TRA UPS $SSB = \sum\limits_{i=1}^{K} \sum\limits_{J=1}^{M} (\bar{y}_i - \bar{y})^2 = M \sum\limits_{i=1}^{K} (\bar{x}_i - \bar{y})^2$ $MSB = \dfrac{SSB}{K-1}$

ENTRO UPS $SSW = \sum\limits_{i=1}^{K} \sum\limits_{J=1}^{M} (x_{iJ} - \bar{y}_i)^2$ $MSW = \dfrac{SSW}{K \cdot M - K} = \dfrac{SSW}{K(M-1)}$

TOTALE $SSTO = \sum\limits_{i=1}^{K} \sum\limits_{J=1}^{M} (x_J - \bar{y})^2$ $S^2 = \dfrac{SSTO}{KM-1}$

$SSTO = SSW + SSB$ $\rho = 1 - \dfrac{M}{M-1} \cdot \dfrac{SSW}{SSTO}$

1) $SSB = 0$ $SSTO = SSW$ $\rho = 1 - \dfrac{M}{M-1} \cdot 1 = \dfrac{M-1-M}{M-1} = -\dfrac{1}{M-1}$ Max Efficienza

2) $SSW = 0$ $SSTO = SSB$ $\rho = 1 - \dfrac{M}{M-1} \cdot 0 = 1$

3) $\rho = 0$ $1 - \dfrac{M}{M-1} \cdot \dfrac{SSW}{SSTO} = 0 \longrightarrow \dfrac{M}{M-1} \cdot \dfrac{SSW}{SSTO} = 1 \implies \dfrac{SSW}{SSTO} = \dfrac{M-1}{M} \cong 1$ se M suff. grande

$SSW \stackrel{\sim}{\cong} SSTO$

① $MSB = f(\rho)$ ② $DEFF = f(\rho)$

$$\rho = 1 - \frac{m}{m-1} \cdot \frac{SSW}{SSTO} = 1 - \frac{m}{m-1} \cdot \frac{(SSTO - SSB)}{SSTO} =$$

$$= 1 - \frac{m}{m-1}\left(1 - \frac{SSB}{SSTO}\right) = 1 - \frac{m}{m-1} + \frac{m}{m-1} \cdot \frac{SSB}{SSTO} =$$

$$= \frac{m-1-m}{m-1} + \frac{m}{m-1}\frac{SSB}{SSTO} = -\frac{1}{m-1} + \frac{m}{m-1} \cdot \frac{(k-1)MSB}{(km-1)\cdot s^2}$$

$$\rho(m-1) = -1 + m \cdot \frac{(k-1)MSB}{(k\cdot m - 1)s^2} \quad \longrightarrow \quad \frac{\rho(m-1)+1}{m\cdot(k-1)} \cdot (km-1)\cdot s^2 = MSB$$

① $\boxed{MSB = \frac{(km-1)}{m(k-1)} \cdot s^2 \left[1 + \rho(m-1)\right]}$

$$DEFF = \frac{Va(\bar{y}_{GR})}{Va(\bar{y}_{ccs})} = \frac{m \cdot s^2_{\bar{y}_u}}{s^2} = \qquad s^2_{\bar{y}_u} = \sum_{i=1}^{k} \frac{(\hat{y}_i - \bar{y})^2}{k-1} = \frac{MSB}{m}$$

$$= \frac{MSB}{s^2} = \frac{\frac{km-1}{m(k-1)} \cdot s^2[1 + \rho(m-1)]}{s^2} = \frac{km-1}{m(k-1)}\left[1 + \rho(m-1)\right] \qquad \begin{array}{l} mk \gg m \\ mk-1 \cong mk - m \\ \frac{km-1}{m(k-1)} \cong 1 \end{array}$$

$$= 1 + \rho(m-1)$$

$$DEFF \simeq 1 + \rho(m-1)$$

$m = 51$ $\qquad$ $\rho = 0.05$ $\qquad$ $DEFF = 1 + 0,05 \cdot 50 = 1 + 2,5 = 3,5$

$m = 31$ $\qquad$ $\rho = 0,2$ $\qquad$ $DEFF = 7$