# Dissimilarity of species interaction networks: quantifying the effect of turnover and rewiring

# Timothée Poisot 1,2,‡

# **Correspondance to:**

Timothée Poisot — timothee.poisot@umontreal.ca

This work is released by its authors under a CC-BY 4.0 license

Last revision: September 28, 2021

<sup>&</sup>lt;sup>1</sup> Université de Montréal <sup>2</sup> Québec Centre for Biodiversity Sciences

<sup>&</sup>lt;sup>‡</sup> These authors contributed equally to the work

Despite having established its usefulness in the last ten years, the decomposition of ecological networks in components allowing to measure their  $\beta$ -diversity retains some methodological ambiguities. Notably, how to quantify the relative effect of mechanisms tied to interaction rewiring vs. species turnover has been interpreted differently by different authors. In this contribution, I present mathematical arguments and numerical experiments that should (i) establish that the decomposition of networks as it is currently done is indeed fit for purpose, and (ii) provide guidelines to interpret the values of the components tied to turnover and rewiring.

- Ecological networks are variable both in time and space (Poisot et al. 2015; Trøjelsgaard & Olesen 2016) -
- this variability motivated the emergence of methodology to compare ecological networks, including in a
- way that meshes with the core concept for the comparison of ecological communities, namely  $\beta$ -diversity
- 4 (Poisot et al. 2012). The need to understand network variability through partitioning in components
- equivalent to  $\alpha$ ,  $\beta$ , and  $\gamma$  diversities is motivated by the prospect to further integrate the analysis of species
- 6 interactions to the analysis of species compositions. Because species that make up the networks do not
- react to their environment in the same way, and because interactions are only expressed in subsets of the
- 8 environments in which species co-occurr, the  $\beta$ -diversity of networks may behave in complex ways, and its
- 9 quantification is likely to be ecologically informative.
- Poisot et al. (2012) and Canard et al. (2014) have suggested an approach to  $\beta$ -diversity for ecological
- 11 networks which is based on the comparison of the number of shared and unique links among species
- within a pair of networks. Their approach differentiates this sharing of links between those established
- between species occurring in both networks, and those established with at least one unique species. This
- framework is expressed as the decomposition  $\beta_{wn} = \beta_{os} + \beta_{st}$ , namely the fact that network dissimilarity
- $_{15}$   $(\beta_{wn})$  has a component that can be calculated directly from the dissimilarity of interactions between
- shared species ( $\beta_{os}$ ), and a component that cannot ( $\beta_{st}$ ). Presumably, the value of these components for a
- pair of networks can generate insights about the mechanisms involved in dissimilarity.
- 18 This approach has been widely adopted since its publication, with recent examples using it to understand
- the effect of fire on pollination systems (Baronio et al. 2021); the impact of rewiring on spatio-temporal
- 20 network dynamics (Campos-Moreno et al. 2021); the effects of farming on rural and urban landscapes on
- species interactions (Olsson et al. 2021); the impact of environment gradients on multi-trophic
- metacommunities (Ohlmann2018MapImp?); and as a tool to estimate the sampling completeness of
- 23 networks (Souza et al. 2021). It has, similarly, received a number of extensions, including the ability to
- 24 account for interaction strength (Magrach et al. 2017), the ability to handle probabilistic ecological
- 25 networks (Poisot et al. 2016), and the integration into the Local Contribution to Beta Diversity (Legendre
- & De Cáceres 2013) approach to understand how environment changes drive network dissimilarity (Poisot
- 27 et al. 2017).
- 28 This later component originates in unique species introducing their unique interactions both between
- 29 themselves, and with species that are common to both networks. CONTINUE WITH BETA ST
- Furthermore, much like the definition of  $\beta$ -diversity in all its forms is a contentious topic amongst

- community ecologists (see e.g. Tuomisto 2010), the  $\beta$ -diversity of networks has been submitted to
- methodological scrutiny over the years. A synthesis of some criticisms, related to the correct denominator
- to use to express the proportion of different links, has recently been published (Fründ 2021). It argues that
- the calculation of network dissimilarity terms as originally outlined by Poisot et al. (2012) is incorrect, as it
- can lead to over-estimating the role of interactions between shared species in a network ("rewiring"), and
- therefore underestimate the importance of species turnover across networks. As mist-understanding
- either of these quantities can lead to biased inferences about the mechanisms generating network
- dissimilarity, it is important to assess how the values (notably of  $\beta_{os}$ , and therefore of  $\beta_{st}$ ) react to
- 39 methodological choices.
- Here, I present a more thorough justification of the methodological choices for the Poisot et al. (2012)
- method, explain how information about species turnover can be extracted from its decomposition, and
- conduct numerical experiments to guide the interpretation of the  $\beta$ -diversity values thus obtained. These
- 43 numerical experiments establish three core facts. First, the decomposition responds to the correct sources
- of network variation; second, the decomposition adequately captures the relative roles of species turnover
- and interaction rewiring; finally, the decomposition adequately captures the role of turnover
- vs. non-turnover (like changes in connectance) processes. Although the alternative normalization
- 47 suggested by Fründ (2021) is not without its uses, which I discuss in conclusion, it is inadequate as a
- network  $\beta$ -diversity measurement, as it introduces many confounding elements that make the
- 49 interpretation of the results more difficult, and should likely not be used as a default.

# 50 Partitioning network dissimilarity

- 51 The approach to quantifying the difference between pairs of networks established in Poisot et al. (2012) is
- 52 a simple extension of the overall method by Koleff et al. (2003) for species dissimilarity baed on
- presence-absence data. The objects to compare,  $X_1$  and  $X_2$ , are partitioned into three values,
- $a = |X_1 \cup X_2|, b = |X_2 \setminus X_1|, \text{ and } c = |X_1 \setminus X_2|, \text{ where } |x| \text{ is the cardinality of set } x, \text{ and } x \text{ is the set}$
- substraction operation. In the perspective of species composition comparison,  $X_1$  and  $X_2$  are the sets of
- species in either community, so that if  $X_1 = \{x, y, z\}$  and  $X_2 = \{v, w, x, y\}$ , we have  $X_1 \cup X_2 = \{v, w, x, y, z\}$ ,
- 57  $X_1 \cap X_2 = \{x, y\}, X_2 \setminus X_1 = \{v, w\}, \text{ and } X_1 \setminus X_2 = \{z\}.$  The core message of Koleff *et al.* (2003) is that the
- overwheling majority of measures of  $\beta$ -diversity can be re-expressed as functions that operate on the
- cardinality (number of elements) of these sets.

#### 60 Re-expressing networks as sets

- 61 Applying this framework to networks requires a few additional definitions. Although ecologists tend to
- think of networks as their adjacency matrix, this representation is far from optimal to get a solid
- understanding of which elements should be counted as part of which set when measuring network
- dissimilarity. For this reason, we need fall back on the definition of a graph as a pair of sets, wherein
- $\mathcal{G} = (V, E)$ . These two components V and E represent vertices (nodes, species) and edges (interactions),
- where V is specifically a set containing the vertices  $\mathcal{G}$ , and E is a set of ordered pairs, in which every pair is
- composed of two elements of V; an element  $\{i, j\}$  in E indicates that there is an interaction from species i
- to species j in the network  $\mathcal{G}$ .
- In the context of networks comparison (assuming the networks to compare are  $\mathcal{M}$  and  $\mathcal{N}$ ), we can further
- 70 decompose the contents of these sets as

$$\mathcal{M} = (V_c \cup V_m, E_c \cup E_{sm} \cup E_{um}),$$

71 and

$$\mathcal{M} = (V_c \cup V_n, E_c \cup E_{sn} \cup E_{un}),$$

- where  $V_c$  is the set of shared species,  $V_k$  are the species belonging only to network k,  $E_c$  are the shared
- edges, and  $E_{sk}$  and  $E_{uk}$  are the interactions unique to k involving, respectively, only species in  $V_c$ , and at
- 14 least one species from  $V_k$ .

## 75 Defining the partitions from networks as sets

- The metaweb (Dunne 2006), which is to say the entire regional species pool and their interaction, can be
- defined as  $\mathcal{M} \cup \mathcal{N}$  (this operation is commutative), which is to say

$$\mathcal{M} \cup \mathcal{N} = (V_c \cup V_m \cup V_n, E_c \cup E_{sm} \cup E_{um} \cup E_{sn} \cup E_{un}).$$

This operation gives us an equivalent to  $\gamma$ -diversity for networks, in that the set of vertices contains all

- <sup>79</sup> species from the two networks, and the set of edges contains *all* the interactions between these species. If,
- 80 further, we make the usual assumption that only species with at least one interaction are present in the set
- of vertices, then all elements of the set of vertices are present at least once in the set of edges, and the set of
- vertices can be entire reconstructed from the set of edges. Although measures of network  $\beta$ -diversity
- operate on interactions (not species), this property is maintained at every decomposition we will describe
- 84 next.
- We can similarly define the intersection (similarly commutative) of two networks:

$$\mathcal{M} \cap \mathcal{N} = (V_c, E_c)$$
.

- The decomposition of  $\beta$ -diversity from Poisot *et al.* (2012) uses these components to measure  $\beta_{os}$  (the
- interaction dissimilarity between shared species, which Fründ (2021) terms "rewiring"), and  $\beta_{wn}$  (the
- overall dissimilarity including non-shared species). We can express the components a, b, and c of Koleff et
- 89 al. (2003) as the cardinality of the following sets:

Component	а	b	С
$eta_{os}$	$E_c$	$E_{sn}$	$E_{sm}$
$eta_{wn}$	$E_c$	$E_{sn} \cup E_{un}$	$E_{sm} \cup E_{um}$

- <sub>90</sub> These decompositions are used to perform the calculations of  $\beta$ -diversity in the EcologicalNetworks.jl
- $_{\rm 91}$   $\,$  package (Banville {\it et al.} 2021) for Julia, which I use for the following numerical experiments.

### 92 Quantifying the importance of species turnover

- The difference between  $\beta_{os}$  and  $\beta_{wn}$  stems from the species dissimilarity between  $\mathcal M$  and  $\mathcal N$ , and it is
- easier to understand the effect of turnover by picking a dissimilarity measure to work as an exemplar. At
- this point, Fründ (2021) introduce a confusin terminology in their work, stating that Sørensen's and
- 96 Whittaker's measures of dissimilarity are the same in the Koleff et al. (2003) framework (they are not; in
- practice,  $\beta_{Sor} = 1 \beta_w$ ), and (ii) noting Whittaker's measure as (b+c)/(2a+b+c), which in the Koleff *et*
- al. (2003) framework is, in fact,  $\beta_t$  (Wilson & Shmida 1984). This does not change the overall conclusions
- 99 as these measures can be re-expressed to converge to the same value. For the sake of consistency, I will use

 $\beta_t$  moving forward; it returns values in [0, 1], with 0 meaning complete similarity, and 1 meaning complete dissimilarity.

### 102 Establishing that $\beta_{wn} \geq \beta_{os}$

Based on a partition between three sets of cardinality a, b, and c,

$$\beta_t = \frac{b+c}{2a+b+c} \,.$$

So as to simplify the notation of the following section, I will introduce a series of new variables. Let

 $A = |E_c|$  be the number of links that are identical between networks;  $S = |E_{sn} \cup E_{sm}|$  be the number of 105 links that are not shared, but only involve shared species (i.e. links from  $\mathcal{M} \cup \mathcal{N}$  established between 106 species from  $\mathcal{M} \cap \mathcal{N}$ ); and  $U = |E_{un} \cup E_{um}|$  the number of links that are not shared, and involve at least 107 one unique species. Adopting the perspective developed in the previous section, wherein networks are 108 sets and the measures of  $\beta$ -diversity operates on these sets, highlights the conceptual issue in the Fründ 109 (2021) alternative normalization: they are using components of the networks that are not part of the networks being compared. 111 There are two important points to note here. First, the number or proportion of species that are shared is not involved in the calculation. Second, the connectance of either network is not involved in the 113 calculation. That all links counted in e.g. U come from  $\mathcal{M}$ , or that they are evenly distributed between  $\mathcal{M}$ 114 and  $\mathcal{N}$ , has no impact on the result. This is a desirable property of the approach: whatever quantitative 115 value of the components of dissimilarity can be interpreted in the light of the connectance and species 116 turnover without any risk of circularity. Therefore the argument of Fründ (2021), whereby the  $\beta_{os}$ 117 component should decrease with turnover, and be invariant to connectance, does not hold: the very point 118 of the approach is to provide measures that can be interpreted in the light of connectance and species turnover. 120 The final component of network dissimilarity in Poisot et al. (2012) is  $\beta_{st}$ , i.e. the part of  $\beta_{wn}$  that is not 121 explained by changes in interactions between shared species ( $\beta_{os}$ ), and therefore stems from species 122 turnover. This fraction is defined as  $\beta_{st} = \beta_{wn} - \beta_{os}$ . 123 The expression of  $\beta_{st}$  does not involve a partition into sets that can be plugged into the framework of

Koleff *et al.* (2003), because the part of  $\mathcal{M}$  and  $\mathcal{N}$  that are composed of their unique species cannot, by definition, share interactions. One could, theoretically, express these as  $\mathcal{M} \setminus \mathcal{N} = (V_m, E_{um})$  and  $\mathcal{N} \setminus \mathcal{M} = (V_v, E_{un})$  (note the non-commutativity here), but the dissimilarity between these networks is trivially maximal for the measures considered.

Using the  $\beta_t$  measure of dissimilarity, we can re-write (using the notation with A, S, and U)

$$\beta_{os} = \frac{S}{2A + S} \,,$$

130 and

$$\beta_{wn} = \frac{S+U}{2A+S+U} \,.$$

Note that  $\beta_{os}$  has the form x/y with x=S and y=2A+S, and  $\beta_{wn}$  has the form (x+k)/(y+k), with k=U. As long as  $k\geq 0$ , it is guaranteed that  $\beta_{wn}\geq \beta_{os}$ , and therefore that  $0\geq \beta_{st}\geq 1$ ; as A, S, and U are cardinalities of sets, they are necessarily satisfying this condition.

We can get an expression for  $\beta_{st}$ , by bringing  $\beta_{os}$  and  $\beta_{wn}$  to a common denominator and simplifying the numerator:

$$\beta_{st} = \frac{2AU}{(2A+S)(2A+S+U)}.$$

Note that this value varies in a non-monotonic way with regards to the number of interactions that are part of the common set of species – this is obvious when developing the denominator into

$$4A^2 + S^2 + 4AS + 2AU + SU$$
,

As such, we expect that the value of  $\beta_{st}$  will vary in a hump-shaped way with the proportion of shared interactions. For this reason, Poisot *et al.* (2012) suggest that  $\beta_{st}/\beta wn$  (alt.  $1 - \beta_{os}/\beta_{wn}$ ) is a better indicator of the *relative* importance of turnover processes on network dissimilarity. This can be calculated as

$$\frac{\beta_{st}}{\beta_{wn}} = \frac{2AU}{(2A+S)(2A+S+U)} \times \frac{S+U}{2A+S+U},$$

which reduces to

151

$$\frac{\beta_{st}}{\beta_{wn}} = \frac{2AU}{(2A+S)(S+U)}.$$

The roots of this expression are A=0 (the turnover of species has no contribution to the difference between  $\beta_{wn}$  and  $\beta_{os}$  if there are no shared species, and therefore no rewiring), and for U=0 (the turnover of species has no contribution if all species are shared).

#### Numerical experiment: response of the components to different sources of network variation

To illustrate the behavior of  $\beta_{st}$ , I conducted a simple numerical experiment in which two networks have the same number of interactions L (recall from the previous section that we do not need to set a number of species yet), and these interactions are partitionned according to proportions  $p_s$  and  $p_r$  into shared (A), rewired (S), and unique (U) links, with  $A = p_s \times L$ ,  $S = (1 - p_s) \times p_r \times L$ , and  $U = (1 - p_s) \times (1 - p_r) \times L$ . The results are represented in fig. 1.

#### [Figure 1 about here.]

The rewiring component  $\beta_{os}$  varies as a function of the proportion of shared links that are rewired; by contrast,  $\beta_{wn}$  varies only as a function of the proportion of links that are shared: that the unshared links are established between common or unique species has no effect on overall network dissimilarity. The quadratic nature of the denominator for  $\beta_{st}$  is clear here, with a maximum reach when there is no re-wiring, and a small number of shared links (*i.e.* the networks are almost entirely dissimilar except for the links between shared species). Althought the raw values of  $\beta_{st}$  may seem low, the normalization using  $\beta_{st}/\beta_{wn}$  magnifies this effect: its values are indeed maximized when the rewiring is lower, *i.e.* all of the network variation stems from turnover processes.

#### Is this decomposition over-estimating the effect of "rewiring?"

One of the arguments put forth by Fründ (2021) is that the decomposition outlined above will overestimate the effect of rewiring; I argue that this is based on a misunderstanding of what  $\beta_{st}$  achieves.

It is paramount to clarify that  $\beta_{st}$  is not a direct measure of the importance of turnover: it is a

quantification of the relative impact of rewiring to overall dissimilarity, which, all non-turnover mechanisms being accounted for in the decomposition, can be explained by turnover mechanisms. In this section, I present two numerical experiments showing (i) that the  $\beta_{os}$  component is in fact an accurate measure of rewiring, and (ii) that  $\beta_{st}$  captures the consequences of species turnover, and of the interactions brought by unique species.

#### 169 Illustrations on arbitrarily small networks are biased

We can re-calculate the illustration of Fründ (2021), wherein a pair of networks with two shared interactions (A = 2) receive either an interaction in S, in U, or in both:

$\overline{A}$	S	U	$eta_{os}$	$eta_{wn}$	$eta_{st}$	$\beta_{st}/\beta_{wn}$
2	0	0	0	0	0	
2	1	0	1/5	1/5	0	0
2	0	1	0	1/5	1/5	0
2	1	1	1/5	1/3	2/15	2/5

The over-estimation argument hinges on the fact that  $\beta_{st} < \beta_{os}$  in the last situation (one interaction as 172 rewiring, one as turnover). Reaching the conclusion of an overestimation from this is based on a mis-interpretation of what  $\beta_{st}$  means. The correct interpretation is that, out of the entire network 174 dissimilarity, only three-fifths are explained by re-wiring. The fact that this fraction is not exactly one-half 175 comes from the fact that the Wilson & Shmida (1984) measure counts shared interactions twice (i.e. it has 176 a 2A term), which over-amplifies the effect of shared interactions as the network is really small. Running the same calculations with A = 10 gives a relative importance of the turnover processes of 47%, and  $\beta_{st}$ 178 goes to 1/2 as A/(S+U) increases. As an additional caveat, the value of  $\beta_{st}$  will depend on the measure of 179 beta-diversity used. Measures that do not count the shared interaction twice are not going to amplify the effect of rewiring.

# Numerical experiment: the decomposition captures the roles of rewiring and turnover accurately

Consider two bipartite networks, each with R species on either side, and each with the same connectance  $\rho$ . We will assume that these networks *share* a proportion p of their species from one side (and share all species from the other), and that the interactions between these species are undergo rewiring with at a rate q. This is sufficient information to calculate the values of A, S, and U required to get the values of  $\beta_{os}$  and  $\beta_{wn}$ . Note that the simplification of assuming that only species from one side can vary is merely for the sake of simplicity, but does not decrease the generality of the argument.

Each network will have  $\rho(1-p)R^2$  interactions that are unique due to species turnover, and so

$$U=2\rho(1-p)R^2.$$

The part of both networks composed of overlapping species has  $\rho pR^2$  interactions, of which  $\rho(1-q)pR^2$  are shared, and  $\rho qpR^2$  underwent rewiring. This leads to

$$A = \rho(1 - q)pR^2,$$

193 and

196

$$S = \rho pqR^2$$
.

Note that we can drop the multiplicative constant  $R^2$ , making the result independent of the size of the network. Based on these components, we can get the values of  $\beta_{os}$  and  $\beta_{wn}$ , as presented in fig. 2.

#### [Figure 2 about here.]

The value of  $\beta_{os}$  is entirely unchanged by variations in p (species sharing), and responds *only* to changes in q (the probability of rewiring), whereas as expected,  $\beta_{wn}$  responded to changes in both of these parameters: the most dissimilar networks have low species sharing (interactions are dissimilar because brought by unique species), and high rewiring (shared species do not share interactions). The relative

changes in  $\beta_{os}$  and  $\beta_{wn}$  lead to predictable changes in  $\beta_{st}$ : its value is maximized when both rewiring and 201 species sharing are low. Increasing rewiring decreases the impact of species turnover (because, for an 202 equal number of interactions, the dissimilarity of interactins in shared species contributes more to  $\beta_{wn}$ ); 203 increasing the chance of sharing species also does decrease  $\beta_{st}$ , trivially because there is no species 204 turnover anymore. Note that when using the correction of  $\beta_{st}/\beta_{wn}$ , the effect of species turnover is 205 magnified for low probabilities of re-wiring. 206 In conclusion, this numerical experiment shows that the decomposition as initially presented by Poisot et 207 al. (2012), i.e. using denominators that make sense from a network composition point of view, succeeds at 208 capturing the relative effect of turnover and rewiring. 209

# Numerical experiment: the decomposition captures the roles of species turnover and connectance accurately

Consider now two bipartite networks, which still have R species on either side, but differ in their connectance ( $\rho_1$  and  $\rho_2$ ) – by maintaining the assumption that species on one side are shared with probability p, and that interactions between shared species are rewired at probability q, we can examine the effect of varying both connectance and turnover on the value of the  $\beta$ -diversity components. Note that, although not presented, we will drop the multiplicative constant  $R^2$  from all calculations, as it is a common factor for all values; again, this implies that the results presented here are independant of network richness.

219 The number of unique links due to species turnover is

$$U = (1 - p)(\rho_1 + \rho_2),$$

which decreases with the proportion of shared species, but increases with connectance. The number of links between shared species takes a little more steps to calculate. First, amongst the  $pR^2$  species in both sub-graphs, network 1 will have  $\rho_1 pR^2$ , and network 2 will have  $\rho_2 pR^2$ . Because  $\rho_1 \neq \rho_2$ , there are only  $\min(\rho_1, \rho_2)pR^2$  links that can be shared, a proportion q of which will undergo re-wiring, and a proportion (1-q) of which will be shared. This leads to the expression (after dropping  $R^2$ ) for the number of shared links:

$$A = p(1-q)\min(\rho_1, \rho_2).$$

The number of unique links due to shared species is the sum of all links in network 1 ( $\rho_1 R^2$ ), minus the sum of the shared links ( $AR^2$ ) and the unique links due to species turnover ( $(1-p)\rho_1 R^2$ ); this same quantity is calculated in the same way for the second networks, leading to (after dropping the multiplicative constant  $R^2$  and some simplifications)

$$S = p(\rho_1 + \rho_2) - 2A.$$

Note that as expected, this last quantity scales with the proportion of shared species (p) and with connectance (as shared species bring more of their interactions), but decreases with the size of the shared links components. The consequences of varying  $\rho_2$  and p are presented in fig. 3.

233

#### [Figure 3 about here.]

Although  $\beta_{os}$  is only responding to changes in connectance (as is expected, seeing that the relative connectances of both networks appear in the expression for S and A),  $\beta_{wn}$  changes in response to both parameters. Specifically, increasing the difference in connectance between the two networks, especially when also increasing the species dissimilarity, results in more dissimilar networks – this is because unique species from both networks bring their own interactions (at rate  $\rho_1$  and  $\rho_2$ ), and therefore contribute to dissimilarity. It is particularly noteworthy that  $\beta_{st}$ , regardless of the differences in connectance, increases with the proportion of unique species. At an equal proportion of shared species,  $\beta_{st}$  decreases with differences in connectance: this is an equally expected result, which indicates that the difference between  $\beta_{os}$  and  $\beta_{wn}$  is in part explained by non-turnover mechanisms (here, changes in connectance). Relying on the  $\beta_{st}/\beta_{wn}$  correction again magnifies this effect, without changing their interpretation.

#### Does the partition of network dissimilarity needs a new normalization?

Based on the arguments presented above, I do not think the suggestion of Fründ (2021) to change the
denominator of  $\beta_{os}$  makes sense as a default; the strength of the original approach by Poisot *et al.* (2012) is
indeed that the effect of turnover is based on a rigorous definition of networks as graphs (as opposed to

networks as matrices), in which the induction of vertices from the edgelist being compared gives rise to biologically meaningful denominators. The advantage of this approach is that at no time does the turnover 249 of species itself (or indeed, as shown in many places in this manuscript, the network richness), or the 250 connectance of the network, enter into the calculation. As such, it is possible to use  $\beta_{os}$  and  $\beta_{wn}$  in 25 relationship to these terms, calculated externally (as was recently done by e.g. Higino & Poisot 2021), 252 without creating circularities. 253 The choice of changing the denominator hinges on what one admits as a definition for  $\beta_{st}$ . If the point of  $\beta_{st}$  is to be a component of overall  $\beta$ -diversity as advocated by Fründ (2021) and Novotny (2009), a change 255 of numerator might be acceptable. Nevertheless, this change of numerator contributes to blurring the 256 frontier between a measure of interaction dissimilarity and a measure of community dissimilarity which starts to add the effect of relative richness; this later case warrants a thorough methodological assessment. 258 Conversely, if as we argue in Poisot et al. (2012),  $\beta_{st}$  is to be meant as a guide to the interpretation of  $\beta_{wn}$ 259 and  $\beta_{os}$ , and related to actual measures of species turnover and network connectance, one must not change the denominator. 261 It is essential to recognize that there are multiple reasons to calculate network dissimilarity, and it is our 262 opinion that the arguments levied by Fründ (2021) against the original partition stem from a 263 misunderstanding of what it intends to do (and does, indeed, do well), not from intrinsic methodological 264 issues in the partition itself. Based on the results presented in this contribution, I argue that the original 265 partition of network  $\beta$ -diversity from Poisot *et al.* (2012) should remain the default.

#### References

Banville, F., Vissault, S. & Poisot, T. (2021). Mangal. il and Ecological Networks. il: Two complementary 268 packages for analyzing ecological networks in Julia. Journal of Open Source Software, 6, 2721. 269 Baronio, G.J., Souza, C.S., Maruyama, P.K., Raizer, J., Sigrist, M.R. & Aoki, C. (2021). Natural fire does not 270 affect the structure and beta diversity of plant-pollinator networks, but diminishes floral-visitor 271 specialization in Cerrado. Flora, 281, 151869. 272 Campos-Moreno, D.F., Dyer, L.A., Salcido, D., Massad, T.J., Pérez-Lachaud, G., Tepe, E.J., et al. (2021). 273 Importance of interaction rewiring in determining spatial and temporal turnover of tritrophic 274 (Piper-caterpillar-parasitoid) metanetworks in the Yucatán Península, México. Biotropica, 53, 275

- 276 1071-1081.
- <sup>277</sup> Canard, E.F., Mouquet, N., Mouillot, D., Stanko, M., Miklisova, D. & Gravel, D. (2014). Empirical evaluation of neutral interactions in host-parasite networks. *The American Naturalist*, 183, 468–479.
- Dunne, J.A. (2006). The Network Structure of Food Webs. In: *Ecological networks: Linking structure and dynamics* (eds. Dunne, J.A. & Pascual, M.). Oxford University Press, pp. 27–86.
- Fründ, J. (2021). Dissimilarity of species interaction networks: How to partition rewiring and species turnover components. *Ecosphere*, 12, e03653.
- Higino, G.T. & Poisot, T. (2021). Beta and phylogenetic diversities tell complementary stories about ecological networks biogeography. *Parasitology*, 1–23.
- Koleff, P., Gaston, K.J. & Lennon, J.J. (2003). Measuring beta diversity for presence–absence data. *Journal of Animal Ecology*, 72, 367–382.
- Legendre, P. & De Cáceres, M. (2013). Beta diversity as the variance of community data: Dissimilarity coefficients and partitioning. *Ecology Letters*, 16, 951–963.
- Magrach, A., Holzschuh, A., Bartomeus, I., Riedinger, V., Roberts, S.P.M., Rundlöf, M., et al. (2017).
- Plant-pollinator networks in semi-natural grasslands are resistant to the loss of pollinators during blooming of mass-flowering crops. *Ecography*, n/a–n/a.
- Novotny, V. (2009). Beta diversity of plant–insect food webs in tropical forests: A conceptual framework. *Insect Conservation and Diversity*, 2, 5–9.
- Olsson, R.L., Brousil, M.R., Clark, R.E., Baine, Q. & Crowder, D.W. (2021). Interactions between plants and pollinators across urban and rural farming landscapes. *Food Webs*, 27, e00194.
- Poisot, T., Canard, E., Mouillot, D., Mouquet, N. & Gravel, D. (2012). The dissimilarity of species
   interaction networks. *Ecology Letters*, 15, 1353–1361.
- Poisot, T., Cirtwill, A.R., Cazelles, K., Gravel, D., Fortin, M.-J. & Stouffer, D.B. (2016). The structure of probabilistic networks. *Methods in Ecology and Evolution*, 7, 303–312.
- Poisot, T., Gueveneux-Julien, C., Fortin, M.-J., Gravel, D. & Legendre, P. (2017). Hosts, parasites and their interactions respond to different climatic variables. *Global Ecology and Biogeography*, n/a–n/a.
- Poisot, T., Stouffer, D.B. & Gravel, D. (2015). Beyond species: Why ecological interaction networks vary

- through space and time. *Oikos*, 124, 243–251.
- Souza, C.S., Maruyama, P.K., Santos, K.C.B.S., Varassin, I.G., Gross, C.L. & Araujo, A.C. (2021).
- Plant-centred sampling estimates higher beta diversity of interactions than pollinator-based sampling across habitats. *New Phytologist*, 230, 2501–2512.
- Trøjelsgaard, K. & Olesen, J.M. (2016). Ecological networks in motion: Micro- and macroscopic variability across scales. *Functional Ecology*, 30, 1926–1935.
- Tuomisto, H. (2010). A diversity of beta diversities: Straightening up a concept gone awry. Part 1. Defining
  beta diversity as a function of alpha and gamma diversity. *Ecography*, 33, 2–22.
- Wilson, M.V. & Shmida, A. (1984). Measuring Beta Diversity with Presence-Absence Data. *Journal of Ecology*, 72, 1055–1064.

figures/numexp1.png

Figure 1: Values of  $\beta_{os}$ ,  $\beta_{wn}$ ,  $\beta_{st}$ , and  $\beta_{st}/\beta_{wn}$  as a function of the proportion of rewired links and the proportion of shared links.

figures/numexp2.png

Figure 2: Response of  $\beta_{os}$  and  $\beta_{wn}$ , and the consequences on  $\beta_{st}$ , to changes in rewiring probability (q) and probability of species sharing (p). As expected,  $\beta_{os}$  is not affected by species turnover, but increases with the rewiring probability. By contrast,  $\beta_{wn}$  increases when the rewiring probability is higher *and* when fewer species are shared. This has important consequences for  $\beta_{st}$ : its value is maximized for low species sharing, and decreases for high rewiring probability.

figures/numexp3.png

Figure 3: Effects of varying the connectance of the second network  $(\rho_2)$  and the proportion of shared species (p) on the values of the  $\beta$ -diversity components. As expected,  $\beta_{os}$  is still independent of species turnover, and  $\beta_{wn}$  increases when species turnover increases, or when the connectances become more dissimilar. These figures have been generated with  $\rho_1=0.25$  and q=0.15, and the results are qualitatively robust to changes in these parameters.