Exercise 1

Kernel Methods in Machine Learning

January 30, 2019

1 Question 1

1.1

- $k_1(\mathbf{x}, \mathbf{y}) = c$ is a kernel, given $\forall \mathbf{x}, \phi(\mathbf{x}) = \sqrt{c}$.
- $k_2(\mathbf{x}, \mathbf{y}) = \langle \mathbf{x}, \mathbf{y} \rangle$ is a kernel, given $\forall \mathbf{x}, \phi(\mathbf{x}) = \mathbf{x}$.
- According to the rule that conic sum of kernel is a kernel, $k_3(\mathbf{x}, \mathbf{y}) = \langle \mathbf{x}, \mathbf{y} \rangle + c$ is a kernel.
- According to the rule that product of kernel functions is a kernel, $K(\mathbf{x}, \mathbf{y}) = (\langle \mathbf{x}, \mathbf{y} \rangle + c)^m$ is a kernel.

1.2

The expansion of $K(\mathbf{x}, \mathbf{y}) = (\langle \mathbf{x}, \mathbf{y} \rangle)^3$ is:

$$\begin{aligned} K(\mathbf{x}, \mathbf{y}) = {} & x_1y_1x_1y_1x_1y_1 + x_1y_1x_2y_2x_1y_1 + x_2y_2x_1y_1x_1y_1 + x_2y_2x_2y_2x_1y_1 + \\ & x_1y_1x_1y_1x_2y_2 + x_1y_1x_2y_2x_2y_2 + x_2y_2x_1y_1x_2y_2 + x_2y_2x_2y_2x_2y_2 \end{aligned}$$

So we get:

$$\begin{aligned} \phi(\mathbf{x}) &= (x_1x_1x_1, x_1x_2x_1, x_2x_1x_1, x_2x_2x_1, x_1x_1x_2, x_1x_2x_2, x_2x_1x_2, x_2x_2x_2) \\ \phi(\mathbf{y}) &= (y_1y_1y_1, y_1y_2y_1, y_2y_1y_1, y_2y_2y_1, y_1y_1y_2, y_1y_2y_2, y_2y_1y_2, y_2y_2y_2) \end{aligned}$$