Robot Manipulators: Chapter 6, Manipulator Dynamics

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Contents

Chapter 6

Manipulator Dynamics

This chapter studies the relationship of manipulator motion to the forces and torques applied to the joints of the manipulator. Forces (especially pertinent to prismatic joints) and torques (rotary joints) are conceptually very similar because they are often thought of "causes" of motion. However, their mathematics tend to be rather different. Since many manipulators are a mix of prismatic and rotary joints, we have to treat lots of special cases. A notation which partially simplifies this solution is a set of generalized joint coordinates q. For example, if the third joint of a six axis manipulator is prismatic, we would have:

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ d_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}$$

and

$$\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \\ \dot{q}_5 \\ \dot{q}_6 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{d}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix}$$

etc.

6.1 Problem Statement and Learning Objectives

Problem Statement

Learning Objectives Upon completing this Chapter, the reader should be able to

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6.2 Basic Gravity Compensation

In this section we will solve a relatively simple dynamics problem, but one of great practical importance: we will develop some equations to compute the torques in each robot joint due to gravity.

First, let's consider a simplified situation in which a rigid shaft in an arbitrary direction supports a point mass at a distance (Figure 6.1). Suppose a point mass, having mass, m, and located at point P, is rigidly connected to the shaft but offset at some distance from the shaft and subject to the force of gravity, mg.

Assume a frame, $\{1\}$, exists whose origin is on the shaft, and in which Z_1 is pointing along the shaft. The moment generated about the origin of frame 1 is

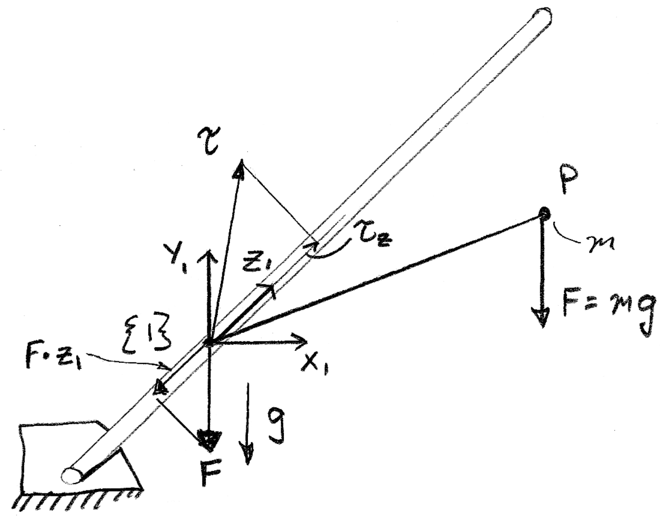

Figure 6.1: A shaft with an off-axis mass generating a torque and a force on the shaft.

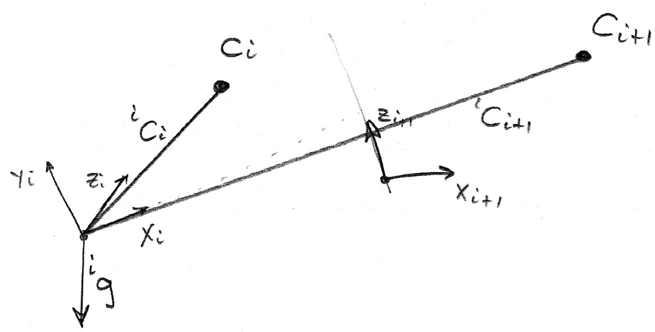

Figure 6.2: Two robot link frames (i, $i+1$) with centers of mass (C_i) and the gravity vector (g) shown for computation of gravity torques.

$$^1\tau = P \otimes F = m(^1P \otimes ^1 g)$$

$$^1\tau = m \begin{bmatrix} 0 & -Pz & P_y \\ P_z & 0 & -P_x \\ -P_y & P_x & 0 \end{bmatrix} \begin{bmatrix} g_x \\ g_y \\ g_z \end{bmatrix} = m \begin{bmatrix} P_y g_z - P_z g_y \\ P_z g_x - P_x g_z \\ P_x g_y - P_y g_x \end{bmatrix}$$

When we apply this to robot links, frame $\{1\}$ will be the link frame and Z_1 will be the axis. Thus the torque we are interested in is the z component of τ

$$\begin{bmatrix} 0 \\ 0 \\ P_x g_y - P_y g_x \end{bmatrix}$$

Now consider the force, $F = mg$ pointing "down". Besides the torque, the force, F, can also be represented in frame $\{1\}$, and it has a projection onto the Z_1 axis:

$$F_z = F \cdot Z_1$$

Applying this to a robot, we assume that the robot is not moving and that the joints are in some pose, θ.

Each link has mass, m_i located at its center of mass: C_i (Figure 6.2). In link i, we wish to compute the z component of torque due to the gravitational force on each center of mass which is more distal to the joint (has a greater subscript, i). The moment on frame i due to the mass in frame j is therefore

$$\tau_{ij} = C_j \otimes m_j g = m_j (C_j \otimes g)$$

where we assume that all quantities are represented in the same frame. For joint i, the natural frame is frame $\{i\}$:

$$\begin{aligned} {}^i\tau &= \sum_{j=i,n} m_j({}^iC_j \otimes {}^ig) \\ &= \left[\sum_{j=i,n} m_j {}^iC_j\right] \otimes {}^ig \end{aligned}$$

The quantity

$$C_{mi} = \sum_{j=i,n} m_j C_j$$

is called the *first moment of mass* which is the center of mass times the total mass for the entire part of the manipulator distal to joint i and it can be expressed in any frame in general.

The torque in the i'th joint, τ_i is therefore:

$$\tau_i = \hat{z} \cdot {}^i\tau = [0 \quad 0 \quad 1]\, {}^i\tau = \begin{bmatrix} 0 \\ 0 \\ {}^i\tau_Z \end{bmatrix}$$

for rotary joints.

For prismatic joints, we have the analogous equation

$$F_{ij} = \sum_{j=i,n} m_j {}^ig$$

and the joint force is just the z component of F_{ij}.

$$F_{Zi} = \hat{z} \cdot F_{ij} = \begin{bmatrix} 0 \\ 0 \\ F_{ijZ} \end{bmatrix}$$