Student Name:	
	Teacher Initials:

Barker College

8<sup>th</sup> November 2022

8:30AM 2 hours 260 copies

ARM/AYH ECB RDG AHP LMD JGD RAS SJB\* DOB JZT

# Year 10 5.3 MATHEMATICS Semester 2 Examination

#### **General Instructions**

- Write your name at the top of this page
- Write using blue or black pen
- Answer in the spaces provided
- NESA approved calculators may be used
- Show ALL necessary working
- Diagrams are NOT to scale
- Marks may not be awarded for careless or poorly arranged working
- A reference sheet is attached to the end of this paper, which may be detached.

Section	Marks
Part A – Common Section Total	/40
Part B – 5.3 Only Section Total	/100
Q1 Surds and Indices	/9
Q2 Interest and Depreciation	/9
Q3 Graphs	/11
Q4 Quadratic Equations and Parabolas	/13
Q5 Coordinate Geometry	/11
Q6 Simultaneous Equations and Inequalities	/6
Q7 Probability	/7
Q8 Analysing Data	/8
Q9 Trigonometry	/10
Q10 Logarithms	/6
Q11 Mixed Questions	/10
Exam Total:	/140

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#### Part A – Common Section (40 marks)

1. Simplify  $3x^3 \times 4x^2$ 

2

2. Expand and simplify 4(a + 5) + 2a

2

3. Fully factorise  $12b^2 - 6b$ 

2

4. **Solve** the following equations:

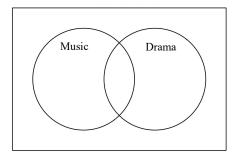
a) 
$$5x + 2 = 2x + 8$$

2

b)  $2d^2 = 98$ 

2

- 5. In a year 10 connect group, 1 student studied both music and drama, 2 studied only music, 4 studied only drama and 13 studied neither music nor drama.
  - i) Complete the Venn diagram below with the above information.



- ii) Calculate the **probability** that a randomly chosen student studies music.
- iii) Hence, in a year group of 360 students, calculate **how many students** would be expected to study music.
- Label each graph with the correct equation, chosen from the list below.
   Two equations will not be used.

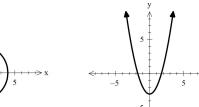
$$y = x^2 - 3$$

Equation:

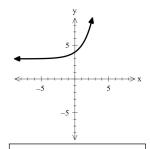
$$x^2 + y^2 = 16$$

$$y = 2^x - 3$$

$$x^2 + y^2 = 4 \qquad y$$







Equation:

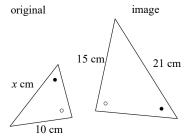
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7. Two similar triangles are shown in the diagram below.



- i) Calculate the scale factor of the two triangles
- ii) Hence, find the length of side length x.
- 8. A back-to-back stem-and-leaf plot for the points scored by two basketball teams during a season is shown below.

Team A	Stem	Team B
1	7	4 7
5	8	5 8
8 2	9	1
4	10	7
9 3 2	11	3 8
6 4	12	2 5
1	13	4

i) Complete the **5-figure summary** for Team A.

ii) The interquartile range for Team B is 37. Which Team has more consistent scores and why? 2

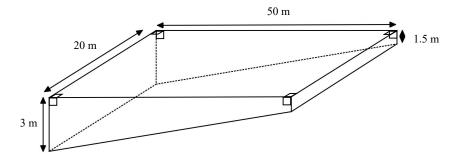
9. Solve simultaneously.

y = 3x + 5y = x - 5

3

3

10. A 50-metre Olympic swimming pool with a shallow and deep end has dimensions as shown on the diagram below. The diagram is not drawn to scale.



Calculate the **capacity** of the pool, in litres.

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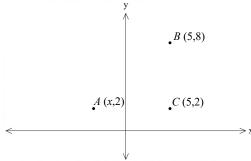
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11. Two banks offer the following deals over a 10-year period.

BANK A: 1.25% p.a. interest, compounding monthly BANK B: 1.5% p.a. simple interest

Which bank deal offers the greatest interest on an investment of \$1000? **Show calculations** for both bank deals in your answer.

12. The points A, B and C are shown on the number plane below.



i) The straight-line distance of the interval *AB* is 10 units. Calculate the *x*-coordinate of point A.

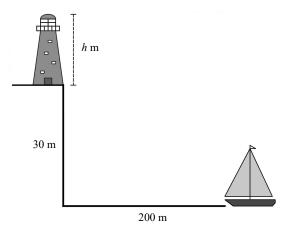
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ii) Hence, calculate the gradient of AB.

2

7

13. A boat is 200m away from the bottom of a 30m high cliff. The angle of elevation from the base of the boat to the top of the lighthouse is 10°.



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Calculate the **height** of the lighthouse, *h*, to 1 decimal place.

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#### PART B – 5.3 Only Section (100 marks)

Question 1 Surds and Indices (9 marks)

(a) **Fully** simplify:  
(a) 
$$3\sqrt{5} - \sqrt{27} + \sqrt{20} + \sqrt{5}$$

$$(\beta) \quad 7\sqrt{2} \times 2\sqrt{5}$$

2

3

(b) Rationalise and simplify:

$$\frac{\sqrt{3}+1}{4\sqrt{3}}$$

(c) Simplify the following, expressing with only positive indices:

$$\frac{9x^6y^{-2}}{(3x^3y)^3}$$

#### Question 2 Interest and Depreciation (9 marks)

(a) A television costing \$4990 is offered at sale at 10% deposit, followed by 24 monthly payments of \$235.78.

2

3

3

(i) Find the <b>total cost</b> of the television to the customer who buys it on terms.
--

(ii) Find the total amount of **interest** paid.

(iii) Find the **flat rate** of interest charged per annum, to the nearest percent.

(b) A video cassette recorder costing \$3950 depreciates at 32% p.a. How many full years would it take for its value to be less than \$1000?

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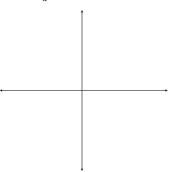
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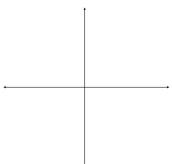
Question 3 Graphs (11 marks)

(a) Sketch  $y = (x - 1)^3$ , showing all intercepts.

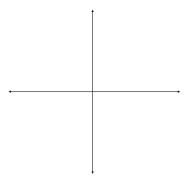


(b) Sketch  $y = \frac{1}{x} + 3$ , showing all asymptotes and intercepts.





(d) Sketch  $y = -2^{-x}$ , showing any asymptotes and intercepts.



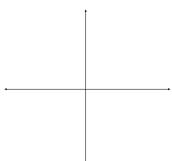
(e) The number of days, D, to complete research is inversely proportional to the number of researchers, R, who are working. Hence  $D = \frac{k}{R}$ . The research takes 125 days to complete with 16 researchers working on it.

(ii) How many researchers would be needed to complete the research within 40 days?

(i) Find the value of *k*.

2

(c) Sketch y = (x + 4)(x + 1)(x - 3), showing all intercepts.



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(d) For the parabola  $y = 2x^2 + 6x - 20$ :

#### Question 4 Quadratic Equations and Parabolas (13 marks)

(a) **Fully** factorise:  $8x^2 - 50$ 

2

2

3

(b) Solve the following quadratic equation using the quadratic formula:

$$5x - 2x^2 = 1$$

Leave your answer in **exact** form.

(c) Solve the following quadratic equation by completing the square:

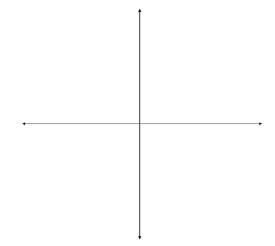
$$x^2 - 8x - 5 = 0$$

Leave your answer in exact form.

(i) Find the **x-intercepts**.

(ii) State the coordinates of the vertex.

(iii) Sketch the graph, labelling all intercepts and the vertex.



2

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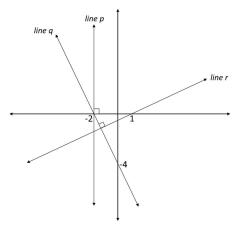
#### **Question 5 Coordinate Geometry (11 marks)**

- (a) A triangle is formed by joining the points P(0,3), Q(1,7) and R(5,8).
  - (i) Find the **exact** length of PQ

(ii) **Prove** that  $\Delta PQR$  is isosceles

(b) The point M(3,7) is the midpoint of the interval joining A(1,12) and B(x,y). Find the **coordinates** of B.

(c) For the following diagram, find the equation of:



(i) line *p* 

(ii) line q in gradient-intercept form 2

(iii) line r in general form 3

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Question 6 Simultaneous Equations and Inequalities (6 marks)

(a) Solve the following equations simultaneously:

$$x - 2y = 8$$
$$2x + y = 1$$

(b) **Solve** the following inequality and **graph the solution** on a number line:

$$1 - 2x \ge 9$$

#### Question 7 Probability (7 marks)

(a) A container holds 11 cards numbered from 3 to 13. A card is selected at random. What is the probability that the selected card is:

(i) even 1

(ii) odd and greater than 6

(iii) the number 6, given that the card selected is known to be even

- (b) A packet contains 5 red lollies and 14 green lollies. Two lollies are selected at random without replacement.
  - (i) Draw a tree diagram to show the possible outcomes. Include the probability on each branch. 2

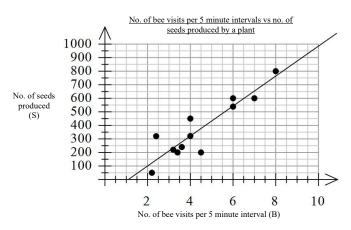
(ii) What is the probability that the two lollies are of **different colours**.

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#### **Question 8 Analysing Data (8 marks)**

(a) The following scatterplot shows how the number of bee visits per 5 minute interval affects the number of seeds a plant produces. A line of best fit is also shown.



(i) Circle ONE of the following that best describes the correlation:

Strong & Negative

Strong & Positive

Weak & Negative

Weak & Positive

3

(ii) Use the two data values (4, 321.41) and (6, 538.15) to find the equation of the line of best fit.

(iii) Use the equation of the line of best fit to approximate the number of seeds produced if there were 9 bee visits in a 5 minute interval.

(b) For a class of 10 students, two tests out of 100 marks were given. In the Latin test, the mean was 68.5 and the standard deviation was 14.2. In the French test, the marks were the following:

(i) For the French test, calculate the mean and standard deviation, correct to one decimal place. 2

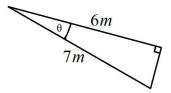
(ii) For which test did the results vary the most? **Justify** your answer.

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#### Question 9 Trigonometry (10 marks)

(a) Find  $\theta$ , correct to the **nearest degree**.

2



- (b) Triangle ABC has an **obtuse angle** at A, AB = 3.2cm, BC = 4.6cm and  $\angle ACB = 32^{\circ}46'$ .
  - (i) Sketch  $\triangle ABC$ , labelling all given information.

1

(ii) Evaluate  $\angle BAC$ , to the **nearest minute**.

3

- (c) Miss Iles sets sail from her home city of Sydney. She sails on a bearing of  $110^{\circ}T$  for 15km before stopping at point A. She then sails on a bearing of  $240^{\circ}T$  for 30km to point B.
  - (i) Draw a diagram, showing all given information.

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(ii) How far is Miss Iles from Sydney? Give your answer to the **nearest kilometre**. 3

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#### Question 10 Logarithms (6 marks)

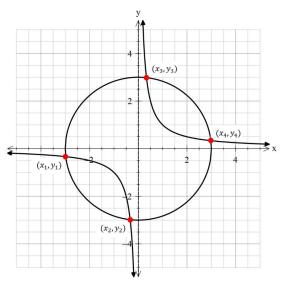
- (a) Write  $r^s = t$  in logarithmic form.
- (b) Evaluate  $\log_2 32$

(c) Solve x, correct to **2 decimal places**.  $10^x = 26$ 

(d) If  $\log_a b = 3.4$  and  $\log_a c = 4.7$ , evaluate  $\log_a \left(\frac{a^2 b}{c}\right)$ 

#### **Question 11 Mixed Questions (10 marks)**

(a) Below are the graphs of  $y = \frac{1}{x}$  and  $x^2 + y^2 = 9$ .



3

Find  $x_1, x_2, x_3$  and  $x_4$ . Round to **2 decimal places**.

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(b) A game involves rolling two six-sided dice, followed by rolling a third six-sided die.

To win the game, the number rolled on the third die must be less than the lowest number or higher than the highest number from the first two rolls.

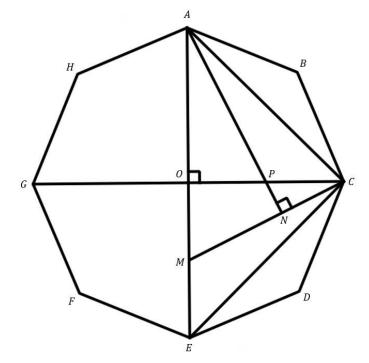
For example, if the first two dice show 2 and 5, the game can only be won by rolling a 1 or 6 with the third die. If a double is rolled on the first two dice, then the game can only be won by rolling any of the other five numbers with the third die.

(i) What is the probability that a player has no chance of winning when rolling the third die?

(ii) What is the probability that a player wins the game?

3

(c) The following figure is a regular octagon.
M is a point that lies on the interval AE and N lies on the interval CM.
The intervals CM and AN are perpendicular to each other.
The intervals AE and GC are perpendicular to each other.



(i) Prove  $\triangle ABC \equiv \triangle CDE$ 

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(ii) List all pairs of similar triangles in the diagram.

Do not include any pairs that are also congruent.

(Make sure your vertices correspond in the named similar triangles)

(iii) List all pairs of congruent triangles.

Year 10 Semoster 2 Examination

PARTA: Common Section

1. 12x<sup>5</sup>

$$2.4a + 20 + 2a = 6a + 20$$

3.6b(2b-1)

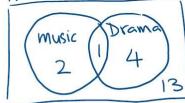
4. a) 
$$5x + 2 = 2x + 8$$

$$3x = 6$$

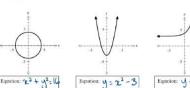
$$d^2 = 49$$

$$d = \pm 1$$

5. i)



ii)



$$7.i)\frac{3}{2}=1.5$$

(ii)

Team A's IQR = 124-92

Team A is more consistent because Team A has smaller value of IQR (22 <37).

$$3x + 5 = x - 5$$

Substitute x = -5 into (2)

$$y = -5 - 5$$

: x = -1010. Area of cross-section=  $\frac{1}{2}x50x(1.5+3) = 112.5m^2$   $V_2 = 112.5 \times 20 = 2250 \text{ m}^3$ 

$$A = 1000 \left(1 + \frac{1.25\%}{12}\right)^{10 \times 12}$$

### Bank B:

.. Bank B offers the greatest interest.

#### 12. i)

$$10=\sqrt{(x-5)^2+(2-8)^2}$$

$$100 = (x-5)^2 + 36$$

$$64 = (\chi - 5)^2$$

$$\pm \sqrt{64} = x - 5$$

$$x = 5 \pm 8$$

$$x = 13$$
 or  $x = 3$ 

ii)

$$M = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 2}{5 + 3} = \frac{6}{8} = \frac{3}{4}$$

13

$$tan 10° = \frac{(h+30)}{200}$$

$$h = 5.26$$

Year 10 5:3 Final Exam Solutions 03 Graphs

Or Surds & Indices

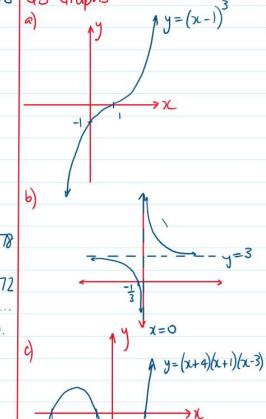
- a) a) 3/5-3/3+2/5+/5 = 6/5-3/3
  - B) 14510
- b)  $\frac{\sqrt{3}+1}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3+\sqrt{3}}{12}$
- c)= $\frac{9x^6y^2}{27x^9y^3} = \frac{1}{3x^3y^5}$

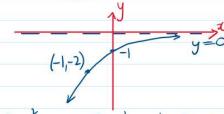
QZ Interest and Depreciation

- a) i) =  $0.1 \times 4990 + 24 \times 235.78$ = \$6157.72
  - 1) 6157.72 4990 = \$1167.72
  - $\frac{1167.72}{(4990-499)} \div 2 = 0.1300...$  = 131. p.q.
- b)  $1000 = 3950(1 0.32)^n$  $0.2531 = 0.68^n$

$$N = \log 0.2531$$
 $\log 0.68$ 

$$n = 3.5626...$$





$$0) D = \frac{k}{R}$$
  $125 = \frac{k}{16}$   $k = 2000$ 

$$) ) = 2000$$

$$R = \frac{2000}{40}$$
  $R = 50$ 

$$a) = 2(4x^2 - 25)$$

$$= 2(2x + 5)(2x - 5)$$

$$2x^{2}-5x+1=0$$

$$x = 5 \pm \sqrt{(-5)^{2}-4x2x1}$$

$$x^{2} - 8x - 5 = 0$$

$$x^{2} - 8x = 5$$

$$x^{2} - 2.4x + 16 = 5 + 16$$

$$x - 4 = \pm \sqrt{2}$$

$$x = 4 \pm \sqrt{2}$$

d) i) 
$$y = 2(x^2 + 3x - 10)$$
  
=  $2(x + 5)(x-2)$   
when  $y=0$ 

 $(x-4)^2 = 21$ 

$$\chi = -5$$
 or  $\chi = 2$ 

$$11) \quad x = -\frac{1}{2a} = -\frac{3}{2}.$$

$$y = 2 \times \left(\frac{3}{2}\right)^{2} + 6 \times \left(\frac{-3}{2}\right) - 20$$

$$= \frac{9}{2} - \frac{18}{2} - 20 = -\frac{19}{2}$$

Verky 
$$\left(\frac{-3}{2}, \frac{-49}{2}\right)$$

a) i) 
$$PQ = \sqrt{(-0)^2 + (7-3)^2}$$
  
=  $\sqrt{1+16}$   
=  $\sqrt{17}$ 

ii) OR = 
$$\sqrt{(-5)^2 + (78)^2}$$
  
=  $\sqrt{10+1}$  =  $\sqrt{17}$ 

b) 
$$\frac{x+1}{2} = 3$$
  $\frac{y+12}{2} = 7$   
 $x = 6 - 1$   $y = 14 - 12$   
 $y = 2$ 

$$B(5,-2)$$

ii) gradient = 
$$-\frac{4}{2}$$
 =  $-2$ 

$$y \cdot int = -4.$$
  
 $y = -2x-4$ 

iii) gradient line 
$$r$$

$$m = \frac{1}{2} = \frac{1}{2}$$

pt 
$$(1,0)$$
 is on the line  
 $y-y_1 = m(x-x_1)$   
 $y-0 = \frac{1}{2}(x-1)$   
 $2y = x-1$   
 $x - 2y - 1 = 0$  general form

## ab Simultaneous Equations.

$$x - 2y = 8 \quad 0$$

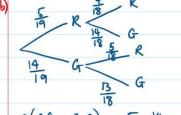
$$@ x2 = 47C+2y = 2$$

sub 
$$x=2$$
 into  $0$   
 $2-2y=8$ 

$$-2y = 6 \qquad y = -3$$
Solution  $x = 2 \quad y = -3$ 

b) 
$$1-2x > 9$$
  
 $-2x > 8$   
 $x < -4$   
Question 7 Pubability

a) i) 
$$P(even) = \frac{5}{11}$$
ii)  $P(odd and greater than b) = \frac{4}{11}$ 
iii)  $P(b|even) = \frac{1}{5}$ 



ii) 
$$M = \frac{538.15 - 321.41}{6-4}$$

$$= 216.74 = 108.37$$

$$S - 321.41 = 108.37(8-4)$$

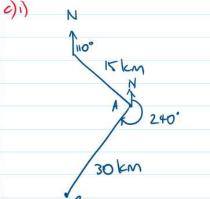
$$S = 108.37B - 112.07$$
ii) When  $B = 9$ 

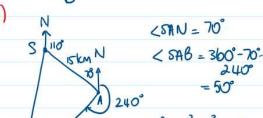
Approx. 
$$\neq$$
 of seeds is 863  
b)  $\pi = 62.5$ 

$$\sigma = 8.913...$$

$$0 = (05^{-1}) = 31^{\circ} \text{ (Nearest degree)}$$

i) 
$$\frac{\sin \angle BAC}{46} = \frac{\sin 32^{\circ} 46'}{3.2}$$





$$58^2 = 15^2 + 30^2 - 2x15x$$
  
 $30 \times 0000$   
 $58^2 = 546 \cdot 4911...$ 

#### 010 Logarithms

$$s = \log_{10} t$$

$$\log_2 2^5 = 5 \log_2 2$$

c) 
$$\log_{10}26 = x$$
  
 $x = 1.41497$   
 $= 1.41(2d.p)$ 

d) 
$$\log_{a}\left(\frac{a^{2}b}{c}\right)$$

= 
$$2\log_a a + \log_a b - \log_a c$$
  
=  $2 + 3.4 - 47$ 

## Q11 Mixed Questions

a) 
$$y = \frac{1}{x}$$
 (1)  $x^2 + y^2 = 9$  (2)

$$\chi^2 + \frac{1}{\chi^2} = 9$$

$$\chi^{4} + 1 = 9\chi^{2}$$
  
 $\chi^{4} - 9\chi^{2} + 1 = 0$ 

$$u^2 - 9u + 1 = 0$$
.  
 $u = 9 + \sqrt{81 - 4}$ 

$$\chi = \pm \sqrt{\frac{9+\sqrt{77}}{2}} \quad \text{or} \quad$$

$$\chi = \pm \sqrt{\frac{9-\sqrt{71}}{2}}$$

$$x = \pm 0.3354...$$
  
 $\pm 0.34 (2.d.p).$ 

$$x_1 = -2.98$$
  $x_3 = 0.34$   $x_4 = 2.98$ 

## no chance: 16 or 61

$$P(\text{no chance}) = \frac{2}{36} = \frac{1}{18}$$

① +000 ways to win 
$$\frac{2}{6} \times \frac{6}{36}$$
 14, 41, 25,52, 36,63

$$P(win) = 4 + 12 + 24 + 40 + 30$$

$$= \frac{110}{216} = \frac{55}{108}$$

$$\triangle AOP = \triangle COM$$

$$\triangle$$
 AOC  $\equiv$   $\triangle$ EOC

△ APC = △CME