Properties of the Temperature Distribution Curves of a Metal Bar

Anand Balivada
Aryan Mishra

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Abstract

We investigate the temperature distribution curves of a metal bar, heated at the centre for a specified time interval, and experimentally establish the conservation of the area under the graphs of these curves. We proceed to compare these to numerical simulations for a one dimensional bar under similar conditions, but with an imposed Neumann Boundary condition to represent insulation (ensuring conservation of area under the graphs).

Contents

1 Introduction

2 Theory and Problem Statement

In a metal bar of length l treated as a one dimensional, closed subset $[0, l]$ of $\mathbb{R}$, the distribution of temperature $T(x, t), x \in [0, l], t > 0$, obeys the equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \ldots (1)$$

where α is the thermal diffusivity of the material of the bar given by $\frac{k}{c\rho}$, where k, c, ρ are the thermal conductivity, specific heat capacity and mass density of the metal bar respectively (Chapter 1.6 of [1]).
We enforce the Neumann Boundary Condition $\frac{\partial T}{\partial x}(0,t) = \frac{\partial T}{\partial x}(l,t) = 0$, whose physical significance is the insulation of the sides of the bar.

Remark. *The heat equation neglects the loss of heat through convection or radiation, and thus the heat equation where the sides of the bar are insulated guarantees no loss of heat.*

Theorem 2.1. $\int_0^l T(x,t)dx$ *is constant for all times* $t > 0$.

Problem Statement: To experimentally verify $Theorem$ 2.1 by measuring the measuring the temperature distribution of a metal bar.

3 Design of the Experiment

4 Results

5 Conclusions

6 References

[1] Churchill, Ruel.V., Ward, James B., Fourier Series and Boundary Value Problems