$$f(x) = (x-r)^m$$
$$f'(x) = m(x-r)^{m-1}$$

$$x_{n+1} = x_n - m\frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} - x_n = -m\frac{f(x_n)}{f'(x_n)}$$

$$s_n = -m\frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{(x_n - r)^m}{m(x_n-r)^{m-1}}$$

$$\overbrace{x_{n+1} - x_n}^{s_n} = -\frac{1}{m}(x_n - r)$$

$$x_{n+1} = x_n - \frac{1}{m}(x_n - r) \Rightarrow s_n = -\frac{1}{m}(x_n - r)$$

$$x_{n+1} - r = x_n - r - \frac{1}{m}(x_n - r)$$

$$x_{n+1} - r = \left(1 - \frac{1}{m}\right)(x_n - r)$$

$$x_{n+1} - r = \frac{m-1}{m}(x_n - r)$$

$$x_{n+1} - r - (x_n - r)$$
$$= x_{n+1} - x_n \cancel{-r} \cancel{+r}$$

$$s_n = x_{n+1} - x_n$$

$$s_n = x_{n+1} - x_n = \frac{m-1}{m}(x_n - r) - (x_n - r)$$

$$s_n = \left(\frac{m-1}{m} - 1\right)(x_n - r)$$

$$s_n = -\frac{1}{m}(x_n - r)$$

$$s_{n+1} = -\frac{1}{m}(x_{n+1} - r)$$

$$S_{n+1} = -\frac{1}{m}\left(x_{n+1} - r\right)$$

$$= -\frac{1}{m}\,\frac{m-1}{m}\left(x_n - r\right)$$

$$\lim_{n\to\infty} \frac{|s_n|}{|s_{n+1}|} = \frac{\cancel{-\frac{1}{m}}\,\cancel{(x_n - r)}}{\cancel{-\frac{1}{m}}\,\frac{m-1}{m}\,\cancel{(x_n - r)}} \qquad \frac{m}{m-1}$$

$$\lim_{n\to\infty} \frac{|S_{n+1}|}{|S_n|} = \frac{m-1}{m}$$