$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i = \frac{1}{n}\sum_{i=1}^{N} Y_i I_i$$

$$I_i = \begin{cases} 1 & u_i \in c \\ 0 & u_i \notin c \end{cases}$$

$I_i \sim$ Bernoulliana (π_i) $\qquad \pi_i = \frac{n}{N}$

$$\rightarrow E[I_i] = \pi_i = \frac{n}{N} \qquad Var[I_i] = \pi_i(1-\pi_i) = \frac{n}{N}\left(1-\frac{n}{N}\right)$$

$$E[\bar{y}] = E\left[\frac{1}{n}\sum_{i=1}^{N} Y_i I_i\right] = \frac{1}{n}\sum_{i=1}^{N} Y_i\, E[I_i] =$$

$$\overset{!}{=} \frac{1}{\cancel{n}}\sum_{i=1}^{N} Y_i \cdot \frac{\cancel{n}}{N} = \frac{1}{N}\sum_{i=1}^{N} Y_i = \bar{Y}$$

$$V[\bar{y}] = V\left[\frac{1}{n}\sum_{i=1}^{N} Y_i I_i\right] =$$

$$= \frac{1}{n^2}\left[\sum_{i=1}^{N} Y_i^2 V[I_i] + \sum_{i=1}^{N}\sum_{j\neq i}^{N} Y_i Y_j \, Cov[I_i, I_j]\right] =$$

$$= \frac{1}{n^2}\left[\sum_{i=1}^{N} Y_i^2 \cdot \frac{n}{N}\left(1-\frac{n}{N}\right) + \sum_{i=1}^{N}\sum_{j\neq i} Y_i \cdot Y_j \left(-\frac{1}{N-1}\right)\frac{n}{N}\left(1-\frac{n}{N}\right)\right] =$$

$$\overset{!}{=} \frac{1}{n^2}\cdot\frac{\cancel{n}}{N}\left(1-\frac{n}{N}\right)\left[\sum_{i=1}^{N} Y_i^2 - \frac{1}{N-1}\sum_{i=1}^{N}\sum_{j\neq i} Y_i \cdot Y_j\right] =$$

$$= \frac{1}{n}\left(1-\frac{n}{N}\right)\cdot\frac{1}{N(N-1)}\left[(N-1)\sum_{i=1}^{N} Y_i^2 - \left(\sum_{i=1}^{N} Y_i\right)^2 + \sum_{i=1}^{N} Y_i^2\right] =$$

$$= \frac{1}{n}\left(1-\frac{n}{N}\right)\cdot\underbrace{\frac{1}{N(N-1)}\left[N\sum_{i=1}^{N} Y_i^2 - \left(\sum_{i=1}^{N} Y_i\right)^2\right]}_{= S^2} = \frac{1}{n}\cdot\left(1-\frac{n}{N}\right)\cdot S^2$$

$$V[I_i] = \pi_i(1-\pi_i) = \frac{n}{N}\left(1-\frac{n}{N}\right)$$

$$Cov[I_i, I_j] = E[I_i I_j] - E[I_i]\cdot E[I_j] =$$

$$\overset{!}{=} \frac{n-1}{N-1}\cdot\frac{n}{N} - \frac{n}{N}\cdot\frac{n}{N} = \frac{n}{N}\left[\frac{n-1}{N-1} - \frac{n}{N}\right] =$$

$$= \frac{n}{N}\cdot\left[\frac{\cancel{Nn} - N - \cancel{nN} + n}{N(N-1)}\right] = \frac{n}{N}\left[\frac{n-N}{N}\cdot\frac{1}{N-1}\right] = \frac{n}{N}\left(-\frac{1}{N-1}\right)\frac{N-n}{N} =$$

$$\overset{!}{=} \boxed{\frac{n}{N}\left(-\frac{1}{N-1}\right)\left(1-\frac{n}{N}\right)}$$

$$\hookrightarrow E[I_i \cdot I_j] = \mathbb{P}[I_i = 1 \wedge I_j = 1] =$$

$$\overset{!}{=} \mathbb{P}[I_j = 1 \mid I_i = 1]\cdot\mathbb{P}[I_i = 1] =$$

$$\overset{!}{=} \frac{n-1}{N-1}\cdot\frac{n}{N}$$

$$P\left(|\bar{y}-\bar{Y}| \leq D\right) = 1-\alpha$$

$$P\left(\underbrace{\frac{|\bar{y}-\bar{Y}|}{\frac{s}{\sqrt{n}}\sqrt{1-f}}}_{Z(0,1)} \leq \frac{D}{\frac{s}{\sqrt{n}}\sqrt{1-f}}\right) = 1-\alpha$$

$$P\left(Z \leq \frac{D}{\frac{s}{\sqrt{n}}\sqrt{1-f}}\right) = 1-\alpha \implies \frac{D}{\frac{s}{\sqrt{n}}\sqrt{1-f}} \cong z_{\alpha/2}$$

$$D \cong z_{\alpha/2}\sqrt{1-f} \cdot \frac{s}{\sqrt{n}}$$

$$n = \frac{z_{\alpha/2}^2 \cdot (1-f)\cdot s^2}{D^2} = \frac{z_{\alpha/2}^2 \left(1-\frac{n}{N}\right)\cdot s^2}{D^2} = \frac{z_{\alpha/2}^2 \cdot s^2}{D^2} - \frac{n}{N}\cdot\frac{z_{\alpha/2}^2 \cdot s^2}{D^2}$$

$$n\left(1+\frac{z_{\alpha/2}^2 \cdot s^2}{N \cdot D^2}\right) = \frac{z_{\alpha/2}^2 \cdot s^2}{D^2}$$

$$n = \frac{\frac{z_{\alpha/2}^2 \cdot s^2}{D^2}}{1+\frac{z_{\alpha/2}^2 \cdot s^2}{D^2 \cdot N}} = \frac{n_0}{1+\frac{n_0}{N}} \longrightarrow$$

DIMENSIONE OTTIMALE IN C.C.S. CON REINSERIMENTO