

# Implementing Approximate inference for Latent Gaussian Markov Random Field Models: the INLA package for R.

# Implementing Integrated Nestad Laplace Approximation

All procedures needed to perform INLA need to be carefully implemented to achieve an optimal speed

- ▶ The GMRFLib-library
- ▶ The inla program
- ▶ The INLA package for R

# Implementing Integrated Nested Laplace Approximation

All procedures needed to perform INLA need to be carefully implemented to achieve an optimal speed

- ▶ The GMRFLib-library
  - ▶ Require a lot of C programming, not user friendly
- ▶ The `inla` program
- ▶ The INLA package for R

# Implementing Integrated Nestad Laplace Approximation

All procedures needed to perform INLA need to be carefully implemented to achieve an optimal speed

- ▶ The GMRFLib-library
- ▶ The inla program
  - ▶ Interface to GMRFLib-library
  - ▶ Avoids all C programming
  - ▶ Requires to write the ini file
  - ▶ Requires to write input files in a special format
  - ▶ Output files can be "tricky" to read
- ▶ The INLA package for R

# Implementing Integrated Nested Laplace Approximation

All procedures needed to perform INLA need to be carefully implemented to achieve an optimal speed

- ▶ The GMRFLib-library
- ▶ The `inla` program
- ▶ The INLA package for R
  - ▶ Interface to the `inla` program
  - ▶ Similar to other R packages
  - ▶ Slightly less flexible than the `inla` program ( but can be used as a starting point.)

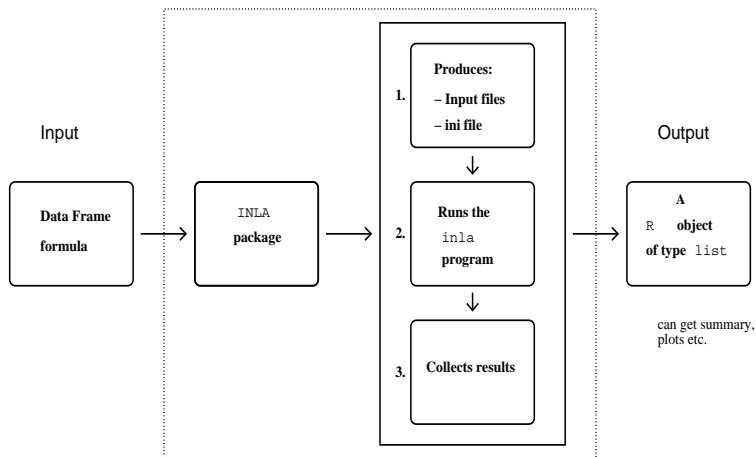
# The INLA package for R

- ▶ Can be downloaded from  
`http://www.math.ntnu.no/~hrue/GMRFLib/R-INLA/`
- ▶ Available for Unix, Windows and Mac
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# The INLA package for R





## Model specification the INLA package I

Assume the following model:

$$\begin{aligned}y &\sim \pi(y|\lambda) \\ \eta &= g(\lambda) = \beta_0 + \beta_1 x_1 + \beta_2 x_1 + f(x_3)\end{aligned}$$

where

$x_1, x_2$  are covariate with linear effect

$$\beta_i \sim \mathcal{N}(0, \tau_1^{-1})$$

$x_3$  can be spatial effect, random effect ....

$$f_1, f_2, \dots \sim \mathcal{N}(0, Q_f^{-1}(\tau_2))$$

## Model specification the INLA package II

The model is specified in R through a formula, similar to the one used in the `glm` routine:

```
> formula <- y ~ x1 + x2 + f(x3)
```

The `f()` function is used to specify non-linear effects in the model.  
The implemented model are :

- ▶ iid random effects
- ▶ rw1 rw2 ar1 smooth effect of covariates or time effect
- ▶ seasonal seasonal effect
- ▶ besag spatial effect (CAR model)
- ▶ generic user defined precision matrix

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The implemented model are :

- ▶ iid random effects
- ▶ `rw1` `rw2` `ar1` smooth effect of covariates or time effect
- ▶ `seasonal` seasonal effect
- ▶ `besag` spatial effect (CAR model)
- ▶ generic user defined precision matrix

## Main functions of the INLA package

- ▶ `f()` : Helps defining non linear effects in the model specification.
- ▶ `inla()` : Performs a Bayesian analysis of additive models
- ▶ `surv.inla()` : Performs a Bayesian analysis of some survival models (experimental!!)

All functions are provided with a help file which can be viewed using

```
> ?inla
```

## Additional functions of the INLA package

- ▶ `summary()` : produces a summary of the main results from a fitted model
- ▶ `plot()` : produces some plots from the fitted model

## Some examples of usage of the INLA package

- ▶ A mixed effect model
- ▶ A model with time series component
- ▶ A model with spatial component
- ▶ A model for survival data

## Mixed model with repeated poisson counts I

Seizure counts in a randomised trial of anti-convulsant therapy in epilepsy. From WinBUGS manual.

Patient	y1	y2	y3	y4	Trt	Base	Age
1	5	3	3	3	0	11	31
2	3	5	3	3	0	11	30
....							
59	1	4	3	2	1	12	37

# Mixed model with repeated poisson counts II

## The model

$$y_{jk} \sim \text{Poisson}(\mu_{jk}); j = 1, \dots, 59; k = 1, \dots, 4$$

$$\begin{aligned} \log(\mu_{jk}) = & \alpha_0 + \alpha_1 \log(\text{Base}_j/4) + \alpha_2 \text{Trt}_j \\ & + \alpha_3 \text{Trt}_j \log(\text{Base}_j/4) + \alpha_4 \text{Age}_j \\ & + \alpha_5 V4 + \text{Ind}_j + \beta_{jk} \end{aligned}$$

$$\alpha_i \sim \mathcal{N}(0, \tau_\alpha) \quad \tau_\alpha \text{ known}$$

$$\text{Ind}_j \sim \mathcal{N}(0, \tau_{\text{Ind}}) \quad \tau_{\text{Ind}} \sim \text{Gamma}(a_1, b_1)$$

$$\beta_{jk} \sim \mathcal{N}(0, \tau_\beta) \quad \tau_\beta \sim \text{Gamma}(a_2, b_2)$$



## Mixed model with repeated poisson counts III

### Fitting the model using INLA

The Epi1 data frame:

y	Trt	Base	Age	V4	rand	Ind
5	0	11	31	0	1	1
3	0	11	31	0	2	1
⋮						

Specifying the model:

```
> formula <- y ~ log(Base/4) + Trt + I(Trt *  
log(Base)) + log(Age) + V4 + f(Ind, model = "iid") +  
f(rand, model = "iid")
```

Running inla

```
> model=inla(formula,family="poisson")
```

## Mixed model with repeated poisson counts III

### Fitting the model using INLA

The Epi1 data frame:

y	Trt	Base	Age	V4	rand	Ind
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Running inla

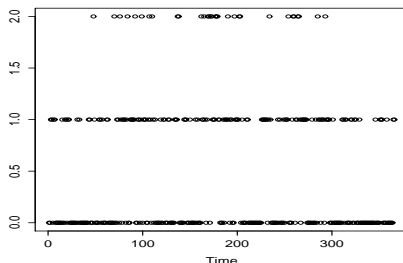
```
> model=inla(formula,family="poisson")
```

## Mixed model with repeated poisson counts IV

Some option of the `inla` function:

- ▶ `verbose=TRUE` shows the steps of the process
- ▶ `keep.data.files=TRUE` keeps the `ini` file and all built input files.
- ▶ `keep.results.files=TRUE` keeps results files (but they're binary files!)

## A model with time series component I



Number of days in Tokyo with rainfall above 1 mm in 1983-84.  
We want to estimate the probability of rain  $p_t$  for calendar day  $t = 1, \dots, 366$

# A model with time series component II

## The model

$$y_t \sim \text{Binomial}(n_t, p_t); \quad t = 1, \dots, 365$$

$$p_t = \frac{\exp(\eta_t)}{1 + \exp(\eta_t)}$$

$$\eta_t = f(t)$$

$$\mathbf{f} = \{f_1, \dots, f_{366}\} \sim \text{cyclic RW2}(\tau)$$

$$\tau \sim \text{Gamma}(1, 0.0001)$$

## A model with time series component III

### Fitting the model using INLA

The Tokyo data frame:

y	n	time
0	2	1
0	2	2
1	2	3
⋮		

## A model with time series component III

### Fitting the model using INLA

The Tokyo data frame:

y	n	time
0	2	1
0	2	2
1	2	3
:		

Specifying the model:

```
> formula <- y ~  
f(time,model="rw2",cyclic=TRUE,param=c(1,0.0001))-1
```



## A model with time series component III

### Fitting the model using INLA

The Tokyo data frame:

y	n	time
0	2	1
0	2	2
1	2	3
:		

Specifying the model:

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:		

Specifying the model:

```
> formula <- y ~  
f(time,model="rw2",cyclic=TRUE,param=c(1,0.0001))-1
```

Running inla

```
> mod.tokyo <-  
inla(formula,family="binomial",Ntrials=n,data=Tokyo)
```

## A model with time series component III

### Fitting the model using INLA

The Tokyo data frame:

y	n	time
0	2	1
0	2	2
1	2	3
:		

Specifying the model:

```
> formula <- y ~  
f(time,model="rw2",cyclic=TRUE,param=c(1,0.0001))-1
```

Running inla

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# Geoadditive model I

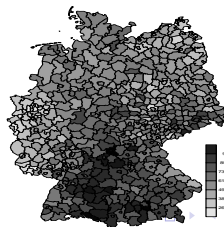
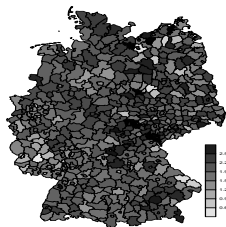
## Disease mapping in Germany

Larynx cancer mortality counts are observed in the 544 districts of Germany from 1986 to 1990 and level of smoking consumption (100 possible values).

$y_i$ ,  $i = 1, \dots, 544$  counts of cancer mortality in Region  $i$

$E_i$ ,  $i = 1, \dots, 544$  known variable accounting for demographic variation in Region  $i$

$c_i$ ,  $i = 1, \dots, 544$  level of smoking consumption registered in Region  $i$



## Geoadditive model II

### The model

$$\begin{aligned}y_i &\sim \text{Poisson}\{E_i \exp(\eta_i)\}; \quad i = 1, \dots, 544 \\ \eta_i &= \mu + f(c_i) + f_s(s_i) + u_i\end{aligned}$$

where:

- ▶  $f(c_i)$  is a smooth effect of the covariate

$$\mathbf{f} = \{f_1, \dots, f_{100}\} \sim \text{RW2}(\tau_f)$$

- ▶  $f_s(s_i)$  is a spatial effect modelled as an intrinsic GMRF

$$f_s(s) | f_s(s'), s \neq s', \lambda_s \sim \mathcal{N}\left(\frac{1}{n_s} \sum_{s \sim s'} f_s(s'), \frac{\tau_{f_s}}{n_s}\right)$$

- ▶  $u_i$  is a random effect

$$\mathbf{u} = \{u_1, \dots, u_{544}\} \sim \mathbf{N}(0, \tau_u \mathbf{I})$$

- ▶  $\mu$  is an intercept term  $\mu \sim \mathcal{N}(0, 0.0001)$

## Geoadditive model II

### The model

$$\begin{aligned}y_i &\sim \text{Poisson}\{E_i \exp(\eta_i)\}; \quad i = 1, \dots, 544 \\ \eta_i &= \mu + f(c_i) + f_s(s_i) + u_i\end{aligned}$$

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## Geoadditive model II

### The model cont.

Prior for the precision parameters:

$$\begin{aligned}\tau_f &\sim \text{Gamma}(1, 0.00005) \\ \tau_{f_s} &\sim \text{Gamma}(1, 0.05) \\ \tau_u &\sim \text{Gamma}(1, 0.001)\end{aligned}$$

## Geoadditive model III

### Fitting the model using INLA

The Germany data frame:

region	E	Y	x
0	7.965008	8	56
1	22.836219	22	65

The model is:

$$\eta_i = \mu + f(c_i) + f_s(s_i) + u_i$$

- ▶ The data set has to contain *one separate column for each term specified through  $f()$*  so in this case we have to add one column..

```
> Germany<-cbind(Germany,region.struct=Germany$region)
```

- ▶ We also need the graph file where the neighbourhood structure is specified `germany.graph`

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## Geoadditive model III

### Fitting the model using INLA

The new data set is:

region	E	Y	x	region.struct
0	7.965008	8	56	0
1	22.836219	22	65	1

Then the formula is

```
formula <-
```

```
Y~f(region.struct,model="besag",graph.file="germany.graph",  
param=c(1,0.00005))+f(x,model="rw2",param=c(1,0.05))+f(region)
```

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param=c(1,0.00005))+f(x,model="rw2",param=c(1,0.05))+f(region)
```

The location of the graph file has to be provided here (the graph file cannot be loaded in R)

## The graph file

	544					
	0	1	11			
The germany.graph file:	1	2	9	10		
	2	4	5	7	14	386
	⋮					

- ▶ Total number of nodes in the graph
- ▶ Identifier for the node
- ▶ Number of neighbours
- ▶ Identifiers for the neighbours

## The graph file

The germany.graph file:

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## Geoadditive model III

### Running the model using INLA

```
mod <-  
inla(formula,family="poisson",data=Germany,E=E,  
control.inla=list(h=0.01),verbose=TRUE)
```

- ▶ Defines the known variable adjusting for demographic variation  $E_i$  (*only used for Poisson likelihood*)
- ▶ Controls some parameters in the `inla` program, in this case the step length to compute the hessian at the mode of  $\pi(\theta|\mathbf{y})$

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## Geoadditive model IV

### COmputing DIC and marginal likelihood

```
mod <-  
inla(formula,family="poisson",data=Germany,E=E,  
control.compute=list(dic=1,mlik=1),  
control.inla=list(h=0.01),verbose=TRUE)
```

## A model for survival data

### Kidney infection data

patient	time	event	age	sex
1	8,16	1,1	28,28	0
2	23,13	1,0	48,48	1
3	22,18	1,1	32,32	0

Times of infection from the time of insertion of catheter on 38 kidney patients using portable dialysis equipment.

2 observation for each patient (38 patients).

Each time can be an *event* (infection) or a *censuring* (no infection)

## Hazard rate and survival function

Density function:

$$y \sim f(y)$$

Survival function:

$$S(y) = 1 - F(y) = \int_y^{\infty} f(u) \, du$$

Hazard function:

$$\begin{aligned} h(y) \, dy &= \text{Prob}(y \leq Y < y + dy | Y > y) \\ h(y) &= \frac{f(t)}{S(t)} \end{aligned}$$

## A model for survival data II

### The model

We model the hazard function for each patient as:

$$h(y_{ij}|w_i, \mathbf{x}_{ij}) = h_0(y_{ij}) w_i \exp(\mathbf{x}_{ij}^T \boldsymbol{\beta}); \quad i = 1, \dots, 38; \quad j = 1, 2$$

where

$h_0(\cdot)$  is the baseline hazard function

$w_i$  is the frailty effect associated with patient  $i$

$\mathbf{x}_{ij}$  is the vector of observed covariates for patient  $i$  at observation  $j$

$\boldsymbol{\beta}$  is a vector of unknown parameters

## A model for survival data II

### The model cont.

We assume the frailty term to be a series of i.i.d. lognormal random variables:

$$b_i = \log(w_i) \sim \mathcal{N}(0, \tau_w)$$

We divide the time axis into  $J$  intervals  $I_k = (s_{k-1}, s_k]$  for  $k = 1, 2, \dots, J$  where  $0 = s_0 < s_1 < \dots < s_J < \infty$ ,  $s_J$  censored time and assume the baseline hazard to be piecewise constant

$$h_0(y) = \lambda_k, \text{ for } y \in I_k.$$

Moreover we assume

$$(\epsilon_1, \dots, \epsilon_J) = \log(\lambda_1, \dots, \lambda_J) \sim \text{RW1}(10^{-4})$$



## A model for survival data II

### The model cont.

The hazard rate can then be written as

$$h(y_{ij}|w_i, \mathbf{x}_{ij}) = \exp(\mathbf{x}_{ij}^T \boldsymbol{\beta} + b_i + \epsilon(y_{ij})); \quad i = 1, \dots, 38; \quad j = 1, 2$$

where

$$\epsilon(y_{ij}) = \epsilon_k \text{ if } y_{ij} \in I_k$$

## A model for survival data III

### Fitting the model using INLA

The Kidney data frame

time	event	age	sex	ID
8	1	28	0	1
16	1	28	0	1
23	1	48	1	2
13	0	48	1	2
22	1	32	0	3
28	1	32	0	3

Specifying the model:

`formula =`

`time~age+sex+f(ID,param=c(0.001,0.001),initial=0.6)`

## A model for survival data III

### Fitting the model using INLA cont.

To run the model we use the function `surv.inla`:

```
> model=surv.inla(formula,family="piecewise.constant",  
n.intervals=10, data=Kidney,event=event,  
control.hazard=list(initial=log(0.0001),fixed=1))
```

## A model for survival data III

### Fitting the model using INLA cont.

To run the model we use the function `surv.inla`:

```
> model=surv.inla(formula,family="piecewise.constant",  
n.intervals=10, data=Kidney,event=event,  
control.hazard=list(initial=log(0.0001),fixed=1))
```

## A model for survival data III

### Fitting the model using INLA cont.

To run the model we use the function `surv.inla`:

```
> model=surv.inla(formula,family="piecewise.constant",  
n.intervals=10, data=Kidney,event=event,  
control.hazard=list(initial=log(0.0001),fixed=1))
```

## A model for survival data III

### Fitting the model using INLA cont.

To run the model we use the function `surv.inla`:

```
> model=surv.inla(formula,family="piecewise.constant",  
n.intervals=10, data=Kidney,event=event,  
control.hazard=list(initial=log(0.0001),fixed=1))
```

## A model for survival data III

### Fitting the model using INLA cont.

To run the model we use the function `surv.inla`:

```
> model=surv.inla(formula,family="piecewise.constant",  
n.intervals=10, data=Kidney,event=event,  
control.hazard=list(initial=log(0.0001),fixed=1))
```

## Other available models for survival data

- ▶ `family=exponential`
- ▶ `family=weibull`