Assignment of ET 4389

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1)

G is the network described in `7.txt`.

- Number of nodes N: 379
- Number of links L: 914
- Link density p: 0.013
- Average degree $E[D]$: 4.82
- Degree variance $Var[D]$: 15.46

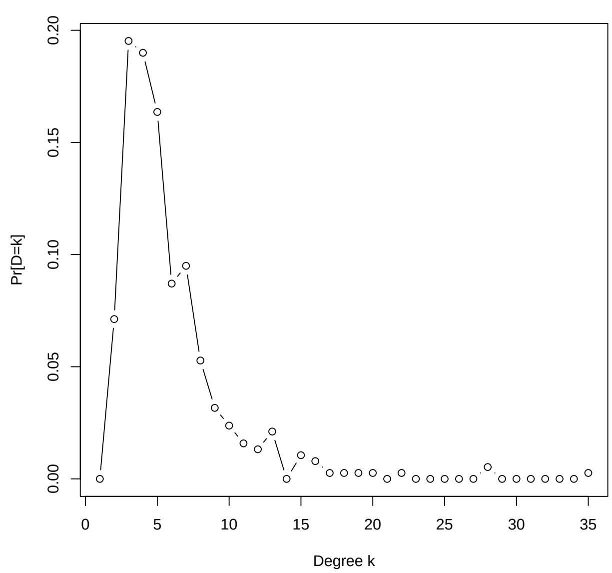

Figure 1: Degree distribution of Graph G

2)

- Degree correlation (assortativity) ρ_D: -0.30

Physical meaning:
Assortativity $\sim$ *Birds of a feather flock together.*
Disassortativity $\sim$ *Opposites attract.*

Networks, in which nodes with a high degree are likely connected to other high-degree nodes are *assortative*; networks in which nodes with a low degree are likely connected to high-degree nodes are *disassortative*.

3)

- Clustering coefficient: 0.17

4)

- Average hopcount $E[H]$: 3.75
- Diameter H_{max}: 7

5)

- Largest eigenvalue (spectral radius) λ_1: 7.44

6)

- Second smallest eigenvalue (algebraic connectivity) of the Laplacian matrix μ_{N-1}: 0.40

7)

Now, we consider the network G_N, described in `NetScience.txt`.

- Number of nodes N: 379
- Number of links L: 914
- Link density p: 0.013
- Average degree $E[D]$: 4.82
- Degree variance $Var[D]$: 15.46
- Clustering coefficient C: 0.80
- Assortativity ρ_D: -0.08
- Average hopcount $E[H]$: 6.04

- Spectral radius λ_1: 10.38
- Algebraic connectivity μ_{N-1}: 0.015
- Diameter H_{max}: 17

8)

I am discussing the metrics in the sense of "which network may allow information to propagte to a larger fraction of the network".

Clustering coefficient C. The clustering coefficient C states how densely the neighbors of a node are connected, that is, are my friends also friends with eeach other. Here, G_N performs much better ($C(G) = 0.17 < C(G_N) = 0.80$), and is therefore better suited, since information can reach me on several channels, and makes the network more robust.

Average degree E[H]. The average degree states the amount of neighbors of a node in a graph, in a communication network, to how many entities a message could be directly sent. Since both networks G and G_N have the same average degree, none of them performs better here.

Diameter H_{max}. The diameter of the communication network states how many edges a message needs to pass between two nodes in the worst case. Since a smaller H_{max} means faster communication in the worst case, G performs better ($H_{max}(G) = 7 < H_{max}(G_N) = 17$).

Spectral radius λ_1. The spectral radius is important for dynamic processes in networks, for example, if, and how fast, a message might go "viral" but also how fast a virus might affect the network. The larger $lambda_1$, the lower the epidemic threshold τ_c. Regarding security, a higher $lambda_1$ is better, therefore, G performs better ($\lambda_1(G) = 7.44 < \lambda_1(G_N) = 10.38$).

Algebraic connectivity μ_{N-1}. This metric states how well connected a graph is and, if the value is greater than zero, that the graph consits of one connected component. Since G has the higher connectivtity, it performs better ($\mu_{N-1}(G) = 0.40 > \mu_{N-1}(G_N) = 0.015$).

All in all, I would recommend design G, and is better suited to convey information across the network.

9)

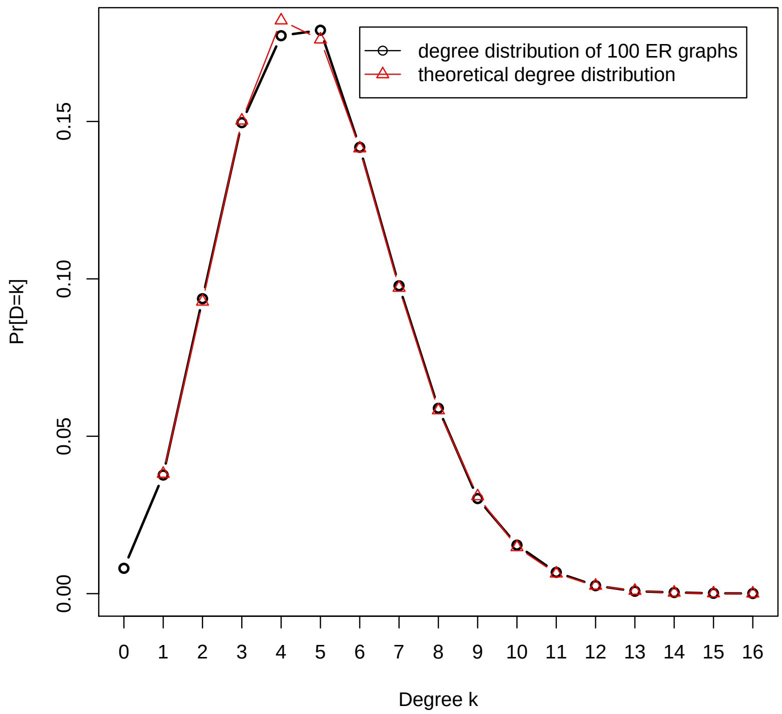

Figure 2: Comparison of the degree distribution of 100 ER instances and the theoretical degree distribution, where $Pr[D = k] = \binom{N-1}{k} p^k (1-p)^{N-1-k} = \binom{378}{k} 0.013^k \cdot 0.987^{378-k}$.

10)

Average of the metrics over the 100 ER random networks:

- Number of nodes N: 379
- Number of links L: 914.33
- Link density p: 0.012764
- Average degree $E[D]$: 4.824960
- Degree variance $Var[D]$: 4.774007

- Clustering coefficient: 0.012574
- Assortativity: -0.003513
- Average hopcount $E[H]$: 3.928829
- Spectral radius λ_1: 6.006430
- Algebraic connectivity μ_{N-1}: 0.025853
- Diameter H_{max}: 7.980000

11)

Now, we have a feeling of which network is better suited for information propagation. However, there are still limitations: for example the speed of information propagation. We know, how many nodes are infected in the $E[n_{R_\infty}]$ state, but not how fast that occured.

Metric	G	G_N	100 ER
N	379	379	379
L	914	914	914.33
p	0.013	0.013	0.013
$E[D]$	4.82	4.82	4.82
$Var[D]$	15.46	15.46	4.77
C	0.17	0.80	0.01
ρ_D	-0.30	-0.08	0.00
$E[H]$	3.75	6.04	3.93
λ_1	7.44	10.38	6.01
μ_{N-1}	0.40	0.015	0.026
H_{max}	7	17	7.98

Table 1: Comparison of all metrics