

YEAR 10

5.3 MATHEMATICS

Semester 2 Examination

RJW* RAS
LMD VAB
JZT ARP
JYR JWH
WMD/AYG
BHC/RAS

1:20 PM MONDAY 5th NOVEMBER

TERM 4, 2018

TOTAL TIME: 2 HOURS

250 COPIES

INSTRUCTIONS TO STUDENTS:

Attempt ALL questions.

Write your name in the spaces provided.

Show ALL necessary working.

Marks may not be awarded for careless or badly arranged work.

Diagrams are NOT necessarily drawn to scale.

Write your answers in the spaces provided on the paper.

A formula sheet is provided with this paper. Please detach this sheet.

This examination consists of TWO Parts.

Part A : COMMON (59 Marks)

Part B : 5.3 SPECIFIC (86 Marks)

TOTAL MARKS: /145

Calculators can be used throughout the examination.

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PART A : COMMON SECTION [59 marks]

*Calculators are permitted

*Show ALL working

*Diagrams are NOT necessarily to scale

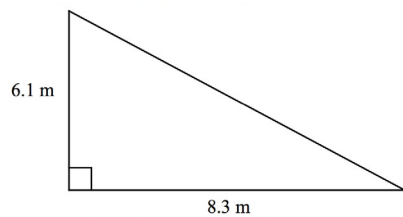
Name

Teacher's initials

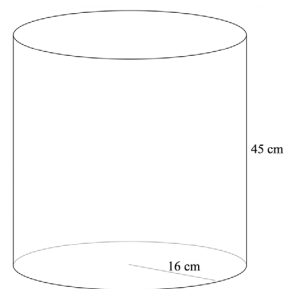
	Marks
1. Simplify	
(a) $2xy + 5yx - 3xy$	1
(b) $k^5 \times k^7$	1

2. Calculate the simple interest earned on \$320 in 10 years if the interest rate is 3% p.a. 2

3. Calculate the length of the hypotenuse of this triangle (correct to 1 decimal place). 2

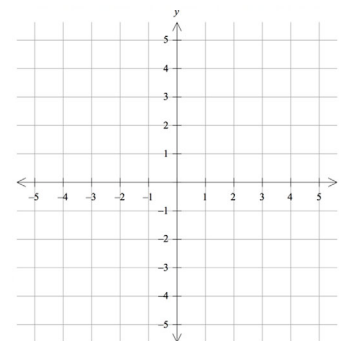


4. Find the volume of this cylinder, correct to 2 decimal places. 2



5. Expand $3x(5 - 2x)$ 1

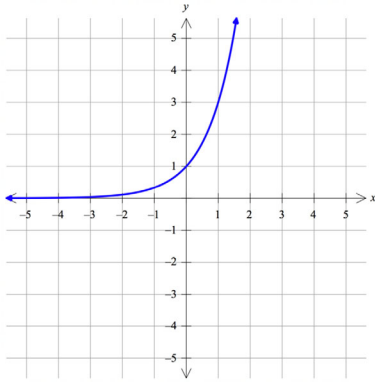
6. Use a pencil and a ruler to sketch the line $y = 3x - 2$ on the number plane below. 2



Name

Teacher's initials

7. Circle the equation that could represent the graph shown below. 1

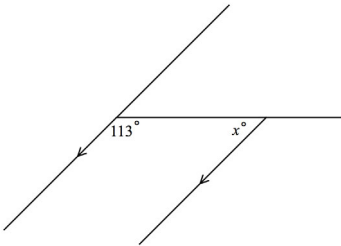


- (A) $y = x + 1$
 (B) $y = x^2 + 1$
 (C) $y = 3^x$
 (D) $x^2 + y^2 = 1$

8. Find the probability of rolling a 4 on a 12-sided die with each side numbered from 1 to 12. 1

9. Factorise $3x^3 - 12x^2$ 2

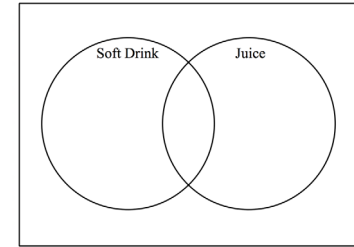
10. Determine the value of x , giving a reason. 2



11. A group of 35 students were asked whether they like to drink Soft Drink or Juice. 14 like Soft Drink, 18 like Juice and 5 like neither Soft Drink nor Juice.

- (i) Complete this Venn diagram using the information provided. 3

Hint: Notice that $14 + 18 + 5 \neq 35$



- (ii) State the probability that a student selected at random likes Juice. 1

- (iii) State the probability that a student selected at random likes both Soft Drink and Juice. 1

- (iv) State the probability that a student who likes Soft Drink will also like Juice. 1

12. Calculate the amount in a bank account after 4 years if \$560 is initially deposited at 6.5% p.a. compounded annually. 2

Name
Teacher's initials

13. For this data set 17, 17, 15, 12, 18, 13, 14, 15, determine the following:
- (i) The mean 2
 - (ii) The median 1
 - (iii) The mode(s) 1
 - (iv) The range 1
 - (v) If each score in the dataset is increased by 2, which of the measures (mean, median, mode or range) will **not** change? 1

14. If $y = x^2 - x$, find the value of y if $x = -3$. 2

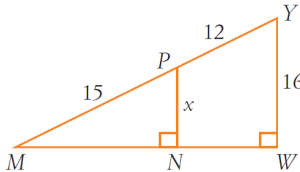
15. If $y = 5x - 3$, find the value of x if $y = 1$. 2

16. This ordered stem-and-leaf plot records the points scored in an IQ test of a sample of adults.

Stem	Leaf									
07	9									
08	1	2	3	6						
09	0	4	4	5	5	7	8	9		
10	1	1	1	2	2	3	X	8		
11	0	1	2	3						
12	1									

- (i) What is a possible value for x ? 1
- (ii) What is the mode of this sample? 1
- (iii) Make a comment on the shape of the distribution. 1

- 17.



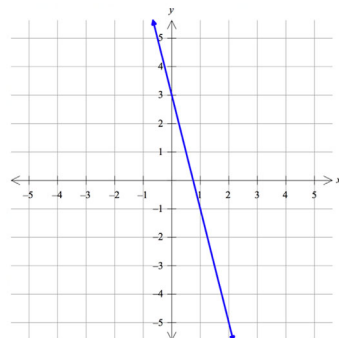
- (i) Mark $\angle MYW$ with an asterisk (*) 1
- (ii) Given $\triangle PMN \sim \triangle YMW$, how long is the interval marked x ? 2

18. Describe an event that has a probability of $\frac{1}{3}$ of happening. 1

Name

Teacher's initials

19. For the line shown

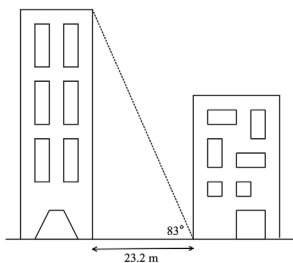


(i) Write down the y -intercept. 1

(ii) Find the gradient. 1

(iii) Write down the equation of the line. 2

20. The distance between two buildings is 23.2 m.
The angle of elevation of the taller building from the base of the shorter building is 83° .
Find the height of the taller building, correct to 2 decimal places. 3



21. Solve

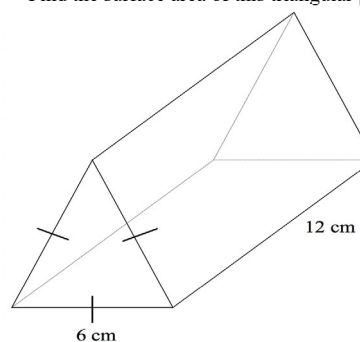
(a) $\frac{3x}{7} + 2 = -1$ 2

(b) $\frac{5k+1}{2} = 8$ 2

(c) $4(h + 1) = 5 - 2h$ 2

(d) $m^2 = 25$ 1

22. Find the surface area of this triangular prism (nearest cm^2). 3



END OF COMMON SECTION

PART B: 5.3 SECTION [86 marks]

Question 1 (16 marks)

Show ALL working
Diagrams are NOT to scale

Name: _____

Teacher Initials: _____

(a) Solve $x^2 - 7x + 10 = 0$ 2

(b) Expand and simplify $3\sqrt{5}(\sqrt{5} + \sqrt{2})$ 2

(c) Solve the inequality and show the solution on a number line $\frac{2x}{5} - 5 \geq 1$ 3

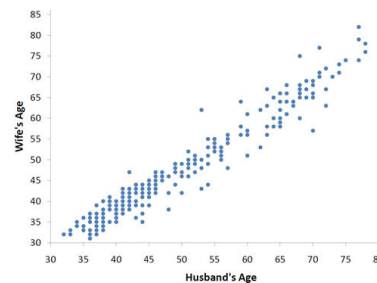


(d) Find the distance between point A (-2, -3) and point B (3, 1) in exact form. 2

(e) Solve the following equations simultaneously: 2

$$\begin{aligned} 2x + 3y &= 24 \\ 5x - 3y &= -3 \end{aligned}$$

(f) 282 married couples were surveyed to help determine whether there is a link between a husband and wife's ages. The scatter plot displays the results from the survey. Choose the **two** appropriate descriptors of the relationship from these four descriptors: strong linear, weak linear, positive, negative 1



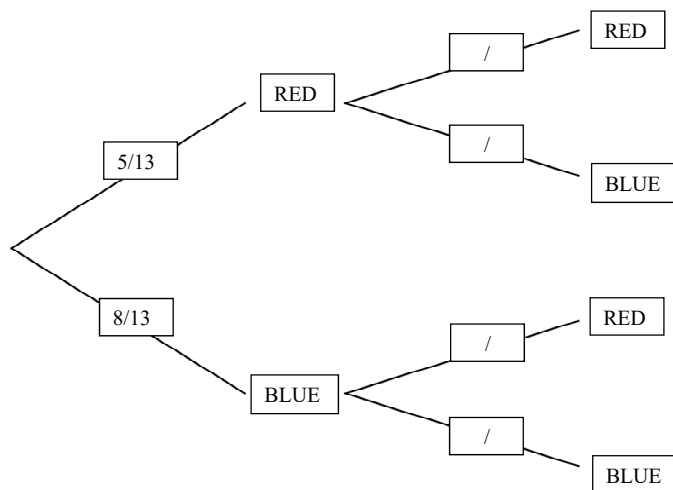
Name: _____

Teacher Initials: _____

- (g) Ben has a marble collection he keeps in a bag. He has 8 blue marbles and 5 red marbles. Two marbles are chosen at random, *without replacement*.

- (i) Complete the probability tree diagram

2



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- (ii) Determine the probability of him choosing two red marbles.

1

- (iii) Determine the probability of him choosing a blue and red marble in any order.

1

End of Question 1

Question 2

(14 marks)

Name: _____

Teacher Initials: _____

Show ALL working

Diagrams are NOT to scale

(a)

O is the centre of the circle below and $AC = BC$. Prove that $\triangle OAC \equiv \triangle OBC$

2

In $\triangle OAC$ and $\triangle OBC$

_____ ()

_____ ()

_____ ()

$\therefore \triangle OAC \equiv \triangle OBC$ ()

(b)

Using the list given in the table, match each equation with its type of curve.

2

(i)

$y = 2x^2 + 4$

(ii)

$y = \frac{3}{x-4}$

(iii)

$y = 6^{-x}$

(iv)

$x^2 + y^2 = \frac{1}{4}$

(v)

$y = 3(x + 1)^3 + 2$

Circle
Parabola
Hyperbola
Exponential
Cubic

(c)

Jack invests his savings of \$3500 for 5 years at 12% p.a. compounded **monthly**. Determine the **total amount** of his investment at the end of the 5 years.

2

(d)

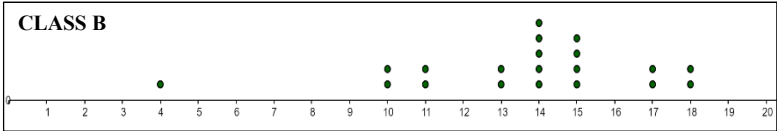
Georgia buys a new laptop for \$2100. It depreciates at a rate of 15% p.a. Determine its value after 4 years.

2

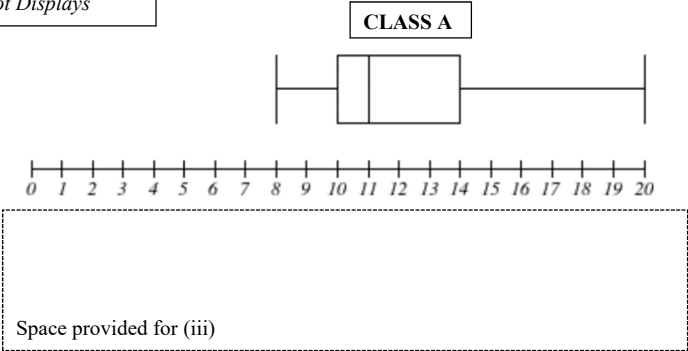
(e)

The Maths test results from two classes are shown. One class has its results displayed as a dot plot and the other as a box plot. The test was out of 20.

Dot Plot Display



Box Plot Displays



- (i)

Calculate the standard deviation for **class B**.

1
- (ii)

Determine the five number summary for **class B**.

2

- (iii)

In the space provided, draw a parallel box-and-whisker plot for class B from its dot plot.

1
- (iv)

Which class has performed better overall? Justify your answer with reference to measures of location and/or spread.

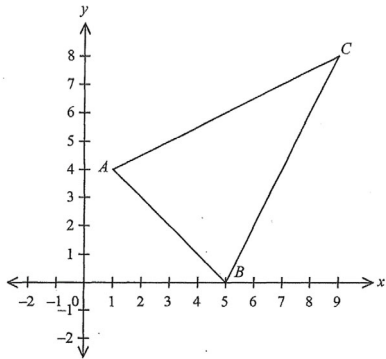
2

Question 3 (14 marks)

Show ALL working
Diagrams are NOT to scale

Name: _____
Teacher Initials: _____

- (a) The points A (1, 4), B (5, 0) and C (9, 8) form the vertices of a triangle.



- (i) Find the coordinates of the midpoints P and Q of AB and AC respectively.

2

- (ii) Show that PQ is parallel to BC.

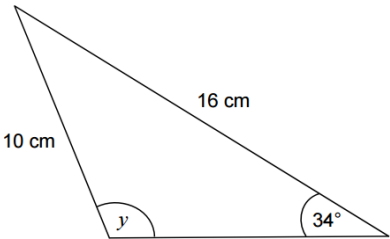
2

- (b) Find the equation of the line passing through points A (1, 4) and C (9, 8).
Give your answer in **general** form.

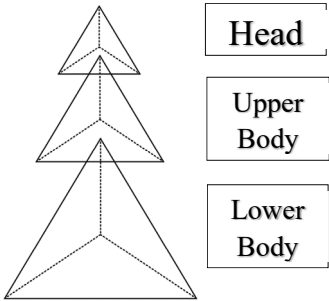
3

- (c) Find, correct to the nearest degree, the value of the obtuse angle y in the diagram.

3



- (d) A robot is constructed from 3 solid triangular pyramids, one pyramid each for the **head**, the **upper body** and the **lower body**.
The upper body's volume is 8 times the volume of the head.
The lower body's volume is 8 times the volume of the upper body.



- (i) The lower body is made using 28m^3 of steel. What volume of steel is required for the head?

1

- (ii) The perpendicular height of the head is 5m. Determine the height of the whole robot.

3

End of Question 3

Question 4 (15 marks)

Show ALL working
Diagrams are NOT to scale

Name: _____
Teacher Initials: _____

- (a) Simplify the following, giving your answer in surd form

2

$$\left(\frac{\frac{2}{3}}{\frac{a}{\frac{4}{b^3}}} \right)^{\frac{3}{4}}$$

- (b) Solve for x using the specified technique:

- (a) By factorisation:

$$3x^2 - 10x - 8 = 0$$

2

- (b) By completing the square:
Leave your answer in surd form.

$$x^2 - 6x + 2 = 0$$

3

- (c) Give the new equations of the graphs after its transformations:

- (a) $y = 2^x$ shifted 2 units down

1

- (b) $y = \frac{3}{x}$ shifted 4 units left

1

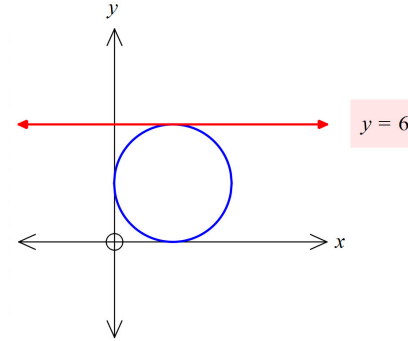
- (c) $y = 5x^3$ reflected across the x-axis, then 7 units up

2

- (d) The lines $y = 6$, the x-axis and the y-axis are all tangents to the circle shown.

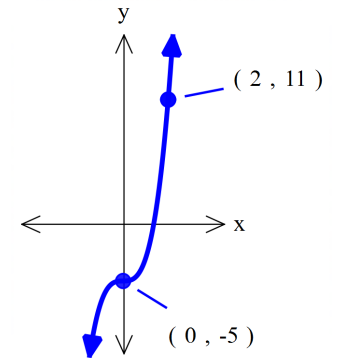
What is the equation of the circle?
(You do not need to expand and simplify your answer)

2



- (e) Given that the graph shown is of the form $y = ax^3 + d$, find its equation.

2



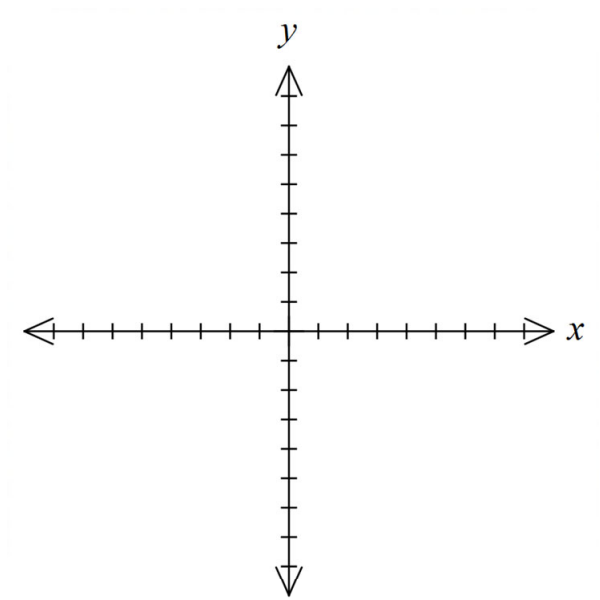
End of Question 4

Question 5 (13 marks)

Show ALL working
Diagrams are NOT to scale

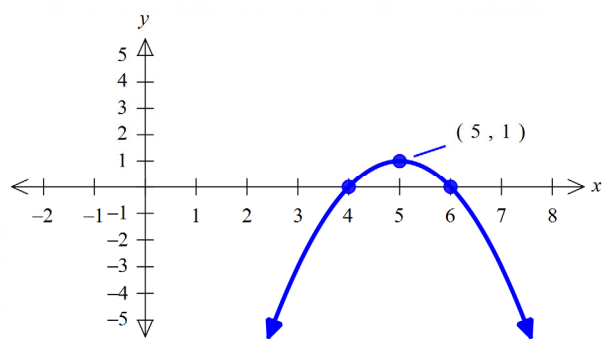
Name: _____
Teacher Initials: _____

- (a) For the parabola $y = x^2 - 6x + 5$
- (i) Find the y intercept 1
- (ii) Find the x intercepts 2
- (iii) Find the coordinates of the vertex of the parabola 2
- (iv) Sketch the graph showing all of the above information 2



- (b) Determine the equation of the graph shown.
(General, vertex or intercept forms of the equation are acceptable)

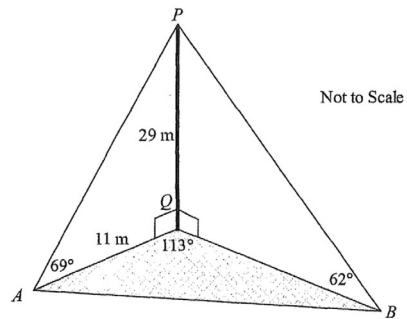
2



Name: _____

Teacher Initials: _____

- (c) Points A , B and Q are on flat ground. The diagram shows a vertical pole PQ of height 29 metres, standing perpendicular to the ground.



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- (i) Show that the distance of B from the foot (Q) of the pole, is approximately 15.4m. **2**

- (ii) Hence, calculate the distance, to the nearest metre, between A and B . **2**

End of Question 5

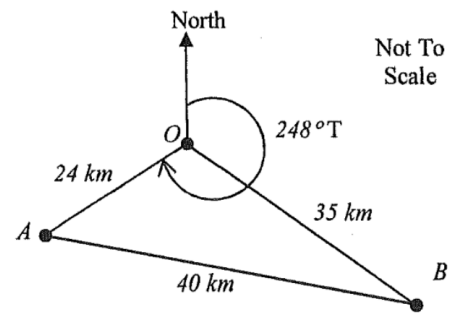
Question 6 (14 marks)

Show ALL working
Diagrams are NOT to scale

Name: _____

Teacher Initials: _____

- (a) A section of rainforest is to be designated for a species count. The shape is shown below. The bearing of landmark A from landmark O is 248°T and is 24km in distance. The distance from landmark A to landmark B is 40km and from landmark B to landmark O is 35km .



- (i)

Show that $\angle AOB \approx 83^\circ$

2
- (ii)

Hence, calculate the area of this section of the rainforest to the nearest km^2 .

2
- (iii)

What is the bearing of landmark O from landmark B ?

2

- (b) By using the substitution of $u = (x - 1)^2$,
Solve for x in the equation: $(x - 1)^4 - 20(x - 1)^2 + 64 = 0$

3

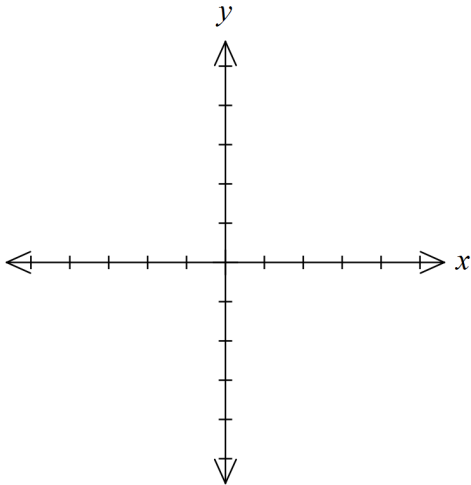
- (c) $A(-1, 1)$, $B(2, 2)$, $C(3, -1)$ and $D(x, y)$ is a kite with vertices in that order. Distance $AB = BC$.
- (i)

Find the equation of the line that point D must lie on.

2
- (ii)

Point D cannot lie on all points on this line. Identify a location of D on this line that does not result in $ABCD$ being a kite.

1



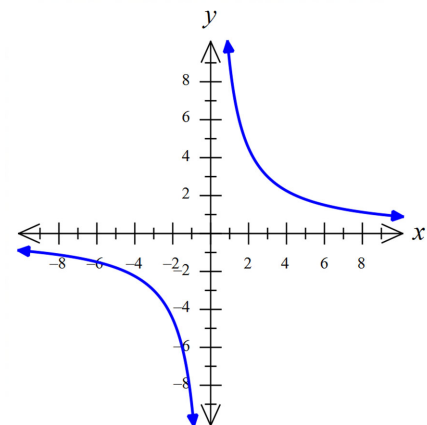
Name: _____

Teacher Initials: _____

- (d) For what values of b do the graphs of the two equations below have **no points of intersection** **2**
The first equation is graphed for you

$$y = \frac{9}{x}$$

$$y = -x + b$$



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End of Question 6
End of Exam Paper

YEAR 10 - FORMULA SHEET

Simple Interest

$$I = Prn$$

P is initial amount
 r is interest rate per period, expressed as a decimal
 n is number of periods

Compound Interest

$$A = P(1 + r)^n$$

A is final amount
 P is initial amount
 r is interest rate per period, expressed as a decimal
 n is number of compounding periods

Depreciation

$$A = P(1 - r)^n$$

A is final value of asset after n periods
 P is initial value of asset
 r is depreciation rate per period,

Gradient-intercept form of a line

$$y = mx + b$$

m is gradient
 b is y-intercept

Slope (gradient) of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Distance between two points

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

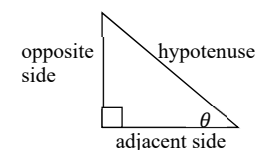
Point-gradient of the equation of a line

$$y - y_1 = m(x - x_1)$$

Solution of a quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Trigonometric Ratios



$$\sin \theta = \frac{\text{opposite side}}{\text{hypotenuse}}$$

$$\cos \theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$$

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Sine rule

In $\triangle ABC$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine Rule

In $\triangle ABC$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

or

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Area of a triangle

In $\triangle ABC$

$$A = \frac{1}{2}ab \sin C$$

Circumference of a circle

$$C = 2\pi r \text{ or } C = \pi D$$

r is radius
 D is diameter

Area

Circle

$$A = \pi r^2$$

r is radius

Sector

$$A = \frac{\theta}{360} \pi r^2$$

r is radius
 θ is number of degrees in central angle

Annulus

$$A = \pi(R^2 - r^2)$$

R is radius of outer circle
 r is radius of inner circle

Trapezium

$$A = \frac{h}{2}(a + b)$$

h is perpendicular height
 a and b are the lengths of the parallel sides

Surface Area

Sphere

$$A = 4\pi r^2$$

r is radius

Closed cylinder

$$A = 2\pi r^2 + 2\pi rh$$

r is radius
 h is perpendicular height

Volume

Prism or cylinder

$$V = Ah$$

A is area of base
 h is perpendicular height

Pyramid or cone

$$V = \frac{1}{3}Ah$$

A is area of base
 h is perpendicular height

Volume and capacity

Unit conversion: $1 \text{ m}^3 = 1000 \text{ L}$

PART B 5-3

2a) In $\triangle OAC, \triangle OBC$

OC is common

OA = OB (radii same)

AC = BC (given)

$\therefore \triangle OAC \cong \triangle OBC$ (SSS)

b) previous page

c) $A = 3500(1+0.05)^{60}$

$= \$63558.44$

d) $A = 2100(1+0.15)^4$

$= \$1096.21$

e) iii)



SD = 3.184

ii) lowest = 4

Q1 = 12

median = 14

Q3 = 15

highest = 18

iv) class 8's median = 14

class 9's median = 11

\therefore class 8 has done better

3 d ii) Let height be h , body = h : H : T

$\therefore h^3 : H^3 : T^3 = 1 : 8 : 64$

$\therefore h : H : T = 1 : 2 : 4$

\therefore total $h = 1+2+4 = 7$

P2

4. a) $\frac{a^{\frac{3}{2}} \times \frac{3}{4}}{b^{\frac{1}{2}} \times \frac{3}{4}} = \frac{a^{\frac{3}{2}}}{b}$

$= \frac{\sqrt{a}}{b}$

b) i) $3x^2 - 10x - 8 = 0$

Multi: -24 Add: -10

$3x^2 - 12x + 2x - 8 = 0$

$3x(x-4) + 2(x-4) = 0$

$(3x+2)(x-4) = 0$

$x = -\frac{2}{3}$ or 4

ii) $x^2 - 6x + 9 = -2 + 9$

$(x-3)^2 = 7$

$x-3 = \pm\sqrt{7}$

$x = 3 \pm \sqrt{7}$

c) i) $y = 2^x - 2$

ii) $y = \frac{3}{x+4}$

iii) $y = -5x^3 + 7$

d) curve = $(3,3)$ $r = 3$

e) $y = 2x^3 - 5$

P2

5. a) i) $y - \text{int} = 5$

ii) $y = (x-5)(x-1)$

$0 = (x-5)(x-1)$

$x = 5, 1$

iii) $V: x = \frac{1}{2}(5+1)$

$y = (3-5)(3-1) = -4$

$y = -(x-4)(x-6)$

or $y = -(x-5)^2 + 1$

c) i) In $\triangle BPA$

$\tan 60^\circ = \frac{29}{AB}$

$AB = 29 \div \tan 60^\circ$

$= 15.4 \text{ m}$

ii) In $\triangle AQB$

$AB^2 = 11^2 + 15.4^2$

$= 211 + 15.4 \cos 113^\circ$

$= 490.5$

$AB = 22 \text{ m}$

(n metre)

Yr 10 B.3 Maths Semester 2 Exam 2018

Common Section - PART A

1. a) $4xy$ or $4yx$

b) k^2m or k^2n

2. $I = P \times R \times T$

$= \$1200 \times 3\% \times 10$

$= \$360$

3. $x^2 \times 6^3 \times 8^2$

$x^2 \times 108 \times 64$

$x^2 \times 6912$

$x^2 \times 10.8 \text{ m}$

4. $V = \pi r^2 h$

$= \pi \times 10^2 \times 45$

$= 36191.15 \text{ cm}^3$

5. $15x - 6x^2$

6.

7. an exponential

$\therefore \odot y = 3^x$

8. $\frac{1}{12}$

9. $3x^2(x-4)$

10. $x = 180^\circ - 113^\circ$

$= 67^\circ$

(conterior \angle is in

parallel lines)

11. i)

PART B 5-3

1. a) $x^2 - 7x + 10 = 0$

$\therefore (x-5)(x-2) = 0$

$x = 2, 5$

b) $3\sqrt{5}(\sqrt{5} + \sqrt{2})$

$= 3\sqrt{5} \times \sqrt{5} + 3\sqrt{10}$

$= 15 + 3\sqrt{10}$

c) $\frac{3x}{5} - 5 \geq 1$

$2x - 25 \geq 5$

$2x \geq 30$

$x \geq 15$

d) $\sqrt{(3-2)^2} + (1-3)^2$

$= \sqrt{1+4} + \sqrt{4}$

$= \sqrt{5} + 2$

e) i) $7x = 21$

$x = 3$

ii) $6 + 3y = 24$

$3y = 18$

$y = 6$

and $x = 3$

f) strong linear

and positive

g) $\frac{4}{12}, \frac{8}{12}, \frac{5}{12}, \frac{7}{12}$

h) $\frac{5}{13} \times \frac{4}{12} = \frac{5}{39}$

iii) $2 \times \frac{5}{13} \times \frac{8}{12} = \frac{20}{39}$

P1

Q2

a) next

page

b) parabola

c) hyperbola

d) exponential

e) circle

f) cubic

6a) In $\triangle AOB$

$$\cos \angle AOB = \frac{24^2 + 35^2 - 40^2}{2 \times 24 \times 35}$$

$$= 0.1196$$

$$\therefore \angle AOB = 83^\circ \text{ (r. degree)}$$

$$ii) A = \frac{1}{2} \times 24 \times 35 \times \sin 83^\circ$$

$$= 417 \text{ km}^2 \text{ (r. km}^2\text{)}$$

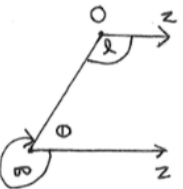
iii)

$$\alpha = 248 - 83$$

$$= 165^\circ$$

$$\theta = 180 - 165$$

$$= 15^\circ$$



bearing of O from B

$$= 360^\circ - 15^\circ = 345^\circ$$

$$b) u^2 - 20u + 64 = 0 \text{ if } u = (x-1)^2$$

$$\therefore (u-16)(u-4) = 0$$

$$\therefore u = 4 \text{ or } u = 16$$

$$\therefore (x-1)^2 = 4 \text{ or } (x-1)^2 = 16$$

$$\therefore x-1 = 2 \text{ or } x-1 = -2 \text{ or } x-1 = 4$$

$$\text{or } x-1 = -4$$

$$\therefore x = 3, -1, 5, -3$$

c) The line must go through B and be perpendicular to AC.

$$\text{gradient AC} = \frac{1-1}{-1-3} = -\frac{1}{2}$$

$$\therefore \text{grad line} = 2$$

equation:

$$y-2 = 2(x-2)$$

$$= 2x-4$$

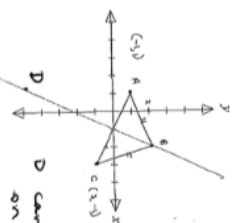
$$\therefore y = 2x-2$$

ii) D can't lie on B(2,2)

as the kite collapsed,

or on (1,0) as ABCD

would be a triangle ABC



D can take most points on this line

$$d) y = \frac{9}{x} - 1$$

$$y = -x + b \text{ --- (2)}$$

$$1) = (2) \therefore \frac{9}{x} = -x + b$$

$$\therefore 9 = -x^2 + bx$$

$$\therefore x^2 - bx + 9 = 0 \text{ --- (3)}$$

For exactly 1 pt of intersection

discriminant must be the

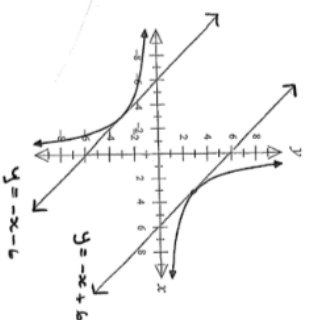
perfect square $(x \pm 3)^2 = 0$

$$\therefore x^2 - bx + 9 = x^2 \pm 6x + 9 \text{ --- (4)}$$

$$\therefore -b = \pm 6$$

$$\therefore b = \pm 6$$

Equ'n (2) becomes $y = -x \pm 6$
Graph these lines above.



Values of b to have no points of intersection:
occur when the line is between these lines
ie $-6 < b < 6$