

New Economic Geography Model, Helpman (1998)

Pol Cosentino*

*Université Paris 1 Panthéon-Sorbonne

Pol.Cosentino@etu.univ-paris1.fr

This note is largely taken from Quantitative spatial economics, Redding and Rossi-Hansberg, 2017, Annual Review of Economics.

Motivation

Canonical Quantitative Spatial Model (used in Hanson (2005) and Redding & Sturm (2008))

- multiregion connected by goods trade and factor mobility
- study the determinants of the spatial distribution of economic activity

Components

1. Preferences

- taste for variety and common preferences
- single traded sector
- no amenities
- residential land use

2. Production Technology

- increasing returns to scale
- exogenous productivity
- no input–output linkages
- no commercial land use

3. Costs of Trading Goods

- iceberg variable trade costs
- symmetric trade costs
- economic and geographic frictions
- no nontraded goods besides residential land use

4. Technology for Idea Flows

- no knowledge externalities or diffusion
- no innovation
- no transferability of ideas

5. Costs of Moving People

- perfectly costless migration

- no commuting
- single worker type with no heterogeneity
- no congestion in transportation

6. Endowments

- homogeneous labor
- exogenous land endowments in regions within a single country
- no capital

7. Equilibrium

- general equilibrium with a single country
- monopolistic competition
- land rents redistributed to residents
- balanced trade in each location

Model set-up

- wide economy of N regions indexed by n
- each region is endowed with an exogenous quality-adjusted supply of land (H_i)
- whole economy is endowed with a measure \bar{L} of workers
- each worker has one unit of labor that is supplied inelastically with zero disutility
- workers are perfectly geographically mobile \Rightarrow in equilibrium, real wages are equalized across all populated regions
- regions are connected by a bilateral transport network that can be used to ship goods subject to symmetric **iceberg trade costs**: $d_{ni} = d_{in} > 1$ units must be shipped from region i in order for one unit to arrive in region $n \neq i$, where $d_{nn} = 1$

Consumer-Preferences

Cobb-Douglas utility function;

$$U_n = \left(\frac{C_n}{\alpha}\right)^\alpha \left(\frac{h_n}{1-\alpha}\right)^{1-\alpha}, \quad 0 < \alpha < 1 \quad (1)$$

with

- C_n : goods consumption, defined over consumption $c_{ni}(j)$ of each horizontally differentiated variety j from the endogenous measures M_i supplied by each region
- h_n : residential land use

$$C_n = \left[\sum_{i \in N} \int_0^{M_i} c_{ni}(j)^\rho dj \right]^{1/\rho}, \quad P_n = \left[\sum_{i \in N} \int_0^{M_i} p_{ni}(j)^{1-\sigma} dj \right]^{1/1-\sigma} \quad (2)$$

Production

Total amount of labor $l_i(j)$ required to produce $x_i(j)$ units of variety j in location i taking into account fixed cost F (monopolistic competition and increasing returns to scale) and location productivity A_i ,

$$l_i(j) = F + \frac{x_i(j)}{A_i} \quad (3)$$

Profit maximization and zero profits imply that equilibrium prices are a constant markup over the marginal cost of supplying a variety to a market,

$$p_{ni}(j) = \left(\frac{\sigma}{1-\sigma} \right) d_{ni} \frac{w_i}{A_i}, \quad (4)$$

and equilibrium output of each variety is equal to a constant that depends on location productivity,

$$x_i(j) = \bar{x}_i = A_i(\sigma - 1)F, \quad (5)$$

which implies that equilibrium employment for each variety is the same for all locations,

$$l_i(j) = \bar{l} = \sigma F \quad (6)$$

Given this constant equilibrium employment for each variety, labor market clearing implies that the total measure of varieties supplied by each location is proportional to the endogenous supply of workers choosing to locate there:

$$M_i = \frac{L_i}{\sigma F} \quad (7)$$

Price Indices and Expenditure Shares

Using equilibrium prices (Equation 4) and labor market clearing (Equation 7), one can express the price index dual to the consumption index (Equation 2) as

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{1}{\sigma F} \right)^{1/1-\sigma} \left[\sum_{i \in N} L_i \left(d_{ni} \frac{w_i}{A_i} \right)^{1-\sigma} \right]^{1/1-\sigma} \quad (8)$$

Using the constant elasticity of substitution (CES) expenditure function, equilibrium prices (Equation 4), and labor market clearing (Equation 7), the share of location n 's expenditure on goods produced in location i is

$$\pi_{ni} = \frac{M_i p_{ni}^{1-\sigma}}{\sum_{k \in N} M_k p_{nk}^{1-\sigma}} = \frac{L_i \left(d_{ni} \frac{w_i}{A_i} \right)^{1-\sigma}}{\sum_{k \in N} L_k \left(d_{nk} \frac{w_k}{A_k} \right)^{1-\sigma}} \quad (9)$$

The model therefore implies a gravity equation for goods trade, where the bilateral trade between locations n and i depends on both bilateral resistance (bilateral trade costs d_{ni}) and multilateral resistance (trade costs to all other locations k , d_{nk}).

Together, Equations 8 and 9 imply that each location's price index can be written in terms of its trade share with itself, so

$$P_n = \frac{\sigma}{\sigma - 1} \left(\frac{L_n}{\sigma F \pi_{nn}} \right)^{1/1-\sigma} \frac{w_n}{A_n} \quad (10)$$

Income and Population Mobility

Expenditure on land in each location is redistributed in a lump sum to the workers residing in that location. Therefore, trade balance at each location implies that per capita income ν_n in each location equals labor income w_n plus per capita expenditure on residential land, $(1 - \alpha)\nu_n$,

$$\nu_n L_n = w_n L_n + (1 - \alpha)\nu_n L_n = \frac{w_n L_n}{\alpha} \quad (11)$$

Land market clearing implies that the supply of quality-adjusted land, H_n , equals the demand for land, $L_n h_n$.

By combining this market clearing condition with the FOC of the consumer problem, we obtain the result that land rents, r_n , are given by

$$r_n = \frac{(1 - \alpha)\nu_n L_n}{H_n} = \frac{1 - \alpha}{\alpha} \frac{w_n L_n}{H_n} \quad (12)$$

Population mobility implies that workers receive the **same** real income in all populated locations,

$$V_n = \frac{\nu_n}{P_n^\alpha r_n^{1-\alpha}} = \bar{V} \quad (13)$$

By using the price index (Equation 10), the assumption that trade is balanced at each location such that income equals expenditure (Equation 11), and land market clearing (Equation 12) in the population mobility condition (Equation 13), one determines that real wage equalization implies that the population L_n and domestic trade share π_{nn} of each location must satisfy

$$\bar{V} = \frac{A_n^\alpha H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)} L_n^{-\frac{\sigma(1-\alpha)-1}{\sigma-1}}}{\alpha \left(\frac{\sigma}{\sigma-1}\right)^\alpha \left(\frac{1}{\sigma F}\right)^{\frac{\alpha}{1-\sigma}} \left(\frac{1-\alpha}{\alpha}\right)^{1-\alpha}} \quad (14)$$

Therefore, the population share of each location ($\lambda_n \equiv L_n/\bar{L}$) depends on its productivity A_n , supply of land H_n , and domestic trade share π_{nn} relative to those of all other locations,

$$\lambda_n = \frac{L_n}{\bar{L}} = \frac{\left[A_n^\alpha H_n^{1-\alpha} \pi_{nn}^{-\alpha/(\sigma-1)} \right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}}{\sum_k \in N \left[A_k^\alpha H_k^{1-\alpha} \pi_{kk}^{-\alpha/(\sigma-1)} \right]^{\frac{\sigma-1}{\sigma(1-\alpha)-1}}} \quad (15)$$

where each location's domestic trade share π_{nn} summarizes its market access to other locations.

General Equilibrium

The properties of the general equilibrium of the model can be characterized analytically by combining the trade share (Equation 9), price index (Equation 2), and population mobility condition (Equation 13). Under the assumption that trade costs are symmetric ($d_{ni} = d_{in}$) one can follow the arguments of Allen & Arkolakis (2014) to show that these three sets of relationships reduce to the following system of N equations in the N populations of each location:

$$L_n^{\tilde{\sigma}\gamma_1} A_n^{-\frac{(\sigma-1)(\sigma-1)}{2\sigma-1}} H_n^{-\frac{\sigma(\sigma-1)(1-\alpha)}{\alpha(2\sigma-1)}} = \bar{W}^{1-\sigma} \sum_{i \in N} \frac{1}{\sigma F} \left(\frac{\sigma}{\sigma-1} d_{ni} \right)^{1-\sigma} \left(L_i^{\tilde{\sigma}\gamma_1} \right)^{\frac{\gamma_2}{\gamma_1}} A_i^{\frac{\sigma(\sigma-1)}{2\sigma-1}} H_i^{\frac{(\sigma-1)(\sigma-1)(1-\alpha)}{\alpha(2\sigma-1)}}, \quad (16)$$

where

- the scalar \bar{W} is determined by the requirement that the labor market clears ($\sum_{n \in N} L_n = \bar{L}$)
- $\tilde{\sigma} \equiv \frac{\sigma-1}{2\sigma-1}$
- $\gamma_1 \equiv \frac{\sigma(1-\alpha)}{\alpha}$
- $\gamma_2 \equiv 1 + \frac{\sigma}{\sigma-1} - \frac{(\sigma-1)(1-\alpha)}{\alpha}$

Wages in turn are implicitly determined by

$$w_n^{1-2\sigma} A_n^{\sigma-1} L_n^{(\sigma-1)\frac{1-\alpha}{\alpha}} H_n^{-(\sigma-1)\frac{1-\alpha}{\alpha}} = \xi, \quad (17)$$

where ξ is a scalar that normalizes wages.

Uniqueness

Allen & Arkolakis (2014) shows that

- there exists a unique L_n for each n that satisfies Equation 16 as long as $\gamma_2/\gamma_1 \in (0, 1)$ given the land area and productivity parameters $\{H_n, A_n\}$ and symmetric bilateral trade frictions $\{d_{ni}\}$ for all locations $n, i \in N$
- one can also guarantee that a solution to Equation 16 can be found by iteration from any initial distribution of populations
- the parameter restrictions that guarantee that an equilibrium exists and is unique amount to imposing conditions that guarantee that congestion forces always dominate agglomeration forces. In our simple model, a sufficient condition for $\gamma_2/\gamma_1 \in (0, 1)$ is $\sigma(1 - \alpha) > 1$
- intuitively, as population concentrates in a location, the measure of varieties produced there expands, which, in the presence of trade costs, makes that location a more attractive residence (an agglomeration force), but at the same time this also bids up land prices (a dispersion force)
- the higher the elasticity of substitution (σ), the weaker is the agglomeration force, and the higher the share of land ($1 - \alpha$), the stronger is the dispersion force
- for parameter values for which $\sigma(1 - \alpha) > 1$, the dispersion force dominates the agglomeration force, and a unique equilibrium distribution of economic activity exists
- the existence of such a unique equilibrium is important because it ensures that counterfactuals for transport infrastructure improvements or other public policy interventions have determinate implications for the spatial distribution of economic activity.

Model Inversion

Suppose that a researcher

- has estimates of the model's two key parameters: the share of residential land in consumer expenditure (α) and the elasticity of substitution between varieties (σ)
- has parameterized symmetric bilateral trade costs (d_{ni}), for example, by assuming that they are a constant elasticity function of distance, and to observe an endogenous population, $\{L_n\}$, and nominal wages, $\{w_n\}$.

There is a one-to-one mapping from the model's parameters and the observed data to the unobserved values of quality-adjusted land $\{H_n\}$ and productivities $\{A_n\}$ (up to a normalization constant)

- the model can be inverted to recover the unique values of unobserved quality-adjusted land and productivities that rationalize the observed data as an equilibrium outcome of the model: using Equations 16 and 17 to solve for $\{A_n, H_n\}$ given $\{L_n, w_n\}$
 1. using Equation 17, we can solve for H_n and substitute this solution in Equation 16
 2. the resulting equation can then be solved for $\{A_n\}$ using information on $\{L_n, w_n\}$
 3. having recovered the unobserved productivities $\{A_n\}$, these can be used, together with the parameterization of trade costs (d_{ni}) and observed wages (w_n) in the trade shares (Equation 9), to generate predictions for unobserved bilateral trade shares (π_{ni}) in the equilibrium observed in the data
- this is exactly the opposite of what we do when we solve for an equilibrium of the model, where we solve for $\{L_n, w_n\}$ given $\{A_n, H_n\}$

Related arguments hold if the researcher directly observes bilateral trade shares (π_{ni}) instead of having to assume values for bilateral trade costs (d_{ni}). In this case, unobserved quality-adjusted land supplies (H_n), productivities (A_n), and bilateral trade costs (d_{ni}) can be recovered from the observed data (up to a normalization or choice of units).

Counterfactuals

- counterfactuals can be undertaken using the observed values of the endogenous variables of the model in an initial equilibrium without having to solve for the unobserved location characteristics, as in the work of Dekle et al. (2007)
- denote the (unknown) value of variables in the counterfactual equilibrium with a prime (x') and the relative value of variables in the counterfactual and observed equilibria by a hat ($\hat{x} = x'/x$)

- suppose that the researcher observes population (L_n), wages (w_n), and trade shares (π_{ni}) in the initial equilibrium and can parameterize the change in bilateral trade costs as a result of the transport infrastructure improvement
- From the trade share (Equation 9), price index (Equation 10), income equal to expenditure (Equation 11), land market clearing (Equation 12), and population mobility (Equation 13), we obtain the following system of equations that can be used to solve for the counterfactual changes in wages, trade shares, and population shares ($\{\hat{w}, \hat{\pi}_{ni}, \hat{\lambda}_n\}$) given only the observed wages, trade shares, and population shares in the initial equilibrium ($\{w, \pi_{ni}, \lambda_n\}$)

$$\hat{w}_i \hat{\lambda}_i (w_i \lambda_i) = \sum_{n \in N} \hat{\pi}_{ni} \hat{w}_n \hat{\lambda}_n \pi_{ni} (w_n \lambda_n) \quad (18)$$

$$\hat{\pi}_{ni} \pi_{ni} = \frac{(\hat{d}_{ni} \hat{w}_i)^{1-\sigma} \hat{L}_i \pi_{ni}}{\sum_{k \in N} (\hat{d}_{nk} \hat{w}_k)^{1-\sigma} \hat{L}_k \pi_{nk}} \quad (19)$$

$$\hat{\lambda}_n \lambda_n = \frac{\hat{\pi}_{nn}^{-\frac{\alpha}{\sigma(1-\alpha)-1}} \lambda_n}{\sum_{k \in N} \hat{\pi}_{kk}^{-\frac{\alpha}{\sigma(1-\alpha)-1}} \lambda_k} \quad (20)$$

- given our assumptions of $\sigma(1 - \alpha) > 1$ and symmetric trade costs, there exists a unique general equilibrium in the model, which ensures that these counterfactuals yield determinate predictions for the impact of the transport infrastructure improvement or another public policy intervention on the spatial equilibrium distribution of economic activity

Welfare

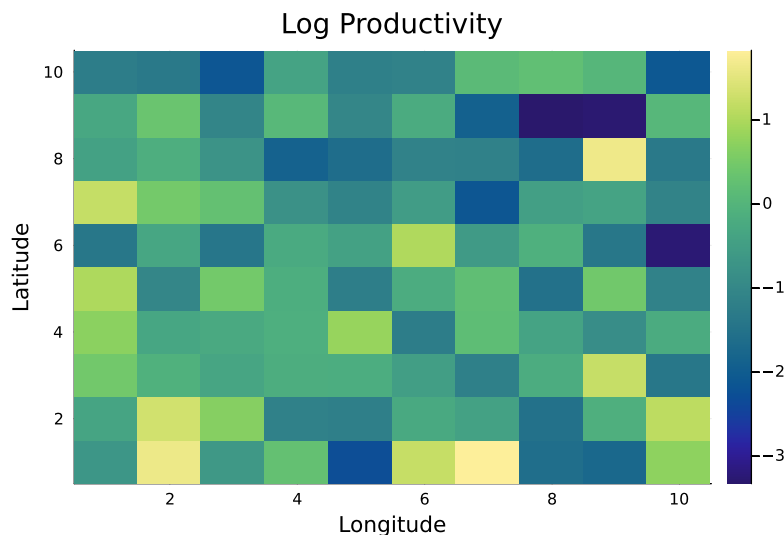
- welfare effects of public policy interventions that change trade costs can be expressed solely in terms of empirically observable sufficient statistics
- considering a transport infrastructure improvement that reduces trade costs between an initial equilibrium (indexed by 0) and a subsequent equilibrium (indexed by 1), perfect population mobility implies that the transport infrastructure improvement leads to reallocations of population across locations until real wages are equalized
- using the population mobility condition (Equation 14), the changes in the domestic trade share (π_{nn}) and population (L_n) for any one location are sufficient statistics for the welfare impact of the transport infrastructure improvement on all locations:

$$\frac{\bar{V}^1}{\bar{V}^0} = \left(\frac{\pi_{nm}^0}{\pi_{nm}^1} \right)^{\alpha/(\sigma-1)} \left(\frac{\lambda_n^0}{\lambda_n^1} \right)^{\frac{\sigma(1-\alpha)-1}{\sigma-1}} \quad (21)$$

- given our assumption of $\sigma(1 - \alpha) > 1$, a larger reduction in a location's domestic trade share must be offset by a larger increase in its population to preserve real wage equalization
- intuitively, if the transport infrastructure improvement decreases trade costs for one location more than for other locations (and thus reduces the domestic trade share of this location), the resulting upwards pressure on its real wage induces a population inflow until the price of the immobile factor land is bid up to restore real wage equalization

Model Illustration

- quantitative model can be used to evaluate the impact of trade frictions (both between countries and between regions within countries) on the spatial distribution of economic activity and welfare
- consider a model economy on a 30×30 latitude and longitude grid
- two countries, one occupying the western half of the grid (West) and the other occupying the eastern half (East)
- assume that labor is perfectly mobile across locations within each country but perfectly immobile across countries
- lowest-cost-route effective distance
- draw a realization for productivity: differ randomly across locations



- same quality-adjusted land area
- share of land in residential consumption expenditure $(1 - \alpha) = 25\%$

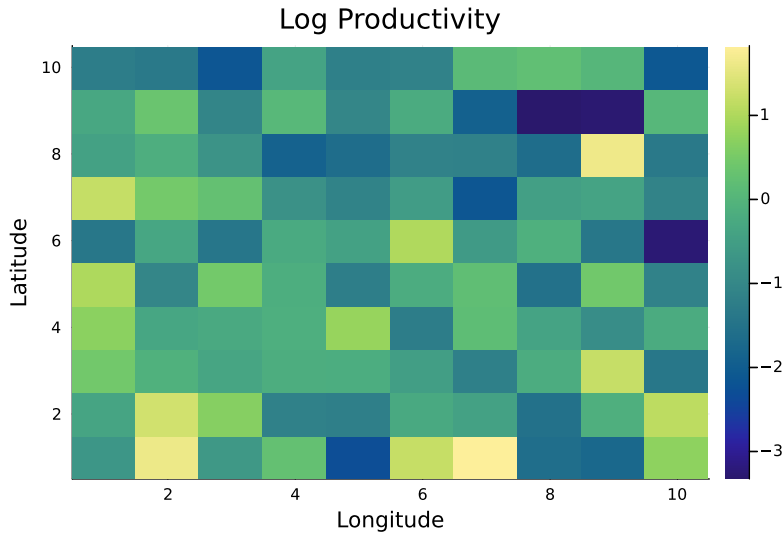
- elasticity of substitution: $\sigma = 5$
- trade costs are a constant elasticity function of effective distance: $d_{ni} = \text{dist}_{ni}^\phi$ with $\phi = 0.375$
- two forms of economic frictions to trade between locations
 1. proportional internal tax on trade with other locations of 100% ($\tau^{in} = 2$)
 2. proportional external tax on trade between the two countries of 100% ($\tau^{out} = 2$)

Initial Equilibrium with both taxes

Stylized facts from initial equilibrium Figure 1:

- areas of high productivity have large population concentrations, high wages, high land prices
- log of the price index is a smooth surface with the gradient governed by trade costs
- prices are lower in areas that produce a large variety of goods, for example, at the two large cities close to the border
- largest agglomerations in this economy are most clearly appreciated in this panel
- border effect created by the tariff between the two countries

Figure 1: Initial Equilibrium

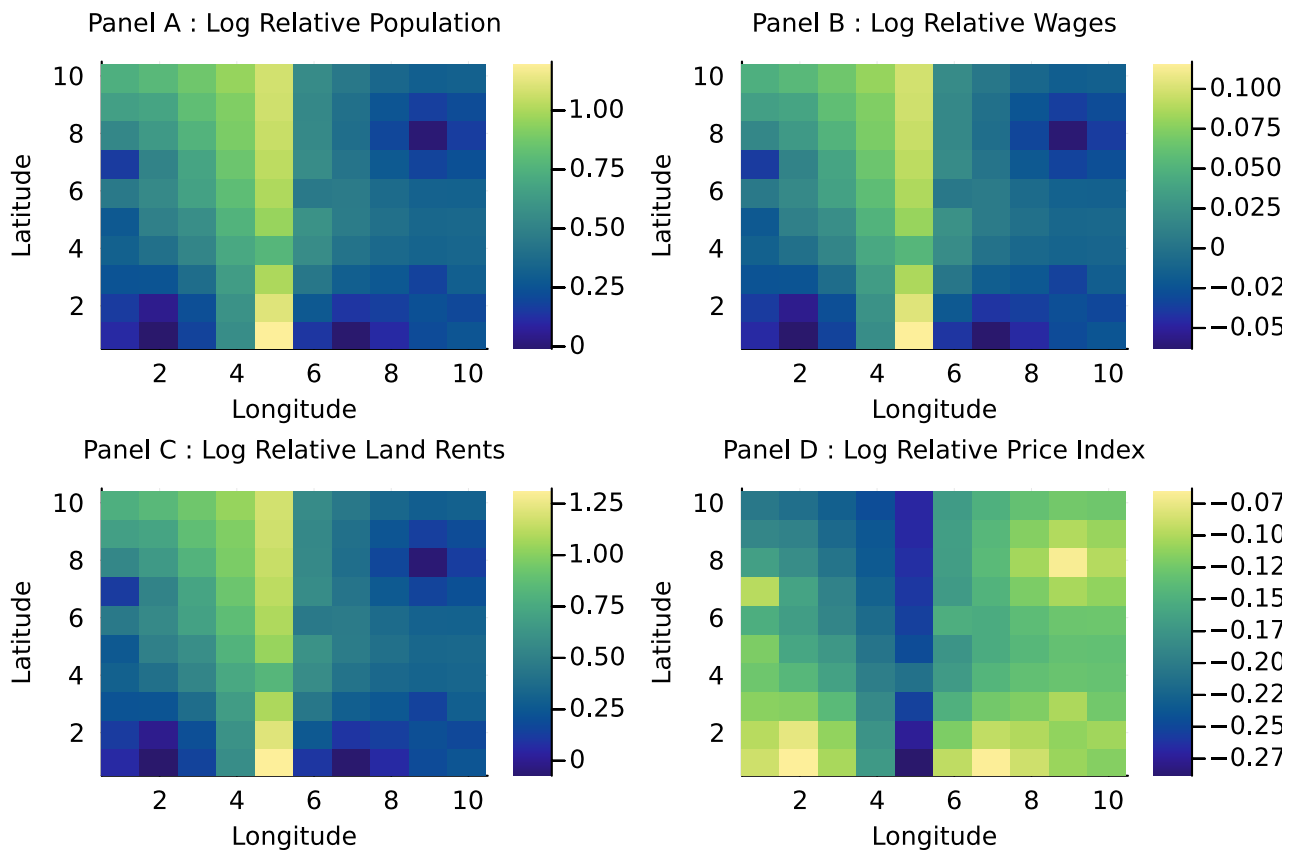


Counterfactual 1: External Liberalization

Figure 2 shows the log relative changes ($\hat{x} = x'/x$) in population ($\log \hat{L}$), wages ($\log \hat{w}$), land prices ($\log \hat{r}$), and price indices ($\log \hat{P}$) due to removal of the proportional tax on trade between the two countries:

- as trade costs between the two countries fall, economic activity reallocates toward the border between them
- areas that benefit the most are the ones close to but on the opposite side of the border from the large cities
- these locations can now trade more cheaply with the large market in those cities and thus experience the largest increases in population, wages, and land rents and the largest reductions in the price index
- in contrast, the largest agglomerations lose relative to these up and coming locations

Figure 2: External Liberalization: relative changes with respect to the initial equilibrium



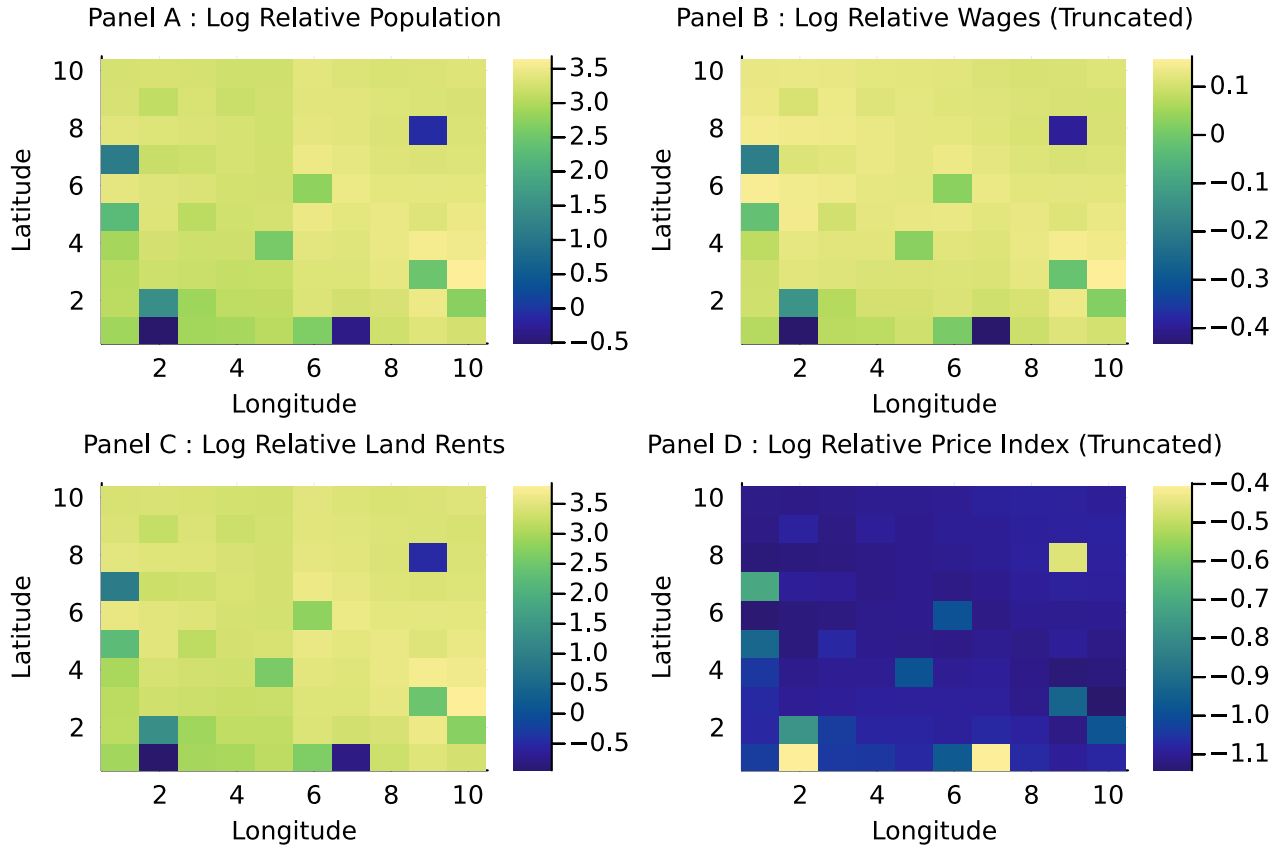
Counterfactual 2: Internal Liberalization

Figure 3 shows the relative changes with respect to the initial equilibrium ($\hat{x} = x'/x$) due to the removal of all internal trade costs (but leave international trade costs as in the initial equilibrium)

- implications of an internal reduction in trade costs are clearly quite different than the ones from an external trade cost reduction

- main effect of the internal liberalization is to reduce the size of the two large cities in favor of rural areas, thereby making economic activity more dispersed
- as trade costs decline, the home market effect reducing local price indexes in large cities weakens, so that prices fall everywhere but less so in areas with larger populations
- wages and land rents also fall in large agglomerations, whereas they increase in all other regions

Figure 3: Internal Liberalization: relative changes with respect to the initial equilibrium



Welfare Effects of Trade Liberalizations

Table 1 reports the resulting impact (in percentage) of trade liberalization policies on the common level of welfare across locations within each country:

- internal trade liberalization raises welfare in West and East much higher than external trade liberalization does
- trade is much larger between regions than between countries, highlighting the greater importance of internal trade frictions relative to external trade frictions

Table 1: Counterfactuals Welfare gains

Welfare Gains in pct		
Trade Liberalization	West	East
External	0.19	0.24
Internal	5.9	10.78