Question 1

$$K_c(x_i, x_j) = \left\langle \phi_c(x_i), \phi_c(x_j) \right\rangle$$

$$= \left\langle \phi(x_i) - \frac{1}{l}\sum_{p=1}^{l} \phi(x_p),\ \phi(x_j) - \frac{1}{l}\sum_{q=1}^{l} \phi(x_q) \right\rangle$$

$$= \langle \phi(x_i), \phi(x_j) \rangle - \left\langle \frac{1}{l}\sum_{p=1}^{l} \phi(x_p), \phi(x_j) \right\rangle - \left\langle \phi(x_i), \frac{1}{l}\sum_{q=1}^{l} \phi(x_q) \right\rangle + \left\langle \frac{1}{l}\sum_{p=1}^{l} \phi(x_p), \frac{1}{l}\sum_{q=1}^{l} \phi(x_q) \right\rangle$$

$$= K(x_i, x_j) - \frac{1}{l}\sum_{p=1}^{l} \langle \phi(x_p), \phi(x_j) \rangle - \frac{1}{l}\sum_{q=1}^{l} \langle \phi(x_i), \phi(x_q) \rangle + \frac{1}{l^2}\sum_{q=1}^{l}\sum_{p=1}^{l} \langle \phi(x_p), \phi(x_q) \rangle$$

$$= K(x_i, x_j) - \frac{1}{l}\sum_{p=1}^{l} K(x_p, x_j) - \frac{1}{l}\sum_{q=1}^{l} K(x_i, x_q) + \frac{1}{l^2}\sum_{q=1}^{l}\sum_{p=1}^{l} K(x_p, x_q)$$

Question 2

The prediction $y_i = \underset{k}{\arg\max}\ P(Y = k \mid X = x_i)$

$$= \underset{k}{\arg\max}\ \exp(\langle w_k, x_i \rangle)$$

To guarantee that $\frac{1}{Z}\exp(\langle w_k, x_i \rangle)$ is a probability,

$$\sum_{k=1}^{K} \frac{1}{Z}\exp(\langle w_k, x_i \rangle) = 1$$

$$Z = \sum_{k=1}^{K} \exp(\langle w_k, x_i \rangle)$$