Student's Name: ……...

Teacher's Initials: …….

Barker College

WMD*	JYR
JZT	VAB
LMD	BHC
RAS	JWH
VAB	RJW
ARP	

YEAR 10

PM WEDNESDAY 16TH MAY

TERM 2, 2018

5.3 MATHEMATICS

TOTAL TIME: 90 MINUTES

Semester 1 Examination

260 COPIES

INSTRUCTIONS TO STUDENTS:

Write your name and teacher's initials on the TOP of EVERY SHEET of PAPER.

Attempt ALL questions.

Show ALL necessary working.

Calculators can be used throughout the examination.

Marks may not be awarded for careless or badly arranged work.

Diagrams are NOT drawn to scale.

Write your answers in the spaces provided on the paper.

A formula sheet is provided on page 2 for use throughout the examination. Detach this sheet.

This examination consists of SIX parts.

PART A:	**COMMON**	**(45 marks)**
PART B:	**SURDS AND INDICES**	**(17 marks)**
PART C:	**INTEREST AND DEPRECIATION**	**(8 marks)**
PART D:	**QUADRATIC EQUATIONS AND PARABOLAS**	**(23 marks)**
PART E:	**SURFACE AREA AND VOLUME**	**(11 marks)**
PART F:	**MIXED QUESTIONS**	**(14 marks)**

TOTAL 118 Marks

FORMULA SHEET

Pythagoras' theorem: $c^2 = a^2 + b^2$

Simple Interest : $I = PRN$

Compound Interest: $A = P\left(1 + r\right)^n$

Depreciation: $A = P\left(1 - r\right)^n$

Circumference of a circle: $C = 2\pi r$

Area of a circle: $A = \pi r^2$

Area of a parallelogram: $A = bh$

Area of a rhombus: $A = \frac{1}{2}xy$

Area of a trapezium: $A = \frac{1}{2}h(a+b)$

Volume of a prism: $V = Ah$

Volume of a cylinder: $V = \pi r^2 h$

Volume of a pyramid: $V = \frac{1}{3}Ah$

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Surface area of a closed cylinder: $SA = 2\pi r^2 + 2\pi rh = 2\pi r(r+h)$

Surface area of a cone: $SA = \pi r^2 + \pi rl$

Surface area of a sphere: $SA = 4\pi r^2$

Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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PART A: COMMON (45 Marks)

Marks

1. Find 15% of $260. **1**

2. 20% of an amount is $40. What is the amount? **1**

3. Using a calculator, or otherwise, evaluate $\dfrac{2.4\times10^{7}}{9.6\times10^{5}}$ **1**

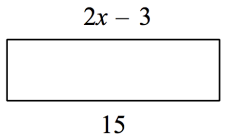
4. Jane and Tom play a game where points are scored as follows: **1**

WIN = +7
LOSS = -3

Jane wins 5 games and loses 3 games, and Tom wins 3 games and loses 5 games.

What is the difference in their final scores?

5. A rectangle is shown. **1**



What is the value of x?

6. A car is travelling at at 68 km/h. How far will it travel in 15 minutes? **1**

7. Simplify the following:

(a) $3a+6b+5a-3b+4$ **1**

(b) $2p\times3q\times(-4q)$ **1**

(c) $9x-4\left[2(1-3x)-2(3-3x)\right]$ **3**

8. Fully factorise: $3ax+9ay-12az$ **1**

9. Given that $a=-2$, $b=3$ and $c=5$, evaluate $(a-b)^{2}\times(c+ab)^{3}$ **2**

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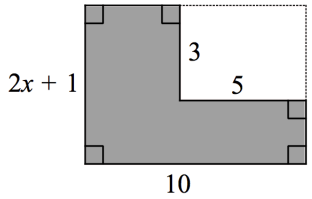
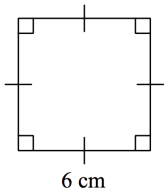
10. Part of a rectangle is shaded. **2**



Find an expression for the shaded area in simplest terms.

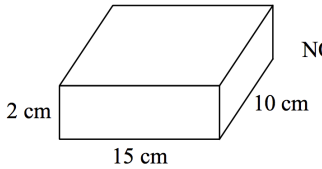
11. What is the area of the largest circle that can fit inside a square of side length 6 cm? **1**

Answer correct to 1 decimal place.



NOT TO SCALE

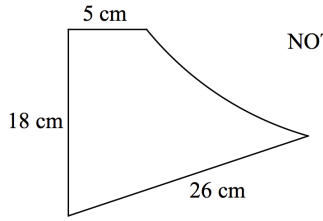
12. The diagram shows a closed rectangular prism. **2**



NOT TO SCALE

Calculate the surface area.

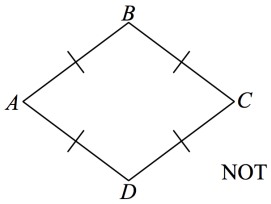
13. The perimeter of the field below is 89 cm. **1**



NOT TO SCALE

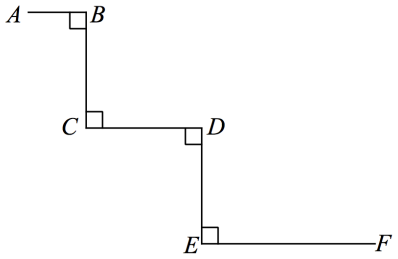
What is the length of the curved part of the perimeter?

14. *ABCD* is a rhombus. Its area is 48 cm^2 and $AC = 8$ cm. Find the length of BD. **2**



NOT TO SCALE

15. In the figure, $AB = 1$ cm, $BC = CD = DE = 2$ cm and $EF = 3$cm. **2**
Find the straight line distance between *A* and *F*, correct to the nearest 0.1 cm.



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16. Simplify $\frac{x}{5}-\frac{x-3}{15}$, giving your answer in its simplest terms. **2**

17. Simplify $\frac{(xy)^2}{x^{-5}y^6}$ and express your answer with positive indices. **2**

18. Jacqui invested \$2750 in an account paying 4.5% p.a. interest, compounding annually. Find:

(i) The final value of her investment after 5 years. **2**

(ii) The amount of interest earned over this period of time. **1**

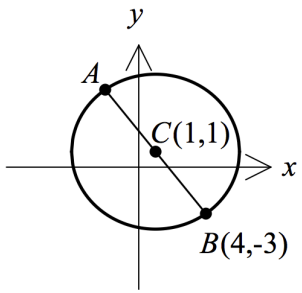
19. Eva is x years old. Tara is 3 years older than Eva. Find an expression for Tara's age in five years time. **1**

20. By selling a particular set of books for \$408, a bookseller suffered a loss of 4%.

(i) Find the original price of the books paid by the bookseller. **2**

(ii) Calculate what the percentage gain or loss would have been if the books had been sold for \$510. **2**

21. The circle has AB as a diameter and centre C. What are the coordinates of A? **2**



A (……. , ……)

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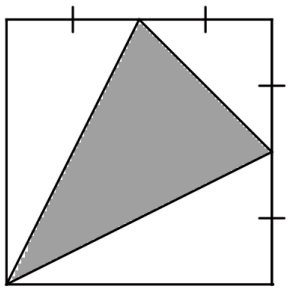
22. What fraction of the square is shaded? **1**



23. During an event, 135 fish balls and 108 chicken wings are catered for the guests.

All the food must be eaten.

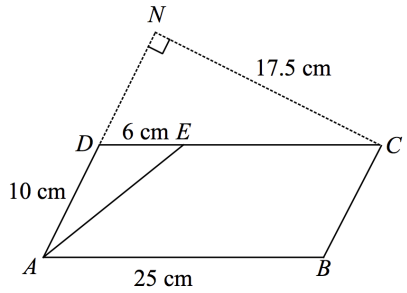
Find:

(i) The largest possible number of guests that can be invited if the guests are to receive the same number of fish balls as each other and the same number of chicken wings as each other. **2**

(ii) The number of chicken wings each guest will receive if (i) is satisfied. **1**

24. In the figure, *ABCD* is a parallelogram, *AB* = 25 cm, *AD* = 10 cm, *DE* = 6 cm and *CN* = 17.5 cm.

AD is extended to *N*.



Find:

(i) the area of parallelogram *ABCD* **1**

(ii) the height of trapezium *ABCE* **1**

(iii) the area of trapezium *ABCE* **1**

End of Part A

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PART B: SURDS AND INDICES (17 Marks)

Marks

Question 1

Consider the numbers stated below.

$81,\ \sqrt{12},\ -6,\ 0,\ \frac{2}{5},\ 9,\ \pi,\ \sqrt{49}$

(i) List the perfect square(s) **1**

(ii) List the integer(s) **1**

(iii) List the irrational number(s) **1**

Question 2

Simplify:

(a) $5\sqrt{7}-\sqrt{63}+2\sqrt{28}$ **2**

(b) $\frac{4\sqrt{3}\times\sqrt{6}}{3\sqrt{2}}$ **2**

(c) $\sqrt[3]{8a^6}-\left(a^{\frac{4}{3}}\right)^{\frac{3}{2}}$ **2**

Part B: Surds and Indices (continued)

Question 3

The hypotenuse of a right-angled triangle is 5 cm in length and one of the shorter sides is $\sqrt{5}$ cm long. **2**

Find the length of the other shorter side, giving your answer in simplest surd form.

Question 4

Simplify $\left(5xt^2\right)^3 \div \left(5x^{-1}t^3\right)^2$ **3**

Question 5

Write the expression $3^x + 3^x + 3^x$ as a single power. **1**

Question 6

Expand and simplify $\left(x+x^{-1}\right)^2$, answering without negative indices. **2**

End of PART B

PART C: INTEREST AND DEPRECIATION (8 marks)

Marks

Question 7

Jill invested an amount for 4 years at a simple interest rate of 3.75% p.a. **2**

If the interest earned was $240 what was the amount of her investment?

Question 8 **3**

A boat was purchased for $100 000. If the value of the boat depreciated by 0.5% per month, find the value of the boat after 10 years.

Question 9 **3**

Philippa invested $10 000 for 10 years in an account paying compound interest, with interest compounding annually. Find the interest rate as a percentage, correct to 1 decimal place, given that the investment had grown to $15 000 at the end of the 10 year period.

End of PART C

PART D: QUADRATIC EQUATIONS AND PARABOLAS (23 marks)

Marks

Question 10

Solve $25t^2 - 16 = 0$. **2**

Question 11

Solve:

(a) $x^2 - x - 20 = 0$ **by factorisation**. **2**

(b) $y^2 - 8y + 11 = 0$ **by the method of completing the square**, giving any answers correct to 1 decimal place. **3**

(c) $3x - 4 - \frac{1}{x} = 0$ by using the **quadratic formula**, leaving any answers in simplest surd form. **3**

Part D: Quadratic Equations and Parabolas (continued)

Question 12

For the parabola $y=(2x-1)(2x+5)$:

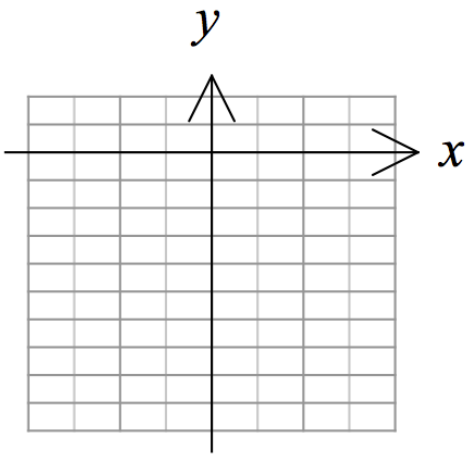
(i) Find the x – intercepts. **1**

(ii) Find the y – intercept. **1**

(iii) Find the equation of the axis of symmetry. **1**

(iv) Find the coordinates of the vertex. **1**

(v) Sketch the parabola on this number plane, showing all of the above features. **2**



Part D: Quadratic Equations and Parabolas (continued)

Question 13

(i) Find the coordinates of the vertex of the parabola $y=x^2+2x+3$. **2**

(ii) The parabola $y=x^2+2x+3$ does not have any x – intercepts. **2**

Use algebra or a graph to explain why this is so.

Question 14

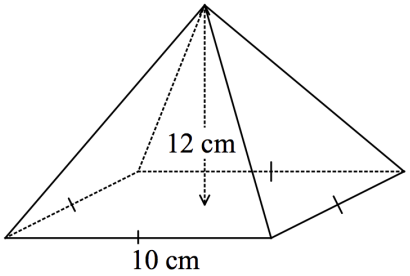
Solve $9^x-10(3^x)+9=0$ using the substitution $u=3^x$. **3**

End of PART D

PART E: SURFACE AREA AND VOLUME (11 marks)

Marks

Question 15



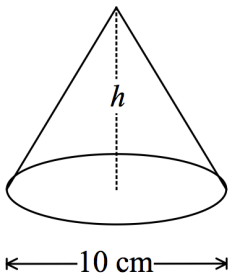
3

The base of a pyramid is a square with side length 10 cm. The perpendicular height is 12 cm.

Find the total surface area of the pyramid.

Question 16

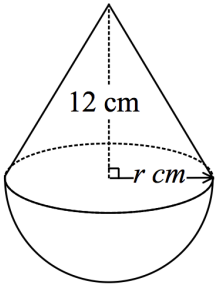
The cone shown in the diagram below has a volume of 100 cm^3. **3**
Calculate the perpendicular height h of the cone, correct to 3 significant figures.



Part E: Surface Area and Volume (continued)

Question 17

The diagram below shows a solid consisting of a hemisphere of radius r cm joined to the bottom of a right circular cone of height 12 cm and radius r cm.
It is given that the volume of the cone is twice the volume of the hemisphere.



(i) Find r. **3**

(ii) Express the volume of the solid in terms of π. **2**

End of PART E

PART F: MIXED QUESTIONS (14 marks)

Marks

Question 18

(i) Factorise $3m^2 - mn - 2n^2$. **2**

(ii) Hence, or otherwise, factorise $3m^2 - mn - 2n^2 - m + n$. **2**

Question 19

Show that $\dfrac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + 2\sqrt{6}$ is a rational number. **3**

Part F: Mixed Questions (continued)

Question 20

The surface area of a closed cylinder, with a height of 2.5 cm, is 63π cm^2. **4**

By forming a quadratic equation and solving it, find the radius of the cylinder.

Question 21

(i) Show that the average of 9 and 4 is greater than the square root of the product of 9 and 4. **1**

(ii) Let x and y be positive numbers. **2**

By starting with the fact that $\left(\sqrt{x}-\sqrt{y}\right)^2 \geq 0$ prove that the average of x and y is always greater than or equal to the square root of the product of x and y.

END OF PAPER

Year 10 | COMMON SECTION – HALF YEARLY 2018 | Page 1 of 2

PART A.

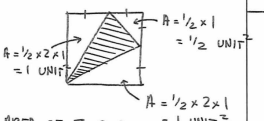
1. $15\% \times \$260 = \$39$
2. $20\% = \$40$ (×5) $\therefore 100\% = \$200$
3. USING CALC ... 25
4. J: $(5\times7)+(3\times-3) = 35-9 = 26$
   T: $(3\times7)+(5\times-3) = 21-15 = 6$
   $\therefore$ DIFFERENCE: $26-6 = 20$
5. OPP. SIDES ARE EQUAL
   $\therefore 2x-3 = 15$
   $2x = 18$
   $x = 9$
6. 15 MINS = 1/4 OF AN HOUR
   $1/4 \times 68 = 17$km
7a. $8a+3b+4$
   b. $-24pq^2$
   c. $9x-4[2(1-3x)-2(3-3x)]$
   $= 9x-4(2-6x-6+6x)$
   $= 9x-4\times(-4)$
   $= 9x+16$
8. $3a(x+3y-4z)$
9. $(-2-3)^2 \times (5-6)^3$
   $= (-5)^2 \times (-1)^3$
   $= 25 \times -1$
   $= -25$
10. AREA = SQUARE – CUT OUT
    $= 10(2x+1) - (3\times5)$
    $= 20x+10-15$
    $= 20x-5$
11. DIAMETER OF CIRCLE = 6cm
    $\therefore$ RADIUS = 3cm
    $A = \pi \times 3^2$
    $= 28.3\text{cm}^2$
12. TOP/BOTTOM = $(15\times10)\times2$
    $= 300\text{cm}^2$
    SIDES = $(2\times10)\times2$
    $= 40\text{cm}^2$
    FRONT/BACK = $(2\times15)\times2$
    $= 60\text{cm}^2$
    TOTAL = 400cm^2
13. $89-5-18-26 = 40$cm
14. AREA = $1/2 \times y$
    $48 = 1/2 \times 8 \times BD$
    $48 = 4 \times BD$
    $\therefore BD = 12$cm
15. ADD HORIZONTAL & VERTICAL ELEMENTS TO MAKE A $\triangle$.

$x^2 = 4^2+6^2$
$= 16+36$
$x^2 = 52$ cm
$\therefore x = 7.2$cm

16. $\frac{3x}{15} - \frac{(x-3)}{15}$ ... COMMON DENOM.
    $= \frac{3x-x+3}{15}$
    $= \frac{2x+3}{15}$
17. $\frac{(xy)^2}{x^{-5}y^6} = x^2y^2 \div x^{-5}y^6$
    $= x^{7}y^{-4}$
    $= \frac{x^7}{y^4}$

Year | | Page 2 of 2

18i. $A = P(1+r)^n$
$= 2750(1.045)^5$
$= \$3427.00$
ii $\$3427 - \$2750 = \$677$
19. NOW: E.. x, T... $x+3$
5 YRS: E.. $x+5$, T.. $x+3+5$
... $x+8$
$\therefore T = x+8$
20i. $96\% = \$408$ (÷96) $1\% = \$4.25$ (×100) $100\% = \$425$
ii. $\frac{\$510}{\$425} \times 100 = 1.2$
$1.2 = 120\%$. $\therefore 20\%$ GAIN
21. A $(-2, 5)$
22.



AREA OF $\square = 2\times2 = 4$
SHADED = $4-(1+1+1/2)$
$= 1\frac{1}{2}$ UNITS^2
FRACTION = $\frac{1\frac{1}{2}}{4} = \frac{3}{8}$
... OTHER METHODS EXIST, TOO!

23i. FIND HCF OF EACH:
$135 = 5\times27$
$108 = 4\times27$
$\therefore$ 27 GUESTS
ii. 4 WINGS, FROM ABOVE.

24i. NC IS PARALLELOGRAM'S HEIGHT
$\therefore A = b\times h$
$= 10\times17.5$
$= 175\text{cm}^2$
ii. HEIGHT OF PARALLELOGRAM IS SAME AS TRAPEZIUM ABCE.
$A = b\times h$ (USING AB AS BASE NOW)
$175 = 25 \times h$
FROM (i) $\frac{175}{25} = h$
$\therefore h = 7$cm
iii. $A = h/2(a+b)$
$= 7/2(25+19)$ (25-16)
$= 154\text{cm}^2$

Year 10 | 5·3 SECTION – HALF YEARLY 2018 | Page 1 of 4

PART B: SURDS & INDICES

1i. 81, 9, 0
ii. 81, −6, 0, 9, 7
iii. $\sqrt{12}$, π
2a. $5\sqrt{7}-\sqrt{63}+2\sqrt{28}$
$= 5\sqrt{7}-3\sqrt{7}+4\sqrt{7}$
$= 6\sqrt{7}$
b. $\frac{4\sqrt{3}\times\sqrt{6}}{3\sqrt{2}} = \frac{4\sqrt{18}}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$
$= \frac{4\sqrt{36}}{6}$
$= \frac{24}{6} = 4$
c. $3\sqrt{8a^6} - (a^{4/3})^{3/2}$
$= (8a^6)^{1/3} - a^{1/6}$
$= 2a^2 - a^2$ ... INDEX LAWS
$= a^2$
3. $5^2-(\sqrt{5})^2 = x^2$
$25-5 = x^2$
$20 = x^2$
$\sqrt{20} = x$
$\therefore x = 2\sqrt{5}$
4. $(5xt^2)^3 \div (5x^{-1}t^3)^2$
$= 125x^3t^6 \div 25x^{-2}t^6$
$= 5x^5$
5. $3^x+3^x+3^x = 3\times3^x$
$= 3^1\times3^x$
$= 3^{x+1}$
6. $(x+\frac{1}{x})^2 = (x+\frac{1}{x})(x+\frac{1}{x})$
$= x^2+1+1+\frac{1}{x^2}$
$= x^2+2+\frac{1}{x^2}$

PART C: INT & DEPRECIATION

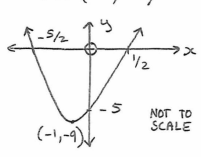
7. $I = Prn$
$240 = P\times0.0375\times4$
$240 = 0.15P$
$P = \frac{240}{0.15} = \$1600$
8. $A = P(r-1)^n$
$= 100\,000(1-0.005)^{120}$
$= \$54\,798.63$
MONTHS IN 10 YRS
9. $\$15\,000 = \$10\,000(1+r)^{10}$
$1.5 = (1+r)^{10}$ ($\sqrt{10}$)
$1.041.. = 1+r$
$\therefore r = 0.041...$
$= 4.1\%$

PART D

10. $25t^2-16 = 0$
$(5t-4)(5t+4) = 0$
$\therefore t = 4/5, -4/5$
... DIFF. OF 2 SQUARES
11a. $x^2-x-20 = 0$
$(x-5)(x+4) = 0$
$\therefore x = 5, -4$
b. $y^2-8y+11 = 0$
$y^2-8y \overset{+16}{} = -11+16$
$(y-4)^2 = 5$
$y-4 = \pm\sqrt{5}$
$y = 4\pm\sqrt{5}$
c. $3x-4-\frac{1}{x}=0$ (×x)
$3x^2-4x-1=0$
$x = \frac{4\pm\sqrt{16-4.3.-1}}{6}$
$= \frac{4\pm\sqrt{28}}{6} = \frac{2\pm\sqrt{7}}{3}$

Year 10 | | Page 2 of 4

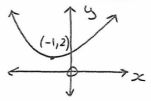
12i. x-INT ... MAKE $y=0$
$0 = (2x-1)(2x+5)$
$\therefore x = 1/2, -5/2$
ii. y-INT ... MAKE $x=0$
$y = (2\times0-1)(2\times0+5)$
$y = -5$
iii. EXPAND: $y = 4x^2+8x-5$
$x = \frac{-b}{2a} = \frac{-8}{8} = -1$
$\therefore x = -1$
iv. SUB. AXIS OF SYMM. INTO ORIGINAL FUNCTION.
$x=-1$ $\therefore y = (2x-1-1)(2x-1+5)$
$= -3\times3$
$= -9$
COORDS: $(-1, -9)$
v.



NOT TO SCALE

13i. AGAIN, USE $x = \frac{-b}{2a}$
$y = x^2+2x+3$
$\therefore x = \frac{-2}{2} = -1$
SUB. INTO ORIGINAL FUNCTION
$y = (-1)^2+(2\times-1)+3$
$= 1-2+3$
$= 2$.
$\therefore$ VERTEX $(-1, 2)$

ii.



CONCAVE UP PARABOLA WITH VERTEX $(-1,2)$ $\therefore$ DOESN'T CROSS x-AXIS.
OR
$x = \frac{-2\pm\sqrt{4-4.1.3}}{2}$
$= \frac{-2\pm\sqrt{-8}}{2}$
CAN'T HAVE NEGATIVE ROOT
$\therefore$ NO x-INTERCEPTS.

14. $9^x-10(3^x)+9=0$
NOTE $9^x = (3^x)^2$
$\therefore u^2-10u+9=0$
$(u-9)(u-1)=0$
$\therefore u = 9, 1$
RE-SUBSTITUTE:
$3^x = 9$ $\quad 3^x = 1$
$x = 2$ $\quad x = 0$

PART E: SA & VOLUME

15.

... TO GET HEIGHT OF EACH TRIANGULAR FACE.
$x^2 = 12^2+5^2$
$= 144+25$
$x^2 = 169$
$x = 13$cm

SURFACE AREA:

BASE = $10 \times 10 = 100cm^2$

SIDES = $(\frac{1}{2} \times 10 \times 13) \times 4$

$= 260cm^2$

TOTAL = $360cm^2$

16. $V = \frac{1}{3}\pi r^2 h$

$100 = \frac{1}{3} \times \pi \times 5^2 \times h$

$100 = \frac{25\pi}{3} \times h$

$\frac{100}{(\frac{25\pi}{3})} = h$ $\therefore h = 3.81...$

$= 3.82cm$

17i. VOL. CONE = 2 x HEMISPHERE

$\frac{1}{3} \times \pi \times r^2 \times 12 = 2 \times (\frac{\frac{4}{3}\pi r^3}{2})$

$4\pi r^2 = \frac{4}{3}\pi r^3$ $\div \pi r^2$

$4 = \frac{4r}{3}$

$12 = 4r$

$\therefore r = 3cm$

ii. CONE: $V = \frac{1}{3} \times \pi \times 3^2 \times 12$

$= 36\pi$

HEMISPHERE: HALF OF CONE AS PER (i)

$\therefore V = 18\pi$

TOTAL = $36\pi + 18\pi$

$= 54\pi\ cm^3$

PART F: MIXED Q'NS

18i. $3m^2 - m - 2$ ... IGNORE n HERE

P: -6, M: -1, F: -3, 2

$3m^2 - 3m + 2m - 2$

$3m(m-1) + 2(m-1)$

$(3m+2)(m-1)$

$\therefore (3m+2n)(m-n)$

RE-INSERT n NOW

ii. $3m^2 - mn - 2n^2 - m + n$

$= (3m+2n)(m-n) - m + n$

$= (3m+2n)(m-n) - 1(m-n)$

$= (3m+2n-1)(m-n)$

... GROUPING BY PAIRS

19. $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + 2\sqrt{6}$

$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + 2\sqrt{6}$

$= \frac{3 - \sqrt{6} - \sqrt{6} + 2}{3-2} + 2\sqrt{6}$

$= 5 - 2\sqrt{6} + 2\sqrt{6}$

$= 5$, WHICH IS RATIONAL.

20. $SA = 2\pi r^2 + 2\pi rh$ {$V = 63\pi$, $r = 2.5$}

$63\pi = 2\pi r^2 + 5\pi r$ $\div \pi$

$63 = 2r^2 + 5r$

$0 = 2r^2 + 5r - 63$

QUAD. FORMULA...

$r = \frac{-5 \pm \sqrt{25 - 4.2.-63}}{4}$

$= \frac{-5 \pm \sqrt{529}}{4}$

$= \frac{-5 \pm 23}{4}$

$= \cancel{-7}$, $4.5cm$

NO! YES!

21i. AVERAGE: $\frac{9+4}{2} = 6.5$

$\sqrt{}$ OF PRODUCT: $\sqrt{9 \times 4} = 6$

$\therefore 6.5 > 6$

ii. $(\sqrt{x} - \sqrt{y})^2 \geq 0$ EXPAND

$x - 2\sqrt{xy} + y \geq 0$

$x + y \geq 2\sqrt{xy}$

$\frac{x+y}{2} \geq \sqrt{xy}$

$\therefore$ ALWAYS TRUE.