Teoria Geoc: Basic Tools

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1 DESCRIBING AND IMPLEMENTING BASIC GEOMETRIC OBJECTS

Homogeneous cordinates are very usefull in practice, allows us to calculate a lot of things.

Point:

$$(x, y) \Leftrightarrow (x : y : 1)$$

Line:

$$ax + bx + c = 0 \Leftrightarrow (a : b : c)$$

Incidence pont-line:

p = (x:y:1) lies in l = (a:b:c) $\Leftrightarrow (a, b, c)(x, y, 1) = 0$

Line through two points:

p and q lies in l $\Rightarrow l = pxq$

Intersecting two lines:

p lies in l1 and l2 $\Rightarrow p = l1xl2$

Parallel lines:

l1 $\parallel$ $l2 \Leftrightarrow l1xl2 = (x, y, 0)$

Identical lines or points:

Make cross product between them is 0: l1 x l2 = (0,0,0)

2 Orientation tests

Given 3 points in the plane p,q,r a robously decide whether r lies to the left, right or oriented to pq line.

$$\begin{vmatrix} q_x - p_x & r_x - p_x \\ q_y - p_y & r_y - p_y \end{vmatrix}$$

The result of this determinant, determine, if lies to left, right or oriented to line.

If determinant menor 0 lies to left

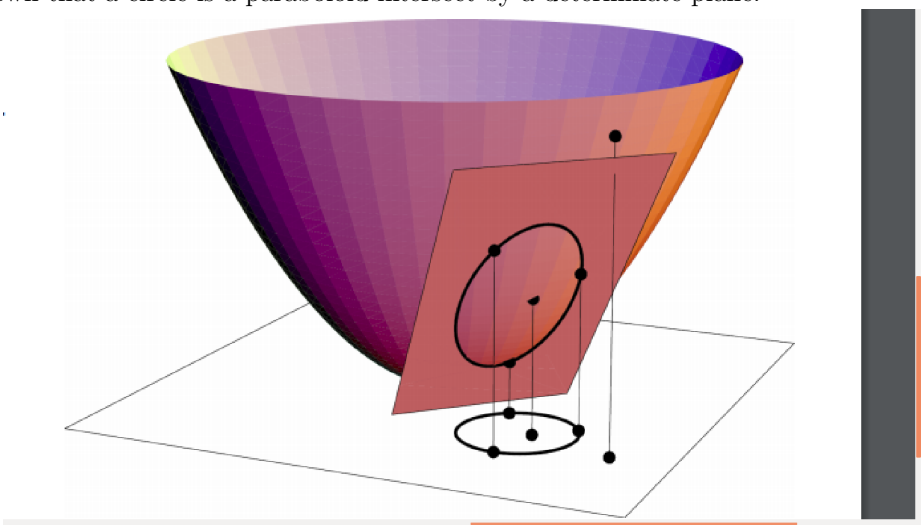
If determinant major 0 lies to right

If determinant = 0 lies oriented to pq.

The same operation can be do it for calculate relative position of point-plane:

det(x,a,b,c), where a,b,c are points of plane π

This can be aplicated to know if a point lies in to circle or not. Cause is known that a circle is a paraboloid intersect by a determinate plane.



Orientation test can be aplicated in test polygons. For know if line intersect whith polygon, if point lies into polygon, or to calculate suporting lines(given a point we draw 2 lines that all points lies to the left, and other that all point lies to the right)
This test have a O(n) cost, but if the polygon is convex the cost is reduced to O(log(n))