



Barker
College

ARM	RDG
LMD	RAS
DOB*	JZT
ECB	AHP
JGD	SJB

YEAR 10

Wednesday, 11th May

TERM 2, 2022

TOTAL TIME: 90 MINUTES

5.3 MATHEMATICS

Semester 1 Examination

260 COPIES

INSTRUCTIONS TO STUDENTS:

Write your name and teacher's initials on the TOP of EVERY marked SHEET of PAPER.

Attempt ALL questions.

Show ALL necessary working.

Calculators can be used throughout the examination.

Marks may not be awarded for careless or badly arranged work.

Diagrams are NOT drawn to scale.

Write your answers in the spaces provided on the paper.

A formula sheet is provided on page 2 for use throughout the examination. Detach this sheet.

This examination consists of SIX parts.

PART A: COMMON SECTION	(36 marks)
PART B: SURDS & INDICES	(15 marks)
PART C: INTEREST & DEPRECIATION	(12 marks)
PART D: QUADRATIC EQUATIONS & PARABOLAS	(29 marks)
PART E: GRAPHS	(16 marks)
PART F: MIXED QUESTIONS	(10 marks)
TOTAL 118 Marks	

Student's Name:

Teacher's Initials:

FORMULA SHEET

Pythagoras' Theorem

$$c^2 = a^2 + b^2$$

Simple interest

$$I = Prn$$

Compound interest

$$A = P(1+r)^n$$

Depreciation

$$A = P(1-r)^n$$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Student's Name:

Teacher's Initials:

PART A: COMMON SECTION (36 Marks)

	Marks
1. Simplify $3a + 9a - 2a$	1
2. If the interest rate is 18% p.a. what is the interest rate per month?	1
3. Solve $6a - 8 = 10$	2
4. Find the simple interest accumulated if \$9500 is invested for 12 years at 7.5% p.a.	2
5. Expand and collect like terms: $x(x + 2) - 2x$	2
6. Simplify fully: $8x^9 \div 2x^3$	2

-
7. Find n if $(a^4)^6 = a^{n+2}$ 2
-
8. Evaluate $3a^0 - 4$ 2
-
9. If $g = 8$, $h = -5$ and $f = 2$, evaluate $h^2 - gf$. 2
-
10. Fully factorise $10x^2 + 8x^3$ 2
-
11. Simplify $\frac{4x-3}{5} + \frac{x-5}{7}$ 3
-
12. \$17600 is invested for 8 years, compounding monthly at 5% p.a. What is the value of the investment at the end of the 8 years? 3

Student's Name:

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13. Penny bought a used car for \$8400. The car depreciates in value annually at a rate of 21% p.a.

Calculate the amount of depreciation after 4 years. Give your answer correct to the nearest cent. 3

14. Fully simplify $\frac{4a^3b^7c^9}{b^3} \div \frac{(a^5b^2)^3}{12c^4}$

3

15. The speed of sound is 332 metres per second in air and 1480 metres per second in water.
There is a bird high in the air and a fish deep under water both 10 000m away from a loud explosion.

If $Speed = \frac{Distance}{Time}$;

- i. How long does it take the bird and the fish to hear the explosion? Answer in seconds to two decimal places. 2

- ii. What is the time gap in-between? Answer in seconds to two decimal places. 1

16. Mark has invested \$x at $y\%$ p.a for 5 years simple interest. 3

In terms of x and y , what is the total worth of the investment, as one fraction, in simplest form?

Student's Name:

Teacher's Initials:

PART B: SURDS & INDICES (15 Marks)

Marks

Question 1

Fully simplify, leaving your answers in simplest surd form.

(a) $\sqrt{80}$

1

(b) $\sqrt{8} \times \sqrt{7} \div \sqrt{28}$

2

(c) $\frac{\sqrt{3}}{2} + \frac{\sqrt{27}}{6}$

2

(d) $(\sqrt{5})^3 + \sqrt{125}$

2

Question 2

Expand & Simplify: $(3\sqrt{7} + \sqrt{6})(4\sqrt{6} + 2\sqrt{7})$

2

Question 3

Prove the following is rational.

3

$$\frac{2}{\sqrt{2}} - \frac{1}{1+\sqrt{2}}$$

Question 4

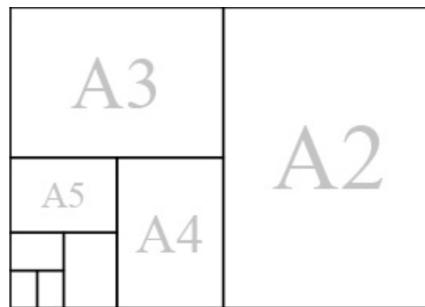
Metric paper, such as A4 sized paper, always has the longer side to shorter side in ratio $\sqrt{2}: 1$.

3

A3 paper is twice as large as A4 paper. The shorter side of A3 paper is the same length as the longer side of A4 paper. This same trend follows from A3 paper to A2 paper. This can be seen in the diagram below.

If A4 paper has a shorter side of 210 mm, what are the dimensions of A2 paper?

Leave your answer in exact form.



End of PART B

Student's Name:

Teacher's Initials:

Question 7

\$50 000 is invested in the local bank and is compounded annually.

3

If after 20 years, the value of the investment is \$81 930, find the annual interest rate, expressed as a percentage correct to 1 decimal place.

PART C: INTEREST & DEPRECIATION (12 marks) Marks

Question 5

A ski trip to Canada is priced at \$8500. Maeve pays a deposit of \$1700. The remainder she repays at \$300 per month for 30 months.

- | | | |
|------|---|---|
| i. | What is the total amount paid for the ski trip? | 1 |
| ii. | Calculate the total interest Maeve paid for the ski trip. | 1 |
| iii. | What was the annual flat interest rate for this purchase, expressed as a percentage correct to 1 decimal place? | 3 |

Question 6

Mark buys a popular Pokémon card for \$500. It doubles in worth every year for 3 years.

- | | | |
|-----|--|---|
| i. | What is the card's value at the end of the third year? | 1 |
| ii. | After the 3 years the card is no longer popular and depreciates at 15% of its current value each year. Mark sells it after a further 10 years. | 3 |

Did Mark make a profit or a loss? Find this value using calculations.

End of PART C

Student's Name:

Teacher's Initials:

Question 10

Use the **quadratic formula** to solve $-1 - 4x - x^2 = 0$ correct to 2 decimal places.

2

PART D: QUADRATIC EQUATIONS & PARABOLAS (29 marks) Marks

Question 8

Fully factorise the following.

(a) $x^2 + 5x + 6$

1

(b) $3x^2 - 13x + 10$

2

Question 11

Use the **completing the square** method to solve $x^2 - 12x - 18 = 0$

Leave your answer in simplest exact form.

3

Question 9

Solve the following equations.

(a) $x^2 + 21x + 20 = 0$

2

(b) $x^2 - 30x = 0$

2

Question 12

Use the substitution $u = x^4$ to solve $x^8 - 97x^4 + 1296 = 0$

3

(c) $3x^2 - 48 = 0$

2

Student's Name:

Teacher's Initials:

Question 13

For the parabola $y = x^2 - 7x + 10$

- i. Find the y -intercept.

1

- ii. Find the x -intercepts.

2

- iii. Find the equation for the axis of symmetry.

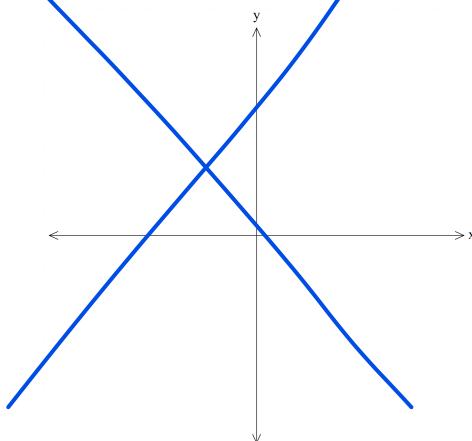
1

- iv. Find the co-ordinates of the vertex

2

- v. Graph the parabola showing the vertex and the intercepts.

2



Question 14

In a right-angled triangle, the second shortest side is 8cm longer than the shortest. The triangle has an area of 52.5cm^2 .

- i. Draw a diagram to represent this information

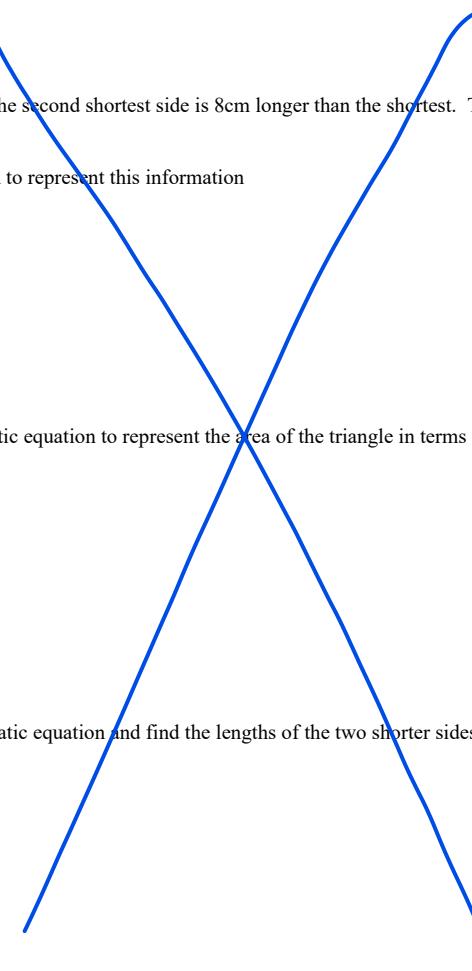
1

- ii. Create a quadratic equation to represent the area of the triangle in terms of x .

1

- iii. Solve the quadratic equation and find the lengths of the two shorter sides of the right-angled triangle.

2



Student's Name:

Teacher's Initials:

PART E: GRAPHS (16 marks)

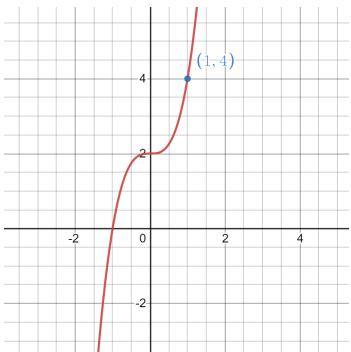
Marks

Question 15

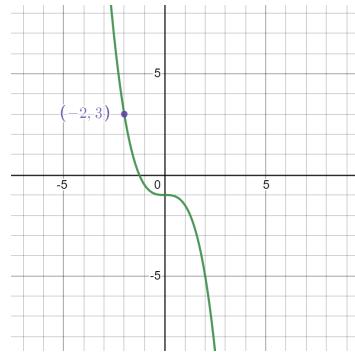
Match the following curves with their respective equations.

3

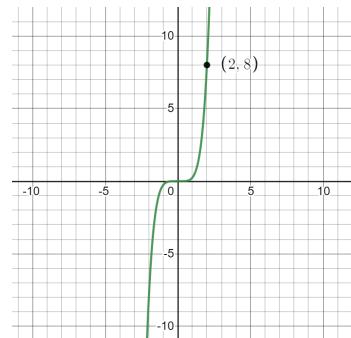
$$y = (x - 2)^3, \quad y = 2x^3 + 2, \quad y = \frac{-x^3}{2} - 1, \quad y = 0.25x^5$$



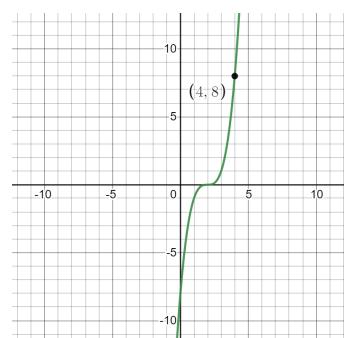
i.



ii.



iii.



iv.

Question 16

Consider the graph $y = -(x + 1)(x - 2)(x - 4)$

- i. What type of curve is this?

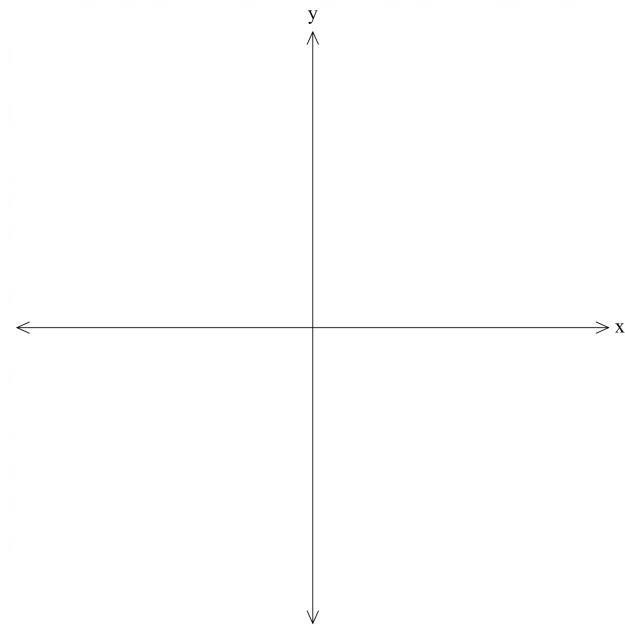
1

- ii. Find the x -intercepts and the y -intercept.

2

- iii. Graph the equation $y = -(x + 1)(x - 2)(x - 4)$ showing all the intercepts.

2



Student's Name:

Teacher's Initials:

Question 18

The graph below is a power curve in the form $y = k(x - b)^6 - 1$

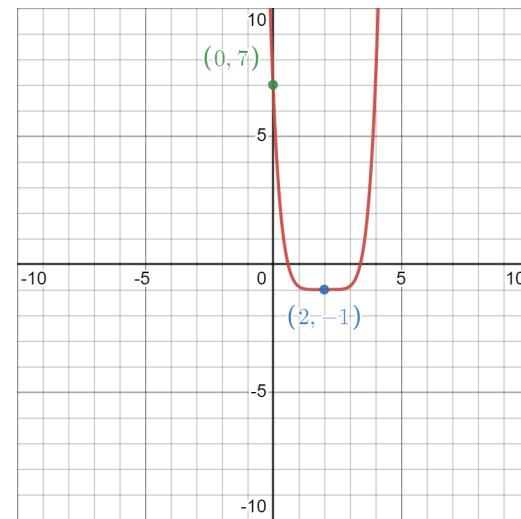
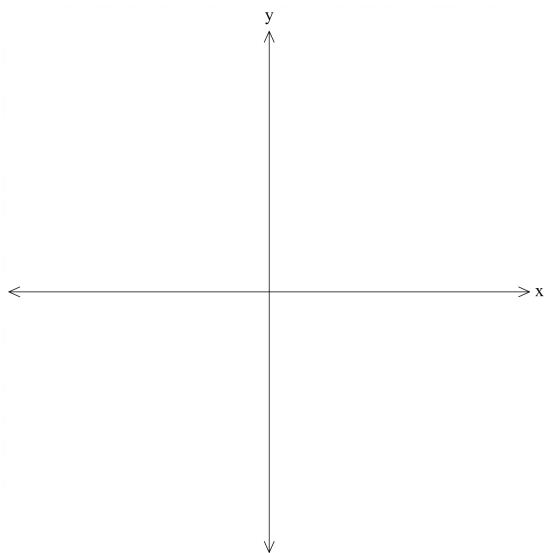
Question 17

Consider the graph $y = \frac{1}{x-3}$

- i. What type of graph is this? 1

- ii. What are the vertical and horizontal asymptotes of this graph? 1

- iii. Sketch the graph $y = \frac{1}{x-3}$ showing the asymptotes and mark the coordinates of one point. 3



- i. What is the value of b? 1

- ii. What is the value of k? 2

End of PART E

Student's Name:

Teacher's Initials:

PART F: MIXED QUESTIONS (10 marks)

Marks

Question 19

$$(\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$$

Use this to find the square root of $(12 + \sqrt{140})$ in the form $\sqrt{a} + \sqrt{b}$

2

Question 20

Solve $x^4 - x^2 + 18x - 81 = 0$

3

Leave your answer in exact form

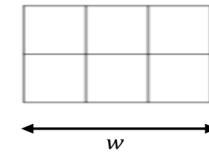
Question 21

If $xy = \frac{1}{6}$ find the value of $\frac{2^{[(3x+2y)^2]}}{2^{[(3x-2y)^2]}}$

2

Question 22

A window frame consisting of 6 identical rectangles is illustrated below with w being the width of the whole window. Only 12 metres of frame is available for construction of all 6 rectangles.



- i. Show that the area of the window is given by $A = 3w - \frac{3w^2}{4}$

2

- ii. Hence or otherwise find the width of the whole window when the area of the window is at its maximum.

1

End of Paper

PART A.	
1. $10a$	
2. $18\% \div 12 = 1.5\%$	
3. $6a = 18$ $a = 3$	
4. $I = 9500 \times 12 \times 7.5\%$ $= \$8550$	
5. $x(x+2) - 2x = x^2 + 2x - 2x$ $= x^2$	
6. $4x^6$	
7. $(a^4)^6 = a^{24} = a^{n+2}$ $\therefore n = 22$	
8. $3a^0 - 4 = 3 \times 1 - 4$ $= 3 - 4 = -1$	
9. $h^2 - gf = (-5)^2 - 8 \times 2$ $= 25 - 16$ $= 9$	
10. $10x^2 + 8x^3 = 2x^2(5 + 4x)$	
11. $\frac{4x-3}{5} + \frac{x-5}{7}$ $= 7(4x-3) + 5(x-5)$ $\frac{35}{35}$ $= 28x - 21 + 5x - 25$ $\frac{35}{35}$ $= \frac{33x - 46}{35}$	
12. $r = 5\% \div 12 = \frac{5}{12}\%$ $n = 8 \times 12 = 96$ $A = 17600 (1 + \frac{5}{12}\%)^{96}$ $= \$262340.30$	

13. $S = 8400 (1 - 21\%)^4$ $= \$3271.81$
$\$8400 - \3271.81 $= \$5128.19$
14. $\frac{4a^3b^7c^9}{b^3} \div \frac{(a^5b^2)^3}{12c^4}$ $= \frac{4a^3b^7c^9}{b^3} \times \frac{12c^4}{a^{15}b^6}$ $= \frac{48a^3b^7c^{13}}{a^{15}b^9}$ $= \frac{48c^{13}}{a^{12}b^2}$
15. BIRD: $332 = \frac{10000}{t}$ $t = 30.12 \text{ secs}$
FISH: $1480 = \frac{10000}{t}$ $t = 6.76 \text{ secs}$
ii. $30.12 - 6.76 = 23.36 \text{ secs}$
16. $I = x \times \frac{4}{100} \times 5$ $= \frac{5xy}{100} = \frac{xy}{20}$
TOTAL $= x + \frac{xy}{20}$ $= \frac{20x}{20} + \frac{xy}{20}$ $= \frac{\$20x + xy}{20}$

PART B: SURDS	
i. $\sqrt{80} = \sqrt{16} \times \sqrt{5} = 4\sqrt{5}$	iii. $I = Prn$ $2200 = (6800) \times r \times \frac{30}{12}$ $\frac{2200}{6800 \times \frac{30}{12}} = r$ $\therefore r = 0.129... = 12.9\% \text{ pa}$ $\times 100$
b. $\sqrt{56} \div \sqrt{28} = \sqrt{2}$	
c. $\frac{\sqrt{3}}{2} + \frac{\sqrt{27}}{6} = \frac{3\sqrt{3}}{6} + \frac{3\sqrt{3}}{6}$ $= \frac{6\sqrt{3}}{6} = \sqrt{3}$	
d. $(\sqrt{5})^3 + \sqrt{125} = 5\sqrt{5} + \sqrt{125}$ $= 5\sqrt{5} + 5\sqrt{5}$ $= 10\sqrt{5}$	vi. $\$500 \times 2 \times 2 \times 2 = \4000
	vii. $A = 4000 (1 - 15\%)^{10}$ $= \$787.50$
	\$787.50 - 500 = \$287.50 PROFIT.
2. $(3\sqrt{7} + \sqrt{6})(4\sqrt{6} + 2\sqrt{7})$ $= 12\sqrt{42} + 42 + 24 + 2\sqrt{42}$ $= 14\sqrt{42} + 66$	7. $81930 = 50000(1+r)^{20}$ $\frac{81930}{50000} = (1+r)^{20}$ $\sqrt[20]{\frac{81930}{50000}} = 1+r$ $\sqrt[20]{\frac{81930}{50000}} - 1 = r$ $0.0249... = r$ $\therefore r = 2.5\% \text{ pa}$ $\times 100$
3. $\left(\frac{2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \right) - \left(\frac{1}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} \right)$ $= \left(\frac{2\sqrt{2}}{2} \right) - \left(\frac{1-\sqrt{2}}{1-2} \right)$ $= \sqrt{2} - (-1+\sqrt{2})$ $= 1$	
4. LONG SIDE : SHORT $\sqrt{2} : 1$ A4: $210\sqrt{2} : 210$ A3: $420 : 210\sqrt{2}$ A2: $420\sqrt{2} : 420$	
	$\therefore 420\sqrt{2} \times 420 \text{ cm}$
PART C: INT & DEPRECIATION	
5i. $1700 + (300 \times 30) = \10700	9i. $x^2 + 5x + 6 = (x+2)(x+3)$
ii. $10700 - 8500 = \$2200$	b. $3x^2 - 13x + 10$ P: 30 $= 3x^2 - 3x - 10x + 10$ M: -13 $= 3x(x-1) - 10(x-1)$ F: -10, -3 $= (3x-10)(x-1)$
	9j. $x^2 + 21x + 20 = 0$ $(x+20)(x+1) = 0$ $\therefore x = -20, -1$
	CONT'D \rightarrow

b. $x^2 - 30x = 0$
 $x(x - 30) = 0$
 $x = 0, 30$

c. $3x^2 - 48 = 0$
 $3(x^2 - 16) = 0$
 $3(x+4)(x-4) = 0$
 $\therefore x = \pm 4$.

10. $-1 - 4x - x^2 = 0$
 $x = \frac{4 \pm \sqrt{16 - 4(-1)}}{-2}$
 $= \frac{4 \pm \sqrt{12}}{-2}$
 $x = -3.73, -0.27$

11. $x^2 - 12x + 36 = 18 + 36$
 $(x-6)^2 = 54$
 $x-6 = \pm \sqrt{54}$
 $x = 6 \pm \sqrt{54}$
 $= 6 \pm 3\sqrt{6}$

12. $x^8 - 97x^4 + 1296 = 0$
Let $u = x^4$
 $u^2 - 97u + 1296 = 0$
 $u = \frac{97 \pm \sqrt{97^2 - 4 \times 1 \times 1296}}{2}$
 $= \frac{97 \pm \sqrt{4225}}{2}$
 $u = 81, 16$
 $\therefore x^4 = 81 \quad x^4 = 16$
 $x = \pm 3 \quad x = \pm 2$

13. i. $y\text{-INT} = 10$
ii. $x\text{-INTS}: x^2 - 7x + 10 = 0$
 $(x-5)(x-2) = 0$
 $\therefore x = 5, 2$

iii. $x = \frac{-b}{2a} = \frac{7}{2} = 3.5$

iv. $y = \left(\frac{7}{2}\right)^2 - 7\left(\frac{7}{2}\right) + 10$
 $= -2\frac{1}{4}$
COORDS = $(3.5, -2.25)$

v.

14. i.

ii. $A = \frac{1}{2} \times b \times h$
 $\frac{1}{2} \times x \times (x+8) = 52.5 \text{ cm}^2$
 $\frac{x^2}{2} + 4x = 52.5 \text{ cm}^2$

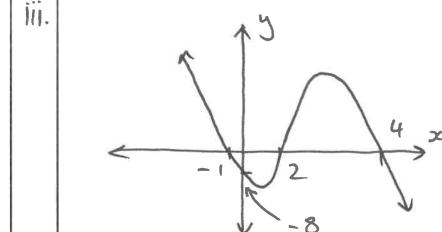
iii. $x^2 + 8x - 105 = 0$
 $x = \frac{-8 \pm \sqrt{64 - 4 \times 1 \times -105}}{2}$
 $= \frac{-8 \pm \sqrt{484}}{2}$
 $x = 7 \text{ cm} \text{ or } -15 \text{ cm}$
DIMENSIONS:
7 cm BASE
15 cm HEIGHT

14. PART E: GRAPHS

i. $2x^3 + 2$ ii. $\frac{-x^3}{2} - 1$
iii. $0.25x^5$ iv. $y = (x-2)^3$

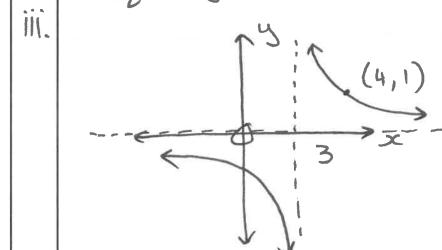
16. i. A CUBIC (OR POLYNOMIAL)

ii. $x\text{-INTS}: -1, 2, 4$
 $y\text{-INT}: \text{MAKE } x=0$
 $-1 \times 1 \times 2 \times 4 = -8$



17. i. A HYPERBOLA

ii. VERT: $x=3$
HORIZ: $y=0$



18. i. $b = 2$ (MOVED RIGHT 2)

ii. $y = k(x-2)^6 - 1$
SUB IN $(0, 7)$

$7 = k(0-2)^6 - 1$
 $8 = k \times 64$
 $\therefore k = 1/8$

PART F: MIXED Q'NS

19. GET IN FORM:
 $x + y + 2\sqrt{xy}$

$12 + \sqrt{140}$
 $= 12 + 2\sqrt{35}$

$\therefore x + y = 12$
 $xy = 35$
 $\therefore x = 5, y = 7$

USING THE FORMULA:
 $12 + \sqrt{140} = \sqrt{5} + \sqrt{7}$

20. $x^4 - x^2 + 18x - 81 = 0$

$x^4 - (x^2 - 18x + 81) = 0$

$x^4 - (x-9)^2 = 0$

DIFF. OF 2 SQUARES:

$(x^2 + (x-9))(x^2 - (x-9)) = 0$
 $(x^2 + x - 9)(x^2 - x + 9) = 0$

$\begin{array}{l} (1) \\ x = \frac{-1 \pm \sqrt{1-4 \times 1 \times -9}}{2} \end{array}$

$\begin{array}{l} (2) \\ = \frac{-1 \pm \sqrt{37}}{2} \end{array}$

$x = \frac{+1 \pm \sqrt{1-4 \times 1 \times 9}}{2}$

$\begin{array}{l} (1) \\ = \frac{1 \pm \sqrt{-35}}{2} \end{array}$

X NO SOL'N!

CONT'D →

$$21. \frac{2^{9x^2+12xy+4y^2}}{2^{9x^2-12xy+4y^2}}$$

. WHEN DIVIDING, SUBTRACT THE INDICES.

$$= 2^{24xy} \quad) \quad xy = 1/6 \\ = 2^4 = 16$$

22. LET h = HEIGHT OF WINDOW

$$\therefore \text{TOTAL VERTICAL FRAMES} \\ = 4h.$$

TOTAL HORIZ. FRAMES

$$= 3w \\ \therefore \text{TOTAL} = 3w + 4h = 12m$$

MAKE h THE SUBJECT:

$$4h = 12 - 3w \\ h = 3 - \frac{3w}{4}$$

$$A = w \times h \\ = w \times \left(3 - \frac{3w}{4} \right) \\ = 3w - \frac{3w^2}{4}$$

ii. THE VERTEX WILL BE THE MAXIMUM VALUE:

$$A = 3w - \frac{3w^2}{4}$$

$$w = \frac{-b}{2a} = \frac{-3}{2 \times \frac{-3}{4}} = \frac{-3}{-1\frac{1}{2}} \\ = 2 \text{ METRES}$$