PoliTo Rocket Team

POLITECNICO DI TORINO

Technical Report

Insights on RPA unclear parameters

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Contents

1 Introduction

In this small report some insights regarding unclear parameters of the RPA software are stated, such as "Divergence Efficiency", "Drag Efficiency", "Relative thickness of near-wall layer" and "Emissivity of gas-side wall".

2 Divergence Efficiency

Divergence Loss For conical nozzle the correction factor is given by following equation [9]:

$$\zeta_d = \frac{1+\cos(\theta_e)}{2} \tag{1}$$

Whereθ_e is the half-angle of the conical nozzle.

For bell nozzles with known pressure field distribution at nozzle exit the correction factor for divergence loss can be calculated using the equation 1:

$$\zeta_d = 1 - \frac{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}}\left[z(\lambda_e)-1\right]-\bar{P}}{\left(\frac{2}{k+1}\right)^{\frac{1}{k-1}} z(\lambda_e)} \tag{2}$$

Where:

$$z(\lambda_e) = 0.5\left(\lambda_e + \frac{1}{\lambda_e}\right) \tag{3}$$

λ_e: characteristic Mach number at nozzle exit obtained for quasi two-dimensional nozzle flow;

$$\bar{P} = \int_1^{\bar{r}_e} \frac{p}{p_{0c}} \bar{r} d\bar{r} \tag{4}$$

$$\bar{r} = \frac{r}{r_i} \text{nozzle exit relative radius} \tag{5}$$

k: specific heats ratio

For bell nozzles with known gas velocity field distribution at nozzle exit the correction factor for divergence loss can be calculated as follows [5]:

$$\zeta_d = \frac{(C_f^{vac})_{2D}}{(C_f^{vac})_{1D}} \tag{6}$$

Where:

- $(C_f^{vac})_{1D}$: thrust coefficient calculated for quasi-one-dimensional nozzle flow;

- $(C_f^{vac})_{2D}$: thrust coefficient calculated for axisymmetric two-dimensional nozzle flow given as:

$$(C_f^{vac})_{2D} = 2\frac{A_e}{A_t}\int_0^1 \left(1-\frac{k-1}{k+1}\lambda^2\right)^{\frac{1}{k-1}}\left[1+\lambda^2\frac{k\left(2\cos^2(\beta)-1\right)+1}{k+1}\right]\bar{r}d\bar{r} \tag{7}$$

Where: λ is the characteristic Mach number at nozzle exit (see Figure 1).

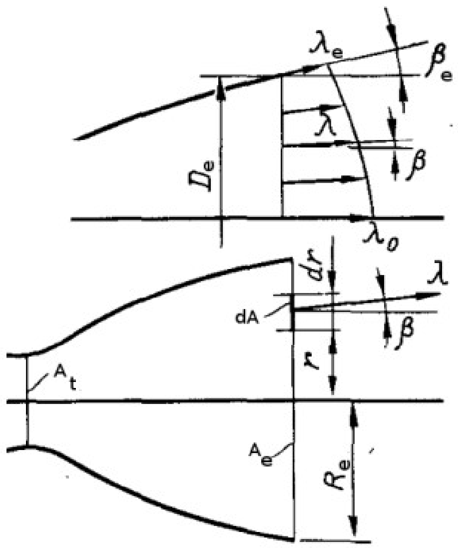

Figure 1: Velocity field distribution atnozzle exit (taken from [5])

Alternatively, the correction factor for bell nozzle can be estimated using the following empirical equation:

$$\zeta_d = 1 - B\frac{e^{n(1-\frac{\bar{r_e}}{\bar{r_a}})}-1}{e^n-1} \tag{8}$$

Where:

- $B = 1.52\cdot(e^{-30(k-1)}+0.1)$
- $n = 1.45\cdot\bar{r_0}^{-0.25} - 0.005\cdot\bar{r_0}$
- $\bar{r_0} = 1+\frac{\bar{r_e}-1}{\bar{L}}$
- $\bar{r_e} = \frac{r_e}{r_i}$ is the nozzle exit relative radius
- $\bar{L}$: is the relative nozzle length
- $\frac{ln\left(\frac{p_{0c}}{p_e}\right)}{ln\left(\frac{p_{0c}(RT)_e}{p_e(RT)_{0c}}\right)}$: is the average isentropic expansion coefficient

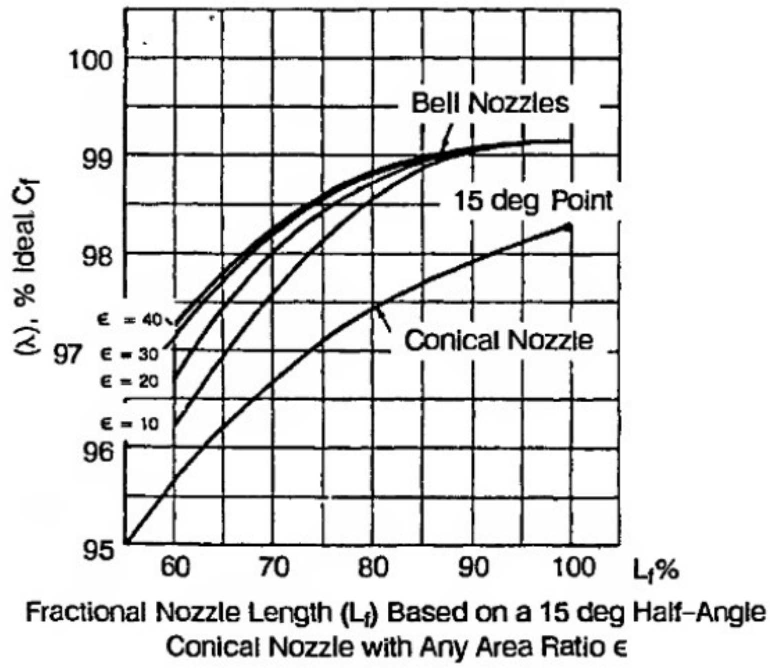

Figure 2: Thrust efficiency vs bell nozzle length (taken from [9])

This equation is applicable for nozzles with $\bar{L} = (0.4 \div 1.0)$ and $k = (1.1 \div 1.25)$. For quick estimation the correction factor can also be obtained from the following diagram:

3 Drag Efficiency

Corrections factors can be grouped into two categories:

- Combustion chamber correction factor ζ_c to correct the performance parameters due to real processes in the combustion chamber
- Nozzle correction factor ζ_n to correct the performance parameters due to real processes in the nozzle

Corrections factors are given by:

$$\zeta_c = \zeta_r \tag{9}$$

$$\zeta_n = \zeta_f \cdot \zeta_d \cdot \zeta_z \tag{10}$$

Where:

- ζ_r: correction factor that represents performance loss due to finite rate kinetics in the combustion chamber;

- ζ_f: correction factor that represents performance loss due to friction in boundary layer;
- ζ_d: correction factor that represents performance loss due to divergence, or two-dimensional flow in the nozzle;
- ζ_d: correction factor that represents performance loss due to multi-phase flow in the nozzle.

Using introduced correction factors, the assessment of delivered performance parameterscan be performed as follows:

$$(I_s^{vac})_d = \zeta_c \cdot \zeta_n \cdot I_s^{vac} \tag{11}$$

$$(I_s^{opt})_d = \zeta_c \cdot \zeta_n \cdot I_s^{vac} - \bar{F}_e P_e \tag{12}$$

$$(I_s^{SL})_d = \zeta_c \cdot \zeta_n \cdot I_s^{vac} - \bar{F}_e P_a^{SL} \tag{13}$$

$$(c^*)_d = \zeta_c \cdot c^* \tag{14}$$

$$(C_f^{vac})_d = \frac{(I_s^{vac})_d}{(c^*)_d} = \zeta_n \cdot C_f^{vac} \tag{15}$$

$$(C_f^{opt})_d = \frac{(I_s^{opt})_d}{(c^*)_d} = \zeta_n \cdot C_f^{opt} \tag{16}$$

$$(C_f^{SL})_d = \frac{(I_s^{SL})_d}{(c^*)_d} = \zeta_n \cdot C_f^{SL} \tag{17}$$

Where $\bar{F}_e = \frac{A_e}{\dot{m}} = \frac{1}{(w\rho)_e}$ is a nozzle exit specific area.

Boundary Layer (Friction) Loss The correction factor due to wall friction in boundary layer can be calculated using following relation:

$$\zeta_f = 1 - \frac{2\bar{\delta_e^{**}}}{1 + \frac{1}{kM_e^2}} \tag{18}$$

Where:

- $\bar{\delta_e^{**}} = \frac{\bar{\delta^{**}}}{r_e}$ is the relative momentum thickness
- r_e is the nozzle exit radius
- M_e is the Mach number at nozzle exit obtained for quasi two-dimensional nozzle flow
- k is the specific heats ratio

This expression is applicable both for laminar and turbulent boundary layers.
In rocket engine nozzles the boundary layer is usually laminar at $Re_{w0} < 1 \cdot 10^7$ and turbulent at $Re_{w0} > 3 \cdot 10^7$. Here Reynolds number is given by:

$$Re_{w0} = \frac{w_e \rho_{0c} L_n}{\mu_w} \tag{19}$$

Where:

- w_e is the gas exhaust velocity in vacuum
- ρ_{0c} is the gas stagnation density at nozzle inlet
- L_n is the nozzle lenght
- μ_w is the dynamic viscosity at T_w

Smaller engines (with thrust below 45000 N) tend to have laminar boundary layers, whereas the large engines are almost always turbulent. For turbulent boundary layer the momentum thickness δ^{**} is given by:

$$\delta^{**} = \frac{\left(\frac{2}{k-1}\right)^{0.1}}{Re_{w0}^{0.2}} \left(\frac{0.015}{\bar{T_w}^{0.5}}\right)^{0.8} \frac{\left(1+\frac{k-1}{2}M_{we}^2\right)^{\frac{k+1}{2(k-1)}}}{M_w^{\nu+1}} \frac{\bar{S}^{0.2}}{\bar{r_e}^2} \left[\int_0^{\bar{S}} \frac{\bar{r}^{-1.25} M^{1+1.25\nu}}{\left(1+\frac{k-1}{2}M_w^2\right)^{\frac{1.36k-0.36}{k-1}}} d\bar{S}\right]^{0.8} \tag{20}$$

Where:

- $\bar{T_w} = \frac{T_w}{T_0}$, for adiabatic nozzles $\bar{T_w} = 0.9$
- $\nu = \frac{18}{7}\bar{T_w} - \frac{2}{7}$
- T_w is the nozzle wall temperature
- T_0 is the gas stagnation temperature
- M_w is the Mach number next to the nozzle wall
- M_{we} is the Mach number at nozzle exit next to the nozzle wall
- $\bar{S} = \frac{S}{r_t}$ is the relative lateral lenght of nozzle
- r_i is the nozzle throat radius

This equation can be used both for convergent and divergent nozzle sections. For laminar boundary layer the momentum thickness δ^{**} can be calculated using equations an other set of equation in reference [1].

For nozzle with know distribution of viscosity stress along the contour (this is usually the case if thermal analysis of the chamber is performed), the correction factor due for friction in boundary layer can be calculated using following relation :

$$\zeta_f = 1 - \frac{2\pi \int_0^{x_e} \tau R \cos(\theta) dx}{C_f A_t p_{0c}} \tag{21}$$

Where:

- ζ_f is the performance correction factor for the friction loss
- τ is the viscosity stress (see section "Gas-Side Heat Transfer" in reference [2])
- C_f is the ideal thrust coefficient
- A_t is the nozzle throat area
- p_{0c} is the stagnation pressure at nozzle inlet
- x,R,θ are size and shape parameters of the thrust chamber as defined in Figure 1 in reference [2]

For quick estimation the performance loss can be estimated using the following diagram:

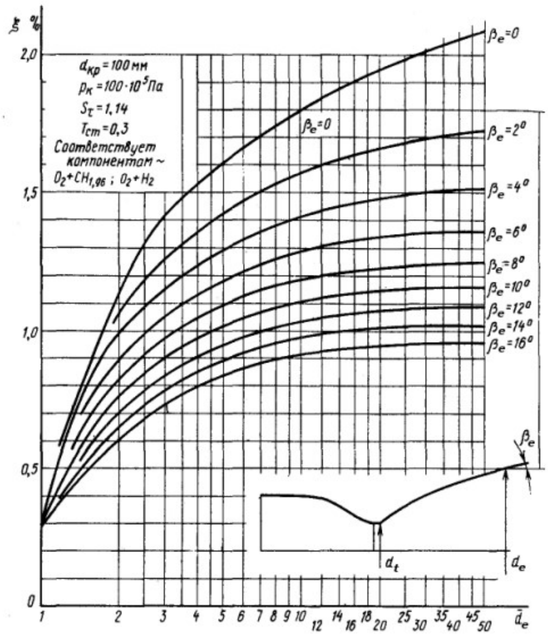

Figure 3: Performance loss coefficient due to friction in boundary layer

4 Convective Heat Transfer

The program implements two different methods of calculating the gas-side heat transfer rates: Ievlev approach for calculation of convective heat transfer in the nozzle and Bartz semi-empirical correlation for gas-side heat transfer coefficient. The user has the possibility to choose either one of two mentioned methods or combined approach, which calculates the resulting heat transfer rate as an average value of that calculated by Ievlev and Bartz methods[3].

In Bartz approach, gas-side heat transfer coefficient h_g is expressed by[4]

$$h_g = \left[\frac{0.026}{D_t^{0.2}} \left(\frac{\mu^{0.2} c_p}{Pr^{0.6}} \right)_{ns} \left(\frac{p_c}{c^*} \right)^{0.8} \left(\frac{D_t}{R} \right)^{0.1} \right] \left(\frac{A_t}{A} \right)^{0.9} \sigma$$

where $R = (R_{tin} + R_{tout})/2$ is the radius of curvature of nozzle contour at throat, D_t is throat diameter, ns means nozzle stagnation condition and σ is the correction factor for property variations across the boundary layer and can be evaluated in terms of nozzle stagnation temperature T_0, local gas-side chamber wall temperature T_w, and local Mach number M[4]:

$$\sigma = \left[0.5 \frac{T_w}{T_0} \left(1 + \frac{\gamma - 1}{2} M^2 \right) + 0.5 \right]^{-0.68} \left(1 + \frac{\gamma - 1}{2} M^2 \right)^{-0.12}$$

On the other side, the approximate formula of Ievlev method is used to calculate the convective heat flux, taking into account the change in the parameters with respect to time, which has the form[4]:

$$q_k(x,t) = B \frac{\tau(\lambda)}{\overline{D}^{1.82}} \frac{P_{cc}^{0.85}(t)}{d_{th}^{0.15}} \frac{S(K_{m_w}(x), T_w(x,t))}{Pr^{0.58}}$$

Detailed insights of Ievlev and Bartz methods are described in software related documentation[2][5] and in cited articles[4].

An input parameter named ”Relative thickness of near-wall layer” is requested by the user, if not specified, the default value 0.025 is assumed.

Consulting software API[6], this parameter is used by the `solve` function as the `wallLayer` variable to *solve cooling problem*. However, it is not specified what it refers to, whether to some geometrical aspects of the nozzle or to the fluid dynamic properties of the flow, such as boundary layer thickness. Therefore, technical support is needed to clarify any doubts.

5 Radiation Heat Transfer

The program can perform the thermal analysis, calculating the heat flux either as a convective heat transfer only or summing the contributions of convective and radiation heat transfers. In the last case, the user shall specify a radiation emissivity coefficient of the wall inner surface[3]. If not specified, the default value 0.8 is assumed for the emissivity of gas-side wall. According to software documentation[5], this value represents the typical emissivity coefficient for oxidized steel in the range of surface temperature of 450-850 K. More typical values of some materials can be found in the following table:

Table 1. Emissivity coefficients of some materials

Material	Surface temperature Tw, K	Emissivity coefficient
Aluminum, oxidized	450 – 850	0.11 – 0.19
Copper alloy (bronze), polished	320	0.1
Copper alloy (bronze), rough	320 – 420	0.55
Copper alloy (brass), oxidized	450 – 850	0.61 – 0.59
Copper, oxidized	450 – 850	0.57 – 0.87
Nickel, oxidized	450 – 850	0.37 – 0.48
Niobium	450 – 650 1250 – 1850	0.17 0.20
Stainless steel	750	0.35
Steel, oxidized	450 – 850	0.8
Steel, heavily oxidized	750	0.98
Chrom-Nickel alloy	400 – 1300	0.64 – 0.76

More generally, for given hot gas temperature T_∞ and wall temperature T_w, the basic correlation for the radiation heat transfer is given by

$$q_r = \varepsilon_e \sigma \left(\varepsilon_r^{T_\infty} T_\infty^4 - \varepsilon_g^{T_w} T_w^4 \right)$$

where

$\sigma = 5.670373 \cdot 10^{-8}\, W/(m^2 K^4)$ is the Stefan-Boltzmann constant;

$\varepsilon_e = \varepsilon_w / \left[1 - (1-\varepsilon_w)(1-\varepsilon_r^{T_w})\right]$ is the wall effective emissivity coefficient;

ε_w is the emissivity coefficient of the wall material (see table above);

$\varepsilon_r^{T_\infty}$ is the emissivity coefficient of the reaction products at temperature T_∞;

$\varepsilon_r^{T_w}$ is the emissivity coefficient of the reaction products at temperature T_w.

A detailed procedure to estimate emissivity coefficients ε_r^T of reaction products at specific temperature T is described in the software documentation[5](page 12).

References

[1] Glushko V.P. and Alemasov V.E. *Thermodynamic and Thermophysical Properties of Combustion Products. Volume 1*. Moscow, 1971.

[2] Alexander Ponomarenko. *RPA – Tool for Rocket Propulsion Analysis*. 2014.

[3] *Rocket Propulsion Analysis 2.3 User Manual*. Cologne, Germany, 2017.

[4] Alexey Gennadievich Vorobyev and Lihui Zhang. *Thermal state calculation of chamber in small thrust liquid rocket engine for steady state pulsed mode*. Department of Rocket Engines, Moscow Aviation Institute, Moscow 125993, Russia; School of Astronautics, Beihang University, Beijing 100191, China, 2019.

[5] Alexander Ponomarenko. *Thermal Analysis of Thrust Chambers*. 2012.

[6] *https://rocket-propulsion.com/downloads/2/docs/scripting_api/index.htm*.