

18K41A05F9

Assignment-05

Mini-Batch Gradient Descent Optimizer

Step-1:

$$[x, y], \eta = 0.1, m = 1, c = -1$$

$$\text{epochs} = 2, \text{batch_size}(bs) = 2$$

X	Y
0.2	3.4
0.4	3.8
0.6	4.2
0.8	4.6

Step-2:

Splitting data

$$nb = 4/2 = 2 \text{ batches}$$

batch-1

x	y
0.2	3.4
0.8	4.6

Step-3: iter = 1, Step-4:

batch = 1

batch-2

x	y
0.4	3.8
0.6	4.2

Step-5:

$$\frac{\partial E}{\partial m} = -\frac{1}{bs} \sum_{i=1}^{bs} [(y_i - mx_i - c)x_i]$$

$$= -\frac{1}{2} \left[(3.4 - (1)(0.2) - (-1))(0.2) + (4.6 - (1)(0.8) - (-1))(0.8) \right]$$

$$= -\frac{1}{2} \left[0.84 + 3.84 \right] = -\frac{1}{2} (4.68)$$

$$= -2.34$$

$$\frac{\partial E}{\partial c} = \frac{-1}{nbs} \sum_{i=1}^{bs} [y_i - mx_i - c]$$

$$= \frac{-1}{2} \left[(3.4 - (1)(0.2) - (-1))(\cancel{0.2}) + (4.6 - (1)(0.8) - (-1))(\cancel{0.8}) \right]$$

$$= \frac{-1}{2} [4.2 + 4.8] = \frac{-1}{2}(9) = -4.5$$

Step-6:

$$\Delta m = -\eta \frac{\partial E}{\partial m} = -(0.1)(-2.34) = 0.234$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(-4.5) = 0.45$$

Step-7:

$$m = m + \Delta m = 1 + 0.234 = 1.234$$

$$c = c + \Delta c = -1 + 0.45 = -0.55$$

Step-8:

$$\text{batch} = \text{batch} + 1 = 2 \leq nb \rightarrow \text{True}$$

Go to step-5

Step-9:

$$\frac{\partial E}{\partial m} = \frac{-1}{2} \left[((3.8) - (1)(0.4) - (-1))(0.4) + (4.2 - (1)(0.6) - (-1))(0.6) \right]$$

$$= -1.97416$$

$$\frac{\partial E}{\partial c} = -3.933$$

Step-10:

$$\Delta m = -\eta \frac{\partial E}{\partial m} = -(0.1)(-1.97416) = 0.197416$$

$$\Delta c = -\eta \frac{\partial E}{\partial c} = -(0.1)(-3.933) = 0.3933$$

Step-11:

$$m = m + \Delta m = 1.234 + 0.197416 = 1.4314$$

$$c = c + \Delta c = 0.3933 + (-0.55) = -0.1567$$

Step-14:

$$\text{batch} = \text{batch} + 1 = 3 \leq nb \text{ — False}$$

Go to Next step i.e., Step-15

Step-15:

$$\text{iter} = \text{iter} + 1 = 2 \leq \text{epochs} \text{ — True}$$

Go to Step-4

Step-16:

$$\text{batch} = 1$$

Step-17:

$$\begin{aligned} \frac{\partial E}{\partial m} &= -\frac{1}{2} \left[\frac{(3.4 - (1.4314)(0.2) - (-0.1567))}{(0.2)} \right. \\ &\quad \left. + (4.6 - (1.4314)(0.8) - (-0.1567))(0.8) \right] \\ &= -1.77167 \end{aligned}$$

~~Step~~

$$\frac{\partial E}{\partial c} = -\frac{1}{2} \left[(3.4 - (1.4314)(0.2) - (-0.1567)) + (4.6 - (1.4314)(0.8) - (-0.1567)) \right]$$

$$= -3.441$$

Step-18:

$$\Delta m = -\eta \frac{\partial \mathcal{E}}{\partial m} = 0.177167$$

$$\Delta c = -\eta \frac{\partial \mathcal{E}}{\partial c} = +0.3441$$

Step-19:

$$m = m + \Delta m = 1.4314 + 0.177167 = 1.60856$$

$$c = c + \Delta c = -0.1567 + 0.3441 = 0.1874$$

Step-20:

$$\text{batch} = \text{batch} + 1 = 2 \leq nb \text{ --- True}$$

Go to step-5

Step-21:

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial m} &= -\frac{1}{2} \left[(3.8 - (1.60856)(0.4) - (0.1874))(0.4) \right. \\ &\quad \left. + (4.2 - (1.60856)(0.6) - (0.1874))(0.6) \right] \\ &= -1.50807 \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial c} &= -\frac{1}{2} \left[(3.8 - (1.60856)(0.4) - (0.1874)) + \right. \\ &\quad \left. (4.2 - (1.60856)(0.6) - (0.1874)) \right] \\ &= -3.00831 \end{aligned}$$

Step-22:

$$\Delta m = -\eta \frac{\partial \mathcal{E}}{\partial m} = 0.150807$$

$$\Delta c = -\eta \frac{\partial \mathcal{E}}{\partial c} = 0.300831$$

Step-23: $m = m + \Delta m = 1.60856 + 0.150807 = 1.759367$

$c = c + \Delta c = 0.1874 + 0.300831 = 0.488231$

Step-24: $batch = batch + 1$

$= 2 + 1 = 3 \leq nb \rightarrow \text{False}$

Go to next step

Step-25:

$iter = iter + 1 = 3 \leq epochs \rightarrow \text{False}$

Go to next step

Step-26:

Print (m, c)

Step-27:

Mean Square Error:

$$mse = \frac{(3.4 - 0.84004)^2 + (3.8 - 1.19185)^2 + (4.2 - 1.54367)^2 + (4.6 - 1.89548)^2}{4}$$

$= \cancel{2068229} 6.91$