# JEE 2023

# LEARN TRIGONOMETRY FORMULAS

**WITH TRICKS**

NEHA MA'AM'S ARMY

# JEE 2023/24 : FULL PREPARATION RESOURCE

tinyurl.com/jeewithnehamam

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# JEE 2023: TRIGONOMETRY FORMULAS with TRICKS


| DATE | DAY | CHAPTER |
|---|---|---|
| 2nd Oct | Sun | Matrices |
| 3rd Oct | Mon | Determinants |
| 5th Oct | Wed | Sequences & Series |
| 8th Oct | Fri | Complex Numbers |
| 10th Oct | Mon | Quadratic Equations |
| 12th Oct | Wed | Limits |
| 14th Oct | Fri | TRIGO formulae with TRICKS |
| 16th Oct | Sun | Continuity & Derivatives |
| 18th Oct | Tues | Application Of Derivatives |

# Measurement of an ANGLE

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# Sexagesimal system

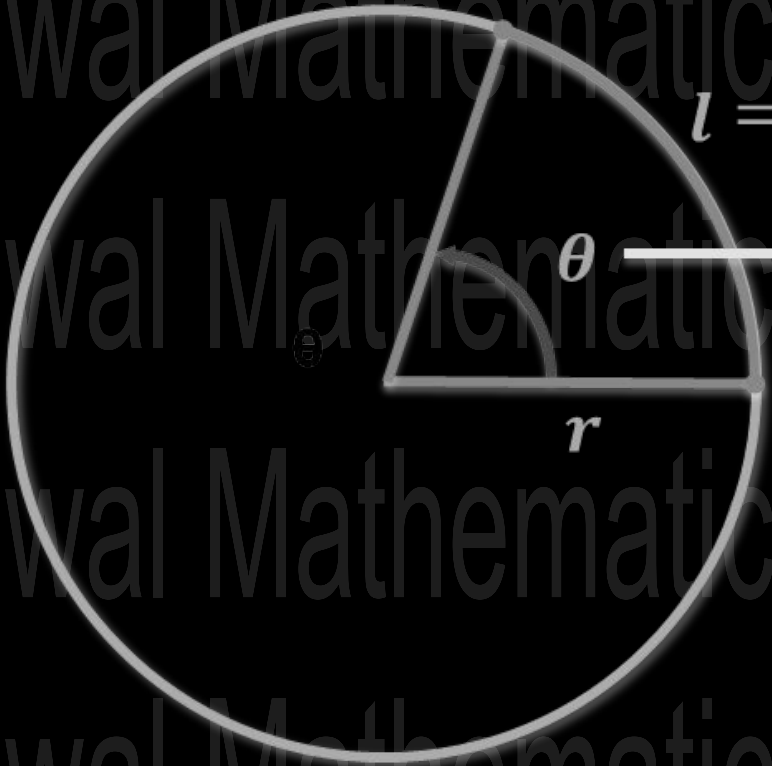
“ DEGREE ”

1 degree = 60 mins

1 min = 60 secs

$$1' = \left(\frac{1}{60}\right)^{th} \text{ of } 1^\circ$$

$$1'' = \left(\frac{1}{60}\right)^{th} \text{ of } 1'$$


$\theta = \dfrac{\text{Arc Length}(l)}{\text{Radius}(r)}$

$$\pi^c = 180^\circ$$

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## Circular system

" RADIAN "

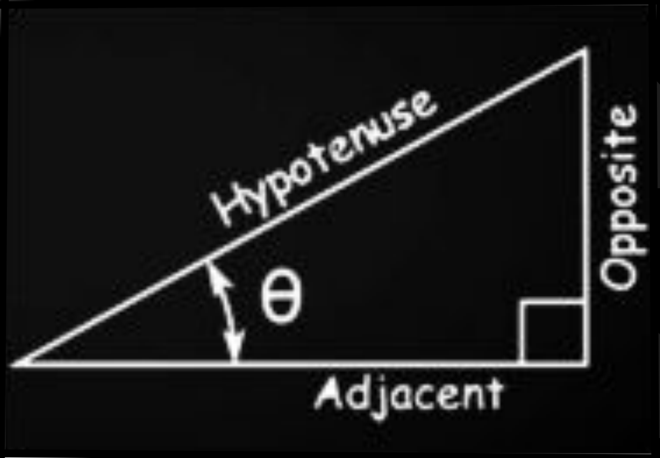
One radian ($1^R$ or $1^C$ ) = measure of an angle subtended at the center of a circle by an arc of length equal o the radius of the circle.

## Introduction to Trigonometry

$$\sin\theta = \frac{\text{Opposite}}{\text{Hypotenuse}}$$

$$cos\theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$\tan\theta = \frac{\text{Opposie}}{\text{Adjacent}}$$



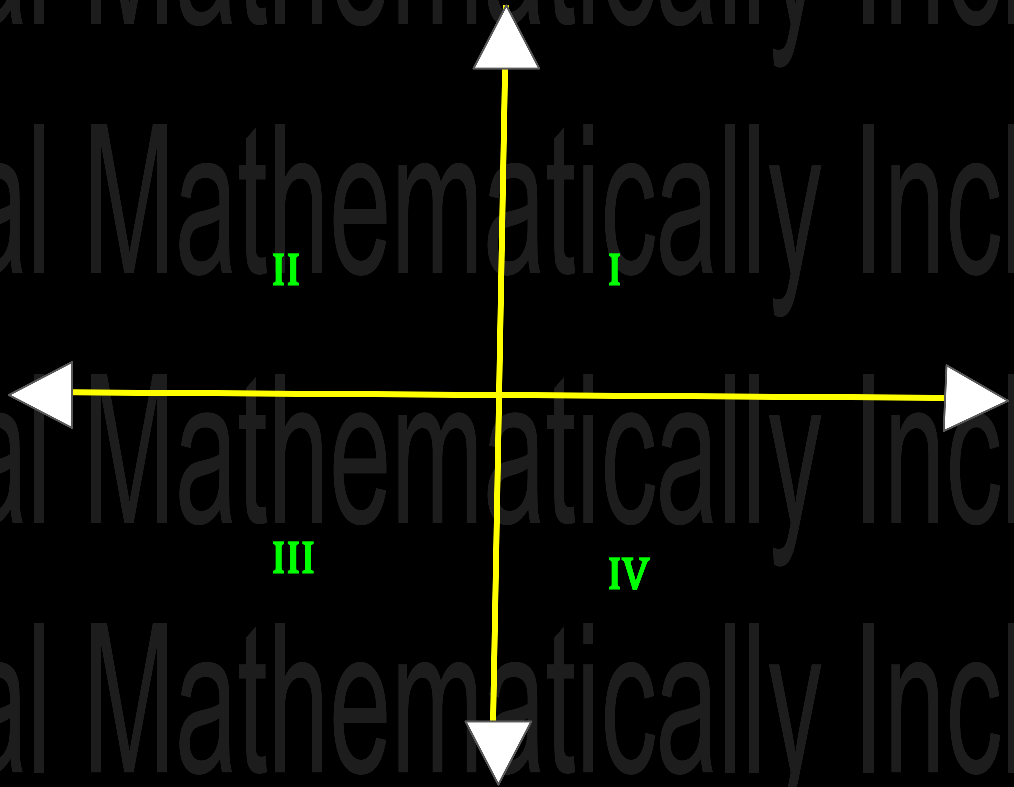
## Trigonometric Identities

1. $\sin^2\theta + \cos^2\theta = 1$

2. $\sec^2\theta - \tan^2\theta = 1$

3. $\text{cosec}^2\theta - \cot^2\theta = 1$

# SIGN of TRIG FUNCTIONS



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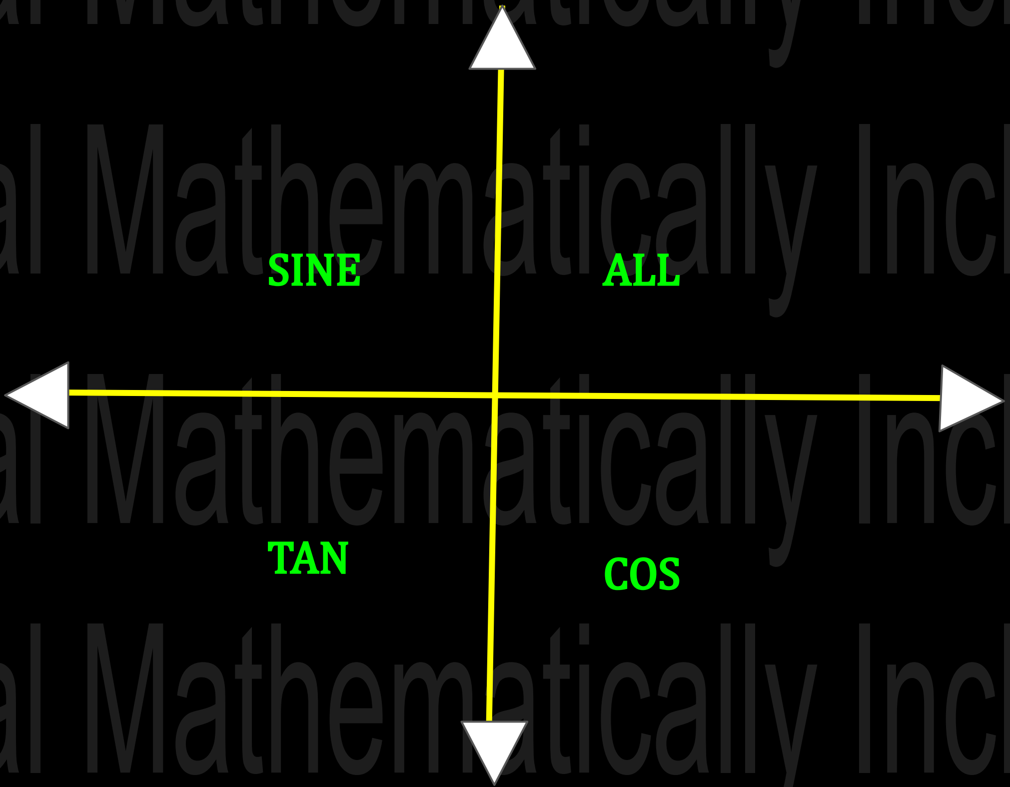
## SIGN of TRIG FUNCTIONS



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## Complementary & Supplementary Angles

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$

$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

$$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$$

$$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$$

$$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$$

## Complementary & Supplementary Angles



$$\sin(\pi + \theta) = -\sin\theta$$

$$\sin(\pi - \theta) = \sin\theta$$

$$\cos(\pi + \theta) = -\cos\theta$$

$$\cos(\pi - \theta) = -\cos\theta$$

$$\tan(\pi + \theta) = \tan\theta$$

$$\tan(\pi - \theta) = -\tan\theta$$

## NEGATIVE of Angles

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

## Compound Angles

(i) $\sin(A+B) = \sin A\cos B + \cos A\sin B$

(ii) $\sin(A-B) = \sin A\cos B - \cos A\sin B$

## Compound Angles

(iii) $\cos(A+B) = \cos A \cos B - \sin A \sin B$

(iv) $\cos(A-B) = \cos A \cos B + \sin A \sin B$

## Compound Angles

(v) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$

(vi) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

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## Factorization Formula:

Factorization Formulae are used to convert Sum and differences into Product.

- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = -2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

## Factorization Formula:

Defactorization Formulae are used to convert product into sum and differences

- $2 \sin A . \cos B = \sin(A+B) + \sin(A-B)$
- $2 \cos A . \sin B = \sin(A+B) - \sin(A-B)$
- $2 \cos A . \cos B = \cos(A+B) + \cos(A-B)$
- $2 \sin A . \sin B = \cos(A-B) - \cos(A+B)$

## Double Angle & Half angle Formula

| Double Angle Formulas | Half Angle Formulas |
|---|---|
| $\sin 2\theta = 2\sin\theta\cos\theta$ | $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$ |
| $\cos 2\theta = \cos^2\theta - \sin^2\theta$ | $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$ |
| $= 2\cos^2\theta - 1$ | $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$ |
| $= 1 - 2\sin^2\theta$ | $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$ |
| $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ | $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$ |

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## Triple Angle Formula

$$\sin 3A = 3\sin A - 4\sin^3 A$$

$$\cos 3A = 4\cos^3 A - 3\cos A$$

$$\tan 3A = \frac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$$

$$\cot 3A = \frac{\cot^3 A - 3\cot A}{3\cot^2 A - 1}$$

## General Solution

If $\sin\theta = \sin\alpha \Rightarrow \theta = n\pi + (-1)^n\alpha$ where $a \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $n \in I$.

If $\cos\theta = \cos\alpha \Rightarrow \theta = 2n\pi \pm \alpha$ where $a \in [0, \pi]$, $n \in I$.

If $\tan\theta = \tan\alpha \Rightarrow \theta = n\pi + \alpha$ where $a \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, $n \in I$.

## General Solution

If $\sin^2\theta = \sin^2\alpha \Rightarrow \theta = n\pi \pm \alpha$ $\quad n \in I$.

If $\cos^2\theta = \cos^2\alpha \Rightarrow \theta = n\pi \pm \alpha$ $\quad n \in I$.

If $\tan^2\theta = \tan^2\alpha \Rightarrow \theta = n\pi \pm \alpha$ $\quad n \in I$.

[ **Note:** $\alpha$ is called the principal angle ]

## Periodic Functions

- According to periodic function definition the fundamental period of a function can be defined as the period of the function which are of the form, $f(x+k) = f(x)$
- $f(x+k) = f(x)$, then k is known as the period of the function and the function f is known as a periodic function.

## Periodic Functions

| Function | Period |
|---|---|
| $\sin x$ | $2\pi$ |
| $\cos x$ | $2\pi$ |
| $\tan x$ | $\pi$ |

| Function | Period |
|---|---|
| $\cot x$ | $\pi$ |
| $\sec x$ | $2\pi$ |
| $\csc x$ | $2\pi$ |

## Periodic Functions

| Function | Period |
|---|---|
| $\sin(ax+b), \cos(ax+b), \sec(ax+b), \operatorname{cosec}(ax+b)$ | $2\pi / a$ |
| $\tan(ax+b), \cot(ax+b)$ | $\pi / a$ |
| $\lvert\sin(ax+b)\rvert, \lvert\cos(ax+b)\rvert, \lvert\sec(ax+b)\rvert, \lvert\operatorname{cosec}(ax+b)\rvert$ | $\pi / a$ |
| $\lvert\tan(ax+b)\rvert, \lvert\cot(ax+b)\rvert$ | $\pi / 2a$ |

# Domain & Range of Trigonometric Functions

| Function | Domain | Range |
|---|---|---|
| sin A | R | [–1, 1] |
| cos A | R | [-1, 1] |
| tan A | $R-\left[\left(2n+1\right)\pi/2, n\in I\right]$ | $R=\left(-\infty, \infty\right)$ |
| cosec A | $R-\left[n\pi, n\in I\right]$ | $\left(-\infty,-1\right]\cup\left[1,\infty\right)$ |
| sec A | $R-\left\{\left(2n+1\right)\pi/2, n\in I\right\}$ | $\left(-\infty,-1\right]\cup\left[1,\infty\right)$ |
| cot A | $R-\left[n\pi, n\in I\right]$ | $\left(-\infty,\infty\right)$ |

We find, $|\sin A|\le 1, |\cos A|\le 1, \sec A\ge 1$ or $\sec A\le -1$ and $\operatorname{cosec} A\ge 1$ or $\operatorname{cosec} A\le -1$

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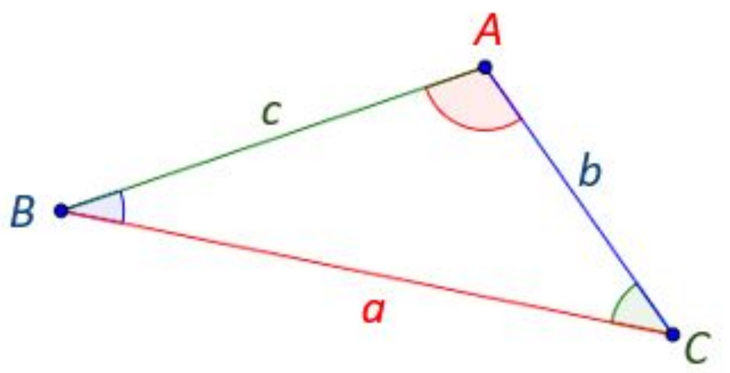
## SINE FORMULA

### Sine Rule or Law of Sines

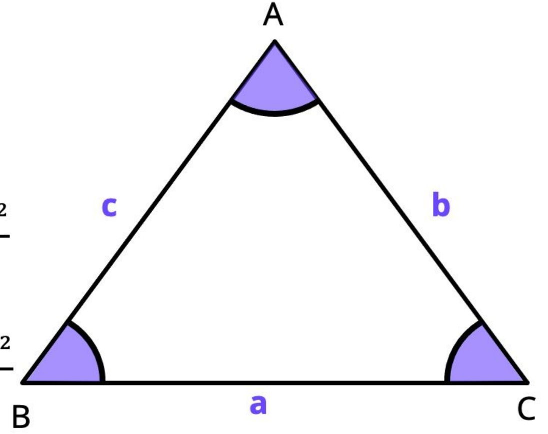



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

## COSINE FORMULA

In any ΔABC, we have

- $a^2 = b^2 + c^2 - 2bc\ \cos A$ **or** $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$
- $b^2 = c^2 + a^2 - 2ac\ \cos B$ **or** $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$
- $c^2 = a^2 + b^2 - 2ab\ \cos C$ **or** $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$



## Some Important Identities

If $A+B+C=\pi$, then

1. $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

2. $\cot B\cot C+\cot C\cot A+\cot A\cot B=1$

3. $\tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{B}{2}=1$

4. $\cot\frac{A}{2}+\cot\frac{B}{2}+\cot\frac{C}{2}=\cot\frac{A}{2}\cot\frac{B}{2}\cot\frac{C}{2}$

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## Some Important Identities

If $A+B+C=\pi$, then

5. $\sin 2A+\sin 2B+\sin 2C=4\sin A\sin B\sin C$

6. $\cos 2A+\cos 2B+\cos 2C=-1-4\cos A\cos B\cos C$

7. $\cos^2 A+\cos^2 B+\cos^2 C=1-2\cos A\cos B\cos C$

8. $\sin A+\sin B+\sin C=4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$

9. $\cos A+\cos B+\cos C=1+4\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$