编程实验2

分类+降维

CLASSIFICATION

Classification Dataset

USPS handwritten digits: 2 classes — digits 3 and 8

Note: You will get part of the data to train your classifiers, the rest is left for us to test your algorithms.

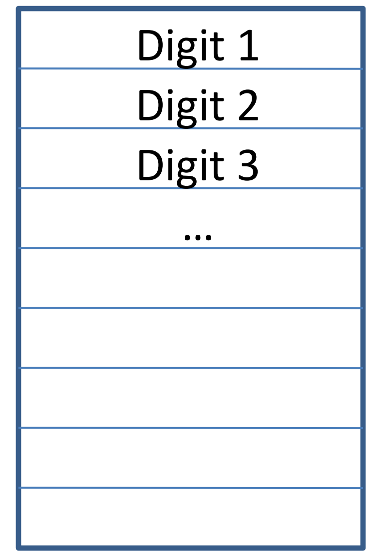
Dataset Representation

All the data is stored in .mat files

In Matlab, type “`load **.mat`” to load the data

Workspace

Name	Value	Min	Max
digits_data	<1000x256 double>	0	255
digits_label	<1000x1 double>	0	1



Experiments

Algorithms	Naïve Bayes	Least Squares	SVM
USPS digits (2 classes)	√	√	√

1. Build a Naïve Bayes classifier

- Write a Matlab function "`nbayesclassifier`" that takes 5 arguments, `training`, `test`, `ytraining`, `ytest`, `threshold` as input, and returns a vector `ypred` as the predictions of the test data, as well as the percentage of prediction accuracy, "`accuracy`"

```
function [ypred,accuracy] = nbayesclassifier(traindata,
trainlabel, testdata, testlabel, threshold)
```

if P(digit=8|image) >threshold, then classify the image to *8*

2. Build a least squares classifier

- Write a Matlab function "`lsclassifier`" that takes 5 arguments, `training`, `test`, `ytraining`, `ytest`, `lambda` as input, and returns a vector `ypred` as the predictions of the test data, as well as the percentage of prediction accuracy, "`accuracy`"

$$\min_{\mathbf{w}} (X\mathbf{w} - \mathbf{y})^2 + \lambda \|\mathbf{w}\|^2$$

```
function [ypred,accuracy] = lsclassifier(traindata,
trainlabel, testdata, testlabel, lambda)
```

3. Build a support vector machine

- Write a Matlab function "`softsvm`" that takes 6 arguments, `training`, `test`, `ytraining`, `ytest`,`C`, `sigma` as input, and returns a vector `ypred` as the predictions of the test data, as well as the percentage of prediction accuracy, "`accuracy`"

```
function [ypred,accuracy] = softsvm(traindata, trainlabel, testdata,
 testlabel, sigma, C)
```

when sigma=0, use linear kernel $K(\mathbf{x}_i,\mathbf{x}_j)= \mathbf{x}_i^{\mathbf{T}}\mathbf{x}_j$,

otherwise use the RBF kernel $K(\mathbf{x}_i,\mathbf{x}_j) = e^{-\frac{\|\mathbf{x}_i - \mathbf{x}_j\|^2}{\sigma^2}}$

4. Cross Validation

- On each dataset:
  - Implement 5 fold cross validation to tune the parameters for each algorithm
  - For each algorithm:
    - Return a matrix: parameter (set) X accuracy on each fold
    - Select the parameter (set) with best average accuracy

Cross-validation

- The improved holdout method: k-fold *cross-validation*
  - Partition data into k roughly equal parts;
  - Train on all but j-th part, test on j-th part

For Naïve Bayes, select threshold from…? (e.g.: threshold=[0.5 0.6 0.7 0.75 0.8 0.85 0.9])

For least squares, select lambda from…?

$$\min_{\mathbf{w}}(X\mathbf{w}-\mathbf{y})^2+\lambda\|\mathbf{w}\|^2$$

For SVM, select (C, sigma) value combination from:

C=[1, 10, 100, 1000], sigma?

5. Testing

Notes on building an SVM

- Make sure you understand the math
- `quadprog` in Matlab
  - Min and max objectives
- Use some simple synthetic data（模拟数据） to verify
- Use the same kernel during training and testing
- When calculating b, remember to use the same kernel!
- Check α_i to debug
  - Do they satisfy the constraints?

```
>> help quadprog
 QUADPROG Quadratic programming.
    X = QUADPROG(H,f,A,b) attempts to solve the quadratic programming
    problem:

             min 0.5*x'*H*x + f'*x   subject to:  A*x <= b
              x

    X = QUADPROG(H,f,A,b,Aeq,beq) solves the problem above while
    additionally satisfying the equality constraints Aeq*x = beq.

    X = QUADPROG(H,f,A,b,Aeq,beq,LB,UB) defines a set of lower and upper
    bounds on the design variables, X, so that the solution is in the
    range LB <= X <= UB. Use empty matrices for LB and UB if no bounds
    exist. Set LB(i) = -Inf if X(i) is unbounded below; set UB(i) = Inf if
    X(i) is unbounded above.

    X = QUADPROG(H,f,A,b,Aeq,beq,LB,UB,X0) sets the starting point to X0.
```

Calculate b in SVM

Dual optimization problem:

$$\max_{\alpha} \sum_{i=1}^{n} \alpha_i - \frac{1}{2} \sum_{i,j=1}^{n} \alpha_i \alpha_j y_i y_j (\mathbf{x}_i^\top \mathbf{x}_j) \quad \text{subject to} \quad 0 \le \alpha_i \le C, \forall i$$

$$\sum_{i=1}^{n} \alpha_i y_i = 0$$

b can be recovered by

$$b = y_i - \sum_{j=1}^{n} \alpha_j y_j K(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for any i that } \alpha_i \neq 0$$

$$b = y_i - \sum_{j=1}^{n} \alpha_j y_j K(\mathbf{x}_i, \mathbf{x}_j) \quad \text{for any i with maximal } \alpha_i$$

$$b = avg_{i:\alpha_i \neq 0} \left(y_i - \sum_{j=1}^{n} \alpha_j y_j K(\mathbf{x}_i, \mathbf{x}_j) \right)$$

DIMENSIONALITY REDUCTION

PCA for Image Denoising

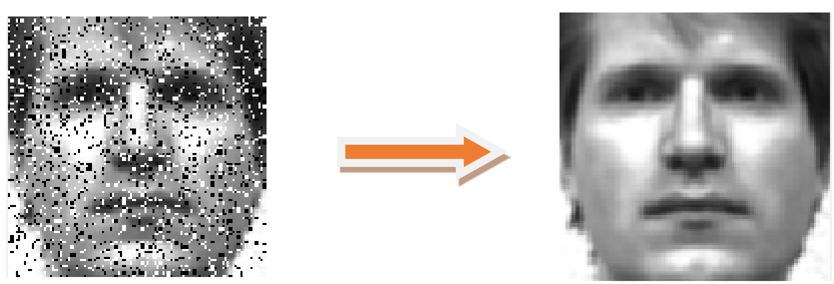
Dataset: YaleFace人脸数据集

Training: 60张正常情况下大小为50x50的人脸图片——60个2500维的向量

Testing: 6个样本，每张均包含了一定的噪声

PCA for Image Denoising

- 在训练过程中通过PCA算法来计算投影矩阵。
- 测试时将带有噪音的图片通过投影矩阵投影至低维空间，保留图片的主要信息，再投影至原空间完成重构，在此过程中会消除噪音的效果。



PCA for Image Denoising

- Write a Matlab function "`reconsPCA`" that takes 4 arguments, `train_data`, `test_data`, `ground_truth`, `threshold` as input, and returns a projection matrix "`proj_matrix`", reconstruction of `test_data`, "`recons_data`", difference to the `ground_truth` , "`recons_error`".

```
function [proj_matrix,recons_data,recons_error] =
reconsPCA(train_data, test_data, ground_truth,
threshold)
```

- Set the number of eigenvectors *m* such that:

  Sum(first m-1 eigenvalues)/Sum(all eigenvalues)<threshold<=Sum(first m eigenvalues)/Sum(all eigenvalues)

  error= $$\frac{\sum_{i=1}^{50}\sum_{j=1}^{50}\left|A_{ij}-B_{ij}\right|^2}{50\times 50}$$