Hi my name is Pol Renau,
and I'm gonna explain some about Robot Arm Motion.

at first if in This problem ther's no restrictions, it is NP-Hard, but We reduce this complexity adding some restrictions.

then Supose a stage with no obstacles, and no angle Restrictions. Then Every Joint can have angle betwen(0,360)
With this the problem will be polinomial. For all presentation we gonna supose this stage as the reality.

Input: arm A, and point P.
Output: Is P reachable from A? If is it give one convinations of Joins Angles that is Solution.

How to define a robot arm?
Every link in arobot arm is defined as Li = length of this link

Robot arm have a Joints that are the points where 2 consecutive links ar conected.
Ji is equals angle at this join. I is between 0 and n.
Ther's n+1 Joints.

Arm A is defined by a list of lenght links.

at first I eplain som Lemas that are important for get the first idea about problem.

First:
The reachability region for an n-link arm is an annulus centred on the origin.

Second:
The reachability region for an n-link arm is an origin-centred annulus with outer radius ro = summ of all links and inner radius ri=0 if the longest link length lM is less than half the total length of the links, and inner radius is equals to longest link minus summ of others, otherwise

Third:
The region of reachability for an arm is independent of the order in which the links are arranged.
As we can see in this image can see every link like a vector, and then arm A reach point P, if the Vector adition of all links reach P. It's know that vector adition is conmutative, then, we shouldn't care about the order of the links.

After this We can start up with the problem

Problem with 2-Links.
Solve this problem is very easy. We define C1 as a circle centred on J0 with radi L1, and C2 centred on P
with radi length of second segment.
The solution of the problem is where C1 and C2 intersect.
We see that there're 4 possible solutions. 1 intersection, 2 intersections, No intersection then ther's no
solution for the problem, and infinite solutions for solve this problem,in this case
we define J0 = 0, but J0 can be any value.

Now Increment dificult of the problem,
Problem with 3 links.
It's possible to reduce every 3 Link problem to 2-link Problem
Aplylling the second lema, we know that 3-link problem have this possible solutions in 2-links
First case:
first segment is l sub 1 plus l two and the other segment is l3
Second case:
first segment is l sub one and other segment is summ o f segments 2 and 3.
Third case:
Jsub 0 is 0, and now the new origin is (origin.x+length link one, origin.y) and the problem have only the Links
lsub 2 and l sub three.

Make problem for general N
Reduce N-link to 3link
And Aplylling the lemma four reduce this to 2 link problem.

Lemma (5) If an n-linked arm A can reach a point, it can reach it with at most two joints “kinked”: only two joints among J1, .. , Jn-1 have nonzero angles. The two joints may be chosen to be those at either end of the “median link”: the link LM such that $\sum i=1\ m-1\ li$ is less than or equal to half the total length of the links, but $\sum i=1\ m\ li$ is more than half.

For proof this lemma we decompose the problem in two cases:

First case internal radi is > 0:
and apling the lema 2, ri = Lm - sum of all except segment M.
cause as we can se here we can freze Longest link and comute others. As we can see in this picture,
we need at most 2 Joints diferent 0.

Second case internal radi equals 0.
For this case it's known tht ther's no link that are longer than the half of total lenght.
Thers a lot of possible solutions for create a 3 segments that noone were longer than

the total half. But we gonna make a easy way. Start from the first segment, summ oll of them
while the summ f them is less than the half, as you can see in this picture.
This will be the first link, the second link will be the next of the last link in the new first link,
in the drat you can see that is the fourth link, with length equals two,
and the third link is the summ of the others.
And Aplylling the lema 2 we know that the reachability region will be the same.