



15 Ecological networks are variable both in time and space (Poisot, Stouffer, and Gravel 2015; Trøjelsgaard  
16 and Olesen 2016) - this variability motivated the emergence of methodology to compare ecological  
17 networks, in a way that meshes with the usual approaches of comparison of ecological communities, *i.e.*  
18  $\beta$ -diversity; although the definition of  $\beta$ -diversity is a contentious topic amongst community ecologists  
19 (see *e.g.* Tuomisto 2010), the need to understand network variability is motivated by the fact that  
20 species that make up the networks do not react to their environment in the same way, and therefore the  
21  $\beta$ -diversity of networks may behave in complex ways.

22 Poiset et al. (2012) and Canard et al. (2014) have suggested an approach to  $\beta$ -diversity for ecological  
23 networks which is based on the comparison of shared and unique links among species, and differentiate  
24 this sharing of links between common and unique species. This framework can be summarized as  
25  $\beta_{un} = \beta_{os} + \beta_{st}$ , namely the fact that overall network dissimilarity ( $\beta_{un}$ ) has a component that can be  
26 calculated directly from the dissimilarity of interactions between shared species ( $\beta_{os}$ ), and a component  
27 that cannot, the later originating in unique species introducing their unique interactions ( $\beta_{st}$ ). This  
28 approach has been widely adopted since its publication, with recent examples using it to understand the  
29 effect of fire on pollination systems (Baronio et al. 2021); the impact of rewiring on spatio-temporal  
30 network dynamics (Campos-Moreno et al. 2021); the effects of farming on rural and urban landscapes  
31 on species interactions (Olsson et al. 2021); and as a tool to estimate the sampling completeness of  
32 networks (Souza et al. 2021). It has, similarly, received a number of extensions, including the ability to  
33 account for interaction strength (Magrath et al. 2017), the ability to handle probabilistic ecological  
34 networks (Poisot et al. 2016), and the integration into the Local Contribution to Beta Diversity (Legendre  
35 and De Cáceres 2013) approach to understand how environment changes drive network dissimilarity  
36 (Poisot et al. 2017).

37 In a recent contribution, Fründ (2021) argues that the calculation of network dissimilarity terms as  
38 outlined by Poiset et al. (2012) is incorrect, as it can lead to over-estimating the role of interactions  
39 between shared species in a network (“rewiring”), and therefore underestimate the importance of species  
40 turnover across networks. Here, I present a more thorough justification of the methodological choices  
41 for the Poiset et al. (2012) method, explain how information about species turnover can be extracted  
42 from its decomposition, and conduct numerical experiments to guide the interpretation of the  $\beta$ -diversity

values thus obtained. These numerical experiments establish three core facts. First, the decomposition responds to the correct sources of network variation; second, the decomposition adequately captures the relative roles of species turnover and interaction rewiring; finally, the decomposition adequately captures the role of turnover vs. non-turnover (like changes in connectance) processes. Although the alternative normalization suggested by Fründ (2021) is not without its uses, which I discuss in conclusion, it is inadequate as a network  $\beta$ -diversity measurement, as it introduces many confounding elements that make the interpretation of the results more difficult, and should likely not be used as a default.

## Partitioning network dissimilarity

The approach to quantifying the difference between pairs of networks established in Poisot et al. (2012) is a simple extension of the overall method by Koleff, Gaston, and Lennon (2003) for species dissimilarity based on presence-absence data. The objects to compare,  $X_1$  and  $X_2$ , are partitioned into three values,  $a = |X_1 \cup X_2|$ ,  $b = |X_2 \setminus X_1|$ , and  $c = |X_1 \setminus X_2|$ , where  $|x|$  is the cardinality of set  $x$ , and  $\setminus$  is the set subtraction operation. In the perspective of species composition comparison,  $X_1$  and  $X_2$  are the sets of species in either community, so that if  $X_1 = \{x, y, z\}$  and  $X_2 = \{v, w, x, y\}$ , we have  $X_1 \cup X_2 = \{v, w, x, y, z\}$ ,  $X_1 \cap X_2 = \{x, y\}$ ,  $X_2 \setminus X_1 = \{v, w\}$ , and  $X_1 \setminus X_2 = \{z\}$ . The core message of Koleff, Gaston, and Lennon (2003) is that the overwhelming majority of measures of  $\beta$ -diversity can be re-expressed as functions that operate on the cardinality (number of elements) of these sets.

## Re-expressing networks as sets

Applying this framework to networks requires a few additional definitions. Although ecologists tend to think of networks as their adjacency matrix, this representation is far from optimal to get a solid understanding of which elements should be counted as part of which set when measuring network dissimilarity. For this reason, we need fall back on the definition of a graph as a pair of sets, wherein  $\mathcal{G} = (V, E)$ . These two components  $V$  and  $E$  represent vertices (nodes, species) and edges (interactions), where  $V$  is specifically a set containing the vertices  $\mathcal{G}$ , and  $E$  is a set of ordered pairs, in which every pair is composed of two elements of  $V$ ; an element  $\{i, j\}$  in  $E$  indicates that there is an interaction *from* species  $i$  to species  $j$  in the network  $\mathcal{G}$ .

69 In the context of networks comparison (assuming the networks to compare are  $\mathcal{M}$  and  $\mathcal{N}$ ), we can  
 70 further decompose the contents of these sets as

$$\mathcal{M} = (V_c \cup V_m, E_c \cup E_{sm} \cup E_{um}),$$

71 and

$$\mathcal{N} = (V_c \cup V_n, E_c \cup E_{sn} \cup E_{un}),$$

72 where  $V_c$  is the set of shared species,  $V_k$  are the species belonging only to network  $k$ ,  $E_c$  are the shared  
 73 edges, and  $E_{sk}$  and  $E_{uk}$  are the interactions unique to  $k$  involving, respectively, only species in  $V_c$ , and  
 74 at least one species from  $V_k$ .

#### 75 **Defining the partitions from networks as sets**

76 The metaweb (Dunne 2006), which is to say the entire regional species pool and their interaction, can  
 77 be defined as  $\mathcal{M} \cup \mathcal{N}$  (this operation is commutative), which is to say

$$\mathcal{M} \cup \mathcal{N} = (V_c \cup V_m \cup V_n, E_c \cup E_{sm} \cup E_{um} \cup E_{sn} \cup E_{un}).$$

78 This operation gives us an equivalent to  $\gamma$ -diversity for networks, in that the set of vertices contains *all*  
 79 species from the two networks, and the set of edges contains *all* the interactions between these species.  
 80 If, further, we make the usual assumption that only species with at least one interaction are present in  
 81 the set of vertices, then all elements of the set of vertices are present at least once in the set of edges,  
 82 and the set of vertices can be entire reconstructed from the set of edges. Although measures of network  
 83  $\beta$ -diversity operate on interactions (not species), this property is maintained at every decomposition we  
 84 will describe next.

85 We can similarly define the intersection (similarly commutative) of two networks:

$$\mathcal{M} \cap \mathcal{N} = (V_c, E_c).$$

86 The decomposition of  $\beta$ -diversity from Poisot et al. (2012) uses these components to measure  $\beta_{os}$  (the  
 87 interaction dissimilarity between shared species, which Fründ (2021) terms “rewiring”), and  $\beta_{wn}$  (the  
 88 overall dissimilarity including non-shared species). We can express the components  $a$ ,  $b$ , and  $c$  of  
 89 Koleff, Gaston, and Lennon (2003) as the cardinality of the following sets:

Component	$a$	$b$	$c$
$\beta_{os}$	$E_c$	$E_{sn}$	$E_{sm}$
$\beta_{wn}$	$E_c$	$E_{sn} \cup E_{un}$	$E_{sm} \cup E_{um}$

90 These decompositions are used to perform the calculations of  $\beta$ -diversity in the `EcologicalNetworks.jl`  
 91 package (Banville, Vissault, and Poisot 2021) for Julia, which I use for the following numerical  
 92 experiments.

### 93 Quantifying the importance of species turnover

94 The difference between  $\beta_{os}$  and  $\beta_{wn}$  stems from the species dissimilarity between  $\mathcal{M}$  and  $\mathcal{N}$ , and it is  
 95 easier to understand the effect of turnover by picking a dissimilarity measure to work as an exemplar.  
 96 At this point, Fründ (2021) introduce a confusing terminology in their work, stating that Sørensen’s and  
 97 Whittaker’s measures of dissimilarity are the same in the Koleff, Gaston, and Lennon (2003) framework  
 98 (they are not; in practice,  $\beta_{Sor} = 1 - \beta_w$ ), and (ii) noting Whittaker’s measure as  $(b + c)/(2a + b + c)$ ,  
 99 which in the Koleff, Gaston, and Lennon (2003) framework is, in fact,  $\beta_t$  (Wilson and Shmida 1984).  
 100 This does not change the overall conclusions as these measures can be re-expressed to converge to the  
 101 same value. For the sake of consistency, I will use  $\beta_t$  moving forward; it returns values in  $[0, 1]$ , with 0  
 102 meaning complete similarity, and 1 meaning complete dissimilarity.

### 103 Establishing that $\beta_{wn} \geq \beta_{os}$

104 Based on a partition between three sets of cardinality  $a$ ,  $b$ , and  $c$ ,

$$\beta_t = \frac{b + c}{2a + b + c}.$$

105 So as to simplify the notation of the following section, I will introduce a series of new variables. Let  
 106  $A = |E_c|$  be the number of links that are identical between networks;  $S = |E_{sn} \cup E_{sm}|$  be the number  
 107 of links that are not shared, but only involve shared species (*i.e.* links from  $\mathcal{M} \cup \mathcal{N}$  established between  
 108 species from  $\mathcal{M} \cap \mathcal{N}$ ); and  $U = |E_{un} \cup E_{um}|$  the number of links that are not shared, and involve at  
 109 least one unique species. Adopting the perspective developed in the previous section, wherein networks  
 110 are sets and the measures of  $\beta$ -diversity operates on these sets, highlights the conceptual issue in the  
 111 Fründ (2021) alternative normalization: they are using components of the networks that are *not* part of  
 112 the networks being compared.

113 There are two important points to note here. First, the number or proportion of species that are shared  
 114 is not involved in the calculation. Second, the connectance of either network is not involved in the  
 115 calculation. That all links counted in *e.g.*  $U$  come from  $\mathcal{M}$ , or that they are evenly distributed between  
 116  $\mathcal{M}$  and  $\mathcal{N}$ , has no impact on the result. This is a desirable property of the approach: whatever  
 117 quantitative value of the components of dissimilarity can be interpreted in the light of the connectance  
 118 and species turnover *without* any risk of circularity. Therefore the argument of Fründ (2021), whereby  
 119 the  $\beta_{os}$  component should decrease with turnover, and be invariant to connectance, does not hold: the  
 120 very point of the approach is to provide measures that can be interpreted in the light of connectance and  
 121 species turnover.

122 The final component of network dissimilarity in Poisot et al. (2012) is  $\beta_{st}$ , *i.e.* the part of  $\beta_{wn}$  that is not  
 123 explained by changes in interactions between shared species ( $\beta_{os}$ ), and therefore stems from species  
 124 turnover. This fraction is defined as  $\beta_{st} = \beta_{wn} - \beta_{os}$ .

125 The expression of  $\beta_{st}$  does not involve a partition into sets that can be plugged into the framework  
 126 of Koleff, Gaston, and Lennon (2003), because the part of  $\mathcal{M}$  and  $\mathcal{N}$  that are composed of their  
 127 unique species cannot, by definition, share interactions. One could, theoretically, express these as  
 128  $\mathcal{M} \setminus \mathcal{N} = (V_m, E_{um})$  and  $\mathcal{N} \setminus \mathcal{M} = (V_v, E_{vn})$  (note the non-commutativity here), but the dissimilarity  
 129 between these networks is trivially maximal for the measures considered.

130 Using the  $\beta_t$  measure of dissimilarity, we can re-write (using the notation with  $A$ ,  $S$ , and  $U$ )

$$\beta_{os} = \frac{S}{2A + S},$$

131 and

$$\beta_{wn} = \frac{S + U}{2A + S + U}.$$

132 Note that  $\beta_{os}$  has the form  $x/y$  with  $x = S$  and  $y = 2A + S$ , and  $\beta_{wn}$  has the form  $(x + k)/(y + k)$ , with  
 133  $k = U$ . As long as  $k \geq 0$ , it is guaranteed that  $\beta_{wn} \geq \beta_{os}$ , and therefore that  $0 \leq \beta_{st} \leq 1$ ; as  $A$ ,  $S$ , and  
 134  $U$  are cardinalities of sets, they are necessarily satisfying this condition.

135 We can get an expression for  $\beta_{st}$ , by bringing  $\beta_{os}$  and  $\beta_{wn}$  to a common denominator and simplifying  
 136 the numerator:

$$\beta_{st} = \frac{2AU}{(2A + S)(2A + S + U)}.$$

137 Note that this value varies in a non-monotonic way with regards to the number of interactions that are  
 138 part of the common set of species – this is obvious when developing the denominator into

$$4A^2 + S^2 + 4AS + 2AU + SU,$$

139 As such, we expect that the value of  $\beta_{st}$  will vary in a hump-shaped way with the proportion of  
 140 shared interactions. For this reason, Poisot et al. (2012) suggest that  $\beta_{st}/\beta_{wn}$  (alt.  $1 - \beta_{os}/\beta_{wn}$ ) is a  
 141 better indicator of the *relative* importance of turnover processes on network dissimilarity. This can be  
 142 calculated as

$$\frac{\beta_{st}}{\beta_{wn}} = \frac{2AU}{(2A + S)(2A + S + U)} \times \frac{S + U}{2A + S + U},$$

143 which reduces to

$$\frac{\beta_{st}}{\beta_{wn}} = \frac{2AU}{(2A + S)(S + U)}.$$

144 The roots of this expression are  $A = 0$  (the turnover of species has no contribution to the difference  
145 between  $\beta_{wn}$  and  $\beta_{os}$  if there are no shared species, and therefore no rewiring), and for  $U = 0$  (the  
146 turnover of species has no contribution if all species are shared).

#### 147 **Numerical experiment: response of the components to different sources of network variation**

148 To illustrate the behavior of  $\beta_{st}$ , I conducted a simple numerical experiment in which two networks  
149 have the same number of interactions  $L$  (recall from the previous section that we do not need to set  
150 a number of species yet), and these interactions are partitionned according to proportions  $p_s$  and  $p_r$   
151 into shared ( $A$ ), rewired ( $S$ ), and unique ( $U$ ) links, with  $A = p_s \times L$ ,  $S = (1 - p_s) \times p_r \times L$ , and  
152  $U = (1 - p_s) \times (1 - p_r) \times L$ . The results are represented in fig. 1.

#### 153 **Is this decomposition over-estimating the effect of “rewiring”?**

154 One of the arguments put forth by Fründ (2021) is that the decomposition outlined above will overesti-  
155 mate the effect of rewiring; I argue that this is based on a misunderstanding of what  $\beta_{st}$  achieves. It is  
156 paramount to clarify that  $\beta_{st}$  is not a direct measure of the importance of turnover: it is a quantification  
157 of the relative impact of rewiring to overall dissimilarity, which, all non-turnover mechanisms being  
158 accounted for in the decomposition, can be explained by turnover mechanisms. In this section, I present  
159 two numerical experiments showing (i) that the  $\beta_{os}$  component is in fact an accurate measure of rewiring,  
160 and (ii) that  $\beta_{st}$  captures the consequences of species turnover, and of the interactions brought by unique  
161 species.

#### 162 **Illustrations on arbitrarily small networks are biased**

163 We can re-calculate the illustration of Fründ (2021), wherein a pair of networks with two shared  
164 interactions ( $A = 2$ ) receive either an interaction in  $S$ , in  $U$ , or in both:



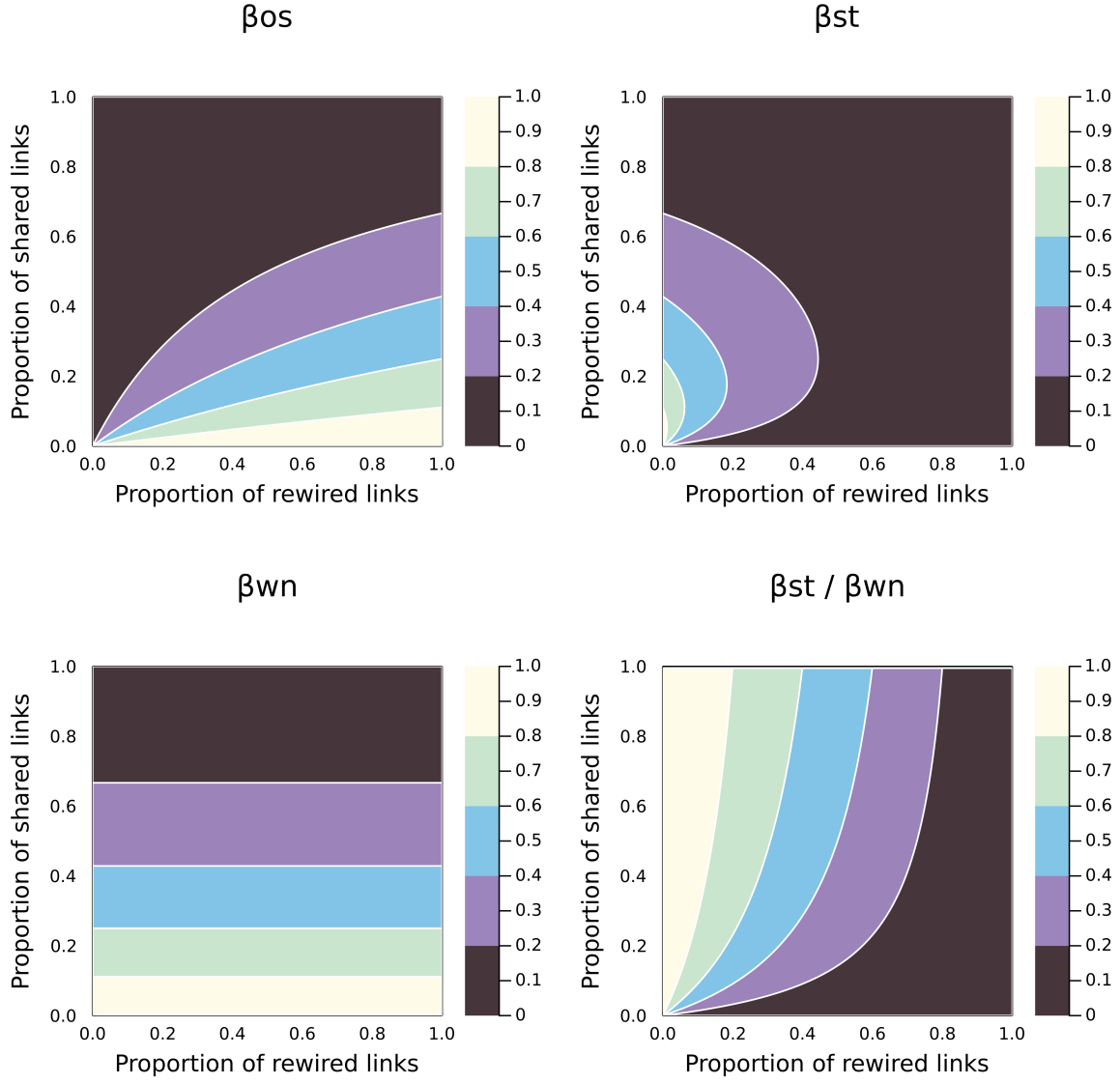


Figure 1: Values of  $\beta_{os}$ ,  $\beta_{wn}$ ,  $\beta_{st}$ , and  $\beta_{st}/\beta_{wn}$  as a function of the proportion of rewired links and the proportion of shared links.

$A$	$S$	$U$	$\beta_{os}$	$\beta_{wn}$	$\beta_{st}$	$\beta_{st}/\beta_{wn}$
2	0	0	0	0	0	
2	1	0	1/5	1/5	0	0
2	0	1	0	1/5	1/5	0
2	1	1	1/5	1/3	2/15	2/5

165 The over-estimation argument hinges on the fact that  $\beta_{st} < \beta_{os}$  in the last situation (one interaction  
 166 as rewiring, one as turnover). Reaching the conclusion of an overestimation from this is based on  
 167 a mis-interpretation of what  $\beta_{st}$  means. The correct interpretation is that, out of the entire network  
 168 dissimilarity, only three-fifths are explained by re-wiring. The fact that this fraction is not exactly  
 169 one-half comes from the fact that the Wilson and Shmida (1984) measure counts shared interactions  
 170 *twice* (*i.e.* it has a  $2A$  term), which over-amplifies the effect of shared interactions as the network is  
 171 really small. Running the same calculations with  $A = 10$  gives a relative importance of the turnover  
 172 processes of 47%, and  $\beta_{st}$  goes to  $1/2$  as  $A/(S + U)$  increases. As an additional caveat, the value of  $\beta_{st}$   
 173 will depend on the measure of beta-diversity used. Measures that do not count the shared interaction  
 174 twice are not going to amplify the effect of rewiring.

#### 175 **Numerical experiment: the decomposition captures the roles of rewiring and turnover accurately**

176 Consider two bipartite networks, each with  $R$  species on either side, and each with the same connectance  
 177  $\rho$ . We will assume that these networks *share* a proportion  $p$  of their species from one side (and share all  
 178 species from the other), and that the interactions between these species are undergo rewiring with at a  
 179 rate  $q$ . This is sufficient information to calculate the values of  $A$ ,  $S$ , and  $U$  required to get the values of  
 180  $\beta_{os}$  and  $\beta_{wn}$ . Note that the simplification of assuming that only species from one side can vary is merely  
 181 for the sake of simplicity, but does not decrease the generality of the argument.

182 Each network will have  $\rho(1 - p)R^2$  interactions that are unique due to species turnover, and so

$$U = 2\rho(1 - p)R^2.$$

183 The part of both networks composed of overlapping species has  $\rho p R^2$  interactions, of which  $\rho(1 - q)p R^2$   
 184 are shared, and  $\rho q p R^2$  underwent rewiring. This leads to

$$A = \rho(1 - q)p R^2 ,$$

185 and

$$S = \rho p q R^2 .$$

186 Note that we can drop the multiplicative constant  $R^2$ , making the result independent of the size of the  
 187 network. Based on these components, we can get the values of  $\beta_{os}$  and  $\beta_{wn}$ , as presented in fig. 2.

188 The value of  $\beta_{os}$  is entirely unchanged by variations in  $p$  (species sharing), and responds *only* to changes  
 189 in  $q$  (the probability of rewiring), whereas as expected,  $\beta_{wn}$  responded to changes in both of these  
 190 parameters: the most dissimilar networks have low species sharing (interactions are dissimilar because  
 191 brought by unique species), and high rewiring (shared species do not share interactions). The relative  
 192 changes in  $\beta_{os}$  and  $\beta_{wn}$  lead to predictable changes in  $\beta_{st}$ : its value is maximized when both rewiring  
 193 *and* species sharing are low. Increasing rewiring decreases the impact of species turnover (because,  
 194 for an equal number of interactions, the dissimilarity of interactins in shared species contributes more  
 195 to  $\beta_{wn}$ ); increasing the chance of sharing species also does decrease  $\beta_{st}$ , trivially because there is no  
 196 species turnover anymore. Note that when using the correction of  $\beta_{st}/\beta_{wn}$ , the effect of species turnover  
 197 is magnified for low probabilities of re-wiring.

198 In conclusion, this numerical experiment shows that the decomposition as initially presented by Poisot et  
 199 al. (2012), *i.e.* using denominators that make sense from a network composition point of view, succeeds  
 200 at capturing the relative effect of turnover and rewiring. ### Numerical experiment: the decomposition  
 201 captures the roles of species turnover and connectance accurately

202 Consider now two bipartite networks, which still have  $R$  species on either side, but differ in their  
 203 connectance ( $\rho_1$  and  $\rho_2$ ) – by maintaining the assumption that species on one side are shared with

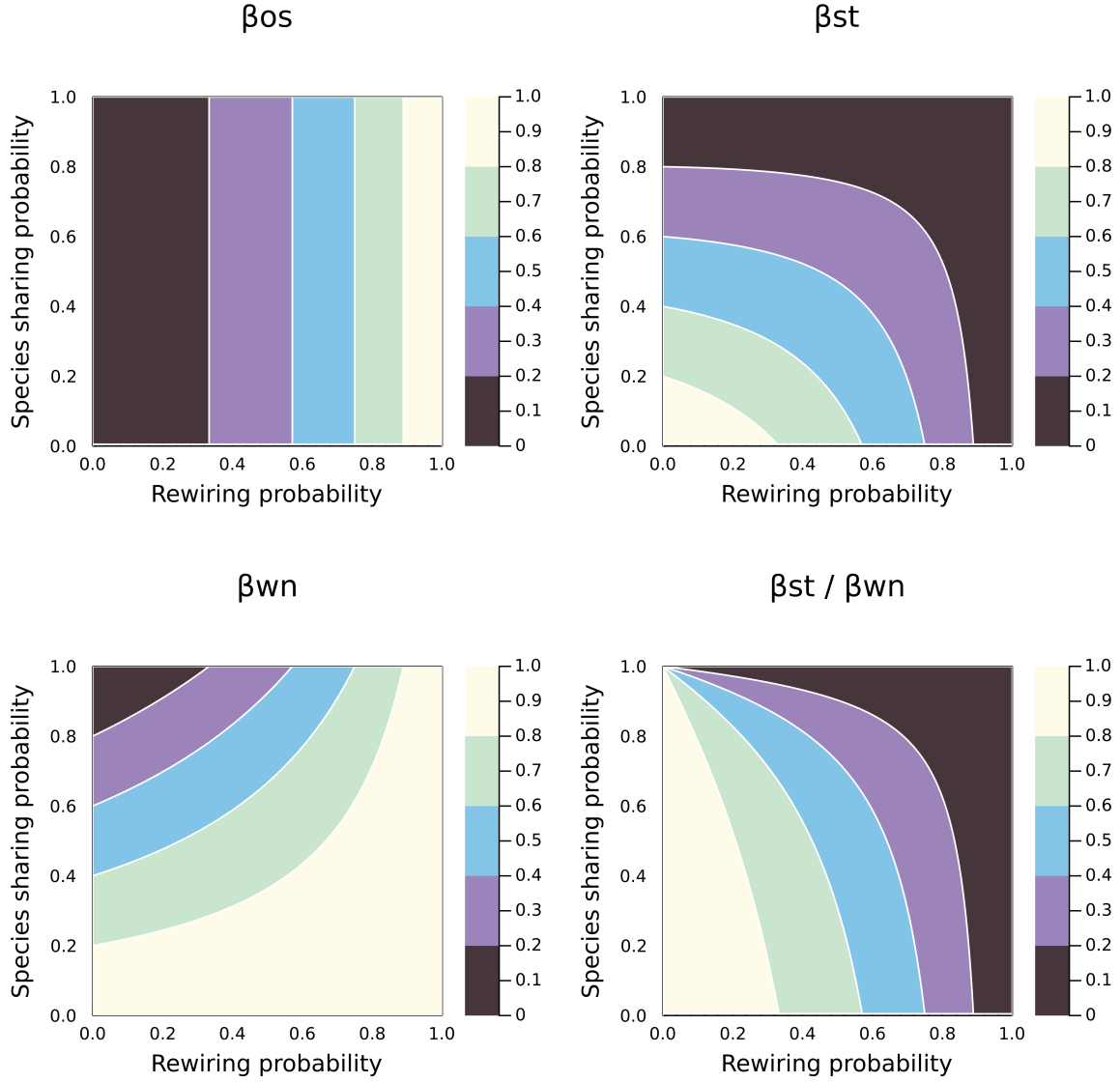


Figure 2: Response of  $\beta_{os}$  and  $\beta_{wn}$ , and the consequences on  $\beta_{st}$ , to changes in rewiring probability ( $q$ ) and probability of species sharing ( $p$ ). As expected,  $\beta_{os}$  is not affected by species turnover, but increases with the rewiring probability. By contrast,  $\beta_{wn}$  increases when the rewiring probability is higher *and* when fewer species are shared. This has important consequences for  $\beta_{st}$ : its value is maximized for low species sharing, and decreases for high rewiring probability.

204 probability  $p$ , and that interactions between shared species are rewired at probability  $q$ , we can examine  
 205 the effect of varying both connectance and turnover on the value of the  $\beta$ -diversity components. Note  
 206 that, although not presented, we will drop the multiplicative constant  $R^2$  from all calculations, as it is  
 207 a common factor for all values; again, this implies that the results presented here are independent of  
 208 network richness.

209 The number of unique links due to species turnover is

$$U = (1 - p)(\rho_1 + \rho_2),$$

210 which decreases with the proportion of shared species, but increases with connectance. The number of  
 211 links between shared species takes a little more steps to calculate. First, amongst the  $pR^2$  species in  
 212 both sub-graphs, network 1 will have  $\rho_1 pR^2$ , and network 2 will have  $\rho_2 pR^2$ . Because  $\rho_1 \neq \rho_2$ , there  
 213 are only  $\min(\rho_1, \rho_2)pR^2$  links that can be shared, a proportion  $q$  of which will undergo re-wiring, and  
 214 a proportion  $(1 - q)$  of which will be shared. This leads to the expression (after dropping  $R^2$ ) for the  
 215 number of shared links:

$$A = p(1 - q)\min(\rho_1, \rho_2).$$

216 The number of unique links due to shared species is the sum of all links in network 1 ( $\rho_1 R^2$ ), minus  
 217 the sum of the shared links ( $AR^2$ ) and the unique links due to species turnover ( $(1 - p)\rho_1 R^2$ ); this  
 218 same quantity is calculated in the same way for the second networks, leading to (after dropping the  
 219 multiplicative constant  $R^2$  and some simplifications)

$$S = p(\rho_1 + \rho_2) - 2A.$$

220 Note that as expected, this last quantity scales with the proportion of shared species ( $p$ ) and with  
 221 connectance (as shared species bring more of their interactions), but decreases with the size of the  
 222 shared links components. The consequences of varying  $\rho_2$  and  $p$  are presented in fig. 3.

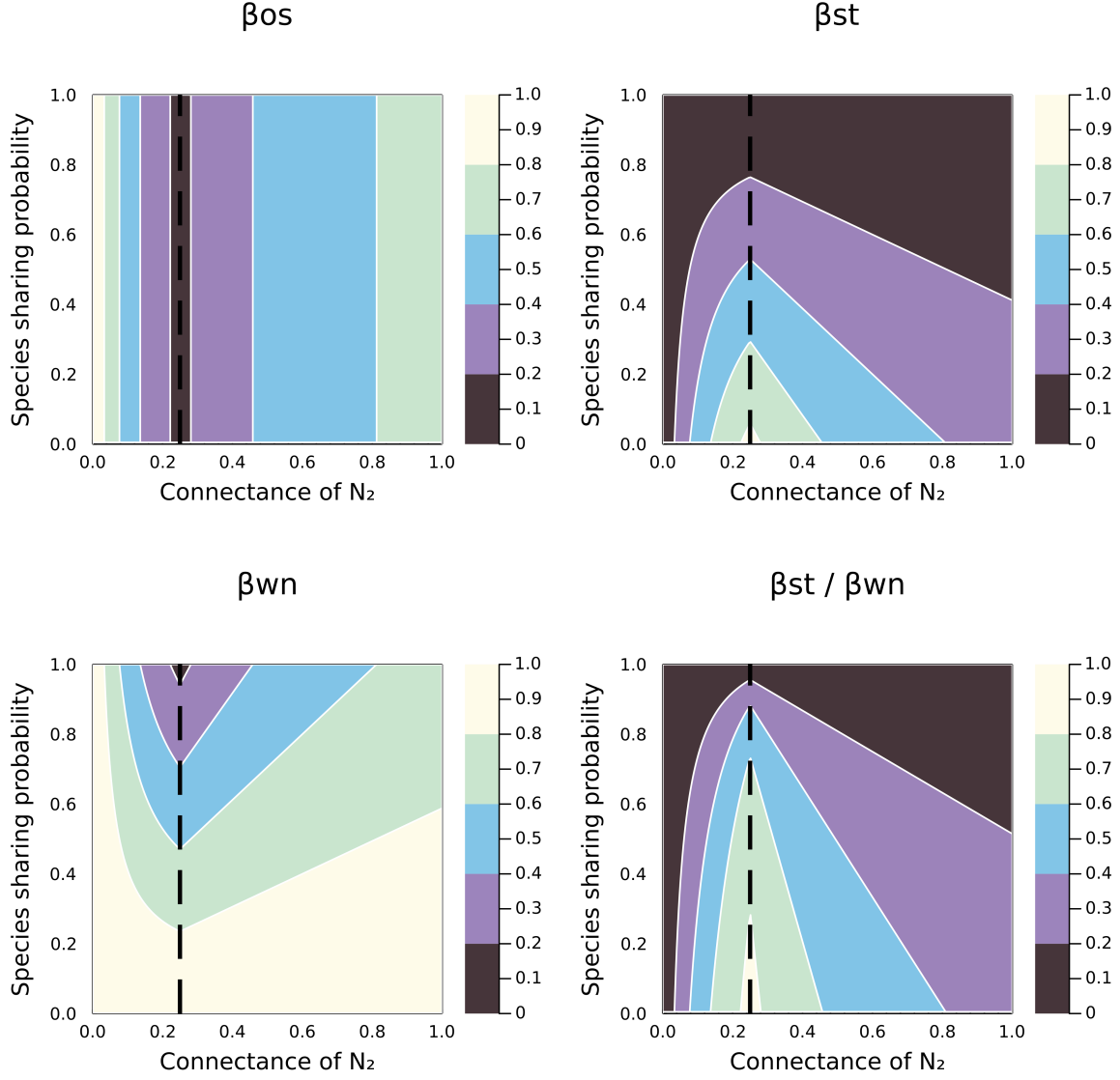


Figure 3: Effects of varying the connectance of the second network ( $\rho_2$ ) and the proportion of shared species ( $p$ ) on the values of the  $\beta$ -diversity components. As expected,  $\beta_{os}$  is still independent of species turnover, and  $\beta_{wn}$  increases when species turnover increases, or when the connectances become more dissimilar. These figures have been generated with  $\rho_1 = 0.25$  and  $q = 0.15$ , and the results are qualitatively robust to changes in these parameters.

Although  $\beta_{os}$  is only responding to changes in connectance (as is expected, seeing that the relative connectances of both networks appear in the expression for  $S$  and  $A$ ),  $\beta_{wn}$  changes in response to both parameters. Specifically, increasing the difference in connectance between the two networks, especially when also increasing the species dissimilarity, results in more dissimilar networks – this is because unique species from both networks bring their own interactions (at rate  $\rho_1$  and  $\rho_2$ ), and therefore contribute to dissimilarity. It is particularly noteworthy that  $\beta_{st}$ , regardless of the differences in connectance, increases with the proportion of unique species. At an equal proportion of shared species,  $\beta_{st}$  decreases with differences in connectance: this is an equally expected result, which indicates that the difference between  $\beta_{os}$  and  $\beta_{wn}$  is in part explained by non-turnover mechanisms (here, changes in connectance). Relying on the  $\beta_{st}/\beta_{wn}$  correction again magnifies this effect, without changing their interpretation.

#### **Does the partition of network dissimilarity needs a new normalization?**

Based on the arguments presented above, I do not think the suggestion of Fründ (2021) to change the denominator of  $\beta_{os}$  makes sense as a default; the strength of the original approach by Poisot et al. (2012) is indeed that the effect of turnover is based on a rigorous definition of networks as graphs (as opposed to networks as matrices), in which the induction of vertices from the edgelist being compared gives rise to biologically meaningful denominators. The advantage of this approach is that at no time does the turnover of species itself (or indeed, as shown in many places in this manuscript, the network richness), or the connectance of the network, enter into the calculation. As such, it is possible to use  $\beta_{os}$  and  $\beta_{wn}$  in relationship to these terms, calculated externally (as was recently done by *e.g.* Higinio and Poisot 2021), without creating circularities.

The choice of changing the denominator hinges on what one admits as a definition for  $\beta_{st}$ . If the point of  $\beta_{st}$  is to be a component of overall  $\beta$ -diversity as advocated by Fründ (2021) and Novotny (2009), a change of numerator *might* be acceptable. Nevertheless, this change of numerator contributes to blurring the frontier between a measure of interaction dissimilarity and a measure of community dissimilarity which starts to add the effect of relative richness; this later case warrants a thorough methodological assessment. Conversely, if as we argue in Poisot et al. (2012),  $\beta_{st}$  is to be meant as

a *guide* to the interpretation of  $\beta_{wn}$  and  $\beta_{os}$ , and related to actual measures of species turnover and network connectance, one must not change the denominator.

It is essential to recognize that there are multiple reasons to calculate network dissimilarity, and it is our opinion that the arguments levied by Fründ (2021) against the original partition stem from a misunderstanding of what it intends to do (and does, indeed, do well), not from intrinsic methodological issues in the partition itself. Based on the results presented in this contribution, I argue that the original partition of network  $\beta$ -diversity from Poisot et al. (2012) should remain the default.

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