Complex Networks

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1 Lecture 1

2 Lecture 2

- a_{ij} is an element from the adjacency matrix A
- we are looking for all node pairs that are within a cluster
- the higher m, the more modular the network is
- X is an eigenvector matrix
- The smallest eigenvalue of the Laplacian is always 0
- The number of zeros in the eigenvalues of Q tells you how your network is connected; if the second eigenvalue is also 0, there are two separate parts of the graph
- If the variance of the degree sequence is large, the eigenvalues are also larger
- if you add more link, the eigenvalue also tend to be larger (or stay the same)
- In a regular graph (where each node has the same degree d), the largest eigenvalue is just the degree
- β the rate of infection (infection can mean anything; e.g. the likelihood of purchase)
- dynamic reaches a stable state; interesting to see how many infections there are in a stable state
- the epidemic threshold can be approximated by the largest eigenvalue
- λ is largest eigenvalue
- x is principal eigenvector. The principal eigenvector states how often a website would be visited in a random walk sceanrio (see PageRank)
- H_{ij}: distance from i to j (hop count matrix)

2.1 Network Model - Why do we want to have one?

Reasons:

1. Find out which properties lead to this state
2. To simplify the real world and just study the properties of interest

$$E[L] = E[\sum_{j>i} a_{ij}] = \sum_{j>i} E[a_{ij}] \tag{1}$$

Choosing a random node, what is the probability that its degree equals k in a Erdös-Renyi graph?

$$Pr[D = k] = \binom{N-1}{k} p^k (1-p)^{N-1-k} \tag{2}$$

$\binom{N-1}{k}$ is the number of possibilities to choose k neighbors.

$$E[D] = (N-1)p \tag{3}$$

Another equation (TODO: look it up):

$$(1-p)^{N-1-k} = \tag{4}$$

$$e^{\log(1-p)^N - 1 - k} \tag{5}$$

2.2 Wigner's semicircle

- histogram or pdf of eigenvalues
- the largest eigenvaluie creates the small peaks

2.3 Other stuff

- Average clustering coefficient should be large for small-world network
- Given N, L we can construct a Erdös-Renyi random graph (algorithm: iteratively distribute the links between the nodes)
- difference between C_{actual} and C_{random}: real networks actually have the small world property, therefore, their number is larger
- regular graph: each node is connected to k nearest nodes
- scale-free means that there is no typical value (e.g. power-law networks)
- $\sum_j d_j(t)$ is the total degree of a graph