Efficienza di Neyman

lunedì 17 maggio 2021 13:26

$$\frac{1-f}{n} = \frac{N-n}{N \cdot n}$$

$$n_h = n \cdot \frac{W_h S_h}{\sum_h W_h \cdot S_h} = n \cdot \frac{N_h \cdot S_h}{\sum_h N_h \cdot S_h}$$

$$V_{prop}(\bar{y}_{str}) - V_{NEY}(\bar{y}_{str}) =$$

$$= \frac{1-f}{n} \sum_{h=1}^{H} W_h S_h^2 - \frac{1}{n}\left(\sum_{h=1}^{H} W_h \cdot S_h\right)^2 + \frac{1}{N}\sum_{h=1}^{H} W_h S_h^2$$

$$= \sum_{h=1}^{H} W_h \cdot S_h^2 \left[\overbrace{\frac{\cancel{N}-\cancel{n}+\cancel{n}}{\cancel{N}n}}^{=1/n}\right] - \frac{1}{n}\left(\sum_{h=1}^{H} W_h \cdot S_h\right)^2$$

$$= \frac{1}{n}\sum_{h=1}^{H} W_h \cdot S_h^2 - \frac{1}{n} \cdot \bar{S}^2 \qquad \bar{S} = \sum_{h=1}^{H} W_h \cdot S_h$$

$$= \frac{1}{n}\sum_{h=1}^{H} W_h \left(S_h - \bar{S}\right)^2$$

$$V_{NEY}(\bar{y}_{str}) = \sum_{h=1}^{H}\left(1 - \frac{n_h}{N_h}\right)\left(\frac{N_h}{N}\right)^2 \frac{S_h^2}{n_h} =$$

$$= \sum_{h=1}^{H}\left(\frac{1}{n_h} - \frac{1}{N_h}\right)\left(\frac{N_h}{N}\right)^2 \cdot S_h^2 =$$

$$= \sum_{h=1}^{H} \frac{W_h^2 \cdot S_h^2}{n_h} - \sum_{h=1}^{H} \frac{W_h^2 \cdot S_h^2}{N_h} =$$

$$= \sum_{h=1}^{H} \left(\frac{N_h}{N^2}\right)^{\cancel{2}} \cdot S_h^{\cancel{2}} \cdot \frac{\sum_h N_h \cdot S_h}{n \cdot \cancel{N_h} \cdot \cancel{S_h}} - \sum_{h=1}^{H} \frac{W_h^2 \cdot S_h^2}{N_h} =$$

$$= \sum_{h=1}^{H} \frac{N_h \cdot S_h}{n \cdot N^2} \cdot \sum_h N_h \cdot S_h - \sum_{h=1}^{H} \frac{W_h^2 \cdot S_h^2}{N_h} =$$

$$= \frac{1}{n}\left(\sum_{h=1}^{H} \frac{N_h \cdot S_h}{N}\right)^2 - \sum_{h=1}^{H}\left(\frac{N_h^{\cancel{2}}}{N^2} S_h^2\right) / \cancel{N_h} =$$

$$= \frac{1}{n}\left(\sum_{h=1}^{H} W_h \cdot S_h\right)^2 - \frac{1}{N} W_h S_h^2$$

lunedì 17 maggio 2021 15:14

$$\frac{1-f}{n} = \frac{1-n/N}{n} = \frac{N-n}{N\cdot n} \qquad + \frac{1}{N}$$

$f_1 \qquad f_2$

$$\frac{N-\cancel{n}+\cancel{n}}{Nn} = \frac{\cancel{N}}{\cancel{N}\cdot n} = \frac{1}{n}$$

$$\frac{1}{n}\sum_{h=1}^{H} W_h\left(S_h-\bar{S}\right)^2 = \frac{1}{n}\sum_{h=1}^{H} W_h\left(S_h^2+\bar{S}^2-2S_h\cdot\bar{S}\right) =$$

$$= \frac{1}{n}\left[\sum_{h=1}^{H} W_h\cdot S_h^2 + \bar{S}^2\underbrace{\sum_{h=1}^{H} W_h}_{=1} - 2\bar{S}\sum_{h=1}^{H} W_h\cdot S_h\right] = \qquad \bar{S} = \sum_{h=1}^{H} W_h\cdot S_h$$

$$= \frac{1}{n}\left[\sum_{h=1}^{H} W_h\cdot S_h^2 + \underbrace{\bar{S}^2 - 2\bar{S}^2}_{-\bar{S}^2}\right] =$$