1 Ultra-simplified linear block diagram

$$\text{H.O.} = \text{Mechanical Admittance} = \frac{\dot{x}_h(s)}{F_h s} = A_{HO}(s) \tag{1}$$

$$\text{Env} = \text{Mechanical Impedance} = \frac{F_E(s)}{\dot{x}_E(s)} = Z_{Env}(s) \tag{2}$$

Loop Gain:

$$G(s) = A_{HO}(s) Z_{Env}(s) H_1(s) H_2(s) \tag{3}$$

Characteristic equation of Closed Loop system:

$$1 + G(s) = 0 \tag{4}$$

Nyquist criteria:

$$|G(s)| \geq 1 \quad \text{and} \quad \angle G(s) \geq 180° \tag{5}$$

Assume:

$$|H_1(s)| = |H_2(s)| = 1, \quad \angle H_1(s) H_2(s) \approx 180° $$

Then

$$|G(s)| = |Z_{Env} A_{HO}| = \left| \frac{Z_{Env}}{Z_{HO}} \right|, \quad \angle G(s) \approx 180° \tag{6}$$

Stability is determined by impedance ratio:

$$\left| \frac{Z_{Env}}{Z_{HO}} \right| \overset{?}{>} 1 \tag{7}$$

Delay:

$$\angle Z_{Env} - \angle Z_{Env} + \angle e^{-sT} \geq 180° \tag{8}$$

What about phase shift assumption? Think of

- Sampling
- Computation delay
- Communication delay
- Phase Lag in mechanisms, environment
- Cognitive, biomechanical and reflex delays in human

2 Stabilization Approaches

2.1 Virtual Coupling

Ed Colgate et al.

2.2 Passivity

A system having one port, with variables $\dot{x}$, and F is passive if

$$\int_{-\infty}^{t} F(t) \dot{x}(t) dt \geq 0 \quad \forall t \tag{9}$$

or

$$\int_{0}^{t} F(t) \dot{x}(t) dt + E(0) \geq 0 \quad \forall t > 0 \tag{10}$$

Where

$$E(0) = \int_{-\infty}^{0} F(t) \dot{x}(t) dt \tag{11}$$

For a two port system, the equation becomes

$$\int_0^t (F_1(t)\dot{x}_1(t)dt + F_2(t)\dot{x}_2(t)dt) + E(0) \geq 0 \quad \forall t > 0 \tag{12}$$

Llewellyn's Absolute stability:
If a two port system has matrix description

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

Then it is stable for any pair of 1-port terminations iff

$$\begin{aligned} \Re(p_{11}) &\geq 0 \\ \Re(p_{22}) &\geq 0 \\ 2\Re(p_{11})\Re(p_{22}) - |p_{12}p_{21}| - \Re(p_{12}p_{21}) &\geq 0 \end{aligned} \tag{13}$$

3 Passivity Observer / Controller

3.1 Passivity Observer

Consider a discrete time one-port system. We can refer to its port variables, $F(k)$ and $\dot{x}(k)$ as conjugate variables (their product is power). The passivity definition of the one port is

$$\sum_{k=1}^{n} F(k)\dot{x}(k) + E(0) \geq 0 \quad \forall n > 0 \tag{14}$$

If the two energy variables $F(k), \dot{x}(k)$ are monitored by sensors and fed into a computer system, we can observe this energy as

$$E_{obsv}(n) = \sum_{k=1}^{n} F(k)\dot{x}(k) + E(0) \tag{15}$$

Software can then determine any instant of time n at which the system violates (14). This "passivity observer" is shown in the figure
A similar observer can be developed for multiple ports. For m ports,

$$E_{obsv} = \sum_{p=1}^{m} \sum_{k=1}^{n} F_p(k)\dot{x}_p(k) + E(0) \tag{16}$$

This is illustrated in figure

3.2 Passivity Controller

Going back to the one-port, since such systems often have control over one of their conjugate variables (i.e. an output), why not modify the output in real time to constrain

$$E_{obsv} > 0$$

To do this we introduce a "Passivity Controller" (PC) into the system which modifies the computed output. There are two forms we can use, the serial PC and the parallel PC.

The Series PC can be described by the following algorithm which computes a modified output force based on desired/computed force and current velocity:

1. $\dot{x}(k)$ is an input measured velocity

2. $F_c(k)$ is the output computed by the system block

3. $E_{obsv}(n) = E_{obsv}(n-1) + [F_c(n)\dot{x}(n) + \alpha(n-1)\dot{x}(n-1)^2]\Delta T$

4. $\alpha(n) = \begin{cases} -E_{obsv}(n)/(\Delta T\dot{x}(n)^2) & E_{obsv}(n) < 0 \\ 0 & E_{obsv}(n) \geq 0 \end{cases}$

5. $F_{out}(n) = F_c(n) + \alpha(n)\dot{x}_n$

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2

4 2017 ACC Workshop

4.1 Hashtrudi-Zaad: (2001)

Position-position
  2-port can be active but absolute stability
  Haykin '70: Stability-Activity Diagram (Graphical version of the inequality (13)
  Limitations: Only passive environments.

4.1.1 "Circle method"

Edwards '92, Haddadi et al 2010, Haddadi et al. 2015
  Scattering Representation / Scattering Matrix / Reflection Coefficients
  Passivity: unit disk in scattering domain
  Environment stability region / all active or passive env's for which combined system is passive.
  ESR a function of frequency: a circle which is function of scattering params.
  (overlap of circles at each frequency) Does ESR cover unit passivity disk?
  Now move from scattering domain to impedance params.
  Application: Transparency Optimized 4-Channel Controller (ToC)
  Absolute stability - requires acceleration estimate

4.1.2 Adding Human Operator Damping

Haddadi et al. 2015.
  With acceleration: As operator adds more damping, ESR region grows.
  Without acceleration: There is no damping for which all passive envs are stable.

4.2 Absolute Stability of multilateral systems and multi-DOF systems

M. Tavakholi
  Application: tele-rehab
  Apply Llewellyn to 3-port terminated by 3 Z_is.
  Definition: Youla Passive N-Port IRE 1959.
  Use Youla's lemmas: reciprocal N-port matrices.
  Raisbeck's criterion (more conservative.)
  Derivation of Counterpart to Llewellyn for 3-ports and counterpart to Raisbeck's criterion.
  N-port, M-DOF system (linearized by Feedback Linearization)
  "Complementary" DOF (IET Control Theory & Applications 2014). Contact tasks constraining some
DOFs.
  Peg-in-hole example: i.e Master one for free motion, master two for peg insertion

4.3 Energy Aware Robotics

Stefano Stramigioli
  "casmir function for control" IFAC 1998
  Port-based thinking
  world is non-linear
  Proper geometry makes multi-dimensional systems as simple as 1-D
  A model is not reality: can't base stability on "environment model"
  "Passivity" is not conservative as often thought – think instead about Energy Awareness.
  Electrical Networks give misleading topology - doesn't represent energy flow.
  Stability is not difficult if we respect physics. / "listening to the math and physics" can avoid problems.
  Contrast non-interactive control (e.g. sattelite dish) vs interactive control (robot holding object). Not
the same as Robust Control.
  example: bi-ped control by "system composition" e.g. bond-graphs
  Examples: DLR grasping

4.4 Stability using Multiplier THeory and IQC

J. Carrasco, H. Tugal, Univ of Manchester
    Assumptions: Human is Passive. Environment = Force = F(V)
    "bi-lateral teleoperation IQC formulation" Ilham Polat 2012
    Assume environment is monotone static nonlinear operator (Passive plus more)
    convex search using "IQC" for controller.
    "Integral Quadratic Constraint" (IQC): Megretski 1997 IQC theorem (cf Multiplier theory, Desoer and
Vidyasagar 1975)
    Zames-Falb Muiltpliers for absolute stability, Carrasco et all EJC 2017.

4.5 Passivity and Life Hearafter

Gunter Niemeyer, Disney Research
    Goal checkup: Transparency ? or User Projection ? Leverage wider robotics community.
    Event based haptics.
    Allure of Passivity. "all natural"
    lack of model is strength but also weakness of passivity.

- HF (>30Hz) open loop

- MF (3Hz) Passivity

- LF (<1Hz) Active may be OK

    Experiment: metal-to-metal contact with 4 sec time delay
    Intent is encoded in human's grasp impedance.
    GN idea/video: Squeeze sensor on gripper driving slave stiffness.