Barker College

Student Name: .....

Teacher's Initials: .....

Staff involved:

Mr Carruthers BHC
Mr Mallam ARM*
Mr Mildren LZM
Mr Peattie DZP
Ms Pham AHP
Mrs Thomas JZT
Mrs Young ALY

YEAR 9

Wednesday, 17th November 2021

5.3 MATHEMATICS

190 copies

Total time: 90 minutes

INSTRUCTIONS TO STUDENTS:

* Write your FULL name and teacher's initials on this page
* Answer ALL questions in the spaces provided
* Show ALL necessary working
* Marks may not be awarded for careless or badly arranged work
* Diagrams are NOT drawn to scale

This examination consists of TWO sections.

SECTION 1 : NON-CALCULATOR (20 minutes)

Calculators must NOT be used in this section.

SECTION 2 : CALCULATOR (70 minutes)

Calculators MAY be used in this section.

* * * *

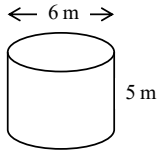
Section	Marks	Your Mark
Section 1	26	
Section 2	92	
Total	118	

SECTION 1: NON-CALCULATOR

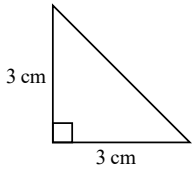
1. Evaluate $(2.0 \times 10^4) \times (5.4 \times 10^7)$ giving your answer in scientific notation. **2**

2. Arrange the following from **smallest to largest**: 5^2, 3^2, 3^3, 2^4 **1**

3. A cylinder has a diameter of 6 m and a height of 5 m. Find its **exact** volume. **2**



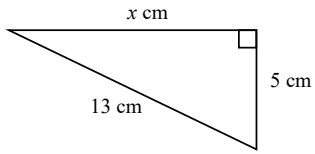
4. Using the following diagram, and showing working, find the value of tan 45°. **2**



5. How many times must 4 be subtracted from 4^3 to get zero? **1**

6. Liam is standing on a bearing of 87° from Oliver. What is the bearing of Oliver from Liam? **1**

7. Find the value of x, showing working. **2**

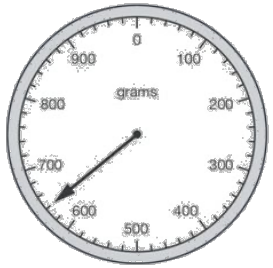


8. One exterior angle of a **regular** polygon is 10°.

(i) What is the sum of all the exterior angles? **1**

(ii) How many sides does the polygon have? **1**

9. Consider the following dial on a set of scales.



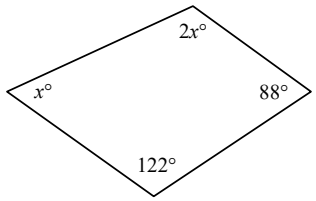
(i) What value is being displayed on the dial? **1**

(ii) What is the absolute error of the scales? **1**

(iii) Between what two values must the true value lie? **1**

10. Evaluate $27^{-\frac{2}{3}}$ giving your answer in simplest form. **2**

11. Find the value of x. Show working, but do not give geometric reasons. **2**



12. Bruce earns a salary of $52 000 p.a. When he takes his 4 weeks of annual leave, he is paid an additional 17.5% leave loading. Calculate his **leave loading only** for the 4 weeks of leave. **2**

13. Two towns are 3.5 cm apart on a map with a scale of 1:100 000. **2**
How many kilometres apart are they in real life?

14. Each score in a set of data is multiplied by 2 then increased by 3.

(i) If the original mean was m, what is the new mean? **1**

(ii) If the original range was r, what is the new range? **1**

End of Section 1

Student Name: .....

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YEAR 9

Wednesday, 17th November 2021

5.3 MATHEMATICS

190 copies **Time: 70 minutes**

SECTION 2: CALCULATOR

INSTRUCTIONS TO STUDENTS:

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* * * *

There are TEN parts to this section.

Part	Topic	Marks	Your Mark
A	Algebra and Indices	8	
B	Products and Factors	10	
C	Equations and Inequations	11	
D	Earning Money	8	
E	Trigonometry	9	
F	Congruence and Similarity	9	
G	Surface Area and Volume	9	
H	Coordinate Geometry	8	
I	Data	8	
J	Mixed Questions	12	
	Total	92	

Part A: Algebra and Indices

15. Round 0.030675 to three significant figures. **1**

16. Simplify and express without negative indices: $\left(\frac{2}{x^8}\right)^{-3}$. **2**

17. Simplify fully: $\frac{10x}{3} - \frac{5x-3}{2}$. **2**

18. Simplify fully: $\frac{6a^3b^5}{b^2c} \div \frac{(3a^4bc^3)^2}{abc}$. **3**

Part B: Products and Factors

19. Circle the correct expansion of $(2a + 3)(2a – 3)$. **1**

A. $4a^2 – 12a – 9$ B. $4a^2 + 12a – 9$ C. $4a^2 + 9$ D. $4a^2 – 9$

20. Fully factorise:

(a) $x^2 + 6x – 7$ **2**

(b) $3x^2 – x – 10$ **2**

(c) $3x + 5xy – 3y – 5y^2$ **2**

21. Fully simplify: $\dfrac{6x+18}{2x^2+8x+6}$. **3**

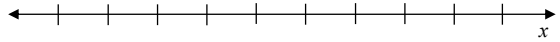
Part C: Equations and Inequations

22. Solve:

(a) $5 – 2x = –12$ **2**

(b) $5+\dfrac{3x}{2}=11$ **2**

23. Solve $3 – 2x > 13$, graphing your answer on the number line below. **4**



24. Solve the following equations simultaneously. **3**

$2x + 3y = –11$

$5x + y = 5$

Part D: Earning Money

25. Hugo is paid $22 per hour. Last week, he worked 30 hours at his normal rate, 5 hours at time-and-a-half, and 3 hours at double time. Calculate his earnings for the week. **2**

26. Eric earns $153 000 p.a. One year, he had allowable tax deductions totalling $1325 and paid PAYG tax of $40 100.

(i) Calculate his taxable income for the year **1**

(ii) Using the tax table, calculate the **income tax** he needs to pay. **2**

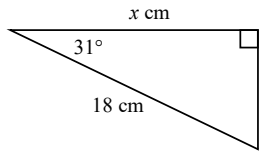
Taxable income	Tax on this income
0 – $18,200	Nil
$18,201 – $45,000	19 cents for each $1 over $18,200
$45,001 – $120,000	$5,092 plus 32.5 cents for each $1 over $45,000
$120,001 – $180,000	$29,467 plus 37 cents for each $1 over $120,000
$180,001 and over	$51,667 plus 45 cents for each $1 over $180,000

(iii) Calculate Eric's **total tax payable** including Medicare Levy of 2% of his taxable income. **1**

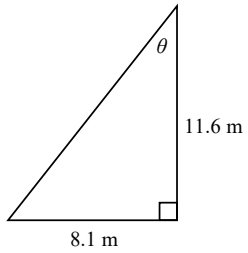
(iv) Does Eric owe extra tax, or will he be given a refund? Calculate the value of the extra tax or refund. **2**

Part E: Trigonometry

27. Find x, correct to 1 decimal place. **2**



28. Find θ, correct to the nearest minute. **3**



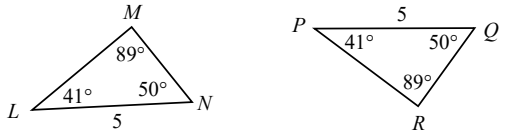
29. Isaac lies on the flat ground 12 m away from a flagpole. The angle of elevation from Isaac to the top of the pole is 53°.

(i) Draw a simple diagram of the situation showing both the given measurements. **1**

(ii) Find the distance from Isaac to the top of the flagpole **to the nearest 10 centimetres**. **3**

Part F: Congruence and Similarity

30. Consider the following pair of congruent triangles.

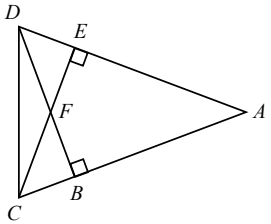


(i) Which congruence test can be used to prove that these triangles are congruent? **1**

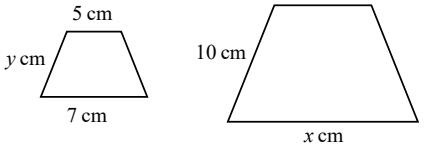
A. SAS B. AAS C. AAA D. RHS

(ii) Which side in $\triangle PQR$ is the same length as MN? **1**

31. In the diagram below, $\angle ACD = \angle ADC$. Prove $\triangle ABD \equiv \triangle AEC$. **4**



32. The trapezium on the left is enlarged to produce a similar figure, shown on the right.



(i) Find the scale factor (enlargement factor). **1**

(ii) Find the values of x and y. **2**

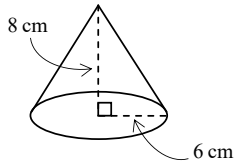
Part G: Surface Area and Volume

33. Circle the expression that gives the **surface area** of a closed cylinder? **1**

A. $\pi r^2 h$ B. $2\pi r^2 + 2\pi rh$ C. $4\pi r^2$ D. $\pi r^2 + \pi rl$ E. $\frac{1}{3}\pi r^2 h$

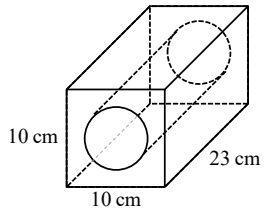
34. A closed cone has a radius of 6 cm and a perpendicular height of 8 cm.

(i) Find the **volume** of the cone correct to 1 decimal place. **2**



(ii) Find the **surface area** of the cone correct to 1 decimal place. **3**

35. The following solid is a square prism with a cylindrical hole of radius 3 cm bored through it. Calculate its **total surface area**, correct to 1 decimal place. **3**

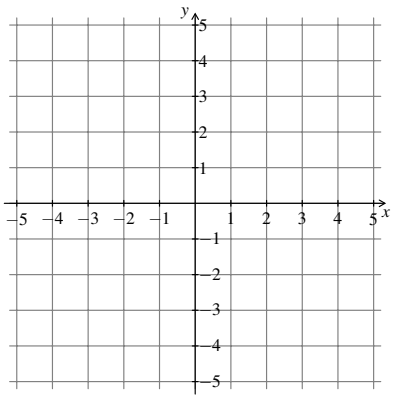


Part H: Coordinate Geometry

36. Find the exact (surd) distance between (2, –1) and (7, –3). **2**

37. Find the equation of the line (in the form $y = mx + c$) with a gradient of 3 that passes through the point (1, 5). **2**

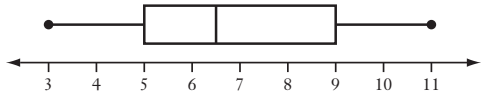
38. Draw a line with a gradient of $-\frac{2}{3}$ through the point (–2, 4). **2**



39. (3, 2) is the midpoint of the points (p, 10) and (1, q). Find the values of p and q. **2**

Part I: Data

40. Consider the box plot below.



(i) Find the interquartile range. **1**

(ii) What percentage of scores lie between 5 and 11? **1**

41. Consider the ordered stem-and-leaf plot below displaying 20 scores.

Stem	Leaf
0	1 3 4 5 6 7 9
1	0 0 1 2 3 3 8 9
2	1 2 4
3	5
4	1

(i) Find the median. **1**

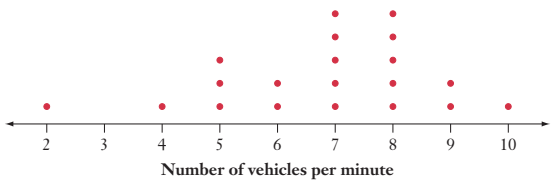
(ii) Find the mode(s). **1**

(iii) Circle the most appropriate description of the shape of the data above. **1**

A. Symmetrical B. Negatively skewed C. Positively skewed D. Normal

42. Complete the frequency table and find the mean of the data in this dot plot. **3**

Score	Frequency	fx
2		
3		
4		
5		
6		
7		
8		
9		
10		



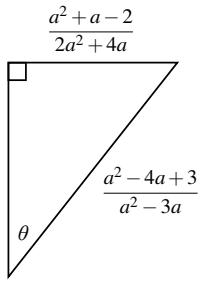
Mean = _____

Part J: Mixed Questions

43. Luke sells electrical goods. He is paid a retainer of $250 per week plus 3% commission on the goods he sells. Last week, he earned $3475. What was the value of electrical goods he sold? **3**

44. Make x the subject of this equation: $c = \dfrac{a+bx}{1+x}$. **3**

45. Find the value of θ in this triangle with side lengths as indicated. **3**



46. The Collatz Conjecture is a famous unsolved problem in mathematics.

It concerns the following procedure being performed on an integer:

- If the integer is **odd**, multiply it by 3 then add 1
- If the integer is **even**, divide it by 2

The procedure is repeated on the result. For example, starting with 3:

$$3 \xrightarrow{\times 3+1} 10 \xrightarrow{\div 2} 5 \xrightarrow{\times 3+1} 16 \xrightarrow{\div 2} 8 \xrightarrow{\div 2} 4 \xrightarrow{\div 2} 2 \xrightarrow{\div 2} 1$$

The Collatz Conjecture proposes that performing the procedure on any positive integer will eventually produce the number 1. In the example above, it took seven steps to reach 1.

(i) Find the integer between 1000 and 2000 that takes the **fewest** steps to reach 1. **1**

Give a brief explanation to justify your answer.

Question 46 continues on page 17

Question 46 (continued)

(ii) Find all integers that are 5 steps **before** your answer in part (i). **2**

For example, 5 is 2 steps before 8 (as in the example above) and 32 is also 2 steps before 8 since $32 \div 2 = 16$ then $16 \div 2 = 8$.

End of Paper

2021 Yr 9 5.3 Sem 2 Student Solutions

Non Calc

1) 1.08×10^{12}

2) $3^2, 2^4, 5^2, 3^3$

3) $V = \pi(3)^2(5)$
$= 45\pi cm^3$

4) $\tan 45 = 1$

5) 16 times

6) 267°

7) $x^2 + 5^2 = 13^2$
$x^2 = 169 - 25$
$x = 12$

8) i) 360°
ii) 36 sides

9) i) 640g
ii) 10g
iii) 630g - 650g

10) $\frac{1}{27^{2/3}}$
$= \frac{1}{9}$

11) $x + 2x + 122 + 88 = 360$
$3x = 150$
$x = 50°$

12) $LL = (\$2000 \div 52) \times 17.5 \times 4$
$= \$700$

13) $3.5cm \rightarrow 3500000cm$
$= 3.5km$

14) i) $2m + 3$
ii) $2r$

Part A

15) 0.0307

16) $\left(\frac{x^8}{2}\right)^3$
$= \frac{x^{24}}{8}$

17) $\frac{20x - 5x + 9}{6}$
$= \frac{15x + 9}{6}$

18) $\frac{6a^3b^3}{c} \times \frac{abc}{9a^8b^2c^6}$
$= \frac{2b^2}{3a^4c^6}$

Part B

19) $4a^2 - 9$ (D)

20a) $(x+7)(x-1)$

b) $(3x+5)(x-2)$

c) $(x-y)(3+5y)$

21) $\frac{6(x+3)}{2(x+3)(x+1)}$
$= \frac{3}{x+1}$

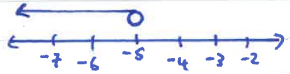
Part C

22a) $-2x = -17$
$x = \frac{17}{2}$

b) $\frac{3x}{2} = 6$
$x = 4$

23) $-2x \geq 10$

$x \leq -5$



24) $2x + 3y = -11 \quad \ldots (1)$

$5x + y = 5 \quad \ldots (2)$

from (2)

$y = 5 - 5x \quad \ldots (2a)$

sub (2a) into (1)

$2x + 3(5 - 5x) = -11$

$x = 2$

sub $x = 2$ into (2a)

$y = 5 - 5(2)$

$y = -5$

$x = 2 \quad y = -5$

Part D

25) $22 \times 30 + 5 \times 22 \times 1.5 + 3 \times 22 \times 2$

$= 957$

26) i) $153000 - 1325 = 151675$

ii) $IT = 29467 + 0.37(151675 - 120000)$

$= 41186.75$

iii) $4186.75 + 2\% \times 151675$

$= 44220.25$

iv) owes extra tax of 4120.25

Part E

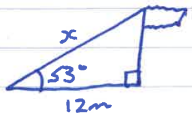
27) $\cos 31 = \frac{x}{18}$

$x = 15.4 \text{ cm}$

28) $\tan\theta = \frac{8.1}{11.6}$

$\theta = 34° 56'$

29) i)



ii) $\cos 53 = \frac{12}{x}$

$x = 19.9 \text{ m}$

Part F

30) i) (B) AAS

ii) QR

31) in $\triangle ABD$ and $\triangle AEC$

$\angle BAE$ is Common

$\angle ABD = \angle AEC$ (given)

$AD = AC$ (equal sides in iscos $\triangle$)

$\therefore \triangle ABD \equiv \triangle AEC$ (AAS)

32) i) factor $= \frac{8}{5}$

ii) $x = 11.2 \quad y = 6.25$

33) $2\pi r^2 + 2\pi rh$ (B)

34) i) $V = \frac{1}{3}\pi(6)^2(8)$

$V = 301.6 \ (1dp) \text{ cm}^3$

ii) $L^2 = 6^2 + 8^2$

$L = 10$

$SA = \pi(6)^2 + \pi(6)(10)$

$= 301.6 \text{ cm}^2$

35) $SA = 4(10 \times 23) + 2(10 \times 10 - \pi(3)^2) + (2\pi(3)(23))$

$= 1497.0 \text{ cm}^2$

Part H

36) $d = \sqrt{(7-2)^2 + (-3--1)^2}$

$= \sqrt{29}$

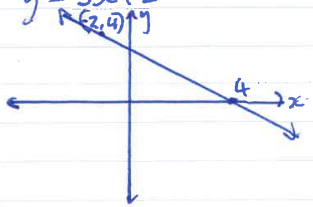
37) $y = 3x + c$

$5 = 3(1) + c$

$c = 2$

$y = 3x + 2$

38)



39) $\frac{p+1}{2} = 3 \qquad \frac{10+q}{2} = 2$

$p = 5 \qquad q = -6$

Part I

40) i) IQR = 4

ii) 75%

41) i) median = 11.5

ii) modes = 10 and 13

iii) positively skewed

42)

Score	frequency	fx
2	1	2
3	0	0
4	1	4
5	3	15
6	2	12
7	5	35
8	5	40
9	2	18
10	1	10
	20	136

mean = 6.8

Part J

43) $3475 = 250 + 3\% \times x$

$x = 107500$ worth of goods

44) $c + cx = a + bx$

$cx - bx = a - c$

$x(c-b) = a - c$

$x = \frac{a-c}{c-b}$

45) $\sin\theta = \frac{\frac{(a+2)(a-1)}{2a(a+2)}}{\frac{(a-3)(a-1)}{a(a-3)}}$

$\sin\theta = \frac{1}{2}$

$\theta = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta = 30°$

46) For fewest steps you would divide by 2 every step.

$\therefore$ the number must be a power of 2

$2^{10} = 1024$

ii)