Barker College

Student's Name: .....

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DXC LMD
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AHP RAS
GPF* JGD
JAI

Wednesday, 10th November 2021
1:20pm
2 hours
245 copies

Year 10
5.3 MATHEMATICS
Semester 2 Examination

General Instructions

- Write your name at the top of this page
- Write using blue or black pen
- Answer in the spaces provided
- NESA approved calculators may be used
- Show ALL necessary working
- Diagrams are NOT to scale
- Marks may not be awarded for careless or poorly arranged working
- A reference sheet is attached to the end of this paper, which may be detached.

Section	Marks
Part A – Common Section Total	**/40**
Part B – 5.3 Only Section Total	**/98**
Q13 Surds and Indices	/9
Q14 Interest and Depreciation	/10
Q15 Graphs	/9
Q16 Quadratic Equations and Parabolas	/12
Q17 Coordinate Geometry	/7
Q18 Simultaneous Equations and Inequalities	/8
Q19 Probability	/5
Q20 Analysing Data	/6
Q21 Trigonometry and Geometry	/16
Q22 Logarithms	/8
Q23 Mixed Questions	/8
Exam Total:	**/138**

Common Section (40 marks)

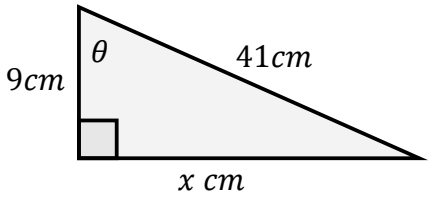
1. Simplify $8a + 3a + 5c - c$ **1**

2. Simplify $(a^4)^9$ **1**

3. Fully factorise $10a + 35c$ **1**

4. Expand $x^3(x^5 - x^2)$ **2**

5. Use the triangle below to answer the following:



   i. Find x **2**

   ii. Find θ, rounded to 1 decimal place. **2**

6. Given that $p = 9$ and $q = -3$, evaluate the following expressions.

i. $p + 2q$ **1**

ii. $pq - \frac{p}{q}$ **2**

7. Solve the following equations.

a. $x + 32 = 51$ **1**

b. $\frac{m}{4} - 5 = -11$ **2**

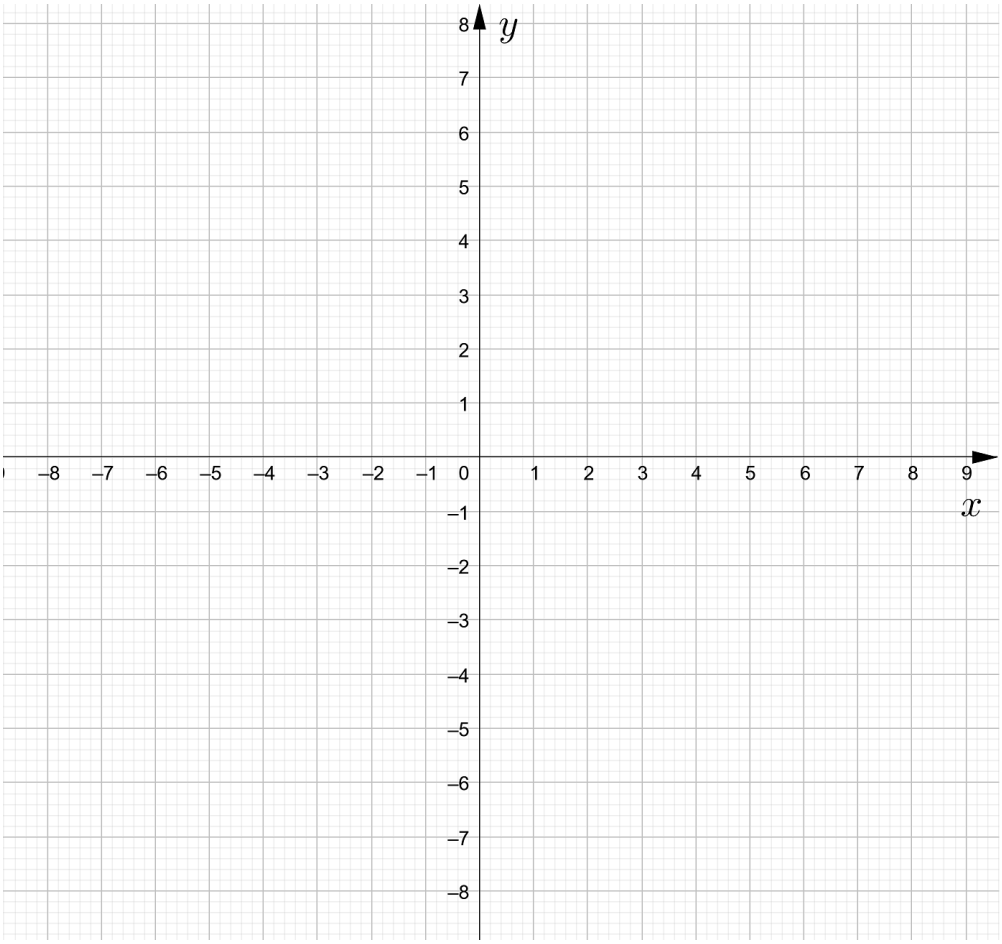
c. $4(p - 5) = 2p + 11$ **3**

8. A straight line has the equation $y = 2x - 2$

i. Complete the table of values for the equation $y = 2x - 2$. **2**

x	-3	-1	2	
y				8

ii. Use the table of values, or otherwise to graph the line on the coordinate plane. **2**



iii. Write down the coordinates of the x-intercept. **1**

iv. If the line $y = 2x - 2$ is shifted 3 units to the right, what will be the equation of the new line? **1**

9. The number of chocolate squares Denzel ate each day over a 10 day period are listed below.
   0 2 4 5 5 7 9 10 11 15

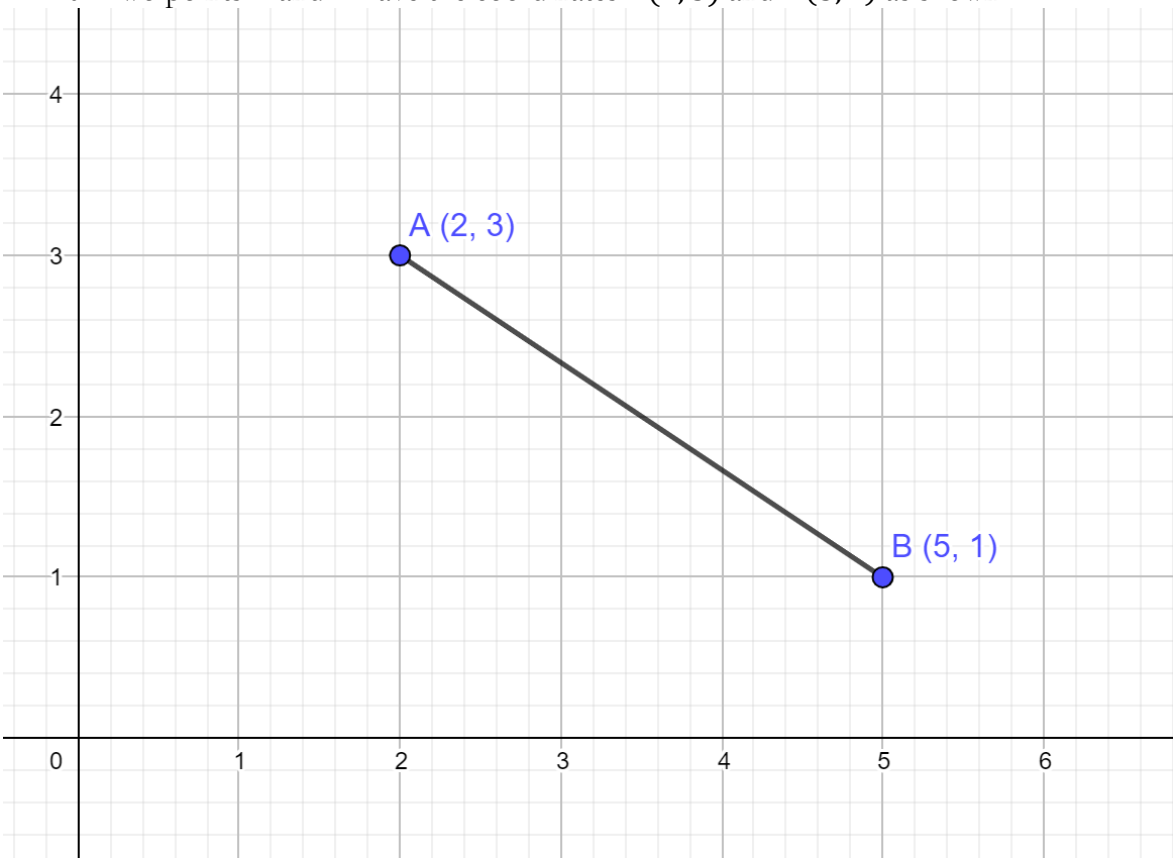
   i. Find the median. **1**

   ii. Find the mean. **1**

   iii. On day 11, Denzel eats x chocolate squares and on day 12 he eats $6x$ chocolate squares. **2**
   The mean after the 12 days is now 8.
   Find the value of x.

   iv. On day 13, Denzel eats y chocolate squares. **1**
   The median remains the same.
   Find the value of y.

10. Two points A and B have the coordinates $A(2,3)$ and $B(5,1)$ as shown.



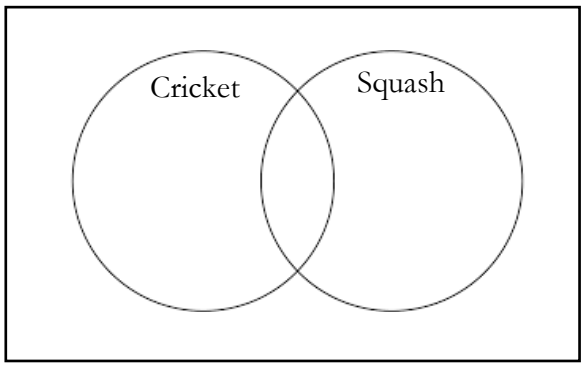
i. Find the coordinates of the midpoint of AB. **2**

ii. Find the gradient of the interval joining AB. **2**

iii. The point C is plotted such that ΔABC has an area of $12\ units^2$. **1**
Find a possible set of coordinates of C.

11. 40 students were asked whether they play cricket or squash.
    28 students play cricket, 15 students play squash.
    3 students **do not** play either sport.

    i. Complete the Venn diagram using the information provided. **2**



    ii. Find the probability that a student selected at random plays one sport only. **2**

12. Arthur is travelling at a constant speed in a straight line from A to B. **2**
    Beatrice is also travelling at a constant speed in a straight line from B to A.

    After a certain time, Arthur and Beatrice pass each other at a point that is $\frac{3}{5}$ of the way from A to B.

    From this time, Beatrice immediately increases her speed and remains at a constant speed, such that Arthur arrives at B at the same time as Beatrice arrives at A. How many times faster is Beatrice's speed after she had passed Arthur than before she passes Arthur?

End of Common Section

PART B 5.3 Only

Question 13 Surds and Indices (9 marks)

Marks

a) Simplify $\sqrt{27} - 5\sqrt{3} + 2\sqrt{32}$ **2**

b) Rationalise the denominator of $\frac{\sqrt{5}+3}{7\sqrt{5}}$ **2**

c) Simplify $3a^0 \times \sqrt{a^{16}}$ **2**

d) Simplify $\frac{(5x^3y)^{-2}}{x^2y^{-4}}$ **3**

Question 14 Interest and Depreciation (10 marks) **Marks**

a) Calculate the amount of interest earned on an investment of $\$\mathbf{6500}$ if it is invested at $\mathbf{5}\%$ p.a. for 7 years, compounded **monthly**. **3**

b) John bought a new surfboard for $\$\mathbf{1099}$. How much will the surfboard be worth in 3 years if it depreciates at a rate of $\mathbf{15}\%$ p.a.? **2**

c) Mike bought a Jet Ski for $\$\mathbf{17000}$. He paid a $\mathbf{10}\%$ deposit and took out a loan for the remaining amount. To repay the loan, Mike needs to pay $\$\mathbf{459}$ every month for 4 years.

Calculate:

i) the size of the loan. **1**

ii) the interest charged. **2**

iii) the flat rate of interest charged per annum. **2**

Question 15 Graphs (9 marks) **Marks**

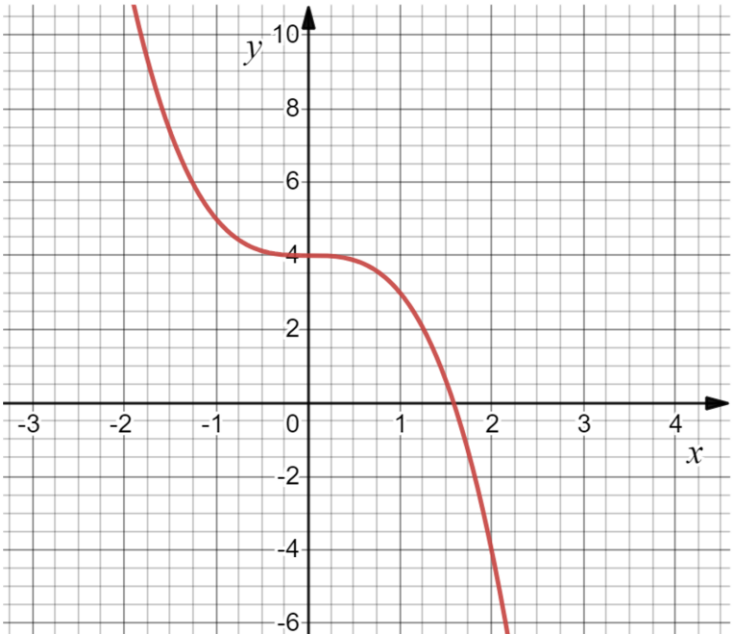
a) The distance (D, in metres) you are from a lightning bolt is directly proportional to the time (T, in seconds) that it takes to hear the thunder.

i) Write a direct proportion equation with k as the constant of variation. **1**

ii) Find the value of k if it takes 3 seconds to hear thunder when you are 1020 metres away from the lightning bolt. **1**

iii) How far from the lightning bolt are you if you count to 30 seconds before you hear any thunder? **1**

b) Write the equation of the following cubic curve which has been drawn to scale. **2**

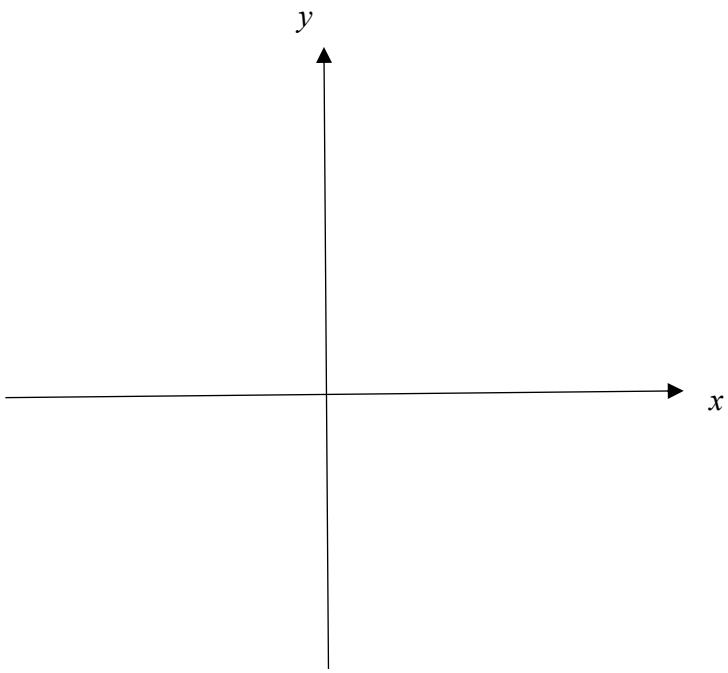


c) Consider the equation $y = \frac{1}{x} - 2$

i) Find its *x*-intercept. **1**

ii) Give the equations of any asymptotes. **2**

iii) Sketch the equation showing the above information. **1**



Question 16 Quadratic Equations and Parabolas (12 marks) **Marks**

a) Solve $3x^2 + 7x - 5 = 0$ by using the **quadratic formula**.
(leave your answer in <u>exact</u> form) **2**

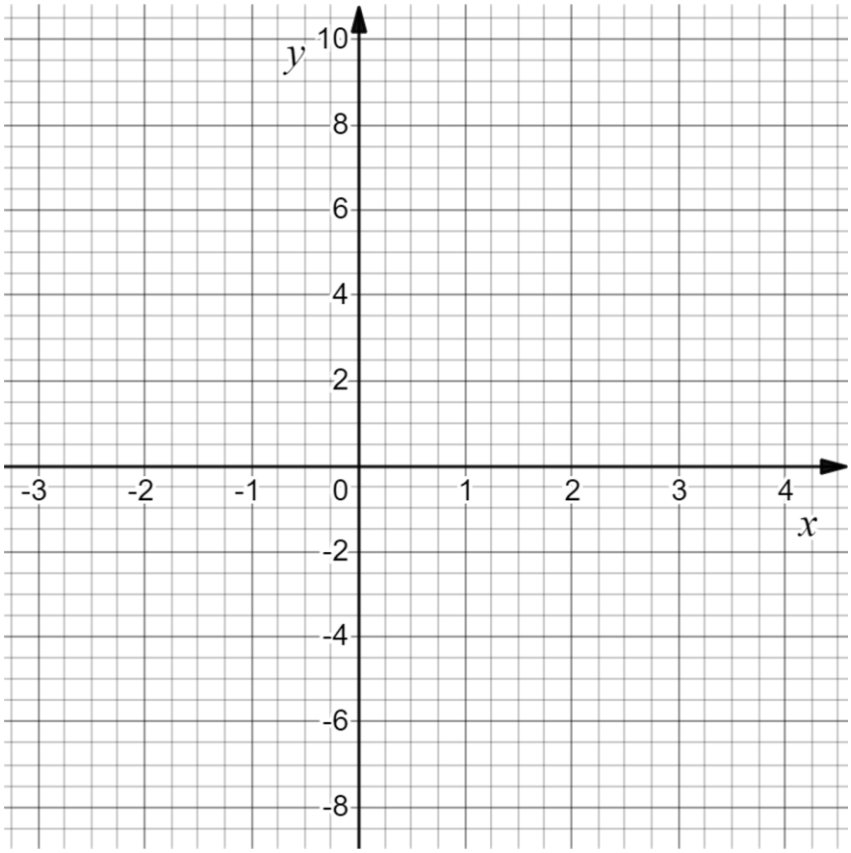
b) Solve $x^2 + 8x - 9 = 0$ by **completing the square**. **3**

c) Consider the parabola $y = x^2 - x - 6$

i) Find the x- and y- intercepts of the parabola. **3**

ii) Find the vertex of the parabola. **2**

iii) Graph the parabola showing the above information and the axis of symmetry. **2**



Question 17 Coordinate Geometry (7 marks) **Marks**

a) Does the point $(-2, 10)$ lie on the line $y = 3x - 4$? **2**
(Justify your answer with appropriate working)

b) Find the equation of the line perpendicular to $y = 2x - 6$ which passes through the point $(-3, 5)$.
Write your answer in **general form.** **3**

c) The midpoint of the interval AB is $(1, -4)$.
Given $A\ (-7, 2)$ find the coordinates of B. **2**

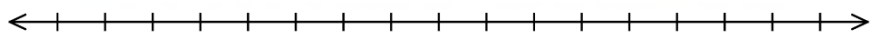
Question 18 Simultaneous Equations and Inequalities (8 marks) **Marks**

a) Solve the following pair of equations simultaneously. **3**

$$2x - 3y = 10$$
$$5x + y = 8$$

b) Solve the following inequality and graph the result on the number line.

$\frac{-2x}{3} > 4$ **2**



c) Show that the line $y = -x - 2$ is a tangent to the circle $x^2 + y^2 = 2$. **3**

Question 19 Probability (5 marks)

Marks

a) The numbers 1, 3 and 8 are written on separate identical cards and placed in a bag. The cards are then drawn at random **without replacement** to form a three-digit number.

What is the probability the number will be

i) greater than 200? **1**

ii) odd? **1**

iii) divisible by 3? **1**

b) Two 6-sided dice are rolled, and the **sum** of the numbers is calculated.

i) What is the probability that the score is **less than** 10? **1**

ii) Given at least one of the dice shows an even number,

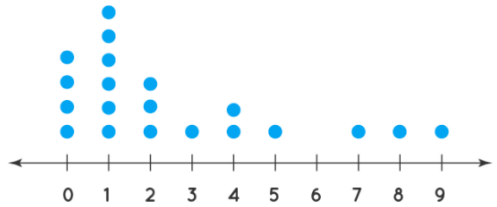
find the probability that the sum is 7. **1**

Question 20 Analysing Data (6 marks) **Marks**

a) Is the following stem and leaf plot best described as symmetrical, positively skewed or negatively skewed? **1**

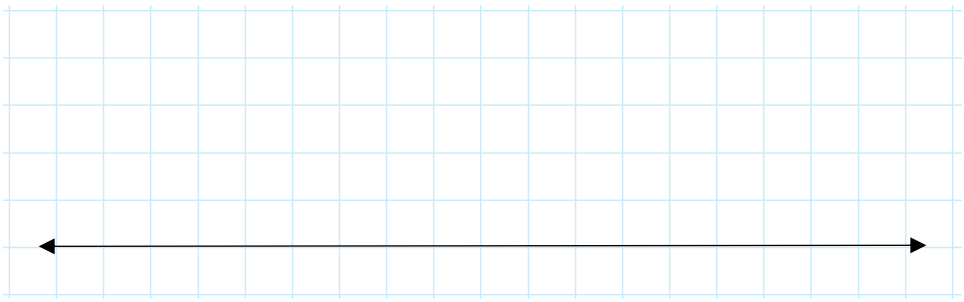
stem	leaf
0	1, 1, 2, 2, 3, 4, 4, 4, 4, 5, 8
1	0, 0, 0, 1, 1, 3, 7, 9
2	5, 5, 7, 7, 8, 8, 9, 9
3	0, 1, 1, 1, 2, 2, 2, 4, 5
4	0, 4, 8, 9
5	2, 6, 7, 7, 8
6	3, 6

b) For the following dot plot:



i) Give a 5-figure summary. **2**

ii) Construct a box plot. **2**



c) The points scored by a rugby league team are recorded in the table below.

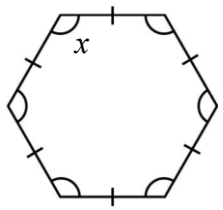
Week	1	2	3	4	5	6	7	8	Standard deviation
Dolphins	24	40	6	18	34	10	12	42	

Fill in the standard deviation for the Dolphins over the first 8 weeks of the season. **1**

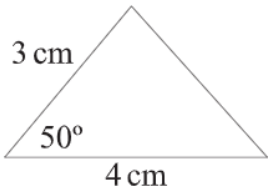
(round your answer correct to 1 decimal place)

Question 21 Trigonometry and Geometry (16 marks) **Marks**

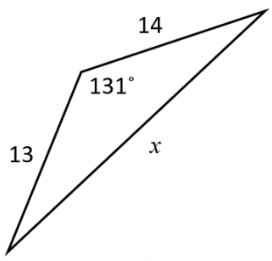
a) Find the size of angle x. **2**



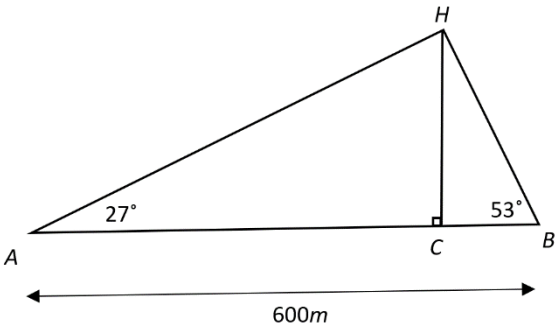
b) Find the area of this triangle, correct to 1 decimal place. **2**



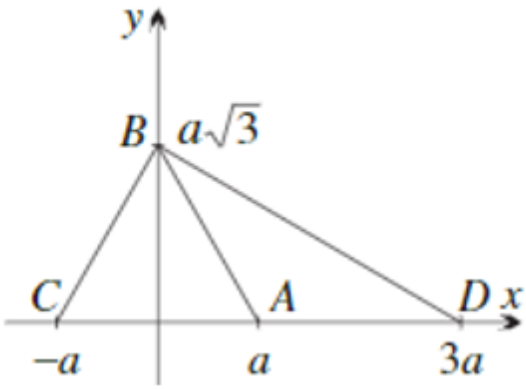
c) Find x, correct to 1 decimal place. **2**



d) A helicopter H is hovering above a straight, horizontal road AB of length 600 metres.
The angles of elevation of H from A and B are $27°$ and $53°$ respectively.
The point C lies on the road directly below H.
Find the height CH of the helicopter above the road, correct to the nearest metre. **3**

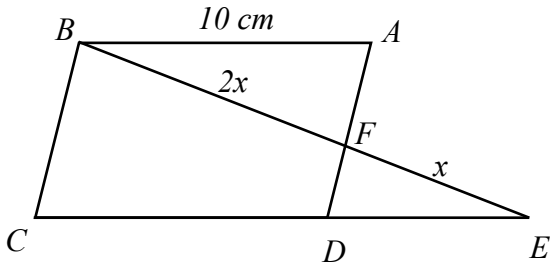


e) Given that ΔABC is equilateral, show $(AB)^2 = \frac{1}{3}(BD)^2$ **3**



f) *ABCD* is a parallelogram, with *CD* produced to *E*.

i) Prove that $\triangle ABF \,|||\, \triangle CEB$. **3**



ii) Hence, find the length *CE*. **1**

Question 22 Logarithms (8 marks)

Marks

a) Evaluate $\log_5 6 - \log_5 150$ using the log laws. **3**

b) Solve $2^x = 11$, correct to 3 decimal places. **2**

c) If $\log_a 3 = 1.099$, find the value of $\log_a(81a)$. **3**

Question 23 Mixed Questions (8 marks)

Marks

a) Line M has a *y*-intercept of -4, and its slope is obtained by multiplying $\frac{1}{7}$ by an integer. Given that Line M passes below $(4, -1)$ and above $(5, -6)$, how many possible slopes could Line M have? **3**

b) Solve: $2\log_{10} x + \log_{10} 3 = \log_{10} 75$ **3**

c) Given $x^2 - \frac{1}{x^2} = 2$,

simplify $(x + \frac{1}{x})^5 (x - \frac{1}{x})^4$, leaving your answer in terms of *x*. **2**

END OF PAPER

YEAR 10 - FORMULA SHEET

Simple Interest

$I = Prn$

P	is initial amount
r	is interest rate per period, expressed as a decimal
n	is number of periods

Compound Interest

$A = P(1 + r)^n$

A	is final amount
P	is initial amount
r	is interest rate per period, expressed as a decimal
n	is number of compounding periods

Depreciation

$A = P(1 - r)^n$

A	is final value of asset after n periods
P	is initial value of asset
r	is depreciation rate per period,

Gradient-intercept form of a line

$y = mx + b$

m	is gradient
b	is y-intercept

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

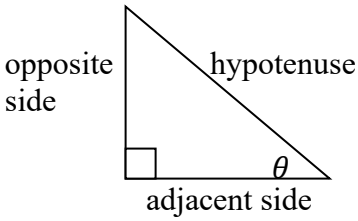
Point-gradient of the equation of a line

$$y - y_1 = m(x - x_1)$$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Ratios



$$\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

In ΔABC

$$c^2 = a^2 + b^2 - 2ab\cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Area of a triangle

In ΔABC

$$A = \frac{1}{2}ab\sin C$$

Circumference of a circle

$C = 2\pi r$ or $C = \pi D$

r is radius
D is diameter

Area

Circle

$A = \pi r^2$

r is radius

Sector

$A = \frac{\theta}{360}\pi r^2$

r is radius
θ is number of degrees in central angle

Annulus

$A = \pi\left(R^2 - r^2\right)$

R is radius of outer circle
r is radius of inner circle

Trapezium

$A = \frac{h}{2}(a + b)$

h is perpendicular height
a and b are the lengths of the parallel sides

Surface Area

Sphere

$A = 4\pi r^2$

r is radius

Closed cylinder

$A = 2\pi r^2 + 2\pi rh$

r is radius
h is perpendicular height

Volume

Prism or cylinder

$V = Ah$

A is area of base
h is perpendicular height

Pyramid or cone

$V = \frac{1}{3}Ah$

A is area of base
h is perpendicular height

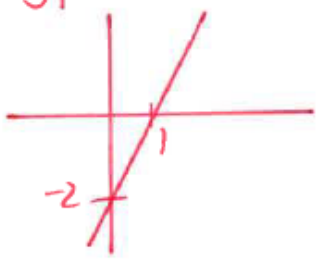
Volume and capacity

Unit conversion: 1 m^3 = 1000 L

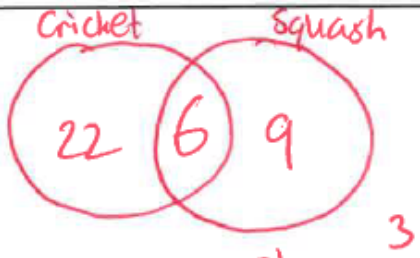
1. $11a+4c$
2. a^{36}
3. $5(2a+7c)$
4. $x^8 - x^5$
5. (i) $x^2 = 41^2 - 9^2$
   $x = 40$
   (ii) $\tan\theta = \frac{40}{9}$
   $\theta = 77.3^\circ$
6. (i) 3
   (ii) $-27 - -3 = -24$
7. a) $x = 19$
   b) $\frac{m}{4} = -6$
   $m = -24$
   c) $4p - 20 = 2p + 11$
   $p = 15.5$ or $\frac{31}{2}$
8. (i)

x	-3	-1	2	5
y	-8	-4	2	8

   (ii)



   (iii) (1, 0)
   (iv) $y = 2x - 8$
9. (i) 6
   (ii) Mean $= \frac{68}{10} = 6.8$
   (iii) $\frac{68+7x}{12} = 8$ $x = 4$
   (iv) $y = 6$
10. (i) (3.5, 2)
    (ii) $-\frac{2}{3}$
    (iii) (-7, 1) or (5, 9) or (17, 1) or (5, -7). Others possible
11. (i)



    (ii) $P(1\text{ sport}) = \frac{31}{40}$
12. If distance between 100km & meet after 1 hour
    Speed A $= 60\frac{km}{hr}$
    B $= 40\frac{km}{hr}$
    ∴ A arrives in 1hr 40mins
    ∴ B needs to travel remaining 60km in 40 mins
    ∴ Speed $= 90\frac{km}{hr}$
    $\frac{90}{40} = 2.25$
    ∴ 2.25 times faster

Yr 10 5.3 Final Exam Solutions

13. a) $3\sqrt{3} - 5\sqrt{3} + 8\sqrt{2}$
$-2\sqrt{3} + 8\sqrt{2}$

b) $\frac{\sqrt{5}+3}{7\sqrt{5}} \times \frac{\sqrt{5}}{5} = \frac{5+3\sqrt{5}}{35}$

c) $3 \times 1 \times (a^6)^{\frac{1}{2}} = 3a^3$

d) $= \frac{y^4}{(5x^3y)^2 \times x^2}$
$= \frac{y^4}{25x^6y^2 \times x^2}$
$= \frac{y^2}{25x^4}$

14. a) $A = 6500\left(1 + \frac{5\%}{12}\right)^{7\times12} = \$9217.23$
$I = 9217.23 - 6500 = \$2717.23$

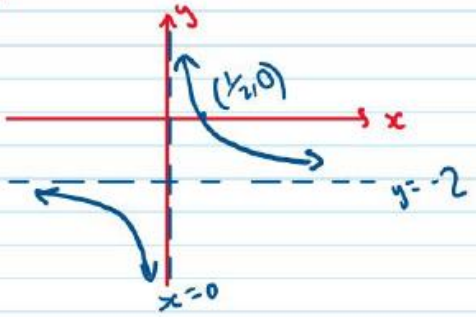
b) $A = 1099(1-15\%)^3 = \$674.92$

c) i) $90\% \times 17000 = \$15300$
ii) repayments $= 459 \times 12 \times 4$
$= \$22032$
$I = 22032 - 15300$
$= \$6732$
iii) $I = PRN$
$6732 = 15300 \times R \times 4$
$R = 0.11$ or 11% pa

15. a) i) $D = kT$
ii) $1020 = k \times 3$
$k = 340$
iii) $D = 340 \times 30$
$= 10200$ m

b) $y = -x^3 + 4$

c) i) set $y=0$ $0 = \frac{1}{2}x - 2$
$2 = \frac{1}{2}x$
$x = \frac{1}{2}$
ii) $x = 0$ & $y = -2$



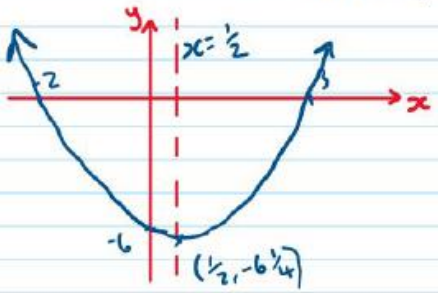
16. a) $x = \frac{-7 \pm \sqrt{49 - 4\times3\times-5}}{2\times3}$
$x = \frac{-7 \pm \sqrt{109}}{6}$

b) $x^2 + 8x + (4)^2 = 9 + 16$
$(x+4)^2 = 25$
$x+4 = \pm5$
$x = -4 \pm 5$
$x = 1, -9$

c) i) y-int: set $x=0$ $y=-6$ $(0,-6)$
x-ints: set $y=0$ $0 = (x-3)(x+2)$
$x = 3, -2$ $(3,0)$ & $(-2,0)$

ii) $x = \frac{-1}{2\times1}$
$x = \frac{1}{2}$
$y = (\frac{1}{2})^2 - (\frac{1}{2}) - 6 = -6\frac{1}{4}$
vertex $(\frac{1}{2}, -6\frac{1}{4})$

iii)



17. a) when $x=-2$ $y = 3\times-2 - 4$
$y = -10 \neq 10$
$\therefore (-2,10)$ not on the line

b) $m = -\frac{1}{2}$ & point $(-3,5)$
$y - 5 = -\frac{1}{2}(x+3)$
$2y - 10 = -x - 3$
$x + 2y - 7 = 0$

c) $\frac{-7+x}{2} = 1$ & $\frac{2+y}{2} = -4$
$x = 9$ $y = -10$
$\therefore B(9,-10)$ or diagrammatically

18. a) $y = 8 - 5x$ (B*)
sub (B*) into (A): $2x - 3(8-5x) = 10$
$17x - 24 = 10$
$17x = 34$
$x = 2$ → $y = 8 - 10$
$y = -2$

b) $-2x > 12$
$x < -6$

18.c) solve simultaneously
one solution → tangent

$y = -x - 2$
$x^2 + y^2 = 2$ $\therefore x^2 + (-x-2)^2 = 2$

$x^2 + x^2 + 4x + 4 = 2$
$2x^2 + 4x + 2 = 0$
$x^2 + 2x + 1 = 0$
$(x+1)^2 = 0$
$x = -1 \rightsquigarrow y = --1 - 2$
$= -1$

∴ touches and at $(-1, -1)$

19.a) i) $\frac{4}{6}$ or $\frac{2}{3}$

ii) $\frac{4}{6}$ or $\frac{2}{3}$

iii) $\frac{6}{6}$ or 1

138
183
318
381
813
831

b) i) $\frac{30}{36} = \frac{5}{6}$

ii) $\frac{6}{27} = \frac{2}{9}$

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

20.a) positively skewed

b) i)

min	Q_1	Q_2	Q_3	max
0	1	1.5	4	9

ii)

c) $\sigma_x = 13.1$

21.a) int $\angle$ sum $= (6-2) \times 180 = 720°$
each int $\angle = 720 \div 6 = 120°$

b) $A = \frac{1}{2} \times 3 \times 4 \times \sin 50°$
$= 4.596\ldots = 4.6\ \text{cm}^2$

c) $x^2 = 13^2 + 14^2 - 2 \times 13 \times 14 \times \cos 131°$
$x^2 = 603.8054\ldots$
$x = 24.57\ldots$
$x = 24.6$ units

21.d) $\angle AHB = 100°$

$\frac{AH}{\sin 53} = \frac{600}{\sin 100°}$

$AH = \frac{600 \sin 53}{\sin 100}$
$= 486.573\ldots$

$\sin 27 = \frac{CH}{486.573\ldots}$
$CH = 220.8997\ldots$
$= 221$ m

e) Applying Pythagoras:

LHS $= (AB)^2$
$= (a\sqrt{3})^2 + (a)^2$
$= 4a^2$

RHS $= \frac{1}{3}(BD)^2$
$= \frac{1}{3}[(3a)^2 + (a\sqrt{3})^2]$
$= \frac{1}{3} \times 12a^2$
$= 4a^2$

∴ LHS = RHS qed.

f) i) In $\triangle ABF$ & $\triangle CEB$
$\angle ABF = \angle CEB$ (alt. $\angle$'s in AB // CE)
$\angle FAB = \angle BCE$ (opp. $\angle$'s // gram ABCD)
∴ $\triangle ABF \,|||\, \triangle CEB$ (equiangular)

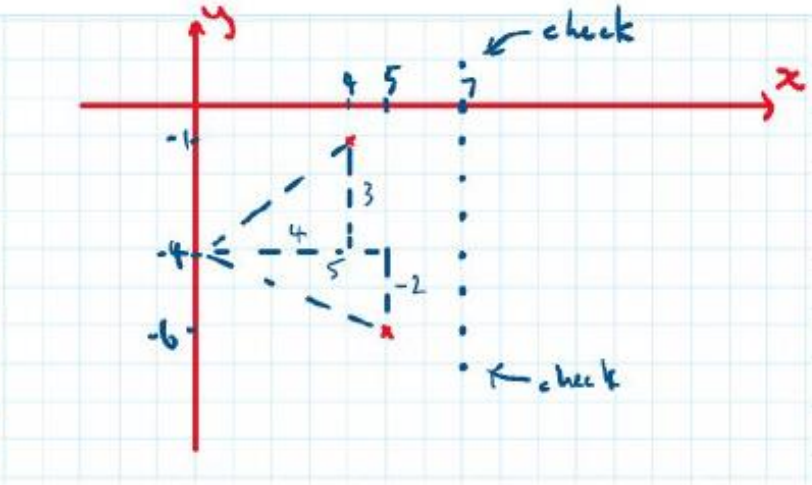
ii) $\frac{CE}{10} = \frac{3x}{2x}$ $\quad CE = 10 \times \frac{3}{2}$
$= 15$

22.a) $\log_5 \left(\frac{5}{250}\right)$
$= \log_5 \left(\frac{1}{25}\right)$
$= -2$

b) $x = \log_2 11$
$= \frac{\log_{10} 11}{\log_{10} 2}$
$= 3.4594\ldots$
$= 3.459$

c) $\log_a 81a = \log_a 81 + \log_a a$
$= \log_a 3^4 + 1$
$= 4 \times 1.099 + 1$
$= 5.396$

23. a)



top check: $y = \frac{5}{7}x - 4$ when $x = 4$ $y = -1.14$ $\therefore$ passes under $(4, -1)$ ✓

bottom check: $y = -\frac{3}{7}x - 4$ when $x = 5$ $y = -6.14$ $\therefore$ does not pass above $(5, -6)$ ×

$\therefore$ 8 solutions

OR

$$-\frac{2}{5} < m < \frac{3}{4}$$

$$-\frac{14}{5} < 7m < \frac{21}{4}$$

$$-2.8 < 7m < 5.25$$

$\therefore$ possible integers $= -2, -1, 0, 1, 2, 3, 4, 5$

8 solutions

b) $\log_{10} x^2 + \log_{10} 3 = \log_{10} 25 + \log_{10} 3$

$$\log_{10} x^2 = \log_{10} 25$$

$$x^2 = 25$$

$$x = \pm 5$$

$x = 5$ as $x = -5$ invalid for $2\log_{10} x$

c) $\left(x + \frac{1}{x}\right)\left(x + \frac{1}{x}\right)^4\left(x - \frac{1}{x}\right)^4$

$= \left(x + \frac{1}{x}\right)\left(x^2 - \frac{1}{x^2}\right)^4$

$= 2^4\left(x + \frac{1}{x}\right)$ or $16\left(x + \frac{1}{x}\right)$