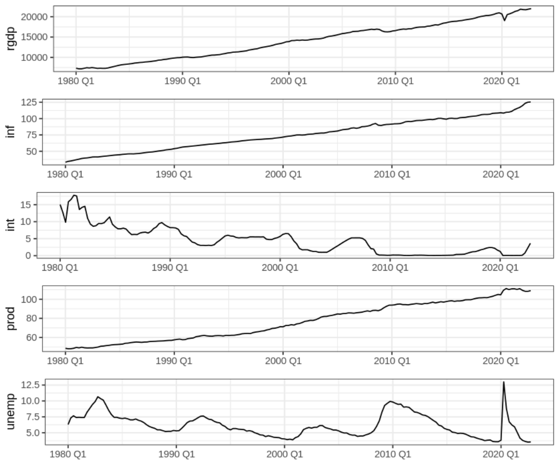
A tbl_ts: 6 × 6

rgdp	inf	int	prod	unemp	date
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>
7341.557	33.30281	15.046667	48.612	6.300000	1980 Q1
7190.289	34.52635	12.686667	48.087	7.333333	1980 Q2
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3
7315.677	36.08742	15.853333	48.626	7.400000	1980 Q4
7459.022	37.04376	16.570000	49.543	7.433333	1981 Q1
7403.745	37.90164	17.780000	49.096	7.400000	1981 Q2

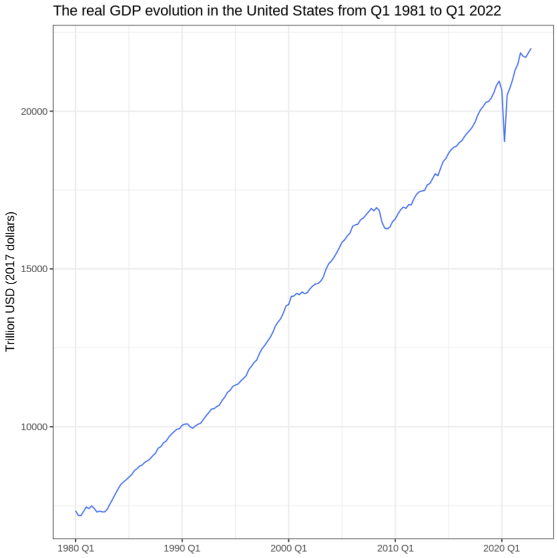
As seen above, we are working with data of quarterly frequency.

Next, we will visualize the evolution of the various variables over time periods.

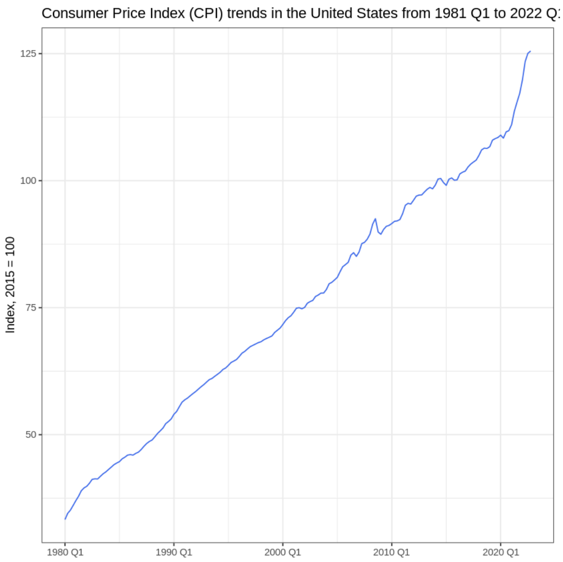
```
rgdp_plot <- ggplot(data, mapping = aes(y=rgdp, x=date)) +
  geom_line() + labs(x=NULL) + theme_bw()

inf_plot <- ggplot(data, mapping = aes(y=inf, x=date)) +
  geom_line() + labs(x=NULL) + theme_bw()

int_plot <- ggplot(data, mapping = aes(y=int, x=date)) +
  geom_line() + labs(x=NULL) + theme_bw()

prod_plot <- ggplot(data, mapping = aes(y=prod, x=date)) +
  geom_line() + labs(x=NULL) + theme_bw()

unemp_plot <- ggplot(data, mapping = aes(y=unemp, x=date)) +
  geom_line() + labs(x=NULL) + theme_bw()

ggarrange(rgdp_plot, inf_plot, int_plot, prod_plot, unemp_plot, ncol = 1, nrow = 6)
```



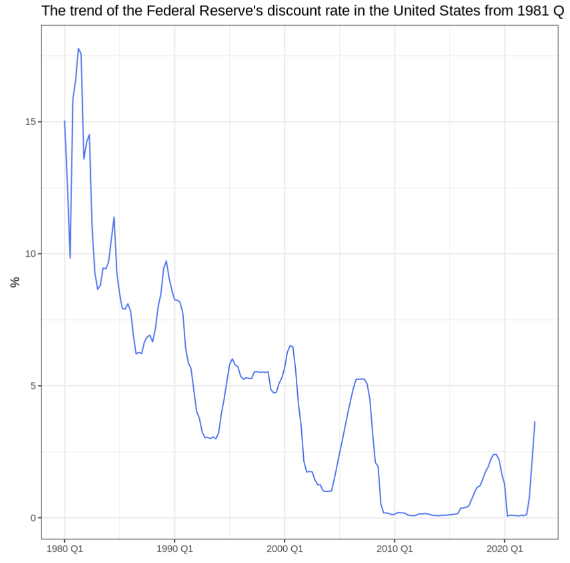

```
rgdp <- data %>%
  ggplot(aes(x = date, y = rgdp)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "Trillion USD (2017 dollars)", color = NULL,
       title = "The real GDP evolution in the United States from Q1 1981 to Q1 2022") +
  theme_bw()
rgdp
```




```
inf <- data %>%
  ggplot(aes(x = date, y = inf)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "Index, 2015 = 100", color = NULL,
       title = "Consumer Price Index (CPI) trends in the United States from 1981 Q1 to 2022 Q1") +
  theme_bw()
inf
```

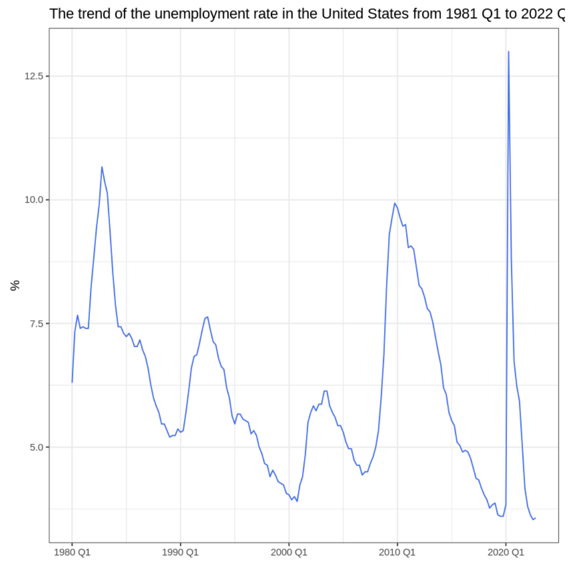



```
int <- data %>%
  ggplot(aes(x = date, y = int)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "%", color = NULL,
       title = "The trend of the Federal Reserve's discount rate in the United States from 1981 Q1 to 2022 Q1") +
  theme_bw()
int
```




```
prod <- data %>%
  ggplot(aes(x = date, y = prod)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "Index, 2017 = 100", color = NULL,
       title = "The trend of productivity in the corporate sector in the United States from 1981 Q1 to 2022 Q1") +
  theme_bw()
prod
```


```
unemp <- data %>%
  ggplot(aes(x = date, y = unemp)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "%", color = NULL,
       title = "The trend of the unemployment rate in the United States from 1981 Q1 to 2022 Q1") +
  theme_bw()
unemp
```



Next, we will add the differenced values of the dimensions to the tsibble.

```
data <-  data %>%
  mutate(ln_unemp = difference(unemp),
         ln_int = difference(int)) %>%
  drop_na()
head(data)
```

A tbl_ts: 6 × 8

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>
7190.289	34.52635	12.686667	48.087	7.333333	1980 Q2	1.03333333	-2.3600000
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.8500000
7315.677	36.08742	15.853333	48.626	7.400000	1980 Q4	-0.26666667	6.0166667
7459.022	37.04376	16.570000	49.543	7.433333	1981 Q1	0.03333333	0.7166667
7403.745	37.90164	17.780000	49.096	7.400000	1981 Q2	-0.03333333	1.2100000
7492.405	38.97048	17.576667	49.772	7.400000	1981 Q3	0.00000000	-0.2033333

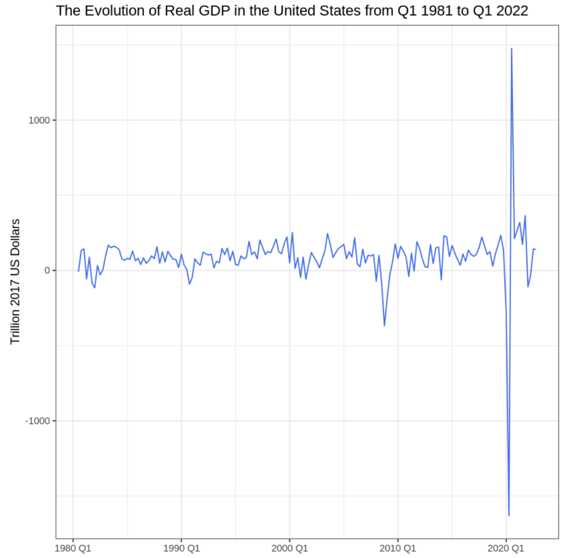
```
data <-  data %>%
  mutate(dif_rgdp = difference(rgdp),
         dif_inf = difference(inf),
         dif_prod = difference(prod),
         dif_unemp = difference(ln_unemp),
         dif_int = difference(ln_int)) %>%
  drop_na()
head(data)
```

A tbl_ts: 6 × 13

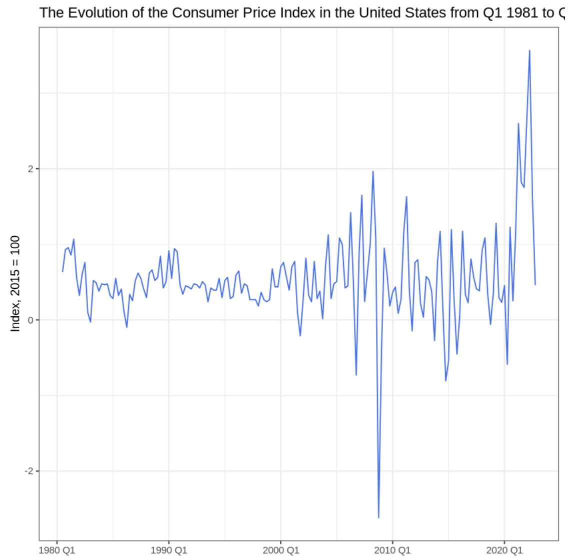
rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.8500000	-8.546	0.6328667	0.060	-0.70000000	-0.4900000
7315.677	36.08742	15.853333	48.626	7.400000	1980 Q4	-0.26666667	6.0166667	133.934	0.9282033	0.479	-0.60000000	8.8666667
7459.022	37.04376	16.570000	49.543	7.433333	1981 Q1	0.03333333	0.7166667	143.345	0.9563333	0.917	0.30000000	-5.3000000
7403.745	37.90164	17.780000	49.096	7.400000	1981 Q2	-0.03333333	1.2100000	-55.277	0.8578867	-0.447	-0.06666667	0.4933333
7492.405	38.97048	17.576667	49.772	7.400000	1981 Q3	0.00000000	-0.2033333	88.660	1.0688400	0.676	0.03333333	-1.4133333
7410.768	39.53303	13.586667	49.171	8.233333	1981 Q4	0.83333333	-3.9900000	-81.637	0.5625467	-0.601	0.83333333	-3.7866667

Visualization:

```
dif_rgdp <- data %>%
  ggplot(aes(x = date, y = dif_rgdp)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "Trillion 2017 US Dollars", color = NULL,
       title = "The Evolution of Real GDP in the United States from Q1 1981 to Q1 2022") +
  theme_bw()
dif_rgdp
```

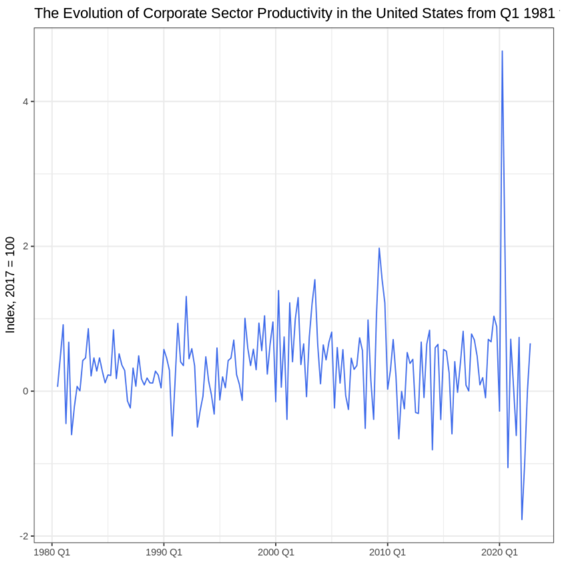



```
dif_inf <- data %>%
  ggplot(aes(x = date, y = dif_inf)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "Index, 2015 = 100", color = NULL,
       title = "The Evolution of the Consumer Price Index in the United States from Q1 1981 to Q1 2022") +
  theme_bw()
dif_inf
```

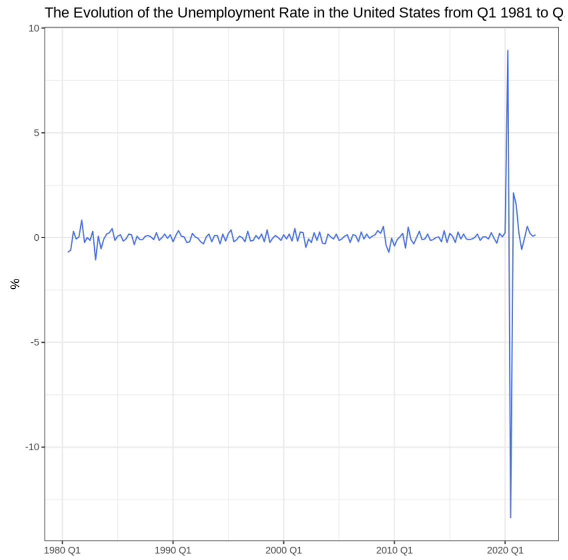



```
dif_int <- data %>%
  ggplot(aes(x = date, y = dif_int)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "%", color = NULL,
       title = "The Evolution of the Federal Reserve's Discount Rate in the United States from Q1 1981 to Q1 2022") +
  theme_bw()
dif_int
```


```
dif_prod <- data %>%
  ggplot(aes(x = date, y = dif_prod)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "Index, 2017 = 100", color = NULL,
       title = "The Evolution of Corporate Sector Productivity in the United States from Q1 1981 to Q1 2022") +
  theme_bw()
dif_prod
```




```
dif_unemp <- data %>%
  ggplot(aes(x = date, y = dif_unemp)) +
  geom_line(color = "royalblue2") +
  labs(x = NULL, y = "%", color = NULL,
       title = "The Evolution of the Unemployment Rate in the United States from Q1 1981 to Q1 2022") +
  theme_bw()
dif_unemp
```




```
head(data)
```

A tbl_ts: 6 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.8500000	-8.546	0.6328667	0.060	-0.70000000	-0.4900000
7315.677	36.08742	15.853333	48.626	7.400000	1980 Q4	-0.26666667	6.0166667	133.934	0.9282033	0.479	-0.60000000	8.8666667
7459.022	37.04376	16.570000	49.543	7.433333	1981 Q1	0.03333333	0.7166667	143.345	0.9563333	0.917	0.30000000	-5.3000000
7403.745	37.90164	17.780000	49.096	7.400000	1981 Q2	-0.03333333	1.2100000	-55.277	0.8578867	-0.447	-0.06666667	0.4933333
7492.405	38.97048	17.576667	49.772	7.400000	1981 Q3	0.00000000	-0.2033333	88.660	1.0688400	0.676	0.03333333	-1.4133333
7410.768	39.53303	13.586667	49.171	8.233333	1981 Q4	0.83333333	-3.9900000	-81.637	0.5625467	-0.601	0.83333333	-3.7866667

Examination of Structural Breaks

H0: There is no structural break in the time series.

H1: There is a structural break in the time series.

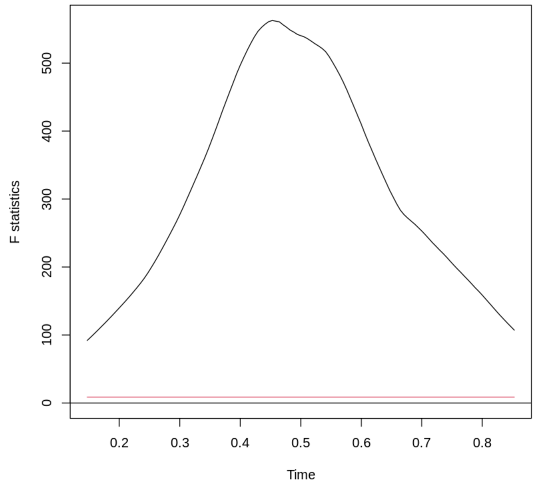
If the significance level (p-value) is less than 0.05, we accept the alternative hypothesis. If it is greater than 0.05, we retain the null hypothesis.

Structural Break Analysis - rgdp

QLR Statistic

```
rgdp_qlr <- Fstats(rgdp ~ 1, data = data)
breakpoints(rgdp_qlr)
sctest(rgdp_qlr, type = "supF")
plot(rgdp_qlr)
```

```
         Optimal 2-segment partition:

Call:
breakpoints.Fstats(obj = rgdp_qlr)

Breakpoints at observation number:
77

Corresponding to breakdates:
0.4470588

        supF test

data:  rgdp_qlr
sup.F = 562.72, p-value < 2.2e-16
```



The p-value is significantly smaller than 0.05, so we reject the null hypothesis. This indicates that there is a structural break in the real GDP time series.

```
data %>%
  slice(rgdp_qlr$breakpoint)
```

A tbl_ts: 1 × 13

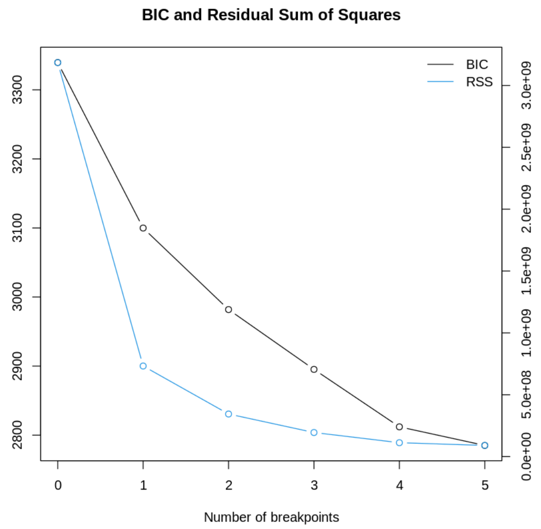
rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
13604.77	70.55752	5.093333	70.334	4.233333	1999 Q3	-0.03333333	0.3466667	178.023	0.4359733	0.65	0	0.3333333

The break occurred in the third quarter of 1999.

BP Statistics

```
rgdp_bp <- breakpoints(rgdp ~ 1, data = data, breaks = 5)
summary(rgdp_bp)
plot(rgdp_bp, breaks = 5)
```

```
	 Optimal (m+1)-segment partition: 

Call:
breakpoints.formula(formula = rgdp ~ 1, breaks = 5, data = data)

Breakpoints at observation number:
                        
m = 1       77          
m = 2       70    132   
m = 3       55 92     137
m = 4    30 67 96     138
m = 5    30 67 93 120 145

Corresponding to breakdates:
                                                               
m = 1                      0.452941176470588                   
m = 2                      0.411764705882353                   0.776470588235294
m = 3                      0.323529411764706 0.541176470588235 
m = 4    0.176470588235294 0.394117647058824 0.564705882352941 
m = 5    0.176470588235294 0.394117647058824 0.547058823529412 0.705882352941176
                          
m = 1                     
m = 2                     
m = 3    0.805882352941176
m = 4    0.811764705882353
m = 5    0.852941176470588

Fit:
                                                       
m   0         1         2         3         4         5        
RSS 3.185e+09 7.323e+08 3.443e+08 1.947e+08 1.124e+08 9.028e+07
BIC 3.340e+03 3.100e+03 2.982e+03 2.895e+03 2.812e+03 2.785e+03
```



Since the minimum point of the BIC curve is at 5, the test identifies 5 structural breaks in the series.

```
data %>%
  slice(rgdp_bp$breakpoint)
```

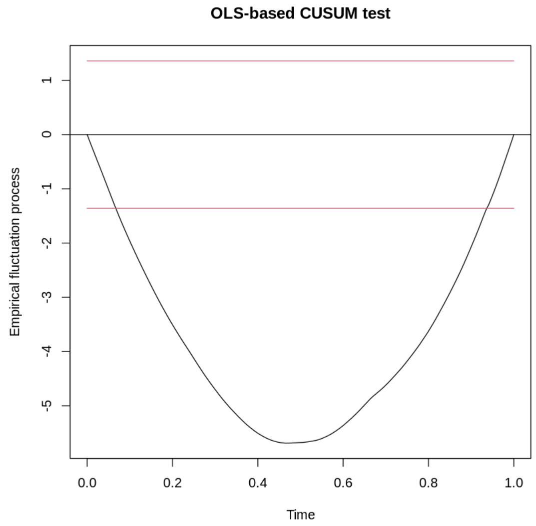
A tbl_ts: 5 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_i
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<db
9319.332	48.67443	6.9166667	55.711	5.833333	1987 Q4	-0.16666667	0.073333333	157.308	0.4078433	0.489	0.10000000	-0.120000
12115.472	67.32288	5.2766667	64.073	5.233333	1997 Q1	-0.10000000	-0.003333333	77.697	0.4500367	-0.127	-0.16666667	0.023333
14988.782	77.87065	1.0166667	81.308	6.133333	2003 Q3	0.00000000	-0.230000000	245.215	0.3797200	1.540	-0.26666667	-0.226666

The occurrence times of the breakpoints can be seen in the "date" column of the table above.

CUSUM Statistics

```
rgdp_cusum <- efp(rgdp ~ 1, data = data, type = "OLS-CUSUM")
plot(rgdp_cusum)
```



There is a structural break in the rgdp time series, as the curve intersects the lines shown in red.

```
data %>%
  slice(rgdp_cusum$datatsp)
```

A tbl_ts: 2 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.850000	-8.546	0.6328667	0.060	-0.7000000	-0.49000000
21989.981	125.52120	3.653333	108.961	3.566667	2022	0.03333333	1.463333	138.847	0.4591333	0.661	0.1333333	0.04333333

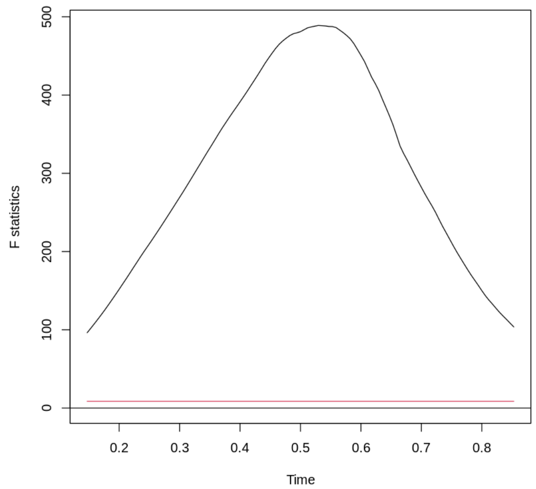
The breakpoints are in the third quarter of 1980 and the fourth quarter of 2022.

Structural Break Analysis - inf

QLR Statistics

```
inf_qlr <- Fstats(inf ~ 1, data = data)
breakpoints(inf_qlr)
sctest(inf_qlr, type = "supF")
plot(inf_qlr)
```

```
         Optimal 2-segment partition:

Call:
breakpoints.Fstats(obj = inf_qlr)

Breakpoints at observation number:
90

Corresponding to breakdates:
0.5235294

        supF test

data:  inf_qlr
sup.F = 488.96, p-value < 2.2e-16
```



The p-value is much smaller than 0.05, so we reject the null hypothesis, indicating that there is a structural break in the inflation time series.

```
data %>%
  slice(inf_qlr$breakpoint)
```

A tbl_ts: 1 × 13

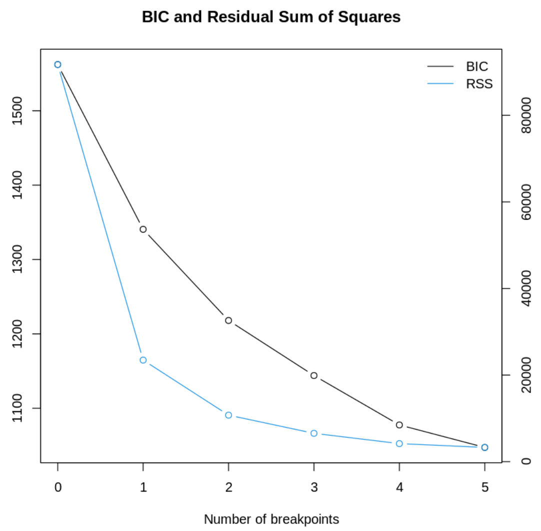
rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
14537.58	76.43615	1.443333	77.811	5.866667	2002 Q4	0.1333333	-0.2966667	17.947	0.23908	-0.076	0.2333333	-0.2866667

The break occurred in the fourth quarter of 2002.

BP Statistics

```
inf_bp <- breakpoints(inf ~ 1, data = data, breaks = 5)
summary(inf_bp)
plot(inf_bp, breaks = 5)
```

```
         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = inf ~ 1, breaks = 5, data = data)

Breakpoints at observation number:

m = 1         90
m = 2      50 107
m = 3   41    90  127
m = 4   38 77 107     145
m = 5   35 63 95  120 145

Corresponding to breakdates:

m = 1                                         0.529411764705882
m = 2                     0.294117647058823 0.629411764705882
m = 3   0.241176470588235                   0.529411764705882 0.747058823529412
m = 4   0.223529411764706 0.452941176470588 0.629411764705882
m = 5   0.205882352941176 0.370588235294118 0.558823529411765 0.705882352941176

m = 1
m = 2
m = 3
m = 4   0.852941176470588
m = 5   0.852941176470588

Fit:

m   0     1     2     3     4     5
RSS 91729 23457 10738  6540  4165  3282
BIC  1562  1341  1218  1144  1078  1047
```



Since the minimum point of the BIC curve is at 5, the test identified 5 structural breaks.

```
data %>%
  slice(inf_bp$breakpoint)
```

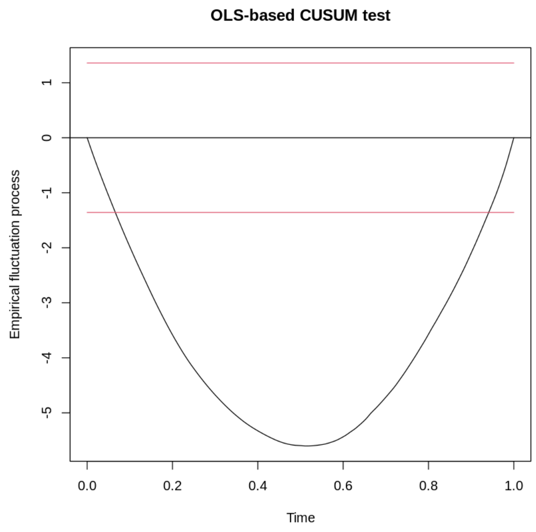
A tbl_ts: 5 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_ir
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl
9771.725	51.33246	9.4433333	56.375	5.200000	1989 Q1	-0.13333333	0.973333333	98.320	0.5625467	0.112	8.881784e-16	0.486666(
11614.418	65.39616	5.3633333	63.171	5.533333	1996 Q1	-0.03333333	-0.356666667	86.351	0.5906767	0.457	6.666667e-02	-0.280000(
15248.680	78.58790	1.0033333	82.062	5.700000	2004 Q1	-0.13333333	0.006666667	85.920	0.7031933	0.102	1.666667e-01	0.026666(

The timing of the breakpoints can be seen in the "date" column of the table above.

CUSUM Statistics

```
inf_cusum <- efp(inf ~ 1, data = data, type = "OLS-CUSUM")
plot(inf_cusum)
```



There is a structural break in the inflation (inf) time series (the curve intersects the lines depicted in red).

```
data %>%
  slice(inf_cusum$datatsp)
```

A tbl_ts: 2 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.850000	-8.546	0.6328667	0.060	-0.7000000	-0.49000000
21989.981	125.52120	3.653333	108.961	3.566667	2022	0.03333333	1.463333	138.847	0.4591333	0.661	0.1333333	0.04333333

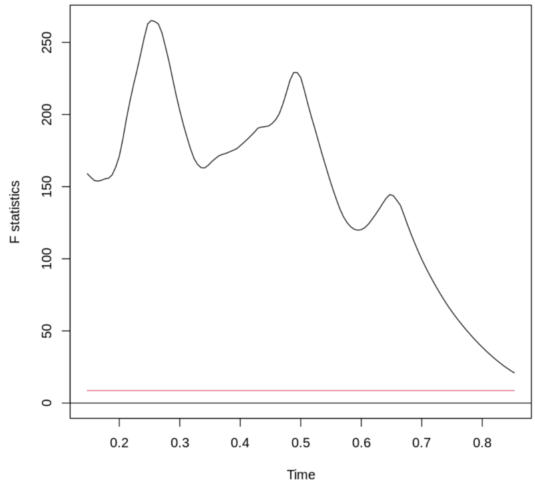
The breakpoints are in the third quarter of 1980 and the fourth quarter of 2022.

Structural Break Analysis - int

QLR Statistics

```
int_qlr <- Fstats(int ~ 1, data = data)
breakpoints(int_qlr)
sctest(int_qlr, type = "supF")
plot(int_qlr)
```

```
         Optimal 2-segment partition: 

Call:
breakpoints.Fstats(obj = int_qlr)

Breakpoints at observation number:
43 

Corresponding to breakdates:
0.2470588 

        supF test

data:  int_qlr
sup.F = 265.22, p-value < 2.2e-16
```



The p-value is significantly less than 0.05, so we reject the null hypothesis, indicating that there is a structural break in the int time series.

```
data %>%
  slice(int_qlr$breakpoint)
```

A tbl_ts: 1 × 13

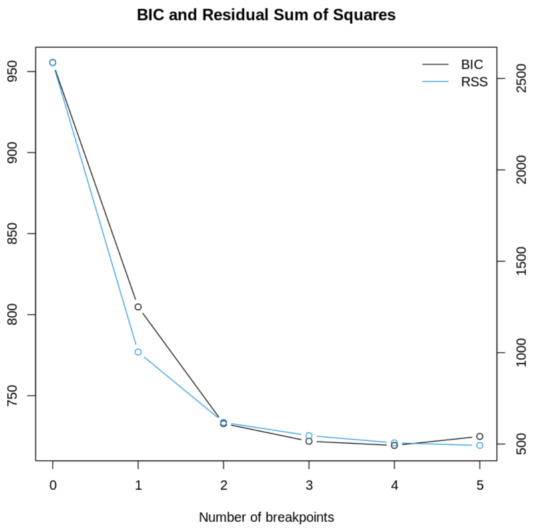
rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
9951.916	56.87356	6.426667	57.756	6.6	1991 Q1	0.4666667	-1.316667	-46.788	0.4641033	0.132	0.03333333	-0.9

The break occurred in the first quarter of 1991.

BP Statistics

```
int_bp <- breakpoints(int ~ 1, data = data, breaks = 5)
summary(int_bp)
plot(int_bp, breaks = 5)
```

```
         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = int ~ 1, breaks = 5, data = data)

Breakpoints at observation number:

m = 1       43
m = 2       42    111
m = 3       42 84 113
m = 4    25 50 84 113
m = 5    25 50 84 113 145

Corresponding to breakdates:

m = 1                      0.252941176470588
m = 2                      0.247058823529412                   0.652941176470588
m = 3                      0.247058823529412 0.494117647058823 0.664705882352941
m = 4   0.147058823529412 0.294117647058823 0.494117647058823 0.664705882352941
m = 5   0.147058823529412 0.294117647058823 0.494117647058823 0.664705882352941

m = 1
m = 2
m = 3
m = 4
m = 5   0.852941176470588

Fit:

m   0      1      2      3      4      5
RSS 2587.2 1003.3  618.5  546.0  506.3  492.4
BIC  955.5  804.8  732.8  721.9  719.3  724.9
```



Since the minimum point of the BIC curve is at 4, the test identified 4 structural breaks.

```
data %>%
  slice(int_bp$breakpoint)
```

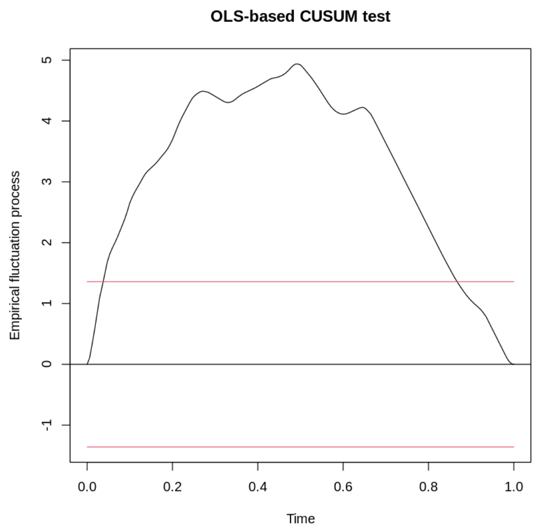
A tbl_ts: 4 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
8872.601	46.32579	6.206667	55.201	6.966667	1986 Q3	-0.2000000	-0.7133333	84.077	0.337530	0.286	-0.3333333	0.1933333
10558.648	59.86912	3.036667	62.143	7.366667	1992 Q4	-0.2666667	-0.2200000	108.975	0.421910	0.349	-0.3000000	0.2933333
14271.694	74.90321	4.326667	74.168	4.400000	2001 Q2	0.1666667	-1.2666667	88.574	0.773500	1.220	-0.1666667	-0.3866667
16854.295	92.51559	1.940000	88.399	6.000000	2008 Q3	0.6666667	-0.1466667	-88.996	1.063637	0.152	0.3333333	0.9433333

The occurrence times of the breakpoints can be seen in the "date" column of the table above.

CUSUM Statistics

```
int_cusum <- efp(int ~ 1, data = data, type = "OLS-CUSUM")
plot(int_cusum)
```



There is a structural break in the int time series (the curve intersects the lines plotted in red).

```
data %>%
  slice(int_cusum$datatsp)
```

A tbl_ts: 2 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.850000	-8.546	0.6328667	0.060	-0.7000000	-0.49000000
21989.981	125.52120	3.653333	108.961	3.566667	2022	0.03333333	1.463333	138.847	0.4591333	0.661	0.1333333	0.04333333

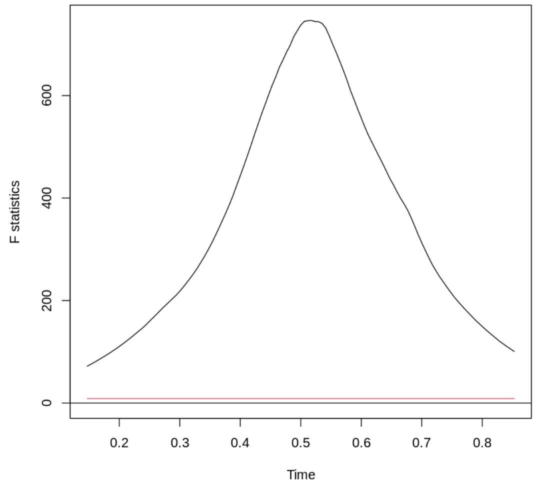
The breakpoints are in the third quarter of 1980 and the fourth quarter of 2022.

Structural Break Analysis - prod

QLR Statistics

```
prod_qlr <- Fstats(prod ~ 1, data = data)
breakpoints(prod_qlr)
sctest(prod_qlr, type = "supF")
plot(prod_qlr)
```

```
         Optimal 2-segment partition: 

Call:
breakpoints.Fstats(obj = prod_qlr)

Breakpoints at observation number:
88 

Corresponding to breakdates:
0.5117647 

        supF test

data:  prod_qlr
sup.F = 746.6, p-value < 2.2e-16
```



The p-value is much smaller than 0.05, so we reject the null hypothesis. There is a structural break in the prod time series.

```
data %>%
  slice(prod_qlr$breakpoint)
```

A tbl_ts: 1 × 13

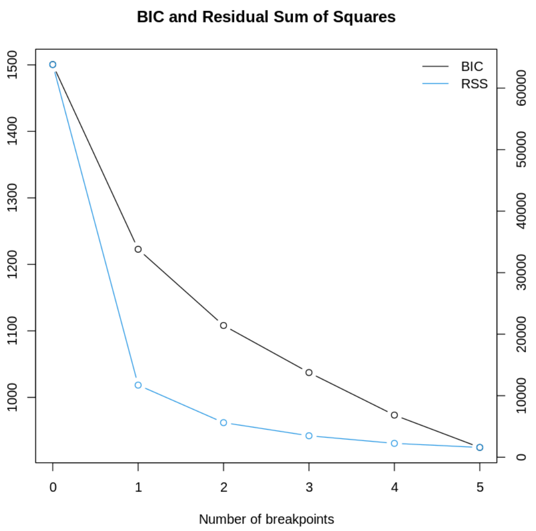
rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
14460.85	75.8736	1.75	77.233	5.833333	2002 Q2	0.1333333	0.01666667	88.063	0.8156933	0.368	-0.06666667	0.4166667

The break occurred in the second quarter of 2002.

BP Statistics

```
prod_bp <- breakpoints(prod ~ 1, data = data, breaks = 5)
summary(prod_bp)
plot(prod_bp, breaks = 5)
```

```
         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = prod ~ 1, breaks = 5, data = data)

Breakpoints at observation number:

m = 1          88
m = 2       74     115
m = 3   46     85 116
m = 4   46     85 115 145
m = 5   36 66 91 116 145

Corresponding to breakdates:

m = 1                                          0.517647058823529
m = 2                     0.435294117647059                      0.676470588235294
m = 3   0.270588235294118                      0.5               0.682352941176471
m = 4   0.270588235294118                      0.5               0.676470588235294
m = 5   0.211764705882353 0.388235294117647 0.535294117647059 0.682352941176471

m = 1
m = 2
m = 3
m = 4   0.852941176470588
m = 5   0.852941176470588

Fit:

m   0       1       2       3       4       5
RSS 63828.3 11724.4  5624.7  3491.5  2256.0  1595.1
BIC  1500.5  1222.7  1108.1  1037.3   973.3   924.7
```



Since the BIC curve's minimum point is at 5, the test identified 5 structural breaks.

```
data %>%
  slice(prod_bp$breakpoint)
```

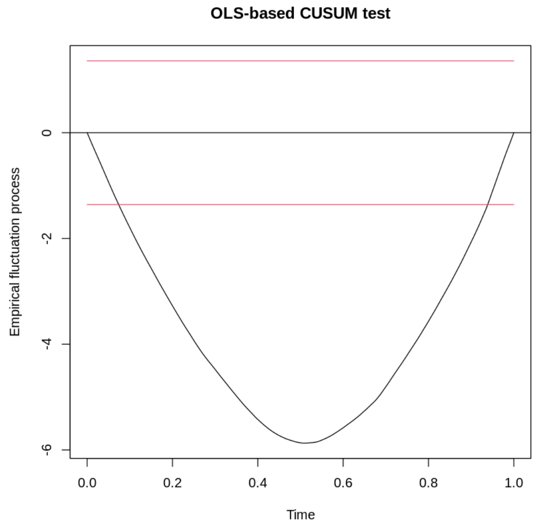
A tbl_ts: 5 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_in
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl
9846.293	52.17628	9.7266667	56.652	5.233333	1989 Q2	0.03333333	0.283333333	74.568	0.8438200	0.277	0.16666667	-0.690000
12037.775	66.87284	5.2800000	64.200	5.333333	1996 Q4	0.06666667	-0.026666667	123.712	0.4781667	0.094	0.30000000	-0.090000
14614.141	77.20965	1.2500000	78.561	5.866667	2003 Q1	0.00000000	-0.193333333	76.561	0.7735067	0.750	-0.13333333	0.103333

The dates of the breakpoints can be seen in the "date" column of the table above.

CUSUM Statistics

```
prod_cusum <- efp(prod ~ 1, data = data, type = "OLS-CUSUM")
plot(prod_cusum)
```



There is a structural break in the prod time series (the curve intersects with the lines shown in red).

```
data %>%
  slice(prod_cusum$datatsp)
```

A tbl_ts: 2 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.850000	-8.546	0.6328667	0.060	-0.7000000	-0.49000000
21989.981	125.52120	3.653333	108.961	3.566667	2022	0.03333333	1.463333	138.847	0.4591333	0.661	0.1333333	0.04333333

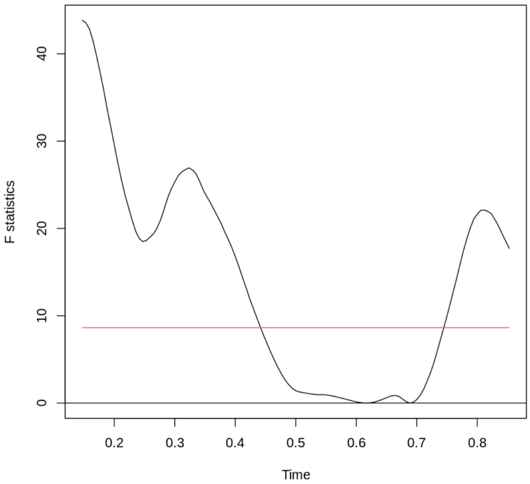
The breakpoints are in the third quarter of 1980 and the fourth quarter of 2022.

Structural Break Analysis - unemp

QLR Statistics

```
unemp_qlr <- Fstats(unemp ~ 1, data = data)
breakpoints(unemp_qlr)
sctest(unemp_qlr, type = "supF")
plot(unemp_qlr)
```

```
         Optimal 2-segment partition:

Call:
breakpoints.Fstats(obj = unemp_qlr)

Breakpoints at observation number:
25

Corresponding to breakdates:
0.1411765

        supF test

data:  unemp_qlr
sup.F = 43.821, p-value = 1.802e-09
```



The p-value is much smaller than 0.05, so we reject the null hypothesis, indicating that there is a structural break in the unemp time series.

```
data %>%
  slice(unemp_qlr$breakpoint)
```

A tbl_ts: 1 × 13

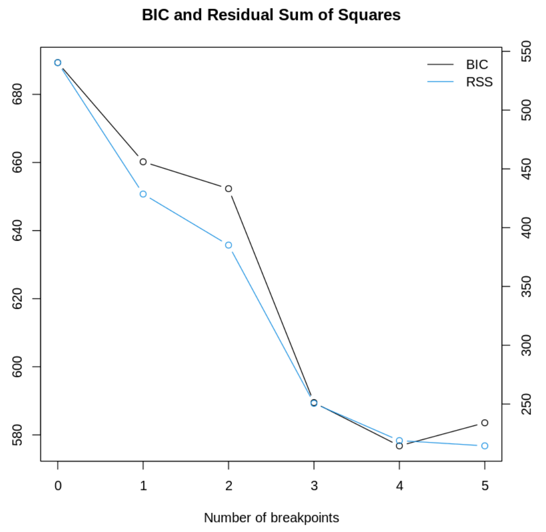
rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
8872.601	46.32579	6.206667	55.201	6.966667	1986 Q3	-0.2	-0.7133333	84.077	0.33753	0.286	-0.3333333	0.1933333

The break occurred in the third quarter of 1986.

BP Statistics

```
unemp_bp <- breakpoints(unemp ~ 1, data = data, breaks = 5)
summary(unemp_bp)
plot(unemp_bp, breaks = 5)
```

```
         Optimal (m+1)-segment partition:

Call:
breakpoints.formula(formula = unemp ~ 1, breaks = 5, data = data)

Breakpoints at observation number:

m = 1   25
m = 2            112 137
m = 3   26       112 137
m = 4   25 57    112 137
m = 5   25 61 86 112 137

Corresponding to breakdates:

m = 1   0.147058823529412
m = 2                                                         0.658823529411765
m = 3   0.152941176470588                                     0.658823529411765
m = 4   0.147058823529412 0.335294117647059                   0.658823529411765
m = 5   0.147058823529412 0.358823529411765 0.505882352941176 0.658823529411765

m = 1
m = 2   0.805882352941176
m = 3   0.805882352941176
m = 4   0.805882352941176
m = 5   0.805882352941176

Fit:

m   0     1     2     3     4     5
RSS 540.4 428.6 385.2 250.6 218.9 214.4
BIC 689.3 660.2 652.3 589.5 576.8 583.6
```



Since the BIC curve has its minimum point at 4, the test identified 4 structural breaks.

```
data %>%
  slice(unemp_bp$breakpoint)
```

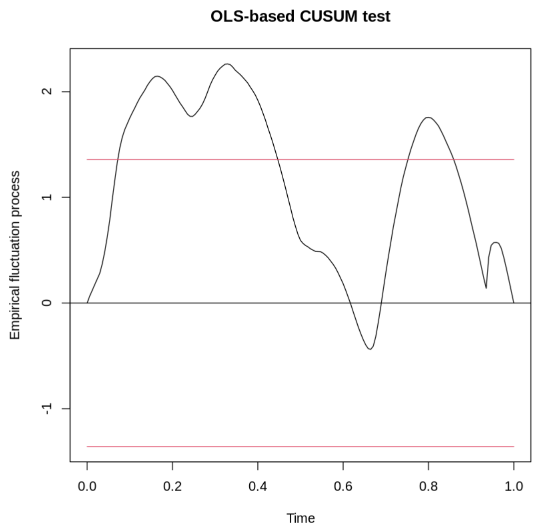
A tbl_ts: 4 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
8872.601	46.32579	6.206667	55.201	6.966667	1986 Q3	-0.2000000	-0.713333333	84.077	0.3375300	0.286	-0.3333333	0.19333333
11152.176	62.83656	4.486667	61.569	6.000000	1994 Q3	-0.2000000	0.546666667	64.815	0.5484833	-0.316	0.1666667	-0.18000000
16943.291	91.45195	2.086667	88.247	5.333333	2008 Q2	0.3333333	-1.090000000	100.288	1.9645567	0.983	0.1333333	0.23000000

The occurrence dates of the breakpoints can be seen in the "date" column of the table above.

CUSUM Statistics

```
unemp_cusum <- efp(unemp~ 1, data = data, type = "OLS-CUSUM")
plot(unemp_cusum)
```



There is a structural break in the unemployment series (the curve intersects the red lines).

```
data %>%
  slice(unemp_cusum$datatsp)
```

A tbl_ts: 2 × 13

rgdp	inf	int	prod	unemp	date	ln_unemp	ln_int	dif_rgdp	dif_inf	dif_prod	dif_unemp	dif_int
<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<qtr>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>	<dbl>
7181.743	35.15922	9.836667	48.147	7.666667	1980 Q3	0.33333333	-2.850000	-8.546	0.6328667	0.060	-0.7000000	-0.49000000
21989.981	125.52120	3.653333	108.961	3.566667	2022	0.03333333	1.463333	138.847	0.4591333	0.661	0.1333333	0.04333333

The breakpoints are in the third quarter of 1980 and the fourth quarter of 2022.

Stationarity Examination

Since the QLR test indicates the presence of structural breaks in the time series, we used the Zivot-Andrews (ZA) test to measure stationarity.

H0: The time series is non-stationary and has a unit root.

H1: The time series is stationary and does not have a unit root.

If the absolute value of the test statistic is greater than the critical value at the 0.05 significance level, we can reject the null hypothesis; otherwise, we must retain it.

Stationarity - rgdp and dif_rgdp

```
rgdp_za <- ur.za(data$rgdp, model = 'both',lag = 2)
summary(rgdp_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
     Min       1Q   Median       3Q      Max 
-1806.12   -62.82    19.82    70.87   772.25 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1341.04341  290.27251   4.620 7.85e-06 ***
y.l1           0.79890    0.04590  17.406  < 2e-16 ***
trend         18.97707    4.23991   4.476 1.44e-05 ***
y.dl1         -0.12176    0.07724  -1.576 0.116894    
y.dl2         -0.03542    0.07569  -0.468 0.640489    
du          -252.61007   69.70742  -3.624 0.000389 ***
dt             3.75488    1.62423   2.312 0.022064 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 182.4 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.9983,	Adjusted R-squared:  0.9982 
F-statistic: 1.522e+04 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -4.3814 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 113 
```

The obtained test statistic value is smaller in absolute terms than the critical value, so we cannot reject the null hypothesis. Therefore, the rgdp time series is non-stationary and has a unit root.

```
dif_rgdp_za <- ur.za(data$dif_rgdp, model = 'both',lag = 2)
summary(dif_rgdp_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
     Min       1Q   Median       3Q      Max 
-1795.68   -35.04    18.47    68.13   894.68 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)   
(Intercept) 112.53799   34.16148   3.294  0.00121 **
y.l1         -0.48271    0.16693  -2.892  0.00437 **
trend         0.11225    0.33032   0.340  0.73443   
y.dl1         0.23798    0.12880   1.848  0.06648 . 
y.dl2         0.03668    0.07802   0.470  0.63886   
du          453.63378  175.95840   2.578  0.01084 * 
dt          -69.65977   32.56946  -2.139  0.03397 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 190.7 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.08143,	Adjusted R-squared:  0.04698 
F-statistic: 2.364 on 6 and 160 DF,  p-value: 0.03246


Teststatistic: -8.8821 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 162 
```

The obtained test statistic value is greater in absolute terms than the critical value, so we can reject the null hypothesis. Therefore, the dif_rgdp time series is stationary and does not have a unit root.

Stationarity - inf and dif_inf

```
inf_za <- ur.za(data$inf, model = 'both',lag = 2)
summary(inf_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.07496 -0.18605 -0.01652  0.17702  1.69979 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  4.92595    1.25005   3.941 0.000121 ***
y.l1         0.87716    0.03561  24.635  < 2e-16 ***
trend        0.05655    0.01652   3.424 0.000784 ***
y.dl1        0.25837    0.07234   3.572 0.000469 ***
y.dl2       -0.42002    0.07612  -5.518 1.36e-07 ***
du          -0.83169    0.33944  -2.450 0.015355 *  
dt           0.39472    0.06228   6.338 2.27e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4485 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.9996,    Adjusted R-squared:  0.9996 
F-statistic: 7.2e+04 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -3.4498 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 159 
```

The obtained test statistic value is smaller in absolute terms than the critical value, so we cannot reject the null hypothesis. Therefore, the inf time series is not stationary and has a unit root.

```
dif_inf_za <- ur.za(data$dif_inf, model = 'both',lag = 2)
summary(dif_inf_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.5821 -0.1678  0.0053  0.1858  1.4652 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.4962341  0.1023935   4.846 2.95e-06 ***
y.l1        -0.0340198  0.1337863  -0.254 0.799602    
trend       -0.0002057  0.0008445  -0.244 0.807924    
y.dl1        0.3355994  0.0959283   3.498 0.000606 ***
y.dl2       -0.1762897  0.0811062  -2.174 0.031208 *  
du          -0.5670939  0.2853760  -1.987 0.048609 *  
dt           0.1889932  0.0395906   4.774 4.05e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.46 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.4314,    Adjusted R-squared:   0.41 
F-statistic: 20.23 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -7.7289 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 156 
```

The obtained test statistic value is larger in absolute terms than the critical value, so we can reject the null hypothesis. Therefore, the dif_inf time series is stationary and does not have a unit root.

Stationarity - int and dif_int

```
int_za <- ur.za(data$int, model = 'both',lag = 2)
summary(int_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.94415 -0.19034  0.03738  0.23227  2.62818 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.126740   0.311208   3.621 0.000394 ***
y.l1         0.876525   0.026385  33.220  < 2e-16 ***
trend       -0.007924   0.002790  -2.840 0.005097 ** 
y.dl1        0.341497   0.074106   4.608 8.25e-06 ***
y.dl2        0.094768   0.062899   1.507 0.133869    
du          -0.352696   0.193412  -1.824 0.070086 .  
dt           0.015692   0.005567   2.819 0.005434 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5767 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.9765,	Adjusted R-squared:  0.9756 
F-statistic:  1109 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -4.6797 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 113 
```

The obtained test statistic value is smaller in absolute terms than the critical value, so we cannot reject the null hypothesis. Therefore, the int time series is not stationary and has a unit root.

```
dif_int_za <- ur.za(data$dif_int, model = 'both',lag = 2)
summary(dif_int_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.70741 -0.15062 -0.00171  0.22122  2.06315 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 14.80964    2.61423   5.665 6.67e-08 ***
y.l1        -1.09416    0.15086  -7.253 1.66e-11 ***
trend       -3.34024    0.50558  -6.607 5.54e-10 ***
y.dl1        0.58022    0.10900   5.323 3.40e-07 ***
y.dl2        0.12920    0.05474   2.360   0.0195 *  
du           5.27897    0.55896   9.444  < 2e-16 ***
dt           3.34011    0.50561   6.606 5.56e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.545 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.4713,	Adjusted R-squared:  0.4514 
F-statistic: 23.77 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -13.8812 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 6 
```

The obtained test statistic value is greater in absolute terms than the critical value, so we can reject the null hypothesis. Therefore, the dif_int time series is stationary and does not have a unit root.

Stationarity - prod and dif_prod

```
prod_za <- ur.za(data$prod, model = 'both',lag = 2)
summary(prod_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.7906 -0.3245 -0.0204  0.2591  4.2922 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 5.568658   1.566608   3.555 0.000498 ***
y.l1        0.883368   0.033371  26.471  < 2e-16 ***
trend       0.035733   0.009653   3.702 0.000294 ***
y.dl1       0.133219   0.077638   1.716 0.088113 .  
y.dl2       0.046404   0.078087   0.594 0.553175    
du          0.870845   0.279651   3.114 0.002187 ** 
dt          0.005886   0.005566   1.057 0.291920    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6082 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.999,	Adjusted R-squared:  0.999 
F-statistic: 2.765e+04 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -3.495 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 83 
```

The obtained test statistic value is smaller in absolute terms than the critical value, so we cannot reject the null hypothesis. Therefore, the prod time series is not stationary and has a unit root.

```
dif_prod_za <- ur.za(data$dif_prod, model = 'both',lag = 2)
summary(dif_prod_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.74834 -0.34869 -0.00863  0.26501  2.68847 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.305079   0.105112   2.902  0.00423 ** 
y.l1        -0.158264   0.141919  -1.115  0.26645    
trend        0.001294   0.001058   1.223  0.22310    
y.dl1        0.118932   0.111536   1.066  0.28789    
y.dl2       -0.022740   0.077957  -0.292  0.77089    
du           1.920233   0.450736   4.260 3.47e-05 ***
dt          -0.332973   0.067583  -4.927 2.07e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.5895 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.1615,    Adjusted R-squared:  0.1301 
F-statistic: 5.137 on 6 and 160 DF,  p-value: 7.455e-05


Teststatistic: -8.1615 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 159 
```

The obtained test statistic value is greater in absolute terms than the critical value, so we can reject the null hypothesis. Therefore, the dif_prod time series is stationary and does not have a unit root.

Stationarity - unemp and dif_unemp

```
unemp_za <- ur.za(data$unemp, model = 'both',lag = 2)
summary(unemp_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.8635 -0.3021 -0.1023  0.1208  9.1127 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.789475   0.453264   3.948 0.000118 ***
y.l1         0.772851   0.052288  14.781  < 2e-16 ***
trend       -0.007522   0.003072  -2.448 0.015434 *  
y.dl1       -0.031403   0.078908  -0.398 0.691185    
y.dl2       -0.007683   0.077111  -0.100 0.920753    
du           1.233699   0.354453   3.481 0.000645 ***
dt          -0.018493   0.007236  -2.556 0.011534 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.813 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.8023,    Adjusted R-squared:  0.7949 
F-statistic: 108.2 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -4.3442 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 112 
```

The obtained test statistic value is smaller in absolute terms than the critical value, so we cannot reject the null hypothesis. Therefore, the unemp time series is not stationary and has a unit root.

```
dif_unemp_za <- ur.za(data$dif_unemp, model = 'both',lag = 2)
summary(dif_unemp_za)
```

```
################################
# Zivot-Andrews Unit Root Test #
################################


Call:
lm(formula = testmat)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.8226 -0.2071 -0.0515  0.0964  8.9707 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.136869   0.136243  -1.005    0.317    
y.l1        -1.773991   0.170855 -10.383  < 2e-16 ***
trend        0.002372   0.001454   1.631    0.105    
y.dl1        0.913719   0.125435   7.284 1.39e-11 ***
y.dl2        0.292519   0.063868   4.580 9.30e-06 ***
du          -4.556207   0.579152  -7.867 5.11e-13 ***
dt           0.606991   0.090943   6.674 3.86e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8257 on 160 degrees of freedom
  (3 observations deleted due to missingness)
Multiple R-squared:  0.6028,	Adjusted R-squared:  0.5879 
F-statistic: 40.47 on 6 and 160 DF,  p-value: < 2.2e-16


Teststatistic: -16.236 
Critical values: 0.01= -5.57 0.05= -5.08 0.1= -4.82 

Potential break point at position: 160 
```

The obtained test statistic value is larger in absolute terms than the critical value, so we can reject the null hypothesis. Therefore, the dif_unemp time series is stationary and does not have a unit root.

ARDL Modeling

```
install.packages("lmtest")
install.packages("dynamac")
install.packages("ARDL")
install.packages("dLagM")
```

```
Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘expm’, ‘aod’, ‘dynlm’, ‘msm’


Installing package into ‘/usr/local/lib/R/site-library’
(as ‘lib’ is unspecified)

also installing the dependencies ‘Formula’, ‘gtools’, ‘operator.tools’, ‘RcppParallel’, ‘nardl’, ‘AER’, ‘formula.tools’, ‘plyr’, ‘wa
```

```
library(lmtest)
library(dynamac)
library(ARDL)
library(dLagM)
```

```
To cite the ARDL package in publications:

Use this reference to refer to the validity of the ARDL package.

  Natsiopoulos, Kleanthis, and Tzeremes, Nickolaos G. (2022). ARDL
  bounds test for cointegration: Replicating the Pesaran et al. (2001)
  results for the UK earnings equation using R. Journal of Applied
  Econometrics, 37(5), 1079-1090. https://doi.org/10.1002/jae.2919
```

```
Use this reference to cite this specific version of the ARDL package.

  Kleanthis Natsiopoulos and Nickolaos Tzeremes (2023). ARDL: ARDL, ECM
  and Bounds-Test for Cointegration. R package version 0.2.4.
  https://CRAN.R-project.org/package=ARDL

Loading required package: nardl


Attaching package: ‘nardl’


The following object is masked from ‘package:dynamac’:

    pssbounds


Loading required package: dynlm


Attaching package: ‘dLagM’


The following object is masked from ‘package:aTSA’:

    forecast


The following objects are masked from ‘package:fabletools’:

    forecast, MAPE, MASE
```

The first task after examining stationarity is determining the integration order of the dimensions.

Each variable has an integration order of I(1), as it required differencing once to make the time series stationary.

Determining the number of lags

AIC Criterion

```
modell1 <- auto_ardl(dif_unemp ~ dif_rgdp + dif_inf + dif_int + dif_prod, data=data, max_order = 3, grid = TRUE)
summary(modell1)
```

```
            Length Class      Mode
best_model 19     dynlm      list
best_order  5     -none-     numeric
top_orders  6     data.frame list
```

```
modell1$top_orders
```

A data.frame: 20 × 6

dif_unemp <int>	dif_rgdp <int>	dif_inf <int>	dif_int <int>	dif_prod <int>	AIC <dbl>
3	3	0	0	2	-21.80234
3	3	1	0	2	-21.10418
3	3	0	0	3	-20.66214
3	3	0	1	2	-20.27605
3	3	1	0	3	-20.06640
3	3	1	1	2	-19.61296
3	3	2	0	2	-19.36948
3	3	0	2	2	-19.25259
3	3	0	1	3	-19.18539
3	3	3	0	2	-19.03450
3	3	0	3	2	-19.00924
3	3	1	2	2	-18.75251
3	3	1	3	2	-18.67379
3	3	1	1	3	-18.63111
3	3	0	2	3	-18.54153
3	3	0	3	3	-18.50173
3	3	3	0	3	-18.41591
3	3	1	3	3	-18.37612
3	3	2	0	3	-18.35890
3	3	1	2	3	-18.21624

```
modell1$best_order
```

dif_unemp: 3 dif_rgdp: 3 dif_inf: 0 dif_int: 0 dif_prod: 2

```
modell1$best_model
```

```
Time series regression with "ts" data:
Start = 4, End = 170

Call:
dynlm::dynlm(formula = full_formula, data = data, start = start,
    end = end)

Coefficients:
    (Intercept)  L(dif_unemp, 1)  L(dif_unemp, 2)  L(dif_unemp, 3)
     -0.0150783       -0.6212023       -0.2042992       -0.0569137
       dif_rgdp   L(dif_rgdp, 1)   L(dif_rgdp, 2)   L(dif_rgdp, 3)
     -0.0039119        0.0010483        0.0017003        0.0007237
        dif_inf          dif_int         dif_prod   L(dif_prod, 1)
      0.0483535       -0.0546517        0.5121898       -0.1743299
 L(dif_prod, 2)
     -0.2568728
```

*The equation for the **ARDL(3,3,3,0,0)** model is:*

dif_unemp(t)=−0.0092−0.6996·dif_unemp(t−1)−0.3035·dif_unemp(t−2)−0.0442·dif_unemp(t−3)−0.0038·dif_rgdp(t)+0.0007·dif_rgdp(t−1)+0.0016·dif_rgdp(t−2)+0.0010·dif_rgdp(t−3)+0.0525·dif_inf(t)−0.0488·dif_int(t)+0.4996·dif_prod(t) −0.1236·dif_prod(t−1)−0.2414·dif_prod(t−2)−0.0854·dif_prod(t−3)+ϵ(t)

BIC Criterium

```
modell2 <- auto_ardl(dif_unemp ~ dif_rgdp + dif_inf + dif_int + dif_prod, data=data, max_order = 3, grid = TRUE, selection = 'BIC')
summary(modell2)
```

```
            Length Class      Mode
best_model 19     dynlm      list
best_order  5     -none-     numeric
top_orders  6     data.frame list
```

```
modell2$top_orders
```

A data.frame: 20 × 6

dif_unemp	dif_rgdp	dif_inf	dif_int	dif_prod	BIC
<int>	<int>	<int>	<int>	<int>	<dbl>
3	3	0	0	2	21.84957
2	3	0	0	2	22.77055
3	3	1	0	2	25.66573
3	3	0	0	3	26.10777
2	3	0	0	3	26.42927
3	3	0	1	2	26.49385
3	2	0	0	2	26.72895
2	3	1	0	2	27.01941
2	3	0	1	2	27.22867
3	2	0	0	3	29.36817
3	3	1	0	3	29.82150
3	3	1	1	2	30.27494
3	3	2	0	2	30.51842
2	3	1	0	3	30.54598
3	3	0	2	2	30.63531
3	3	0	1	3	30.70251
2	3	0	1	3	30.82442
3	2	1	0	2	31.23472
2	3	1	1	2	31.43730
3	2	0	1	2	31.63642

```
modell2$best_order
```

dif_unemp: 3 dif_rgdp: 3 dif_inf: 0 dif_int: 0 dif_prod: 2

The model is ARDL(2,3,0,2).

```
modell3 <- modell2$best_model
summary(modell3)
```

```
Time series regression with "ts" data:
Start = 4, End = 170

Call:
dynlm::dynlm(formula = full_formula, data = data, start = start,
    end = end)

Residuals:
     Min       1Q   Median       3Q      Max
-0.93148 -0.12238 -0.00027  0.15307  0.52724

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)     -0.0150783  0.0387835  -0.389 0.697975
L(dif_unemp, 1) -0.6212023  0.0653628  -9.504  < 2e-16 ***
L(dif_unemp, 2) -0.2042992  0.0440029  -4.643 7.31e-06 ***
L(dif_unemp, 3) -0.0569137  0.0238999  -2.381 0.018471 *
dif_rgdp        -0.0039119  0.0001021 -38.313  < 2e-16 ***
L(dif_rgdp, 1)   0.0010483  0.0002903   3.612 0.000411 ***
L(dif_rgdp, 2)   0.0017003  0.0002907   5.849 2.87e-08 ***
L(dif_rgdp, 3)   0.0007237  0.0002348   3.082 0.002434 **
dif_inf          0.0483535  0.0310363   1.558 0.121295
dif_int         -0.0546517  0.0235976  -2.316 0.021878 *
dif_prod         0.5121898  0.0308879  16.582  < 2e-16 ***
L(dif_prod, 1)  -0.1743299  0.0447921  -3.892 0.000148 ***
L(dif_prod, 2)  -0.2568728  0.0445225  -5.770 4.23e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2171 on 154 degrees of freedom
Multiple R-squared:  0.9736,    Adjusted R-squared:  0.9715
F-statistic: 472.8 on 12 and 154 DF,  p-value: < 2.2e-16
```

Where the p-value is greater than the chosen significance level (0.05), those variables can be excluded from the model.

We can remove dif_inf and dif_int from the equation, leaving only three variables in the model: **ARDL(2,3,2)**.

We will re-estimate the model with only the remaining variables.

```
modellARDL <- ardl(dif_unemp ~ dif_rgdp + dif_prod, data=data, order=c(2,3,2))
summary(modellARDL)
```

```
Time series regression with "ts" data:
Start = 4, End = 170

Call:
dynlm::dynlm(formula = full_formula, data = data, start = start,
    end = end)

Residuals:
     Min       1Q   Median       3Q      Max
-0.95910 -0.10956 -0.00327  0.14554  0.68342

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)     -0.0256946  0.0341850  -0.752 0.453397
L(dif_unemp, 1) -0.6220954  0.0669564  -9.291  < 2e-16 ***
L(dif_unemp, 2) -0.2269877  0.0446334  -5.086 1.03e-06 ***
dif_rgdp        -0.0038986  0.0001010 -38.583  < 2e-16 ***
L(dif_rgdp, 1)   0.0010861  0.0002973   3.653 0.000353 ***
L(dif_rgdp, 2)   0.0016033  0.0002964   5.409 2.32e-07 ***
L(dif_rgdp, 3)   0.0011481  0.0001883   6.097 8.07e-09 ***
dif_prod         0.5035566  0.0311988  16.140  < 2e-16 ***
L(dif_prod, 1)  -0.1703882  0.0459525  -3.708 0.000289 ***
L(dif_prod, 2)  -0.2427603  0.0453345  -5.355 2.99e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2231 on 157 degrees of freedom
Multiple R-squared:  0.9715,	Adjusted R-squared:  0.9699
F-statistic: 595.3 on 9 and 157 DF,  p-value: < 2.2e-16
```

*The equation for the **ARDL(2,3,2)** model is:*

$dif\_unemp(t)=-0.0163-0.6273\times dif\_unemp(t-1)-0.2256\times dif\_unemp(t-2)-0.0038\times dif\_rgdp(t)+0.0010\times dif\_rgdp(t-1)+0.0016\times dif\_rgdp(t-2)+0.0010\times dif\_rgdp(t-3)+0.4896\times dif\_prod(t)-0.1581\times dif\_prod(t-1)-0.2487\times dif\_prod(t-2)+\epsilon(t)$

```
library(knitr)
```

```
kable(tidy(modellARDL), caption="Parameters of the ARDL Model")
```

```
Warning message:
“The `tidy()` method for objects of class `dynlm` is not maintained by the broom team, and is only supported through the `lm` tidier

This warning is displayed once per session.”


Table: Parameters of the ARDL Model

|term            |   estimate| std.error|  statistic|   p.value|
|:---------------|----------:|---------:|----------:|---------:|
|(Intercept)     | -0.0256946| 0.0341850|  -0.751633| 0.4533971|
|L(dif_unemp, 1) | -0.6220954| 0.0669564|  -9.291049| 0.0000000|
|L(dif_unemp, 2) | -0.2269877| 0.0446334|  -5.085606| 0.0000010|
|dif_rgdp        | -0.0038986| 0.0001010| -38.583218| 0.0000000|
|L(dif_rgdp, 1)  |  0.0010861| 0.0002973|   3.653037| 0.0003527|
|L(dif_rgdp, 2)  |  0.0016033| 0.0002964|   5.408749| 0.0000002|
|L(dif_rgdp, 3)  |  0.0011481| 0.0001883|   6.096998| 0.0000000|
|dif_prod        |  0.5035566| 0.0311988|  16.140231| 0.0000000|
|L(dif_prod, 1)  | -0.1703882| 0.0459525|  -3.707922| 0.0002894|
|L(dif_prod, 2)  | -0.2427603| 0.0453345|  -5.354870| 0.0000003|
```

Az ARDL modell hibakorrekciós alakja (UECM)

```
uecm <- uecm(modellARDL)
summary(uecm)
```

```
Time series regression with "ts" data:
Start = 4, End = 170

Call:
dynlm::dynlm(formula = full_formula, data = data, start = start,
    end = end)

Residuals:
     Min       1Q   Median       3Q      Max
-0.95910 -0.10956 -0.00327  0.14554  0.68342

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)        -2.569e-02  3.419e-02  -0.752   0.4534
L(dif_unemp, 1)    -1.849e+00  8.885e-02 -20.810  < 2e-16 ***
L(dif_rgdp, 1)     -6.111e-05  2.342e-04  -0.261   0.7945
L(dif_prod, 1)      9.041e-02  4.925e-02   1.836   0.0683 .
d(L(dif_unemp, 1))  2.270e-01  4.463e-02   5.086 1.03e-06 ***
d(dif_rgdp)        -3.899e-03  1.010e-04 -38.583  < 2e-16 ***
d(L(dif_rgdp, 1))  -2.751e-03  3.019e-04  -9.114 3.57e-16 ***
d(L(dif_rgdp, 2))  -1.148e-03  1.883e-04  -6.097 8.07e-09 ***
d(dif_prod)         5.036e-01  3.120e-02  16.140  < 2e-16 ***
d(L(dif_prod, 1))   2.428e-01  4.533e-02   5.355 2.99e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2231 on 157 degrees of freedom
Multiple R-squared:  0.9907,    Adjusted R-squared:  0.9901
F-statistic:  1852 on 9 and 157 DF,  p-value: < 2.2e-16
```

The equation for the **UECM** model is:

$\Delta y(t)=-0.0163-1.8530\cdot y(t-1)-0.0001\cdot x1(t-1)+0.0827\cdot x2(t-1)+0.2256\cdot\Delta y(t-1)-0.0038\cdot\Delta x1(t)$
$-0.0026\cdot\Delta x1(t-1)-0.0010\cdot\Delta x1(t-2)+0.4896\cdot\Delta x2(t)+0.2487\cdot\Delta x2(t-1)+u(t)$

Interval Boundary Tests

Cointegration signifies a long-term relationship between variables, meaning that they move together over time despite experiencing short-term fluctuations.

Bounds Test

H0: There is no long-term cointegrating relationship.

H1: There is a long-term cointegrating relationship.

```
tbounds <- bounds_t_test(uecm , case = 3, alpha = 0.01)
tbounds
```

```
        Bounds t-test for no cointegration

data:  d(dif_unemp) ~ L(dif_unemp, 1) + L(dif_rgdp, 1) + L(dif_prod,     1) + d(L(dif_unemp, 1)) + d(dif_rgdp) + d(L(dif_rgdp, 1))
+     d(L(dif_rgdp, 2)) + d(dif_prod) + d(L(dif_prod, 1))
t = -20.81, Lower-bound I(0) = -3.4250, Upper-bound I(1) = -4.0972,
p-value = 1e-06
alternative hypothesis: Possible cointegration
null values:
   k    T
   2 1000
```

```
fbounds <- bounds_f_test(uecm, case = 3)
fbounds
```

```
        Bounds F-test (Wald) for no cointegration

data:  d(dif_unemp) ~ L(dif_unemp, 1) + L(dif_rgdp, 1) + L(dif_prod,     1) + d(L(dif_unemp, 1)) + d(dif_rgdp) + d(L(dif_rgdp, 1))
+     d(L(dif_rgdp, 2)) + d(dif_prod) + d(L(dif_prod, 1))
F = 146.5, p-value = 1e-06
alternative hypothesis: Possible cointegration
null values:
   k    T
   2 1000
```

Since the t-statistic value falls outside the lower and upper bounds, we can reject the null hypothesis and accept the alternative hypothesis, indicating that there is a cointegrating relationship between the variables.

Additionally, the p-value of the F-statistic is less than 0.05, which also indicates the presence of cointegration.

RECM Model

```
recm <- recm(uecm, case=3)
summary(recm)
```

```
Time series regression with "zooreg" data:
Start = 4, End = 170

Call:
dynlm::dynlm(formula = full_formula, data = data, start = start,
    end = end)

Residuals:
     Min       1Q   Median       3Q      Max
-0.95910 -0.10956 -0.00327  0.14554  0.68342

Coefficients:
                     Estimate Std. Error t value Pr(>|t|)
(Intercept)        -2.569e-02  1.721e-02  -1.493    0.137
d(L(dif_unemp, 1))  2.270e-01  4.244e-02   5.348 3.05e-07 ***
d(dif_rgdp)        -3.899e-03  8.575e-05 -45.466  < 2e-16 ***
d(L(dif_rgdp, 1))  -2.751e-03  2.721e-04 -10.112  < 2e-16 ***
d(L(dif_rgdp, 2))  -1.148e-03  1.802e-04  -6.371 1.94e-09 ***
d(dif_prod)         5.036e-01  2.643e-02  19.054  < 2e-16 ***
d(L(dif_prod, 1))   2.428e-01  3.943e-02   6.156 5.85e-09 ***
ect                -1.849e+00  8.765e-02 -21.097  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2217 on 159 degrees of freedom
  (0 observations deleted due to missingness)
Multiple R-squared:  0.9907,    Adjusted R-squared:  0.9903
F-statistic:  2411 on 7 and 159 DF,  p-value: < 2.2e-16
```

Significant (the p-value for the ECT is less than 0.05), indicating that there is a long-term effect. Additionally, the coefficient is negative, which reflects the speed of adjustment towards long-term equilibrium.

$\Delta y(t) = -1.853 \cdot ECT(t-1) + u(t)$

Model Diagnostics Tests

Ramsey RESET Test

With this test, we evaluate the model specification.

H0: The model specification is correct.

H1: The model specification is incorrect.

```
library(lmtest)
```

```
resettest(modellARDL)
```

```
        RESET test

data:  modellARDL
RESET = 33.979, df1 = 2, df2 = 155, p-value = 5.798e-13
```

The p-value is less than 0.05, so we reject the null hypothesis. This indicates that the model specification is incorrect.

At this point, either the ARDL model needs to be re-specified, or an alternative model should be chosen for forecasting.

The ARDL model is also inadequate because the variables are of the same order.

VAR or VECM?

Based on the previous tests, we know that the original time series are not stationary, only their once-differenced versions are. Therefore, we need to conduct cointegration tests to determine whether we should estimate a VAR model with the differenced time series or a VECM model if cointegration is present.

The order of integration for each variable is **I(1)**.

Selecting the Order of the VAR(P) Model

```
vardata <- data[,c('unemp', 'rgdp', 'inf', 'int', 'prod')]
```

```
lagselect <- VARselect(vardata, lag.max = 5, type = 'trend')
lagselect$selection
```

```
AIC(n):    4 HQ(n):    2 SC(n):    2 FPE(n):    4
```

Optimal lag: lagselect$selection - 1, so 4 - 1 = 3

Cointegration Testing - Johansen Test - Trace Test

If the number of cointegration relationships is 0:

H0: There is no cointegration.

H1: There is more than 0 cointegration relationships.

If the number of cointegration relationships is 1:

H0: There is at most 1 cointegration relationship.

H1: There is more than 1 cointegration relationship.

If the number of cointegration relationships is 2:

H0: There is at most 2 cointegration relationships.

H1: There is more than 2 cointegration relationships.

And so on.

```
koint_t <- ca.jo(vardata, type = "trace", ecdet = "trend", K = 4, spec = "transitory")
summary(koint_t)
```

```
######################
# Johansen-Procedure #
######################

Test type: trace statistic , with linear trend in cointegration

Eigenvalues (lambda):
[1] 2.317585e-01 1.478985e-01 8.851145e-02 6.028403e-02 3.081863e-02
[6] 3.768512e-17

Values of teststatistic and critical values of test:

           test 10pct  5pct  1pct
r <= 4 |   5.20 10.49 12.25 16.26
r <= 3 |  15.52 22.76 25.32 30.45
r <= 2 |  30.90 39.06 42.44 48.45
r <= 1 |  57.47 59.14 62.99 70.05
r = 0  | 101.24 83.20 87.31 96.58

Eigenvectors, normalised to first column:
(These are the cointegration relations)

             unemp.l1      rgdp.l1      inf.l1       int.l1      prod.l1
unemp.l1  1.000000000  1.000000000  1.00000000   1.00000000  1.000000000
rgdp.l1   0.004981187 -0.002082444  0.00328223  -0.03939908  0.001133463
inf.l1   -0.128711154 -0.614509394 -0.18336007 -12.61860744 -0.927773585
int.l1   -0.793325388 -0.392960952  0.04924735   4.13016705  0.315176492
prod.l1  -0.706718856  0.013920920 -0.54596055  -5.91204264 -0.062326835
trend.l1 -0.131087690  0.426322464  0.02352291  11.98784730  0.424452751
            trend.l1
unemp.l1   1.0000000
rgdp.l1    0.0206871
inf.l1   -12.6041892
int.l1    -2.9144797
prod.l1    0.5258200
trend.l1   3.5971506

Weights W:
(This is the loading matrix)

           unemp.l1      rgdp.l1      inf.l1        int.l1      prod.l1
unemp.d -0.02286149 -0.122954768 -0.05106377 -1.491477e-05 -0.021920063
rgdp.d   5.76130155 21.718736437 16.34642321  7.424161e-01  6.976452469
inf.d   -0.02641315  0.007275585 -0.10617684 -1.221579e-03  0.035129803
int.d    0.17739915  0.018906195 -0.15729273  4.342559e-04  0.010467543
prod.d   0.05734127 -0.067606826  0.17239339  2.646615e-03  0.009330876
             trend.l1
unemp.d -5.499664e-15
rgdp.d   5.458566e-14
inf.d    3.854249e-14
int.d   -1.914991e-14
prod.d  -6.631298e-14
```

r = 0: The test statistic value is greater than the 5% critical value, so we accept the alternative hypothesis, indicating that there is more than 0 cointegration relationship.

r = 1: The test statistic value is smaller than the 5% critical value, so we cannot reject the null hypothesis, indicating that there is at most 1 cointegration relationship.

Based on this, we can conclude that there is exactly 1 cointegration relationship among the time series.

Since there is a cointegration relationship, we will use a **VECM** for forecasting, taking into account the long-term relationship as well.

VECM Modeling

```
install.packages('tsDyn')
library(tsDyn)
```

```
Installing package into '/usr/local/lib/R/site-library'
(as 'lib' is unspecified)

also installing the dependencies 'deSolve', 'iterators', 'mnormt', 'tseriesChaos', 'foreach'


Attaching package: 'tsDyn'


The following object is masked from 'package:dLagM':

    MAPE
```

```
The following object is masked from ‘package:fabletools’:

    MAPE
```

We need to specify the number of cointegration relationships (r = 1), as determined in the previous step, and the number of lags (3).

```
modelVECM <- tsDyn::VECM(vardata, 3, r = 1, estim = "ML")
summary(modelVECM)
```

```
#############
###Model VECM
#############
Full sample size: 170   End sample size: 166
Number of variables: 5  Number of estimated slope parameters 85
AIC 575.987     BIC 852.9539    SSR 4353091
Cointegrating vector (estimated by ML):
   unemp        rgdp        inf        int       prod
r1     1 0.004298898 -0.2882033 -0.6988964 -0.6865166


               ECT                  Intercept              unemp -1
Equation unemp -0.0427(0.0486)      -0.2268(0.5426)        -1.5060(0.3201)***
Equation rgdp  10.9384(10.6100)     165.3460(118.3536)     343.3801(69.8117)***
Equation inf   -0.0288(0.0301)      -0.2260(0.3363)        0.0074(0.1984)
Equation int   0.1983(0.0346)***    2.0367(0.3862)***      -0.0165(0.2278)
Equation prod  0.0601(0.0352).      1.3151(0.3928)**       -0.4095(0.2317).
               rgdp -1              inf -1               int -1
Equation unemp -0.0060(0.0013)***   0.0111(0.1320)       -0.2229(0.1073)*
Equation rgdp  1.1351(0.2800)***    -6.0299(28.7871)     56.0173(23.3958)*
Equation inf   -0.0003(0.0008)      0.6323(0.0818)***    0.0105(0.0665)
Equation int    1.9e-05(0.0009)     0.1218(0.0939)       0.3653(0.0763)***
Equation prod  -0.0029(0.0009)**    -0.0767(0.0955)      0.0049(0.0776)
               prod -1               unemp -2              rgdp -2
Equation unemp 0.3534(0.2028).       0.2711(0.3332)        0.0006(0.0013)
Equation rgdp  -68.3792(44.2344)     -34.2807(72.6754)     -0.0510(0.2934)
Equation inf   0.1133(0.1257)        0.1327(0.2065)        0.0013(0.0008)
Equation int    5.2e-05(0.1443)      -0.2485(0.2371)       -0.0006(0.0010)
Equation prod  0.1583(0.1468)        0.1059(0.2412)        -8.7e-05(0.0010)
               inf -2               int -2               prod -2
Equation unemp -0.0680(0.1363)      -0.0623(0.1141)      -0.0526(0.1923)
Equation rgdp  17.6498(29.7326)     18.0021(24.8916)     -9.8775(41.9392)
Equation inf   -0.5182(0.0845)***   0.0389(0.0707)       -0.0965(0.1192)
Equation int   0.0921(0.0970)       -0.1701(0.0812)*     0.0126(0.1368)
Equation prod  -0.0256(0.0987)      -0.0091(0.0826)      0.0032(0.1392)
               unemp -3              rgdp -3             inf -3
Equation unemp 0.5145(0.3165)        0.0026(0.0013)*    -0.0314(0.1445)
Equation rgdp  -98.6471(69.0409)     -0.5008(0.2748).   -18.4846(31.5179)
Equation inf   0.0129(0.1962)        0.0002(0.0008)     0.4663(0.0896)***
Equation int   -0.3986(0.2253).      -0.0018(0.0009).   0.1127(0.1028)
Equation prod  0.1373(0.2291)        0.0004(0.0009)     -0.1759(0.1046).
               int -3              prod -3
Equation unemp -0.0259(0.0897)     -0.3101(0.1926)
Equation rgdp  8.8980(19.5741)     88.3618(42.0135)*
Equation inf   0.0002(0.0556)      0.1198(0.1194)
Equation int   0.2180(0.0639)***   0.1216(0.1371)
Equation prod  0.0304(0.0650)      -0.0047(0.1394)
```

The equation:

r1=unemp(t)+0.0042×rgdp(t)−0.2875×inf(t)−0.7038×int(t)−0.6763×prod(t)

The negative signs indicate a long-term relationship between the given variable and unemployment.

The ECT describes the short-term dynamics of the system, indicating how quickly the variables adjust to changes and return to their long-term equilibrium.

Unemp Equation: The **ECT has a negative coefficient** – unemployment (unemp) adjusts upwards if it falls below the long-term equilibrium level and adjusts downwards if it rises above the long-term equilibrium level.

RGDP Equation: The **ECT has a positive coefficient** – real GDP (rgdp) adjusts upwards when it deviates from the long-term equilibrium.

Inf Equation: The **ECT has a negative coefficient** – inflation (inf) adjusts upwards if it falls below the long-term equilibrium level and adjusts downwards if it rises above the long-term equilibrium level.

Int Equation: The **ECT has a positive coefficient** – the interest rate (int) adjusts upwards when it deviates from the long-term equilibrium.

Prod Equation: The **ECT has a positive coefficient** – productivity (prod) adjusts upwards when it deviates from the long-term equilibrium.

Δunemp(t)=−0.0489·ECT(t−1)−0.2962−1.5254·unemp(t−1)−0.0061·rgdp(t−1)+0.0174·inf(t−1)−0.2271·int(t−1)+0.3671·prod(t−1)+0.3482·unemp(t−2)+0.0009·rgdp(t−2)−0.0946·inf(t−2)−0.0604·int(t−2)−0.1025·prod(t−2)+0.6263·unemp(t−3)+0.0031·rgdp(t−3)−0.0507·inf(t−3)−0.0225·int(t−3)−0.3819·prod(t−3)

Δrgdp(t)=12.0056·ECT(t−1)+177.8284+345.4536·unemp(t−1)+1.1404·rgdp(t−1)−8.3745·inf(t−1)+56.8658·int(t−1)−71.3676·prod(t−1)−62.3818·unemp(t−2)−0.1622·rgdp(t−2)+25.0369·inf(t−2)+19.9037·int(t−2)+11.1579·prod(t−2)−102.4128·unemp(t−3)−0.5188·rgdp(t−3)−16.0928·inf(t−3)+9.1578·int(t−3)+88.6211·prod(t−3)

Δinf(t)=−0.0293·ECT(t−1)−0.2328+0.0043·unemp(t−1)−0.0003·rgdp(t−1)+0.6322·inf(t−1)+0.0106·int(t−1)+0.1103·prod(t−1)+0.1152·unemp(t−2)+0.0012·rgdp(t−2)−0.5202·inf(t−2)+0.0376·int(t−2)−0.0791·prod(t−2)+0.0240·unemp(t−3)+0.0002·rgdp(t−3)+0.4680·inf(t−3)+0.0003·int(t−3)+0.1101·prod(t−3)

Δint(t)=0.2017·ECT(t−1)+2.0874+0.0084·unemp(t−1)+0.0001·rgdp(t−1)+0.1216·inf(t−1)+0.3701·int(t−1)−0.0152·prod(t−1)−0.2385·unemp(t−2)
−0.0006·rgdp(t−2)+0.0972·inf(t−2)−0.1658·int(t−2)+0.0109·prod(t−2)−0.4002·unemp(t−3)−0.0018·rgdp(t−3)+0.1202·inf(t−3)+0.2199·int(t−3)+0.1245·prod(t−3)

Δprod(t)=0.0579·ECT(t−1)+1.2882−0.3829·unemp(t−1)−0.0029·rgdp(t−1)−0.0696·inf(t−1)+0.0065·int(t−1)+0.1183·prod(t−1)+0.0870·unemp(t−2)−0.0002·rgdp(t−2)−0.0122·inf(t−2)−0.0179·int(t−2)+0.0429·prod(t−2)+0.2683·unemp(t−3)+0.0011·rgdp(t−3)−0.2209·inf(t−3)+0.0327·int(t−3)−0.0867·prod(t−3)

Model Diagnostic Tests

To check the model assumptions, the VECM model needs to be transformed into a VAR model.

```
modelVAR <- vec2var(koint_t, r = 1)
```

Testing Cross-Correlation

H0: There is no cross-correlation among the residuals.

H1: There is cross-correlation among the residuals.

```
serial <- serial.test(modelVAR, lags.pt = 5, type = "PT.asymptotic")
serial
```

```
	Portmanteau Test (asymptotic)

data:  Residuals of VAR object modelVAR
Chi-squared = 68.414, df = 30, p-value = 7.89e-05

$serial

	Portmanteau Test (asymptotic)

data:  Residuals of VAR object modelVAR
Chi-squared = 68.414, df = 30, p-value = 7.89e-05
```

The p-value is less than 0.05, so we can reject the null hypothesis. This indicates that there is cross-correlation among the residuals.

Testing for ARCH Effects:

H0: The model is homoscedastic, meaning the variance of the residuals is constant.

H1: The model is not homoscedastic, meaning the variance of the residuals is not constant.

```
Arch <- vars::arch.test(modelVAR, lags.multi = 10, multivariate.only = TRUE)
Arch
```

```
        ARCH (multivariate)
```

The p-value is greater than 0.05, so we cannot reject the null hypothesis. Therefore, the model is homoscedastic, meaning the variance of the residuals is constant.

Testing for Normality of Residuals

```
data:  Residuals of VAR object modelVAR
```

H0: The residuals are normally distributed.

H1: The residuals are not normally distributed.

```
norm <- normality.test(modelVAR, multivariate.only = TRUE)
norm
```

```
$JB

        JB-Test (multivariate)

data:  Residuals of VAR object modelVAR
Chi-squared = 20287, df = 10, p-value < 2.2e-16


$Skewness

        Skewness only (multivariate)

data:  Residuals of VAR object modelVAR
```