

# User Manual for UGKS1D and UGKS2D Code

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# Chapter 1

## Unified Gas-Kinetic Scheme

This chapter describes the Unified Gas-Kinetic Scheme published in [1, 2]. This is a 1D formulation. The 2D formulation with directional splitting is described in [3], and can be extended to a truly multidimensional formulation.

### 1.1 Model equation

The model equation is the BGK-Shakhov model. In one dimension case,

$$\frac{\partial f}{\partial t} + u \frac{\partial f}{\partial x} = \frac{f^+ - f}{\tau} \quad (1.1)$$

where  $f$  is the distribution function,  $u$  is partial velocity,  $\tau$  is particle collision time and  $f^+$  is the modified equilibrium distribution function.

The collision time is given by  $\tau = \mu/p$ , where  $\mu$  is the dynamic viscosity and  $p$  is the pressure

The modified equilibrium distribution is

$$f^+ = g \left[ 1 + (1 - \text{Pr}) \mathbf{c} \cdot \mathbf{q} \left( \frac{c^2}{RT} - 5 \right) / (5pRT) \right] = g + g^+ \quad (1.2)$$

where  $g$  is the Maxwellian distribution,  $\text{Pr}$  is the Prandtl number,  $\mathbf{c} = \mathbf{u} - \mathbf{U}$  is the random velocity,  $\mathbf{U}$  is the macroscopic velocity,  $\mathbf{q}$  is heat flux,  $R$  is gas constant and  $T$  is the temperature.

The Maxwellian distribution for 1D problem is

$$g = \rho \left( \frac{\lambda}{\pi} \right)^{\frac{\kappa+1}{2}} e^{-\lambda((u-U)^2 + \xi^2)} \quad (1.3)$$

where  $\rho$  is density,  $\lambda = m/2kT$ ,  $m$  is molecule mass,  $k$  is Boltzmann constant,  $K$  is the number of internal degree of freedom and  $\xi^2 = \xi_1^2 + \xi_2^2 \dots + \xi_K^2$ . For example, a monatomic gas at 1D problem has  $K = 2$  to account for the motion in  $y, z$  direction, and  $\xi^2 = v^2 + w^2$ , where  $v, w$  are particle velocity in  $y, z$  direction.

The collision invariants are  $\psi = (1, u, 1/2(u^2 + \xi^2))^T$ , and the macroscopic variables can be calculated via

$$w = \begin{pmatrix} \rho \\ \rho U \\ \rho E \end{pmatrix} = \int \psi_\alpha f d\Xi, \quad \alpha = 1, 2, 3 \quad (1.4)$$

$$p = \frac{1}{3} \int [(u - U)^2 + \xi^2] f d\Xi \quad (1.5)$$

$$q = \frac{1}{2} \int (u - U)[(u - U)^2 + \xi^2] f d\Xi \quad (1.6)$$

where  $E$  is total energy and  $d\Xi = dud\xi$

## 1.2 Reduced distribution function

## 1.3 Nondimensionalization

## 1.4 Interface flux

## 1.5 Update cell averaged value

## 1.6 Boundary condition

## Chapter 2

# Usage

This chapter describes how to compile the source code and documentation. Currently, it's only tested under Linux.

### 2.1 Compile under \*nix

### 2.2 Compile under windows





## Chapter 3

# UGKS1D Code

This chapter describes the structure and the included shock structure test case at  $\text{Ma}=8.0$  in the UGKS1D code.



## Chapter 4

# UGKS2D Code

This chapter describes the structure and the included lid-driven cavity test case in the UGKS2D code.

### 4.1 test

test



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