Closed-loop Amplify-and-Forward Relaying in Bursty Interference

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***Abstract*—Inherent interfering signals generated by the underlying elements found in power substation have been known to span over consecutive noise samples, resulting in bursty interfering noise samples. Considering a Wireless Sensor Network (WSN) system, we adopt a two-state Markov-Gaussian noise model and explore closed-loop MIMO transmissions in Rayleigh channel corrupted by such noise. For uncoded transmissions, we show how sensor nodes transmission energy can be reduced by implementing a sub-optimum nodes selection technique for cooperative closed-loop MIMO and the corresponding Maximum Likelihood (ML) and Maximum a Posteriori (MAP) decoders linked to the noise parameters.**

***Index Terms*—Cooperative communications, Euclidean distance, impulsive noise model, Markov Gaussian, precoder, WSN.**

I. Introduction

The ubiquity of ad hoc wireless system such as the wireless sensor network (WSN) has found application in many fields, providing sensing, monitoring, and measurements to name but a few. In power substation environment, the inherent background Additive White Gaussian Noise (AWGN) is constantly present, but this classical observation is no longer relevant at an instant when the occurrence of an impulse becomes noticeable. Such impulse features an amplitude which supersedes the background AWGN. In such realistic environment, noise signals generated by the underlying elements span over several samples [1], giving rise to bursty appearances of impulses.

Due to the autonomy of nodes in WSN, a WSN operating in a power substation environment could allow nodes to cooperatively transmit by employing the Multiple-Input Multiple-Output (MIMO) technique. Linear closed-loop MIMO precoding [2], [3] is a technique involving multi-antenna transmissions, but with optimal distribution of resources on multiple transmit antennas, leading to improved system performance. Closed-loop cooperative transmission ensures that the source node cooperates with its idle neighbors to provide spatial diversity. Each cooperating node then precodes the data before it transmits over the diverse subchannels to the receiver where data is combined and detected. The advantages of the traditional precoded MIMO technique are inherited in this distributed system though with added overhead. This work aims to adapt WSN transmissions to the bursty interferences found in the power substation environment.

In [4], the performances of different diversity combining methods in impulsive interferers modeled by Class A noise has been studied, followed by the derivation of bounds for the bit error rate (BER). Cooperative diversity in the presence of Class A noise [5] has been investigated with the MAP detection derived for the Alamouti-based (open-loop) scheme. Neither of these works considered distributed closed-loop MIMO transmissions, nor a channel with memory. Fertonani and Colavolpe [6] studied the performance/latency tradeoff of a low-density parity check (LDPC) coded Single-Input Single-Output (SISO) transmissions in a channel perturbed by bursty interferers modeled by two-state Markov-Gaussian noise model. The study established a relationship between the channel memory and the system information rate using a MAP symbol detection without exploiting the statistics of the interferers. The work in [7] exploits cooperative closed-loop MIMO transmissions in memoryless channel with the Class A noise model using two closed-loop MIMO transmission techniques with nodes selection.

While being motivated by developing a wireless communication network that adapts to the power substation environments, the contributions of this paper are as follow:

(1) Reduce the complexity of nodes selection technique assuming full and limited channel information.
(2) Reduce the overall nodes transmission energy in bursty impulsive interferers.
(3) Extend the expression of ergodic capacity of a closed-loop MIMO system in bursty impulsive noise.

Section II considers the theory and the model of the implemented bursty impulsive noise model, while section **??** presents the considered MIMO precoding and cooperative system model. Presented in section V are the MAP and ML optimal decoders. Section VI highlights and discusses the obtained results while section VII concludes the paper.

II. Bursty Impulsive Noise Model

We adopt a two-state Markov-Gaussian impulsive noise model as shown in Figure 1 [6], in which, the channel in good (G) state is impaired by the background Gaussian noise while in the bad (B) state, the noise is impaired by the impulsive interferers. The noise state s_k which defines the noise sample n_k at each time epoch k is either good or bad such that $s_k \in \{G, B\}$. The probability density functions of the resultant noise n_k given the state s_k are given as

$$p(n_k|s_k = G) = \frac{1}{2\pi\sigma_G^2} \exp\left(-\frac{|n_k|^2}{2\sigma_G^2}\right) \tag{1}$$

$$p(n_k|s_k = B) = \frac{1}{2\pi\vartheta\sigma_G^2} \exp\left(-\frac{|n_k|^2}{2\vartheta\sigma_G^2}\right) \tag{2}$$

here $\vartheta \geq 1$ denotes the ratio between the average noise power in the bad channel and that in the good channel. The transition probabilities can be derived from the knowledge of average number of samples in state G (n_G) and state B (n_B) as

$$n_G = \frac{1}{p_{GB}} \text{ and } n_B = \frac{1}{p_{BG}} \tag{3}$$

where P_{GB} and P_{BG} are the transition probabilities from G state to B state and from B state to G state respectively. The

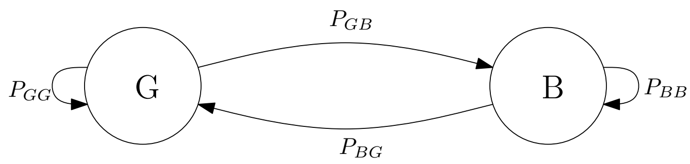

Fig. 1: The Two-state Markov Gaussian model.

knowledge of the statistical parameter pair of P_{GB} and P_{BG} is enough to describe the system state process. The parameter ς which, quantifies the channel memory can be defined [6] as:

$$\varsigma = \frac{1}{p_{GB} + p_{BG}} \tag{4}$$

We are interested in the channel with bursty impulsive noise, which is characterized by $\varsigma \geq 1$, hence the average duration of the bursts can be varied by modifying the parameter ς. The probability of being in each state is given as

$$p_G = p(s_k = G) = \frac{p_{BG}}{p_{GB} + p_{BG}} \tag{5}$$

$$p_B = p(s_k = B) = \frac{p_{GB}}{p_{GB} + p_{BG}} \tag{6}$$

and the the transition matrix of the state evolution is

$$T = \begin{pmatrix} p_{GG} & p_{GB} \\ p_{BG} & p_{BB} \end{pmatrix} \tag{7}$$

By implementing the two-state Markov Gaussian noise model, the correlation of the impulsive noise can be characterized.

III. System Model

Our model assumes a scenario where nodes are closer to each other at some distance d_{loc}, and must cooperate to transmit to the DGN located at a greater direct distance d, where $d \gg d_{loc}$. We then introduce a power gain factor parameter K_R of the original signal at the relays, where $K_R = \left(\frac{d}{d_{loc}}\right)^{PL}$, and PL is the path loss exponent. We consider a cooperative MIMO communication over Rayleigh flat-fading channels consisting of n_r direct paths and $n_r \times N$ amplify-and-forward (AF) hop MIMO relay channel as shown in figure 2. In the first phase, the source broadcasts the symbols to the relays R_1 to R_N, where N is the number of relays, and the data gathering node (DGN). The signals received at $i-$th relay R_i $(i = 1, 2, \ldots, N)$ and the DGN are given by

$$y_{SR_i} = \sqrt{P} K_R h_{SR_i} x + n_{SR_i} \tag{8}$$

$$y_{SD_i} = \sqrt{P} h_{SD_l} x + n_{SD_l}, \quad l = 1, 2, \ldots n_r \tag{9}$$

where h_{SR_i} and h_{SD_l} are the channel gains between the source and the $i-$th relay and the $l-$th receiving antenna of the DGN; the DGN is a multi-antenna single node, x is the transmitted symbol from the source. We assume that the channel coefficient h_q is Rayleigh distributed (where $q \in \{SR_i, SD_l\}$); n_q is the noise term for the q link which is impulsive and modeled as two-state MG process as described in the previous section. The Rayleigh fading is modeled as zero mean and variance σ_{h_q}, where $\sigma_{h_q} = E[|h_q|^2]$.

In the second phase, N relay terminals amplify their received signals, precode and forward them to n_r antennas

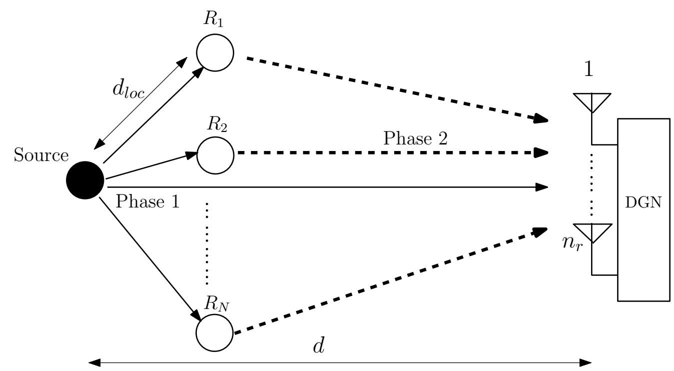

Fig. 2: System model: Cooperative AF relaying.

at the DGN. With the normalized signal $\mathbf{x}'$ at the relay node, where $\mathbf{x}' = \frac{\mathbf{y}_{SR}}{K_R}$, the received signal at the DGN is

$$\mathbf{y}_{RD} = \mathbf{HFx}' + \mathbf{n}_{RD} = \mathbf{HF}(\mathbf{h}_{SR}\mathbf{x} + \frac{\mathbf{n}_{SR}}{K_R}) + \mathbf{n}_{RD} \tag{10}$$

This is a MIMO transmission, where $\mathbf{H}$ is a $[n_r \times N]$ Rayleigh non-correlated i.i.d. channel matrix and $\mathbf{F}$ is a precoding matrix; $\mathbf{n}_{RD}$ is a noise vector which is impulsive and modeled as two-state MG process as described in the previous section. At the DGN, the MRC combining can be expressed as

$$\mathbf{y}_{DGN} = \mathbf{h}^*_{SD}\mathbf{y}_{SD} + (\mathbf{HF})^*\mathbf{y}_{RD} \tag{11}$$

IV. BER Analysis

A. Maximal Ratio Combining (MRC)

Assuming the implementation of an MRC scheme at the DGN, with the DGN having the full knowledge of the channel state information such that the relayed paths can be coherently combined. Having known that the channel gains are random variables (RV) which follow Rayleigh distribution, then the signal-to-noise ratio (SNR) can be modeled as a chi-squared RV. The SNR of each link will be i.i.d. exponentially distributed. Then for the source-DGN (S-D) link, the pdf of the instantaneous SNR, Γ_{SD}, is given by

$$f_{\Gamma_{SD}}(\Gamma) = \frac{\Gamma^{n_r - 1}}{\overline{\Gamma}^{n_r}_{SD}(n_r - 1)!} e^{-\Gamma/\overline{\Gamma}_{SD}} \tag{12}$$

where the mean SNR, $\overline{\Gamma}_{SD} = E[\Gamma_{SD}]$ and the instantaneous SNR, $\Gamma_{SD} = \frac{\sum_{i=1}^{n_r} |h_{SD_i}|^2 P}{\sigma^2_{SD_{i,v}}}$; $\sigma^2_{SD_{i,v}}$ is the variance of the two-state impulsive noise of the S-D link, where $v \in \{G, B\}$. For the source-relay (S-R) link, the pdf of the instantaneous SNR, Γ_{SR}, is given by

$$f_{\Gamma_{SR}}(\Gamma) = \frac{\Gamma^{N-1}}{\overline{\Gamma}^{N}_{SR}(N - 1)!} e^{-\Gamma/\overline{\Gamma}_{SR}} \tag{13}$$

where the mean SNR $\overline{\Gamma}_{SR} = E[\Gamma_{SR}]$ and the instantaneous SNR $\Gamma_{SR} = \frac{\sum_{l=1}^{N} |h_{SR_l}|^2 P}{\sigma^2_{SR_{l,v}}}$. For the relay to the DGN (R-D) MIMO link, the receiver's SNR between the $l-$th transmitting relay and n_r receiving antennas at the DGN are chi-squared variables with the pdf of Γ_{RD} given by

$$f_{\Gamma_{RD}}(\Gamma) = \frac{\Gamma^{n_r - 1} e^{-\Gamma/\overline{\Gamma}_{RD}} N}{\overline{\Gamma}^{n_r}_{RD}(n_r - 1)!} \left[1 - e^{-\Gamma/\overline{\Gamma}_{RD}} \sum_{i=1}^{n_r} \frac{(\Gamma/\overline{\Gamma}_{RD})^{i-1}}{(i-1)!}\right]^N \tag{14}$$

where the mean SNR $\overline{\Gamma}_{RD} = E[\Gamma_{RD}]$ and the instantaneous, SNR $\Gamma_{RD} = \sum_{l=1}^{N}\sum_{i=1}^{n_r}\Gamma_{R_lD_i} = \frac{\|\mathbf{H}_v\mathbf{F}_d\|_F^2}{\sigma_{RD,v}^2}$, where $E[\cdot]$ is the expectation operator, and $\|\cdot\|_F^2$ is the frobenius norm.

At the DGN, the end-to-end SNR, Γ_{DGN} is

$$\Gamma_{DGN} = \Gamma_{SD} + \frac{\Gamma_{RD}\Gamma_{SR}}{\Gamma_{RD} + \Gamma_{SR} + 1} \tag{15}$$

The second term in equation (15) is the instantaneous SNR of the S-R-D link. In order to find the pdf of Γ_{DGN} in equation (15), we adopt the approach in [8] with the aim of simplifying the system analysis, by assuming that Γ_{SD_i}, Γ_{SR_l} and Γ_{R_lD} are independent. Then the CDF of $\Gamma_l = \min(\Gamma_{SR_l}, \Gamma_{R_lD_i})$ can be expressed as

$$\begin{aligned} F_{\Gamma_l}(\Gamma) &= 1 - [1 - Pr(\Gamma_{SR_l} \le \Gamma)][1 - Pr(\Gamma_{R_lD} \le \Gamma)] \\ &= 1 - [1 - F_{\Gamma_{SR_l}}][1 - F_{\Gamma_{R_lD}}] \\ &= 1 - e^{\Gamma/\overline{\Gamma}_{Q_l}} \end{aligned} \tag{16}$$

where the CDF of the SNR is given by $F_{\Gamma_q}(\Gamma) = 1 - e^{-\frac{\Gamma}{\overline{\Gamma}_q}}$ and $\overline{\Gamma}_{Q_l} = \frac{\overline{\Gamma}_{SR_l}\overline{\Gamma}_{R_lD}}{\overline{\Gamma}_{SR_l}+\overline{\Gamma}_{R_lD}}$

1) **Best Relay:** Since the we have assumed i.i.d. Rayleigh distributed channels, the noise variances are identical, i.e. $\sigma_{SR_l}^2 = \sigma_{SD_i}^2 = \sigma_{R_lD_i}^2 = \sigma_v^2$. In this case, the SNR of each link will be exponentially distributed with the mean, $\overline{\Gamma}_q = P\sigma_{h_q}^2/\sigma_v^2$.

By setting $E[|h_q|] = (\frac{d}{d_q})^{PL}$, the mean SNR for the S-D link is

$$\overline{\Gamma}_{SD_i} = E[\Gamma_{SD_i}] = \frac{P}{\sigma_v^2} = \overline{\Gamma}_v \tag{17}$$

For the S-R link, the corresponding mean SNR is given by

$$\overline{\Gamma}_{SR_l} = K_R\frac{P}{\sigma_v^2} = K_R\overline{\Gamma}_v \tag{18}$$

The (post-processing) instantaneous SNR on the R-D link can further be written as $\Gamma_{RD} = \frac{\|\mathbf{H}_v\|_F^2 P}{\sigma_v^2} = \sum_{i=1}^{b}\frac{(H_{v_i})^2 P}{\sigma_v^2}$, where H_{v_i} is the diagonal element of $\mathbf{H}_v$. Note that $\|\mathbf{F}_d\|_F^2 = P$, and since the max-$d_{\min}$ precoder employed on the R-D MIMO link uses b=2 sub-streams, then the mean SNR is given by

$$\overline{\Gamma}_{R_lD} = b\overline{\Gamma}_v \tag{19}$$

From equations (18) and (19), $\overline{\Gamma}_{Q_l} = \dfrac{bK_R}{K_R + b}\overline{\Gamma}_v = K'\overline{\Gamma}_v$, where $K' = \dfrac{bK_R}{K_R + b}$. With N relays, the idea is to select the best relay as

$$B = \arg\max_{l\in\mathscr{R}}\{\Gamma_l\} \tag{20}$$

with $\mathscr{R} = \{1, \ldots, N\}$. The instantaneous SNR for the best relay is given by

$$\Gamma_B = \max_{l\in\mathscr{R}}\{\Gamma_l\} = \max_{l\in\mathscr{R}}\{\min(\Gamma_{SR_l}, \Gamma_{R_lD})\} \tag{21}$$

Having assumed i.i.d. links, i.e. $\overline{\Gamma}_{Q_l} = \overline{\Gamma}_Q = K'\overline{\Gamma}_v$, we re-write the CDF in equation (16) as $F_{\Gamma_l}(\Gamma) = 1 - e^{-\Gamma/\overline{\Gamma}_Q}$, then the CDF for the best relay can be expressed as

$$F_{\Gamma_B}(\Gamma) = [F_{\Gamma_l}(\Gamma)]^N = \left[1 - e^{-\Gamma/\overline{\Gamma}_Q}\right]^N \tag{22}$$

By taking the derivative of the CDF in equation (22) with respect to Γ, using the product rule we obtain the pdf of the instantaneous SNR of the S-R-D link as

$$f_{\Gamma_B}(\Gamma) = \frac{N}{\overline{\Gamma}_Q}e^{-\Gamma/\overline{\Gamma}_Q}\left[1 - e^{-\Gamma/\overline{\Gamma}_Q}\right]^{N-1} \tag{23}$$

For the S-D link, the instantaneous SNR is

$$f_{\Gamma_{SD_i}}(\Gamma) = \frac{1}{\overline{\Gamma}_{SD_i}}e^{-\Gamma/\overline{\Gamma}_{SD_i}} \tag{24}$$

2) **The Moment Generating Function of end-to-end SNR at the DGN :** The Moment Generating Function (MGF) of Γ_{SD_i} is $M_{\Gamma_{SD}}(s) = \prod_{i=1}^{n_r} M_{\Gamma_{SD_i}} = \prod_{i=1}^{n_r}(1 + \overline{\Gamma}_{SD_i}s)^{-1}$, and by equation (17), we have

$$M_{\Gamma_{SD,v}}(s) = (1 + \overline{\Gamma}_v s)^{-n_r} \tag{25}$$

where the definition of MGF is given as

$$M_{\Gamma_q}(s) = \int_0^\infty e^{-s\Gamma}f_{\Gamma_q}(\Gamma)d\Gamma$$

Similarly, by applying binomial expansion from equation (1.111) of [9] to the third term in equation (23), i.e. $\left[1 - e^{-\Gamma/\overline{\Gamma}_Q}\right]^{N-1} = \sum_{k=1}^{N}\binom{N}{k}(-1)^{k-1}e^{-(k-1)\Gamma/\overline{\Gamma}_Q}$, we can now write equation (23) as

$$f_{\Gamma_B}(\Gamma) = \frac{1}{\overline{\Gamma}_Q}\sum_{k=1}^{N}\binom{N}{k}k(-1)^{k-1}e^{-k\Gamma/\overline{\Gamma}_Q} \tag{26}$$

and the respective MGF is

$$\begin{aligned} M_{\Gamma_{B,v}}(s) &= \sum_{k=1}^{N}\binom{N}{k}\frac{k(-1)^{k-1}}{(k + \overline{\Gamma}_Q s)} \\ &= \sum_{k=1}^{N}\binom{N}{k}\frac{k(-1)^{k-1}}{(k + K'\overline{\Gamma}_v s)} \end{aligned} \tag{27}$$

The MGF expression of the end-to-end SNR at the DGN is therefore

$$\begin{aligned} M_{\Gamma_{DGN,v}}(s) &= M_{\Gamma_{SD}}(s)M_{\Gamma_B}(s) \\ &= \sum_{k=1}^{N}\binom{N}{k}\frac{k(-1)^{k-1}}{(k + K'\overline{\Gamma}_v s)(1 + \overline{\Gamma}_v s)^{n_r}} \end{aligned} \tag{28}$$

We now derive the expression for the average symbol error rate (SER), $\overline{P}_e^{mrc}$ based on the MGF approach for the BPSK, which is given as [8]

$$\overline{P}_e^{mrc} = \frac{1}{\pi}\int_0^{\pi/2} M_{\Gamma_{DGN,v}}\left(\frac{g_{BPSK}}{\sin^2\theta}\right)d\theta \tag{29}$$

where $g_{BPSK} = 1$.

$$\begin{aligned} \overline{P}_{e,v}^{mrc} &= \frac{1}{2\pi K'\overline{\Gamma}_v^2}\sum_{k=1}^{N}\binom{N}{k}k(-1)^{k-1}B\left(\frac{1}{2}, \frac{5}{2}\right) \\ &\times F_1\left(\frac{5}{2}, 1, n_r, 3; -\frac{k}{K'\overline{\Gamma}_v}, -\frac{1}{\overline{\Gamma}_v}\right) \end{aligned} \tag{30}$$

where $B(\cdot)$ is the Euler integral of the first kind and $F_1(\cdots,\cdots)$ is the Appell hypergeometric function [10], equation (3.211). See appendix A for the derivation of equation (30).

The average SER of the MRC technique for the two-state Markov-Gaussian noise can be expressed as

$$\overline{P}_e^{mrc} = (1-p_B)\overline{P}_{e,G}^{mrc} + p_B\overline{P}_{e,B}^{mrc} \quad (31)$$

The mean SNR for the G and B state are given as $\overline{\Gamma}_G = \frac{P}{\sigma_G^2}$ and $\overline{\Gamma}_B = \frac{P}{\vartheta\sigma_G^2}$ respectively.

V. Performance Measures

Assuming ML detection at the DGN, the average probability of error limited to the nearest $d_{\min}$ neighbors [11] can be approximated as

$$P_e \approx \frac{\overline{N_e}}{2}\,\mathrm{erfc}\left(\sqrt{\frac{(d_{min}^{BPSK})^2 E_T}{4\sigma^2}}\right) \quad (32)$$

where $\overline{N_e}$ is the average number of all nearest neighbors. The bit error probability is

$$P_b \approx \frac{\overline{N_e}}{2b\log 2(M)}\,\mathrm{erfc}\left(\sqrt{\frac{(d_{min}^{BPSK})^2 E_T}{4\sigma^2}}\right) \quad (33)$$

where M=2 for BPSK modulation and b=2. The probability of bit error for the two-state Markov Gaussian noise can be written as

$$\begin{aligned} P_b^{MG} &= p_G P_b^G + p_B P_b^B \quad (34)\\ P_b^{MG} &= p_G\frac{\overline{N_e}}{4}\,\mathrm{erfc}\left(\sqrt{\frac{(d_{min}^{BPSK})^2 E_T}{4\sigma_G^2}}\right)\\ &\quad + p_B\frac{\overline{N_e}}{4}\,\mathrm{erfc}\left(\sqrt{\frac{(d_{min}^{BPSK})^2 E_T}{4\vartheta\sigma_G^2}}\right) \quad (35)\end{aligned}$$

where P_b^G and P_b^B are the probabilities of error in state G and state B respectively. Similarly, the closed-form bound for the two-state Markov Gaussian noise probability of error is

$$\begin{aligned} P_e^{MG} &\le p_G\frac{M-1}{\sqrt{\pi}}\exp\left(-\frac{(d_{min}^{BPSK})^2 E_T}{4\sigma_G^2}\right) +\\ &\quad p_B\frac{M-1}{\sqrt{\pi}}\exp\left(-\frac{(d_{min}^{BPSK})^2 E_T}{4\vartheta\sigma_G^2}\right) \quad (36)\end{aligned}$$

The probability of error in (35) and (36) are averaged over the statistics of d_{min}^2 and are shown in figure 3.

The Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm can be used to compute the log-likelihood ratio (LLR) in the MAP detector at the DGN. Similar detector implemented in [12] is considered but with uncoded transmission over a flat faded MIMO Rayleigh channel corrupted by two-state additive Markov Gaussian noise. The received sample vector $\mathbf{y}_k$ at time epoch k is

$$\mathbf{y}_k = \mathbf{H}_v\mathbf{F}_d\mathbf{x} + \mathbf{G}_v\mathbf{n}\ ,\ k = 0, 1, \ldots, N-1 \quad (37)$$

Note that subscript k has been omitted from the right-hand terms of equation (37) for the sake of clarity, where $\mathbf{x}$ denotes the transmitted symbol vector with length b taken from the BPSK constellation, $\mathcal{C}$ and N is the size of the whole sequence $\mathbf{y}^N = \{\mathbf{y}_0, \mathbf{y}_1, \ldots \mathbf{y}_{N-1}\}$ to be detected.

The decoding decision of the BCJR algorithm is based on likelihood ratios of the input bits given by

$$L(x_k|\mathbf{y}) = \ln\left[\frac{p(x_k = 1|\mathbf{y}^N)}{p(x_k = -1|\mathbf{y}^N)}\right] \quad (38)$$

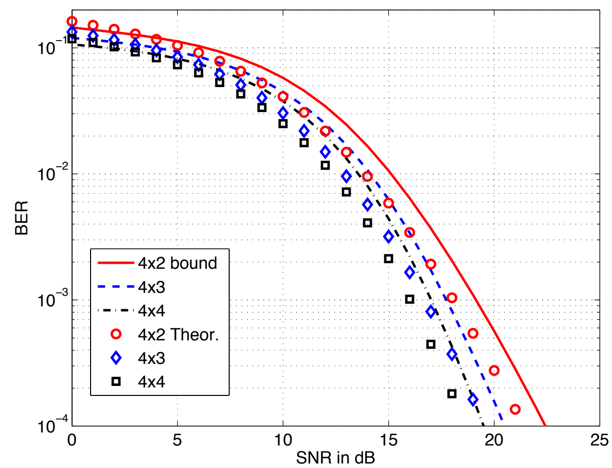

Fig. 3: Approximate and bound of the BER with BPSK modulation.

A sub-optimal MAP detector for symbol detection in a channel corrupted by two-state Markov Gaussian noise [12] is considered at the DGN. It evaluates the posteriori probability $p(x_k = u|\mathbf{y}^N)$ for each symbol x_k belonging to the binary modulation alphabet $u \in \{1, -1\}$ as

$$p(x_k = u|\mathbf{y}^N) \propto p(x_k = u, \mathbf{y}^N) = \sum_{s_k, s_{k+1}} p(x_k{=}u, \mathbf{y}^N, s_k, s_{k+1}) \quad (39)$$

where s_k and s_{k+1} are the noise states at the time epochs k and $k+1$ respectively. The proportionality in equation (39) indicates that the two operands are proportional with a positive multiplicative factor which has no effect on the decoding process. We begin by defining the following quantities:

$$\begin{aligned} \alpha_k(s_k) &= p(y_0, y_1, \ldots y_{k-1}, s_k) \quad (40)\\ \beta_k(s_k) &= p(y_k, y_{k+1}, \ldots y_{N-1}|s_k)\\ \mu_k(x_k, s_k, s_{k+1}) &= p(s_{k+1}|s_k)p(n_k = \|\mathbf{y}_k - \mathbf{H}_v\mathbf{F}_d\mathbf{s}\|^2|s_k)\end{aligned}$$

where $\alpha(s_k)$ and $\beta_k(s_k)$ are the the forward and backward filters respectively, the term $\mu_k(x_k, s_k, s_{k+1})$ is the branch metrics . For the two-state Markov Gaussian noise $p(n_k = \|\mathbf{y}_k - \mathbf{H}_v\mathbf{F}_d\mathbf{s}\|^2|s_k)$ is conditionally defined depending on s_k as

$$p(n_k{=}\|\mathbf{y}_k - \mathbf{H}_v\mathbf{F}_d\mathbf{s}\|^2|s_k{=}G) = \frac{1}{\sqrt{2\pi\sigma_G^2}}\exp\left(-\frac{n_k}{2\sigma_G^2}\right) \quad (41)$$

$$p(n_k{=}\|\mathbf{y}_k - \mathbf{H}_v\mathbf{F}_d\mathbf{s}\|^2|s_k{=}B) = \frac{1}{\sqrt{2\pi\vartheta\sigma_G^2}}\exp\left(-\frac{n_k}{2\vartheta\sigma_G^2}\right) \quad (42)$$

The BCJR algorithm computes the LLR of each bit using two recursive processes; the forward recursion computes the forward metric $\alpha(s_k)$ while the backward recursion computes the backward metric $\beta_k(s_k)$ at k-th time on the trellis. The term $p(x_k = u, \mathbf{y}^N, s_k, s_{k+1})$ from (39) can be decomposed as

$$\begin{aligned} p(x_k{=}u, \mathbf{y}^N, s_k, s_{k+1}) &= p(x_k = u)\sum_{s_k, s_{k+1}} \alpha_k(s_k)\beta_{k+1}(s_{k+1})\\ &\quad \times\mu_k(x_k, s_k, s_{k+1}) \quad (43)\end{aligned}$$

The forward and backward filters can be recursively computed thus:

$$\alpha_{k+1}(s_{k+1}) = \sum_{s_k, x_k} \alpha_k(s_k) p(x_k) \mu_k(x_k, s_k, s_{k+1}) \quad (44)$$

$$\beta_k(s_k) = \sum_{s_{k+1}, x_k} \beta_{k+1}(s_{k+1}) p(x_k) \mu_k(x_k, s_k, s_{k+1}) \quad (45)$$

and can be initialized with the probabilities of being in each state, i.e.

$$\alpha_0(s_0 = S) = p_S \quad \text{and} \quad \beta_N(s_N = S) = 1 \quad (46)$$

where $S \in (G, B)$

Ultimately the LLR of each bit is

$$L(x_k|\mathbf{y}) = \ln \frac{\sum_{s_k, s_{k+1}} \alpha_k(s_k) \mu_k(x_k = 1, s_k, s_{k+1}) \beta_{k+1}}{\sum_{s_k, s_{k+1}} \alpha_k(s_k) \mu_k(x_k = -1, s_k, s_{k+1}) \beta_{k+1}} \quad (47)$$

A. Maximum Likelihood (ML) and Maximum a Posteriori (MAP) detection for bivariate noise

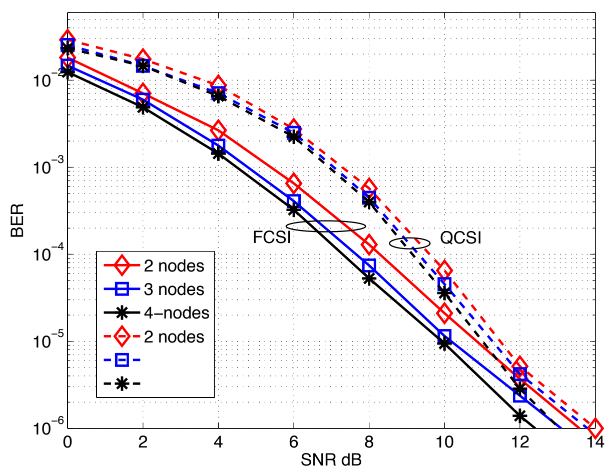

Fig. 4: Random nodes transmission, $\varsigma = 10$, $\vartheta = 100$.

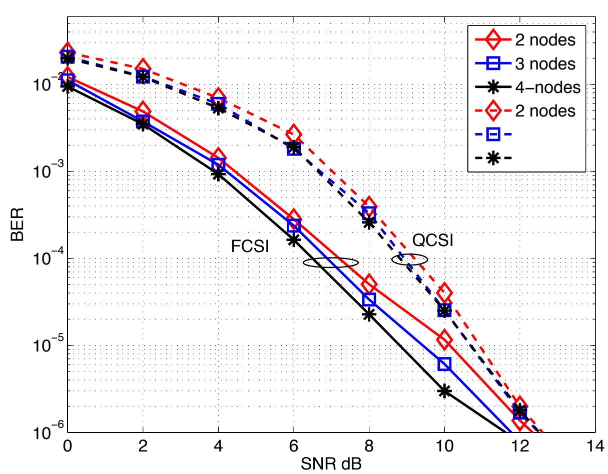

Fig. 5: Selected nodes transmission, $\varsigma = 10$, $\vartheta = 100$.

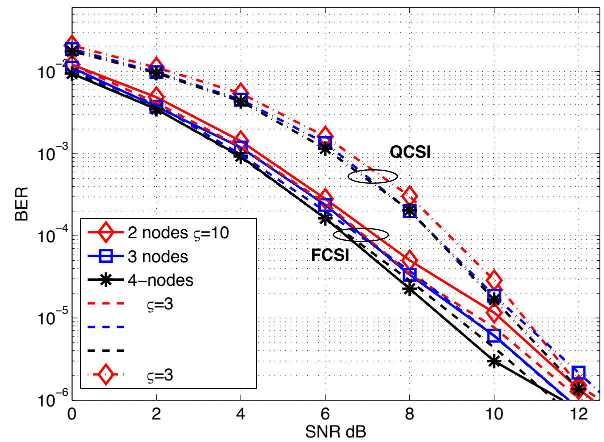

Fig. 6: Selected nodes transmission for $\varsigma = 3$ and $\varsigma = 10$.

VI.

VII. Conclusion

Appendix A
Derivation of (30)

Substituting equation (28) into equation (29), we have

$$\begin{aligned}\overline{P}_{e,v}^{mrc} &= \sum_{k=1}^{N} \binom{N}{k} \frac{k(-1)^{k-1}}{\pi} \int_0^{\pi/2} \frac{1}{\left(k + \frac{K'\overline{\Gamma}_v}{\sin^2\theta}\right)\left(1 + \frac{\overline{\Gamma}_v}{\sin^2\theta}\right)^{n_r}} d\theta \\ &= \sum_{k=1}^{N} \binom{N}{k} \frac{k(-1)^{k-1}}{\pi} \int_0^{\pi/2} \frac{(\sin^2\theta)^2}{(k\sin^2\theta + K'\overline{\Gamma}_v)(\sin^2\theta + \overline{\Gamma}_v)} d\theta \end{aligned} \quad (48)$$

Let $\sin^2\theta = x$ and $\frac{d\theta}{dx} = \frac{1}{2\sqrt{x(1-x)}}$, then substitute into equation (48):

$$\begin{aligned}\overline{P}_{e,v}^{mrc} &= \sum_{k=1}^{N} \binom{N}{k} \frac{k(-1)^{k-1}}{\pi} \int_0^1 \frac{x^2}{(kx + K'\overline{\Gamma}_v)(x + \overline{\Gamma}_v)^{n_r}} \\ &\times \frac{dx}{2\sqrt{x(1-x)}} \end{aligned} \quad (49)$$

$$\begin{aligned}&= \frac{1}{2\pi K'\overline{\Gamma}_v^2} \sum_{k=1}^{N} \binom{N}{k} k(-1)^{k-1} \\ &\times \int_0^1 x^{3/2}(1-x)^{-\frac{1}{2}} \left(1 + \frac{kx}{K'\overline{\Gamma}_v}\right)^{-1} \left(1 + \frac{x}{\overline{\Gamma}_v}\right)^{-n_r} dx \end{aligned} \quad (50)$$

Using the integral of the form $\int_0^1 x^{\lambda-1}(1-x)^{\mu-1}(1-ux)^{-\varrho}(1-vx)^{-\sigma}dx = B(\mu,\lambda)F_1(\lambda,\varrho,\sigma,\lambda+\mu;u,v)$ from [10], equation (3.211), the closed form expression for $\overline{P}_{e,v}^{mrc}$ is given in equation (30).

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