

RPA: Tool for Rocket Propulsion Analysis

Estimation of Engine Mass

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Abstract

Rocket Propulsion Analysis (RPA) is a multi-platform analysis tool intended for use in conceptual and preliminary design (design phases 0/A/B1).

This report describes a model used in RPA to estimate the mass of the engine.

Contents

Nomenclature.....	3
Introduction.....	4
Numerical Model.....	4
Engine Mass.....	4
Thrust Chamber Mass.....	4
Original model.....	4
RPA model.....	7
Turbopump Mass.....	8
Original model.....	8
RPA model.....	8
Mass of Other Components.....	9
Original model.....	9
RPA model.....	9
References.....	10

Nomenclature

A cross-sectional area, m^2

\bar{A} relative cross-sectional area, $\bar{A} = A/A_t$

c^* characteristic velocity, m/s

p pressure, Pa

ρ density, kg/m^3

\dot{m} mass flow rate, kg/s

Further symbols will be introduced and explained in the text.

Introduction

Rocket Propulsion Analysis (RPA) is a multi-platform analysis tool intended for use in conceptual and preliminary design (design phases 0/A/B1). On early design steps many parameters of the engine are not known, or estimated using (semi)empirical methods. Therefore the exact calculation of the engine mass is not possible.

An engine mass estimation is based on semi-empirical model published by A. Kozlov et al [2]. The model has been developed from physical equations using available data for historical engines such as F-1, RS-25 (SSME), RS-27, RS-89, HM-7, LR-10, TR-201, AJ-10, and some other.

In order to better match the data for historical engines, the slightly modified model (mainly by adjusting coefficients) has been implemented in software tool RPA.

Numerical Model

Engine Mass

Engine mass m_e is represented by mass of its components:

$$m_e = \sum_i (m_c)_i + \sum_j (m_{tp})_j + m_a$$

where

m_c - mass of thrust chamber

m_{tp} - mass of turbopump unit

m_a - mass of other components

Thrust Chamber Mass

Original model

Mass of thrust chamber is a sum of mass its components:

$$m_c = m_g + m_{inj} + m_{cyl} + m_{inl} + m_e$$

where m_g is a mass of hot gas duct (applicable for gas generator cycle only), m_{inj} is a mass of injector head, m_{cyl} is a mass of cylindrical part of combustion chamber, m_{inl} is a mass of nozzle from inlet to throat, and m_e is a mass of nozzle from throat to nozzle exit.

For simplicity, all components of thrust chamber can be replaced by surfaces of revolution with various specific mass. Then the mass of components can be expressed as a product of surface area and specific mass.

Hence, the mass of combustion chamber and nozzle inlet can be written as:

$$m_{cyl} + m_{inl} = \gamma_c (S_{cyl} + S_{inl}) = \frac{\dot{m}_c c^*}{p_c} \gamma_c (\bar{S}_{cyl} + \bar{S}_{inl})$$

where γ_c is a specific mass of combustion chamber (kg/m^2), $\bar{S}_{cyl} = S_{cyl}/A_t$ is a relative surface area of cylindrical part of combustion chamber, $\bar{S}_{inl} = S_{inl}/A_t$ is a relative surface area of nozzle inlet.

The mass of injector head can be written as:

$$m_{inj} = A_c \delta_{inj} \rho_{inj} = A_t \bar{A}_c \delta_{inj} \rho_{inj}$$

where A_c is a cross-section area of combustion chamber and $\bar{A}_c = A_c/A_t$ is a relative cross-section area, δ_{inj} is an effective thickness (height) of injector head, and ρ_{inj} is an effective density of injector head.

The mass of hot gas duct is given by:

$$m_g = S_g \delta_g \rho_g = \pi D_g L_g \delta_g \rho_g$$

where D_g is an average diameter of gas duct, L_g is a length of gas duct, δ_g is an effective wall thickness of gas duct, and ρ_g is an effective density of gas duct.

The mass of nozzle exit can be written as:

$$m_e = \gamma_e S_e + \gamma_{ext} S_{ext} = \frac{\dot{m}_c c^*}{p_c} (\gamma_e \bar{S}_e + \gamma_{ext} \bar{S}_{ext})$$

where γ_e and γ_{ext} are specific masses of cooled nozzle exit and uncooled nozzle extension (kg/m^2), $\bar{S}_e = S_e/A_t$ and $\bar{S}_{ext} = S_{ext}/A_t$ are relative surface areas of cooled nozzle exit and uncooled nozzle extension correspondingly.

From given expressions and using available data for historic engines, Kozlov et al derive the following equations for mass of the thrust chamber:

- for chambers of engines with staged-combustion cycle:

$$m_c = A_t \left\{ \gamma_c (\bar{S}_{cyl} + \bar{S}_{inj}) + \gamma_e \bar{S}_e + \frac{3.962 \cdot 10^6}{(p_c \dot{m}_c c^*)^{0.25}} + 17.58 (p_c \dot{m}_c c^*)^{0.125} - \frac{13.3}{A_t} \right\}$$

- for chambers of engines with other cycles:

$$m_c = A_t \left\{ \gamma_c (\bar{S}_{cyl} + \bar{S}_{inj}) + \gamma_e \bar{S}_e + \frac{1.63 \cdot 10^6}{(p_c \dot{m}_c c^*)^{0.25}} - \frac{8.5}{A_t} \right\}$$

Equations for coefficients γ_c , γ_e , \bar{S}_{cyl} , \bar{S}_{inj} , and \bar{S}_e are listed in the table 1.4 (p.44) in the textbook by Kozlov et al [2] (here equation for γ_c is on row 1, equation for γ_e is on row 2, equation for \bar{S}_{inj} is on row 3, equation for \bar{S}_{cyl} is on row 4, and equation for \bar{S}_e is on row 5:

Величина и размер- ность	Расчетная формула	Диапазон изменения параметров и их размерность
$\gamma_K, \frac{\text{кг}}{\text{м}^2}$	$\gamma_K = 3,03 \frac{p_K \cdot 10^{-6}}{\sqrt{d_{кр}}} - 17$	$1,581 \cdot 10^6 \leq \frac{p_K}{\sqrt{d_{кр}}} \leq 5,85 \cdot 10^6$ $p_K, \text{Па}; d_{кр}, \text{м}$
$\gamma_c, \frac{\text{кг}}{\text{м}^2}$	$\gamma_c = 5,894 \cdot 10^{-2} \left(\frac{p_K}{\sqrt{\epsilon d_{кр}}} \right)^{0,475} - 23,58$	$0,0632 \cdot 10^6 \leq \frac{p_K}{\sqrt{\epsilon d_{кр}}} \leq$ $\leq 0,31623 \cdot 10^6$ $p_K, \text{Па}; d_{кр}, \text{м}$
$\bar{S}_{c.c}$	$\bar{S}_{c.c} = \frac{2}{\bar{q}_K \beta} + \frac{0,818}{\sqrt{\bar{q}_K \beta}} - 0,974$	$\bar{q}_K, \text{с/м}, \beta, \text{м/с}$
$\bar{S}_{ц}$	$\bar{S}_{ц} = 3,544 L_{прив} \times$ $\times \sqrt{\frac{p_K \bar{q}_K}{\dot{m}_K}} - 2 \frac{1}{\sqrt{\bar{q}_K \beta}} + \sqrt{\bar{q}_K \beta} + 1$	$\bar{q}_K, \text{с/м}, \beta, \text{м/с}$ $p_K, \text{Па}, \dot{m}_K, \frac{\text{кг}}{\text{с}}$ $L_{прив}, \text{м}$
\bar{S}_c	$\bar{S}_c = S_0 \left[1 - \left(1,415 - \frac{0,274}{\sqrt{\bar{R}_a}} \right) \times \right.$ $\times f(z) \left. \right],$ где $S_0 = (32 - 10n)(\bar{R}_a - 1) +$ $+ (2,1 + 1,6n^4)(\bar{R}_a - 1)^{2,25};$ $\bar{R}_a = \frac{r_a}{r_{кр}} = \frac{\left(\frac{2}{n+1} \right)^{\frac{0,5}{n-1}} \left(\frac{n-1}{n+1} \right)^{0,25}}{\sqrt{\frac{2}{\epsilon n} - \frac{n+1}{\epsilon n}}};$ $f(z) = 1 - \exp(-\sqrt[3]{1-z});$ $z = 1 - \left\{ \frac{\sin \beta_a}{0,6 - (0,018n - 0,0175)} \dots \rightarrow \right.$ $\left. \leftarrow \dots \frac{1}{(R_a + 24)} \right\}^{4/3}$	$\beta_a, \text{градус}$ $50 \leq \epsilon \leq 3000$ $1,13 \leq n \leq 1,21$

RPA model

For RPA, the model has been updated as follows:

- for chambers of engines with staged-combustion cycle:

$$\gamma_c = 2.03 \cdot 10^{-6} \frac{p_c}{\sqrt{D_t}} - 17$$

$$\gamma_e = 5.494 \cdot 10^{-2} \left(\frac{p_c}{\sqrt{A_e D_t}} \right)^{0.472} - 23.58$$

$$m_c = A_t \left\{ \gamma_c (\bar{S}_{cyl} + \bar{S}_{inj}) + \gamma_e \bar{S}_e + \frac{3.962 \cdot 10^6}{(p_c \dot{m}_c c^*)^{0.25}} - \frac{13.3}{A_t} \right\} + A_t 17.58 (p_c \dot{m}_c c^*)^{0.125} \log(1 + P \cdot 10^{-5}) n$$

where P is a thrust of the engine (N) and n is a number of thrust chambers.

- for chambers of engines with gas-generator and pressure-fed cycles:

$$\gamma_c = \max \left(20 \mid 3.691 \cdot 10^{-6} \frac{p_c}{\sqrt{D_t}} - 17 \right)$$

$$\gamma_e = \max \left(20 \mid 0.054 \left(\frac{p_c}{\sqrt{A_e D_t}} \right)^{0.49} - 5 \right) \quad \text{for } P \leq 50 \cdot 10^3$$

$$m_c = A_t \left\{ \gamma_c (\bar{S}_{cyl} + \bar{S}_{inj}) + \gamma_e \bar{S}_e + \frac{3.368 \cdot 10^5}{(p_c \dot{m}_c c^*)^{0.25}} - \frac{10.042}{A_t} \right\} \quad \text{for } P \leq 50 \cdot 10^3$$

$$\gamma_e = \max \left(20 \mid 0.05894 \left(\frac{p_c}{\sqrt{A_e D_t}} \right)^{0.475} - 23.58 \right) \quad \text{for } P > 50 \cdot 10^3$$

$$m_c = A_t \left\{ \gamma_c (\bar{S}_{cyl} + \bar{S}_{inj}) + \gamma_e \bar{S}_e + \frac{1.63 \cdot 10^6}{(p_c \dot{m}_c c^*)^{0.25}} - \frac{8.5}{A_t} \right\} \quad \text{for } P > 50 \cdot 10^3$$

where P is a thrust of the engine (N).

- for chambers of engines with expander cycle:

$$\gamma_c = 3.691 \cdot 10^{-6} \frac{p_c}{\sqrt{D_t}} - 17$$

$$\gamma_e = 0.054 \left(\frac{p_c}{\sqrt{A_e D_t}} \right)^{0.5} - 5$$

$$m_c = A_t \left\{ \gamma_c (\bar{S}_{cyl} + \bar{S}_{inj}) + \gamma_e \bar{S}_e + \frac{3.368 \cdot 10^5}{(p_c \dot{m}_c c^*)^{0.25}} - \frac{10.042}{A_t} + 25.667 (p_c \dot{m}_c c^*)^{0.125} \right\}$$

Turbopump Mass

Original model

For purposes of mass estimation, the turbopump consists of set of discs (representing impellers and turbines wheels) and cylinders (representing housing elements), which effective size depends on density and mass flow rate of propellant components, as well as on main design parameters of turbopump (such as rotation speed and outlet pressure).

From these assumptions, and from analysis of data for historical engines, the mass of turbopump can be written as:

$$m_{tpu} = A + B \sum_i \frac{\dot{m}_i H_i^{1.5}}{\omega} = A + B \sum_i \frac{\dot{m}_i}{\omega} \left(\frac{\Delta p_i}{\rho_i} \right)^{1.5} = A + B \cdot D$$

where

$$D = \sum_i \frac{\dot{m}_i}{\omega} \left(\frac{\Delta p_i}{\rho_i} \right)^{1.5}$$

$\Delta p_i = p_{out\ i} - p_{in\ i}$ - increase in pressure of propellant component i .

Coefficients A and B are given for different types of engine cycles as follows:

- for staged-combustion cycle:

$$A=19 \text{ and } B=0.232 \cdot 10^{-3} \quad \text{for } 2.93 \cdot 10^4 \leq D \leq 1.82 \cdot 10^6$$

- for other cycles:

$$A=6.29 \text{ and } B=0.981 \cdot 10^{-3} \quad \text{for } 1170 \leq D \leq 3.22 \cdot 10^4$$

$$A=21 \text{ and } B=0.54 \cdot 10^{-3} \quad \text{for } 2.93 \cdot 10^4 \leq D \leq 7.52 \cdot 10^5$$

RPA model

For RPA, the model has been updated as follows:

- for staged-combustion cycle:

$$A=19 \text{ and } B = \frac{0.232 \cdot 10^{-3}}{e^{1.3 \cdot 10^{-7} P}}$$

$$B_f = 0.35 B \text{ for } \rho_f < 200 \text{ kg/m}^3 \text{ where } \rho_f \text{ is a density of the fuel}$$

- for gas-generator cycle:

$$A=6.29 \text{ and } B=0.981 \cdot 10^{-3} \quad \text{for } D < 3 \cdot 10^4$$

$$A=21 \text{ and } B=0.82 \cdot 10^{-3} \quad \text{for } 3 \cdot 10^4 \leq D \leq 8 \cdot 10^5$$

$$A=80 \text{ and } B=0.3 \cdot 10^{-3} \quad \text{for } D > 8 \cdot 10^5$$

$$B_f = 0.7 B \text{ for } \rho_f < 200 \text{ kg/m}^3 \text{ where } \rho_f \text{ is a density of the fuel}$$

- for expander cycle:

$$A=15 \text{ and } B_{ox}=4.403 \cdot 10^{-4}, \quad B_f=4.269 \cdot 10^{-5}$$

- for booster turbopumps:

$$A=1.171 \text{ and } B=0.013$$

- for booster turbopumps in engines with expander cycle:

$$A=1.171 \text{ and } B=1.981 \cdot 10^{-3} \quad \text{for } D < 3 \cdot 10^4$$

$$A=1.171 \text{ and } B=1.82 \cdot 10^{-3} \quad \text{for } 3 \cdot 10^4 \leq D \leq 8 \cdot 10^5$$

Mass of Other Components

Original model

Mass of other engine components is given as follows:

- for staged-combustion cycle:

$$m_a = 0.235 P + 57 \quad \text{for } 14.7 \leq P \leq 981$$

$$m_a = 0.396 P - 73.1 \quad \text{for } 883 \leq P \leq 1678$$

- for other cycles:

$$m_a = 0.217 P + 57.5 \quad \text{for } 14.7 \leq P \leq 981$$

$$m_a = 0.377 P - 93.1 \quad \text{for } 883 \leq P \leq 1678$$

where P is a thrust of the engine (kN).

RPA model

For RPA, the model has been updated as follows:

- for staged-combustion cycle:

$$m_a = \frac{0.235 \cdot 10^{-3} P}{\log(1 + P \cdot 10^{-5})} + 57 \quad \text{for } P \leq 980 \cdot 10^3$$

$$m_a = 0.3 \cdot 10^{-3} P - 73.1 \quad \text{for } P > 980 \cdot 10^3$$

$$m_a = (0.3 \cdot 10^{-3} P - 73.1) e^{1.6 \cdot 10^{-7} P} \quad \text{for } P > 980 \cdot 10^3$$

with more than 1 main turbopump

- for other cycles:

$$m_a = 0.217 P \cdot 10^{-3} + 57.5 \quad \text{for } P \leq 980 \cdot 10^3$$

$$m_a = 0.377 \cdot 10^{-3} P - 93.1 \quad \text{for } 980 \cdot 10^3 \leq P \leq 2000 \cdot 10^3$$

$$m_a = 1.2(0.377 \cdot 10^{-3} P - 93.1) \quad \text{for } 980 \cdot 10^3 \leq P \leq 2000 \cdot 10^3 \text{ and } \rho_f < 200 \text{ kg/m}^3$$

$$m_a = (0.377 \cdot 10^{-3} P - 93.1) e^{1.6 \cdot 10^{-7} P} \quad \text{for } 980 \cdot 10^3 \leq P \leq 2000 \cdot 10^3$$

$$m_a = 1.2(0.377 \cdot 10^{-3} P - 93.1) e^{1.6 \cdot 10^{-7} P}$$

with more than 1 main turbopump

for $980 \cdot 10^3 \leq P \leq 2000 \cdot 10^3$ and $\rho_f < 200 \text{ kg/m}^3$

$$m_a = 0.477 P \cdot 10^{-3} + 93.1$$

with more than 1 main turbopump

for $P > 2000 \cdot 10^3$

where P is a thrust of the engine (N).

References

1. Ponomarenko A. RPA: Tool for Liquid Propellant Rocket Engine Analysis. 2010.
2. Kozlov A.A., Noviliv V.N., Solovjev Y.V. Propellant Feed and Control of Liquid-Propellant Rocket Systems. Moscow, Mashinostroenie, 1988 (in Russian).