CSU11001 Homework I

Zicheng Liang 23331250

17th Oct. 2023

Q1: Id Number:

$$\left.\begin{matrix} 2\,3\,3\,3\,1\,2\,5\,0 \\ \quad\quad\quad x_1\, x_2\, x_3 \\ y_3\, y_2\, y_1 \quad\quad \\ z_1\, z_2\, z_3 \quad \end{matrix}\right\} \Rightarrow \begin{cases} \vec{x} = \begin{pmatrix}2\\5\\0\end{pmatrix} \\ \vec{y} = \begin{pmatrix}1\\3\\3\end{pmatrix} \\ \vec{z} = \begin{pmatrix}3\\1\\2\end{pmatrix} \end{cases}$$

Q2: (i) $\vec{x}\cdot\vec{y} =$

$$\begin{pmatrix}2\\5\\0\end{pmatrix}\cdot\begin{pmatrix}1\\3\\3\end{pmatrix} = 2\times1 + 5\times3 + 0\times3 = 17$$

(ii) $\boxed{\vec{x}\times(\vec{x}\times\vec{y})}^{\times}$ $\vec{x}\times(\vec{y}\times\vec{z}) =$

$$\vec{x}\times\left(\begin{pmatrix}1\\3\\3\end{pmatrix}\times\begin{pmatrix}3\\1\\2\end{pmatrix}\right) = \vec{x}\times\left(\begin{pmatrix}2\times3-1\times3\\3\times3-1\times2\\1\times1-3\times3\end{pmatrix}\right) = \vec{x}\times\begin{pmatrix}3\\7\\-8\end{pmatrix}$$

$$= \begin{pmatrix}2\\5\\0\end{pmatrix}\times\begin{pmatrix}3\\7\\-8\end{pmatrix} = \begin{pmatrix}5\times(-8)-0\times7\\0\times3-2\times(-8)\\2\times7-3\times5\end{pmatrix} = \begin{pmatrix}-40\\16\\-1\end{pmatrix}$$

$$\vec{x}\times(\vec{y}\times\vec{z}) = \begin{pmatrix}-40\\16\\-1\end{pmatrix}$$

(iii) $(\vec{x}\times\vec{y})\times\vec{z} =$

$$\left(\begin{pmatrix}2\\5\\0\end{pmatrix}\times\begin{pmatrix}1\\3\\3\end{pmatrix}\right)\times\vec{z} = \begin{pmatrix}5\times3-0\times3\\0\times1-2\times3\\2\times3-5\times1\end{pmatrix}\times\vec{z} = \begin{pmatrix}15\\-6\\1\end{pmatrix}\times\begin{pmatrix}3\\1\\2\end{pmatrix}$$

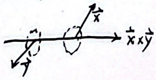
$$= \begin{pmatrix}-6\times2-1\times1\\1\times3-15\times2\\15\times1-(-6)\times3\end{pmatrix} = \begin{pmatrix}-13\\-27\\33\end{pmatrix}$$

therefore, $\begin{cases}(\vec{x}\times\vec{y})\times\vec{z} = \begin{pmatrix}-13\\-27\\33\end{pmatrix} \\ \vec{x}\times(\vec{y}\times\vec{z}) = \begin{pmatrix}-40\\16\\-1\end{pmatrix}\end{cases}$, obviously $(\vec{x}\times\vec{y})\times\vec{z}$ is not equal to $\vec{x}\times(\vec{y}\times\vec{z})$

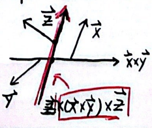
From observation we see that

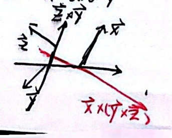
$\vec{x}\times\vec{y} = \begin{pmatrix}15\\-6\\1\end{pmatrix}$, while $\begin{cases}\vec{x}\cdot(\vec{x}\times\vec{y}) = \begin{pmatrix}2\\5\\0\end{pmatrix}\cdot\begin{pmatrix}15\\-6\\1\end{pmatrix} = 30-30+0 = 0 \\ \vec{y}\cdot(\vec{x}\times\vec{y}) = \begin{pmatrix}1\\3\\3\end{pmatrix}\cdot\begin{pmatrix}15\\-6\\1\end{pmatrix} = 15-18+3 = 0\end{cases}$

therefore the cross product of $\vec{x}$ and $\vec{y}$ creates a vector that is prependicular to both $\vec{x}$ and $\vec{y}$



, given three vectors $\vec{x}, \vec{y}, \vec{z}$, it will be like



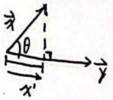


There fore $\vec{x}\times(\vec{y}\times\vec{z}) \neq (\vec{x}\times\vec{y})\times\vec{z}$

P2 $\vec{x}=\begin{pmatrix}2\\5\\0\end{pmatrix}$ $\vec{y}=\begin{pmatrix}1\\3\\3\end{pmatrix}$ $\vec{z}=\begin{pmatrix}3\\1\\2\end{pmatrix}$

Zicheng Liang
23331250

Q2 (iv)



given the illustration, the vector (orthogonal) projection of $\vec{x}$ onto $\vec{y}$ can be obtained by:

$$\vec{x}' = \|\vec{x}\| \cdot \cos\theta \cdot \frac{\vec{y}}{\|\vec{y}\|}$$

$\cos\theta$ can be obtained by:

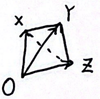
$$\begin{cases} \vec{x}\cdot\vec{y} = \|\vec{x}\|\cdot\|\vec{y}\|\cdot\cos\theta = 17 \\ \|\vec{x}\| = \sqrt{29} \\ \|\vec{y}\| = \sqrt{19} \end{cases} \Rightarrow \cos\theta = \frac{17}{\|\vec{x}\|\cdot\|\vec{y}\|} = \frac{17}{\sqrt{551}}$$

therefore $\vec{x}' =$

$$\left(\sqrt{29} \times \frac{17}{\sqrt{551}} \times \frac{1}{\sqrt{19}}\right)\begin{pmatrix}1\\3\\3\end{pmatrix} = \frac{17}{19}\begin{pmatrix}1\\3\\3\end{pmatrix} = \begin{pmatrix}\frac{17}{19}\\ \frac{51}{19}\\ \frac{51}{19}\end{pmatrix}$$

$$\vec{x}' = \begin{pmatrix}\frac{17}{19}\\ \frac{51}{19}\\ \frac{51}{19}\end{pmatrix}$$

Q2 (v) The Parametric Form:



Assuming $\begin{cases}\vec{x} = \overrightarrow{OX} \\ \vec{y} = \overrightarrow{OY} \\ \vec{z} = \overrightarrow{OZ}\end{cases}$, we can obtain $\begin{cases}\overrightarrow{YX} = \vec{x} - \vec{y} = \begin{pmatrix}1\\2\\-3\end{pmatrix} \\ \overrightarrow{YZ} = \vec{z} - \vec{y} = \begin{pmatrix}2\\-2\\-1\end{pmatrix}\end{cases}$, where $\overrightarrow{YZ}, \overrightarrow{YX}$ are both vectors on the given plane; $Y(1,3,3)$ is a point on the plane.

Then set a point $P(p_1, p_2, p_3)$, $P = Y + s\overrightarrow{YX} + t\overrightarrow{YZ}$ $(s,t \in \mathbb{R})$

$$P = \begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix} = \begin{pmatrix}1\\3\\3\end{pmatrix} + s\begin{pmatrix}1\\2\\-3\end{pmatrix} + t\begin{pmatrix}2\\-2\\-1\end{pmatrix}$$

Then $\begin{cases} p_1 = 1 + s + 2t \\ p_2 = 3 + 2s - 2t \\ p_3 = 3 - 3s - t \end{cases}$ $(s,t \in \mathbb{R})$, the set of equations represents plane XYZ.

Q3 (i)

$$A = \begin{pmatrix}2&5&0\\1&3&3\\3&1&2\end{pmatrix} \quad B = \begin{pmatrix}1&2&3\\3&5&1\\3&0&2\end{pmatrix}, \quad AB = \begin{pmatrix}2&5&0\\1&3&3\\3&1&2\end{pmatrix}\cdot\begin{pmatrix}1&2&3\\3&5&1\\3&0&2\end{pmatrix}$$

$$AB = \begin{pmatrix} 2\times1+5\times3+0 & 2\times2+5\times5+0 & 2\times3+5\times1+0 \\ 1\times1+3\times3+3\times3 & 1\times2+3\times5+0 & 1\times3+3\times1+3\times2 \\ 3\times1+3\times1+2\times3 & 3\times2+1\times5+0 & 3\times3+1\times1+2\times2 \end{pmatrix}$$

$$AB = \begin{pmatrix}17&29&11\\19&17&12\\12&11&14\end{pmatrix}$$

(go to P3)

 $\vec{x} = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$ $\vec{y} = \begin{pmatrix} 1 \\ 3 \\ 3 \end{pmatrix}$ $\vec{z} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

Zicheng Liang
23331250

Q3(i) (continue from P2)

$$BA = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 5 & 1 \\ 3 & 0 & 2 \end{pmatrix} \begin{pmatrix} 2 & 5 & 0 \\ 1 & 3 & 3 \\ 3 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 1\times2+2\times1+3\times3 & 1\times5+2\times3+3\times1 & 1\times0+2\times3+3\times2 \\ 3\times2+5\times1+1\times3 & 3\times5+5\times3+1\times1 & 3\times0+5\times3+1\times2 \\ 3\times2+1\times0+2\times3 & 3\times5+0\times3+2\times1 & 3\times0+0\times3+2\times2 \end{pmatrix}$$

$$BA = \begin{pmatrix} 13 & 14 & 12 \\ 14 & 31 & 17 \\ 12 & 17 & 4 \end{pmatrix}, \quad AB = \begin{pmatrix} 17 & 29 & 11 \\ 19 & 17 & 12 \\ 12 & 11 & 14 \end{pmatrix}$$

Q3(ii)

BA is not equal to AB.

Assume that A, B are both n×n matrices,

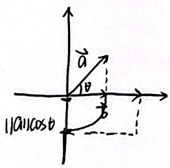
$$AB = \begin{pmatrix} R_{A1}C_{B1} & R_{A1}\cdot C_{B2} & \cdots & R_{A1}C_{Bn} \\ R_{A2}C_{B1} & & & \\ \vdots & & \ddots & \\ R_{An}C_{B1} & \cdots & \cdots & R_{An}C_{Bn} \end{pmatrix}$$

(R_{A1} means Row1 of Matrix A
C_{B1} means Coloumb 1 of Matrix B)

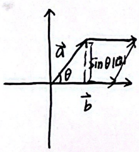
$$BA = \begin{pmatrix} R_{B1}C_{A1} & R_{B1}C_{A2} & \cdots & R_{B1}C_{An} \\ R_{B2}C_{A1} & & & \vdots \\ \vdots & & \ddots & \\ R_{Bn}C_{A1} & \cdots & \cdots & R_{Bn}C_{An} \end{pmatrix}$$

No item in BA and AB neccessarily equals to another item.

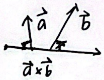
Also I had some thoughts on the ~~dot~~ vector product, but they do not help solving the problem.



$\vec{a}\cdot\vec{b} = \|\vec{a}\|\|\vec{b}\|\cos\theta$, the result equals to the area of a rectangular which consists of $\|a\|\cos\theta$ and $\|b\|$ as its two sides.
i.e. $\|b\|$, and $\vec{a}$'s orthogonal projection onto $\vec{b}$



known $\|\vec{a}\times\vec{b}\| = \|\vec{a}\|\|\vec{b}\|\sin\theta$
that is the area of the parallelgoram consists of $\vec{a}$ and $\vec{b}$ as its sides.
also from Q2.(iii), the vector $(\vec{a}\times\vec{b})$ is a new vector prependicular to both $\vec{a}$ and $\vec{b}$



I do not have any other thoughts, but I will be happy to learn more about vector products' meaning in linear space!

14 $\quad x = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \quad y = \begin{pmatrix} 3 \\ 3 \end{pmatrix} \quad z = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$

Zicheng Liang
23331250

Q4: $x=0,\ y=5,\ z=2$

Then
$$\begin{cases} 2x+y-2z = 2\times 0 + 5 - 2\times 2 = 1 = b_1 \\ 2x+3y+z = 2\times 0 + 3\times 5 + 2 = 17 = b_2 \\ 3x+2y+2z = 3\times 0 + 2\times 5 + 2\times 2 = 14 = b_3 \end{cases}$$

$$\Leftrightarrow \begin{cases} 2x+y-2z = 1 \\ 2x+3y+z = 17 \\ 3x+2y+2z = 14 \end{cases}$$

Q5: the set of equations can be rewritten into an augmented matrix:

(a)
$$\left(\begin{array}{ccc|c} 2 & 1 & -2 & 1 \\ 2 & 3 & 1 & 17 \\ 3 & 2 & 2 & 14 \end{array}\right) \xrightarrow{R_1 \leftrightarrow R_3} \left(\begin{array}{ccc|c} 3 & 2 & 2 & 14 \\ 2 & 3 & 1 & 17 \\ 2 & 1 & -2 & 1 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 2 & 3 & 1 & 17 \\ 2 & 1 & -2 & 1 \end{array}\right)$$

$$\xrightarrow{R_2 - R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 2 & 3 & 16 \\ 2 & 1 & -2 & 1 \end{array}\right) \xrightarrow{R_3 - 2R_1} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 2 & 3 & 16 \\ 0 & 3 & -4 & 7 \end{array}\right) \xrightarrow[R_3 \leftrightarrow R_2]{R_3 - R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 1 & -7 & -9 \\ 0 & 2 & 3 & 16 \end{array}\right)$$

$$\xrightarrow{R_3 - 2R_2} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 1 & -7 & -9 \\ 0 & 0 & 17 & 34 \end{array}\right) \xrightarrow{\frac{R_3}{17}} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 1 & -7 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right) \xrightarrow{R_2 + 7R_3} \left(\begin{array}{ccc|c} 1 & -1 & 1 & -3 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$\xrightarrow{R_1 + R_2 - R_3} \left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right) \Leftrightarrow \begin{cases} x = 0 \\ y = 5 \\ z = 2 \end{cases}$$

(b) First the solutions for x, y, z match the values I obtained in Question 4.

Second compare the matrix to the set of equations:

say these steps:

$$\left(\begin{array}{cccc} 3 & 2 & 2 & 14 \\ 2 & 3 & 1 & 17 \\ 2 & 1 & -2 & 1 \end{array}\right) \xrightarrow{R_1 - R_2} \left(\begin{array}{cccc} 1 & -1 & 1 & -3 \\ 2 & 3 & 1 & 17 \\ 2 & 1 & -2 & 1 \end{array}\right)$$

$$\begin{array}{l} 3x+2y+2z=14 \\ 2x+3y+z=17 \end{array} \xrightarrow{R_1 - R_2} \begin{array}{l} (3x-2x) + (2y-3y) + (2z-z) = 14-17 \\ \Leftrightarrow \cancel{(2x+3)}\ x - y + z = -3 \end{array}$$

$$\left(\begin{array}{cccc} 1 & -1 & 1 & -3 \\ 0 & 1 & -7 & -9 \\ 0 & 0 & 17 & 34 \end{array}\right) \xrightarrow{\frac{R_3}{17}} \left(\begin{array}{cccc} 1 & -1 & 1 & -3 \\ 0 & 1 & -7 & -9 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$17z = 34 \xrightarrow{\frac{R_3}{17}} z = 2$$

, the left hand side equals to the right hand side, the right hand side, so the answer given by Gaussian elimination is equal to the solutions to the original set of equations.