Computational assignment

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1 Exact solution

Given differential equation is (variant 7)

$$y' = -x + \frac{y(2x+1)}{x}$$

$$y' - \frac{y(2x+1)}{x} = -x$$

Solve complementary equation

$$y' - \frac{y(2x+1)}{x} = 0$$

$$y' = \frac{y(2x+1)}{x}$$

$$\frac{y'}{y} = \frac{2x+1}{x}$$

It is separable equation, so we are able to integrate both parts

$$\frac{dy}{y} = \frac{2x+1}{x}dx$$

$$\int \frac{dy}{y} = \int \frac{2x+1}{x}dx$$

$$\ln|y| = 2x + \ln|x| + \ln C$$

$$y = e^{2x}xC$$

Use variation of parameters

$$y = e^{2x}xC(x)$$

$$y' = (e^{2x}x)'C(x) + (e^{2x}x)C'(x)$$

$$y' = (2e^{2x}x + e^{2x})C(x) + (e^{2x}x)C'(x)$$

Substitute to initial equation

$$(2e^{2x}x + e^{2x})C(x) + (e^{2x}x)C'(x) - e^{2x}C(x)(2x+1) = -x$$

$$e^{2x}xC'(x) = -x$$

$$C'(x) = -e^{-2x}$$

$$C(x) = \frac{e^{-2x}}{2} + \lambda$$

$$y = e^{2x}x(\frac{e^{-2x}}{2} + \lambda)$$

$$y = \frac{x}{2} + \lambda e^{2x}x$$

Initial value problem

$$x0 = 1, y0 = 3$$

$$3 = \frac{1}{2} + \lambda e^{2}$$

$$\lambda = \frac{5}{2e^{2}}$$

$$y = \frac{x}{2} + \frac{5e^{2x}x}{2e^{2}}$$

$$y = \frac{x + 5e^{2x-2}x}{2}$$

$$y = \frac{x(1 + 5e^{2x-2})}{2}.$$