Formation control for a team of rotorcraft with biased actuators

Guidance Navigation and Control course

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I. Problem definition

This final assignment is solving the following task: Consider three rotorcraft Q_1, Q_2 and Q_3. Each rotorcraft has a biased velocity command, i.e.,

$$\dot{p}_i = u_i + b_i, \tag{1}$$

where $b_i \in \mathbb{R}$ is an arbitrary **unknown** bias. All the rotorcraft can measure their absolute positions ${}^N p_i$, and they have the capability of sharing information, e.g., one rotorcraft can communicate its position to another one. Please, design an algorithm that both, estimate the biases, and exploit displacement-based consensus to achieve an equilateral triangle.

II. The Solution

A. Bias estimation

To estimate the bias of the velocity command, a Kalman filter is employed. The state vector q consists of the north and east position (p_N and p_E) and the bias itself (b). To assemble the state observer, we need the following assumption

Assumption 2.1: The velocity of the rotorcraft is considered equal to the velocity command.

The estimated evolution of our states can be then described as

$$\hat{q}(k+1) = F\hat{q}(k) + G\hat{u}(k) \tag{2}$$

$$F = \begin{bmatrix} 1 & 0 & \Delta T \\ 0 & 1 & \Delta T \\ 0 & 0 & 1 \end{bmatrix} \qquad \hat{q} = \begin{bmatrix} \hat{p_N} \\ \hat{p_E} \\ \hat{b} \end{bmatrix} \qquad G = \begin{bmatrix} \Delta T & 0 \\ 0 & \Delta T \\ 0 & 0 \end{bmatrix} \qquad \hat{u} = \begin{bmatrix} u_{v_N} \\ u_{v_E} \end{bmatrix} \tag{3}$$

For the covariance matrix of the state vector q, we choose the following initial values (no correlations between the states):

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

and the evolution of the covariance of q can be described as

$$P(k+1) = FP(k)F^T + GQ(k)G^T, \tag{5}$$

where Q represents a covariance matrix of the inputs. Since there is no correlation between the inputs and the inputs are simulated, we choose the following covariance matrix:

$$Q = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}. \tag{6}$$

The measurement $\hat{y}_m$ with covariance

$$P_{y_m} = \begin{bmatrix} 0.0001 & 0 \\ 0 & 0.0001 \end{bmatrix}. \tag{7}$$

will then be fused with the observation function of expected value

$$\hat{y} = H\hat{q} \tag{8}$$

where

$$H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \tag{9}$$

and of covariance

$$P_y = HPH^T. \tag{10}$$

After calculating Kalman gain

$$K = PH^T(P_y + P_{y_m})^{-1}, \tag{11}$$

we can obtain the fused mean for state q as

$$\hat{q}_f = \hat{q} + K(\hat{y}_m - \hat{y}) \tag{12}$$

and the fused covariance

$$P_f = P - KHP. \tag{13}$$

If the covariance P_f is positive definite, we can assume that the inversion in 11 was free of numerical errors and the fused mean and covariance replace the predicted ones.

During the simulation, the fusion with measurements is carried out only on every 10th iteration (100ms), since fusion on every iteration significantly increases the computational time but has negligible impact on the result.

B. Formation control

Fig. 1 shows an undirected graph comprising of nodes ν and edges ϵ:

$$\nu = \{1, 2, 3\} \quad \epsilon = \{(1, 3), (3, 2)\} \tag{14}$$

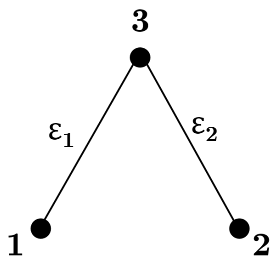

Fig. 1: Undirected graph representing equilateral triangle

The graph can be described by a following incidence matrix B:

$$B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \tag{15}$$

If the equilateral triangle is given specific dimensions and orientation, we can turn the edges ϵ into displacements δ and thus define error signals as

$$e = B^T p - \delta \tag{16}$$

where p are the absolute positions of the quadrotors

$$p = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \end{bmatrix}. \tag{17}$$

Given the assumption 2.1 and considering the bias to be already corrected for, we propose the following controller

$$u = k(-Be) \tag{18}$$

***Theorem** 2.2:* The origin of the displacement error signal e with control law 18 is globally asymptotically stable for any $k >= 0$.

Proof: Consider the following Lyapunov function

$$V = \frac{1}{2}||e||^2, \tag{19}$$

whose time derivative satisfies by including 18 and using assumption 2.1

$$\frac{\mathrm{d}V}{\mathrm{d}t} = e^T \dot{e} \tag{20}$$

$$= e^T B^T \dot{p} \tag{21}$$

$$= e^T B^T u \tag{22}$$

$$= -k e^T B^T B e \tag{23}$$

which is clearly non-increasing for any $k >= 0$ in the compact set

$$\mathcal{Q} \triangleq \Big\{ e : ||e||^2 \leq 2V(0) \Big\}. \tag{24}$$

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In simulation, the formation is controlled for 2 axes, thus the control input is split into north (u_n) and east (u_e) direction as follows:

$$\begin{aligned} u_n &= k(-B(B^T p_n - \delta_n)) \\ u_e &= k(-B(B^T p_e - \delta_e)), \end{aligned} \tag{25}$$

where p_n and p_e are column vectors containing the respective coordinates of the 3 quadrotors (similar to 27), while δ_n and δ_e are the desired displacements in the respective directions. For the simulation, an equilateral triangle with a side length of 4 was chosen as shown in fig. 2 and corresponding displacements for sides ϵ_1 and ϵ_2 are

$$\delta_n = \begin{bmatrix} 3.464 \\ -3.464 \end{bmatrix} \qquad \delta_e = \begin{bmatrix} 2 \\ 2 \end{bmatrix}. \tag{26}$$

The calculations of the control inputs are carried out on each multirotor and based on the number assigned to it, the corresponding row of u_n and u_e is used for the velocity command.

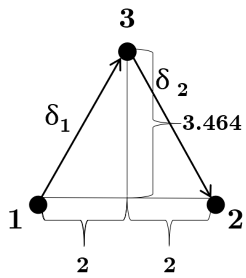

Fig. 2: Displacements of the equilateral triangle

III. Simulation

Since the formation control is dependent on the assumption 2.1 ($\dot{p} = u$), the bias needs to be estimated first and be compensated for. When giving a velocity command to a vehicle, the estimated bias $\hat{b}$ is subtracted from the system input u. If the system input is equal to zero, the aircraft starts moving in the direction of the bias with the velocity given by the bias and as the estimate is getting closer to the true value b, the aircraft slows down to eventual standstill.

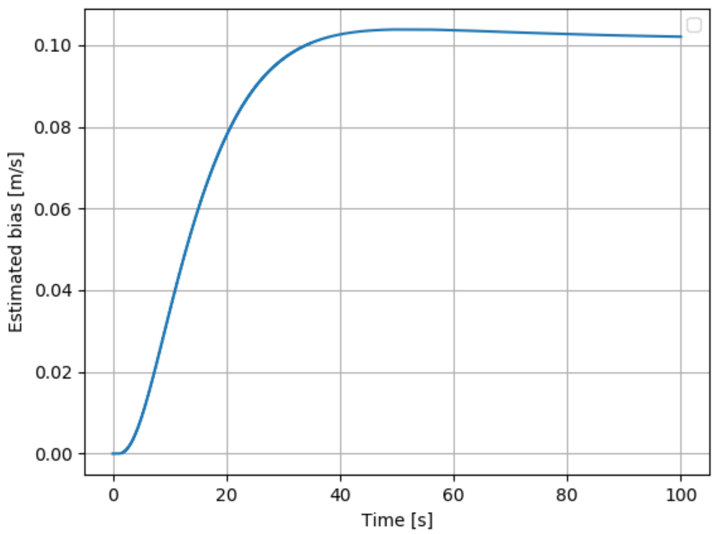

Fig. 3: Evolution of the estimated bias for bias $b = 0.1m/s$

Based on early simulations and fig. 3, the time derivative of $\hat{b}$ was designated as a criterion that the bias can be considered well estimated and the formation control can be engaged. More specifically, after initial period of 10s (take-off), the time derivation of $\hat{b}$ is monitored until its absolute value falls below 0.0001 (cannot be compared to 0 because of zero-crossing detection issues). When this condition is fulfilled, the formation control is turned on.

simulation length [s]	400	contr. gain k [-]	0.015
time step dt [s]	0.01	bias b [m/s]	0.1

TABLE I: Simulation settings

The table I lists the settings used for the simulation. The initial positions are specified below and the initial values of states $\hat{p_N}$ and $\hat{p_E}$ are filled out with those values.

$$p_n = \begin{bmatrix} 1.0 \\ 1.2 \\ -1.1 \end{bmatrix} \qquad p_e = \begin{bmatrix} 1.2 \\ 2.0 \\ 2.6 \end{bmatrix} \tag{27}$$

Fig. 4 shows the trajectories of the quadrotors from the simlation. The final positions are marked with small circles and the sides of the formed triangle with dashed line. There is an apparent change in the flight direction of all three aircrafts in the moment the formation control was engaged.

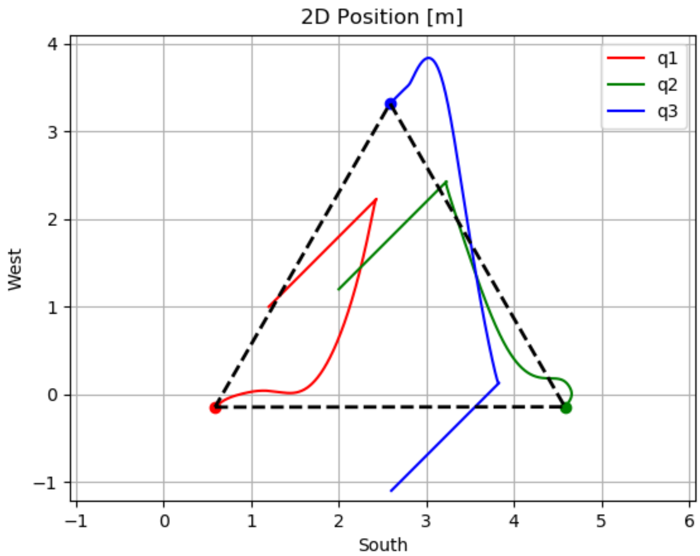

Fig. 4: Trajectories recorded during the simulation

Similar phenomenon can be observed in fig. 5 right before $t = 50$s where the time derivation of the estimated bias goes through zero. The formation controller commands velocity significantly different from the vehicle velocity at that moment. Due to inertia, the model in the bias estimation algorithm is temporarily flawed and the assumption 2.1 is not applicable. The estimated bias changes in the direction opposite to the direction of flight, suggesting that the inertia is mistaken for a bias.

As can be seen in fig. 6, the lengths of all three sides of the formed triangle converge to 4m, forming an equilateral triangle.

The simulation code, as well as the figures and this report can be found at `https://github.com/Pokornz/GNC`

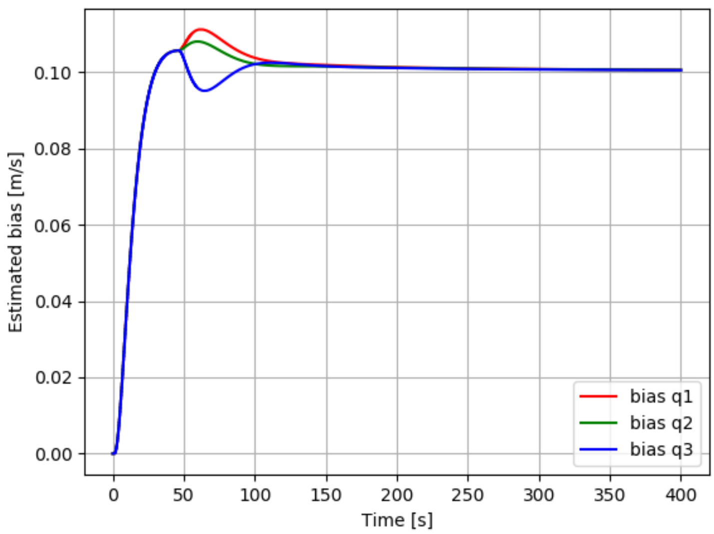

Fig. 5: Bias estimates

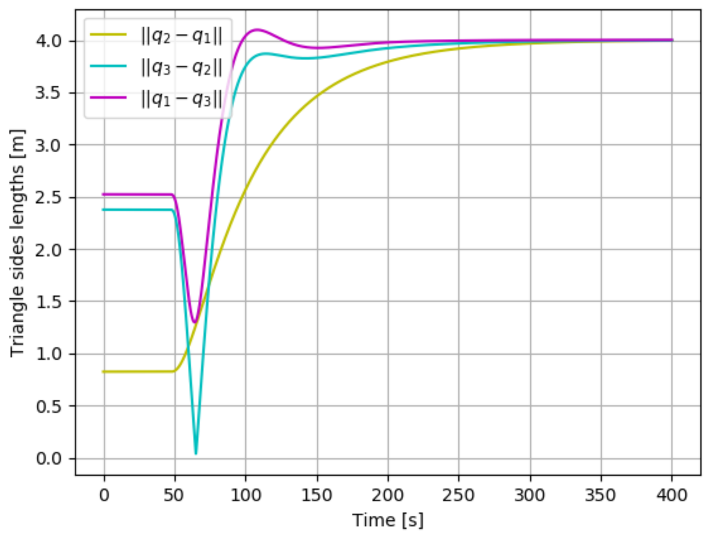

Fig. 6: Triangle dimensions in time

REFERENCES