বিপরীত ত্রিকোনমিতিক ফাংশন, ত্রিকোনমিতিক সমীকরণ

বিপরীত বৃত্তীয় ফাংশন

সূত্র:

(a) (i) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

(ii) $\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

(iii) $\sec^{-1}x + \mathrm{cosec}^{-1}x = \frac{\pi}{2}$

(b) (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$

(ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\frac{x-y}{1+xy}$

(iii) $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1}\frac{x+y+z+xyz}{1-yz-zx-xy}$.

(c) $2\tan^{-1}x = \tan^{-1}\frac{2x}{1-x^2} = \sin^{-1}\frac{2x}{1+x^2} = \cos^{-1}\frac{1-x^2}{1+x^2}$

(d) (i) $\sin^{-1}x \pm \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\}$

(ii) $\cos^{-1}x \pm \cos^{-1}y = \cos^{-1}\{xy \mp \sqrt{1-x^2}\sqrt{1-y^2}$

(e) (i) $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^{2)}}$

(ii) $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$

Type-1: গানিতিক প্রমাণ

উদাহরণ-1: Prove that, $\tan^{-1}\frac{3}{4} - 2\tan^{-1}\frac{1}{5} = \cos^{-1}\frac{63}{65}$

L. S $= \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{2\times\frac{1}{2}}{1-\frac{1}{25}} = \tan^{-1}\frac{3}{4} - \tan^{-1}\frac{5}{12} = \tan^{-1}\frac{\frac{3}{4}-\frac{5}{12}}{1+\frac{3}{3}\times\frac{5}{12}} = \tan^{-1}\frac{(9-5)/12}{21/16} = \tan^{-1}\frac{4/12}{21/13}$

$= \tan^{-1}\frac{1}{3}\times\frac{16}{21} = \tan^{-1}\frac{16}{63} = \cos^{-1}\frac{36}{\sqrt{16^2+36^2}} = \cos^{-1}\frac{63}{65}$

উদাহরণ-2: prove that, $\cos^{-1}\{1+\cos(2\tan^{-1}\frac{x}{a})\}^{\frac{1}{2}} = \sin^{-1}\sqrt{\frac{x^2-a^2}{x^2+a^2}}$

L.H.S $= \cos^{-1}\{1+\cos\cos\text{-}1\frac{1-(\frac{x}{a})^2}{1+(\frac{x}{a})^2})\}^{\frac{1}{2}}$

$= \cos^{-1}\{1+\frac{a^2-x^2}{a^2+x^2}\}^{\frac{1}{2}}+$

$= \cos^{-1}\frac{\sqrt{2a^2}}{a^2+x^2} = \cos^{-1}\sqrt{\frac{2}{1+(\frac{x}{a})^2}} = \cos^{-1}(\sqrt{2}cos\theta) = A$(ধরি)

তাহলে, $\cos A = \sqrt{2}cos\theta$

$\sqrt{1-2cos^2\theta} = (1-\frac{2}{sec^2\theta})^{1/2}l = (1-\frac{2}{1+tan^2\theta})^{1/2}$

$= (1 - \frac{2}{1+\frac{x^2}{a^2}})^{1/2}$

$= (\frac{a^2+x^2-2a^2}{a^2+b^2})^{1/2}$

$\sin A = (\frac{x^2-b^2}{x^2+b^2})^{1/2}$

$A = \sin^{-1} (\frac{x^2-b^2}{x^2+b^2})^{1/2}$

নিজে চেষ্টা কর: prove that, $\sin^{-1}\{1-\cos(2\tan^{-1}\sqrt{\frac{x}{a}})\}^{\frac{1}{2}} = \cos^{-1}\sqrt{\frac{a-x}{a+x}}$

Hint: $\sin^{-1}(\sqrt{2}\sin\theta) = A$; $\sin A = \sqrt{2}.\frac{\sqrt{x}}{\sqrt{x+a}}$. $\therefore \cos A = (\frac{x+a-2a}{x+a})^{1/2}$

উদাহরণ-3: prove that, $\sin^{-1}\frac{3}{5} + \frac{1}{2}\cos^{-1}\frac{5}{13} - \cot^{-1}2 = \tan^{-1}\frac{28}{29}$.

L. S

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3} - \cot^{-1}2$

$= \tan^{-1}\frac{\frac{3}{4}+2/3}{1-\frac{3}{4}\times 2/3} - \cot^{-1}2$

$= \tan^{-1}\frac{9+8}{6} - \tan^{-1}\frac{1}{2}$

$= \tan^{-1}\frac{\frac{17}{6}-\frac{1}{2}}{1+\frac{17}{3}\times\frac{1}{2}}$

$= \tan^{-1}\frac{14\times 2}{29}$

$= \tan^{-1}\frac{28}{29}$

= R. H.S Proved

$Let, \frac{1}{2}\cos^{-1}\frac{5}{13} = \theta$

$\Rightarrow \cos^{-1}\frac{5}{13} = 2\theta$

$\Rightarrow \frac{5}{13} = \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$\Rightarrow 5 + 5\tan^2\theta = 13 - 13\tan^2\theta$

$\Rightarrow 18\tan^2\theta = 13 - 5$

$\Rightarrow 18\tan^2\theta = 8$

$\Rightarrow \tan\theta = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}$ (সুক্ষ্মকোণ)

$\therefore \theta = \tan^{-1}\frac{2}{3}$, but $\theta = \frac{1}{2}\cos^{-1}\frac{5}{13}$.

উদাহরণ-4: prove that, $\cos^{-1}\sqrt{2/3} - \cos^{-1}\frac{\sqrt{6}+1}{2\sqrt{3}} = \frac{\pi}{6}$.

L.H .S= $\cos^{-1}\sqrt{2/3} - \left[\cos^{-1}\sqrt{2/3} - \cos^{-1}\frac{\sqrt{3}}{2}\right]$ [$2\sqrt{3}$ হর]

$= \frac{\pi}{6}$ = R. S Proved

নিজে চেষ্টা কর: prove that, $2\tan^{-1}\{\sqrt{\frac{a-b}{a+b}}\tan\frac{\theta}{2}\} = \cos^{-1}\frac{b+a\cos\theta}{a+b\cos\theta}$.

উদাহরণ-5: prove that, $\sin^2(\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b}) = \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2}$

ধরি, $\cos^{-1}\frac{x}{a} + \cos^{-1}\frac{y}{b} = \theta$

$\Rightarrow \cos^{-1}\{\frac{xy}{ab} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}\} = \theta$

$\Rightarrow \cos\theta = \frac{xy}{ab} - \sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$ $\Rightarrow \cos\theta - \frac{xy}{ab} = -\sqrt{1-\frac{x^2}{a^2}}\sqrt{1-\frac{y^2}{b^2}}$

$\Rightarrow \cos^2\theta - 2\frac{xy}{ab}\cos\theta + \frac{x^2y^2}{a^2b^2} = -\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{x^2y^2}{a^2b^2} + 1$

$\Rightarrow \therefore \sin^2\theta = \frac{x^2}{a^2} - \frac{2xy}{ab}\cos\theta + \frac{y^2}{b^2}$

উদাহরণ-6: prove that ,$\sec^{-1}x = \operatorname{cosec}^{-1}y$ হলে $\frac{1}{x^2} + \frac{1}{y^2} = 1.$

$\sec^{-1}x = \operatorname{cosec}^{-1}y$

$\Rightarrow \cos^{-1}\frac{1}{x} = \sin^{-1}\frac{1}{y}$

$\Rightarrow \frac{1}{x} = \sqrt{1-(\frac{1}{y})^2}$

$\Rightarrow \frac{1}{x^2} + \frac{1}{y^2} = 1$ Proved.

নিজে চেষ্টা কর: $\tan^{-1}\frac{xy}{z} + \tan^{-1}\frac{yz}{x} + \tan^{-1}\frac{zx}{y} = \frac{\pi}{2}$ দেখাও যে, $x^2 + y^2 + z^2 = 1$.

উদাহরণ-7: $\sin^2(\cos^{-1}\frac{2}{3}) - \cos^2(\sin^{-1}\frac{\sqrt{2}}{\sqrt{3}}) = ?$

L.H.S $= 1 - \frac{4}{9} - 1 + \frac{2}{3} = \frac{-4+6}{9} = \frac{2}{9}$

উদাহরণ-8: $\sin^{-1}(\sqrt{2}\sin\theta) + \sin^{-1}\sqrt{\cos 2\theta} = \frac{\pi}{2}$

L. H.S $= \cos^{-1}\sqrt{1-(\sqrt{2}\sin\theta)^2} + \sin^{-1}\sqrt{\cos 2\theta}$

$= \cos^{-1}\sqrt{1-2\sin^2\theta} + \sin^{-1}\sqrt{\cos 2\theta}$

$= \cos^{-1}\sqrt{\cos 2\theta} + \sin^{-1}\sqrt{\cos 2\theta}$

$= {}^{\pi}/_{2}$

= R.H.S Proved

উদাহরণ-9: $\tan^{-1}x + \frac{1}{2}\sec^{-1}\frac{1+y^2}{1-y^2} + \frac{1}{2}\operatorname{cosec}^{-1}\frac{1+z^2}{2z} = \pi, Prove\ that, x + y + z = xyz$

$\tan^{-1}x + \frac{1}{2}\sec^{-1}\frac{1+y^2}{1-y^2} + \frac{1}{2}\operatorname{cosec}^{-1}\frac{2z}{1+z^2} = \pi$

$\Rightarrow \tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow \tan^{-1}\frac{x+y+z-xyz}{1-xy-yz-zx} = \pi$

$\Rightarrow \frac{x+y+z-xyz}{1-xy-yz-zx} = \tan\pi = 0$

$\therefore x + y + z = xyz$

নিজে চেষ্টা কর: $A + B + C = \pi$, $A = \tan^{-1}2$, $B = \tan^{-1}3$ হলে $C = ?$

উদাহরণ-10: $\sin(\pi cos\alpha) = \cos(\pi sin\alpha)$ হলে প্রমান কর, $\alpha = \pm\frac{1}{2}\sin^{-1}\frac{3}{4}$

$\Rightarrow \sin(\pi cos\alpha) = \sin(\frac{\pi}{2} \pm \pi sin\alpha)$

$\Rightarrow \pi cos\alpha \mp \pi sin\alpha = \frac{\pi}{2}$

$\Rightarrow cos\alpha \mp sin\alpha = \frac{1}{2}.$

$\Rightarrow 1 \mp sin2\alpha = \frac{1}{4}$

$\Rightarrow \mp sin2\alpha = \frac{1}{4} - 1 = \frac{-3}{4}$

$\Rightarrow \alpha = \pm\frac{1}{2} sin^{-1}\frac{3}{4}$

or,

$\frac{1}{\sqrt{2}} cos\alpha \pm \frac{1}{\sqrt{2}} sin\alpha = \frac{1}{2\sqrt{2}}.$

$\Rightarrow \cos\alpha.\cos45^0 \pm \sin\alpha \sin45^0 = \frac{1}{2\sqrt{2}}.$

$\Rightarrow \cos(\alpha \mp 45^0) = \frac{1}{2\sqrt{2}}$

$\Rightarrow \alpha = \pm 45^0 + \cos^{-1}\frac{1}{2\sqrt{2}} = \pm\frac{\pi}{4} + \cos^{-1}\frac{1}{2\sqrt{2}}$

or,

$\sin45^0, \cos\alpha \pm \cos45^0 \sin\alpha = \frac{1}{2\sqrt{2}}$

$\Rightarrow \sin(45^0 \pm \alpha) = \frac{1}{2\sqrt{2}}$

$\Rightarrow \sin(45^0 \pm \alpha) = \frac{1}{2\sqrt{2}}$

$\Rightarrow \alpha = \frac{\pi}{4} \pm \sin^{-1}\frac{1}{2\sqrt{2}}$

নিজে চেষ্টা কর: $\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-a^2}{1+b^2} = 2\tan^{-1}x$ হলে দেখাও যে, $x = \frac{a-b}{a+b}$

Type-2: সমাধান

উদাহরণ-1: সমাধান কর: $\sin\cot^{-1}\cos\cos^{-1}\frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{5}}$

$$\Rightarrow \sin\cot^{-1}\frac{1}{\sqrt{1-x^2}} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \sin\sin^{-1}\sqrt{\frac{1+x^2}{2+x^2}} = \frac{2}{\sqrt{5}}$$

$$\Rightarrow \frac{1+x^2}{2+x^2} = \frac{4}{5}$$

$$\Rightarrow x^2 = 3$$

$$\therefore x = \pm\sqrt{3}$$

উদাহরণ-2: সমাধান কর: $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x.$

$$\Rightarrow \sin^{-1}x + \cos^{-1}x + \sin^{-1}(1-x) = 2\cos^{-1}x$$

$$\Rightarrow \frac{\pi}{2} + \sin^{-1}(1-x) = 2\cos^{-1}x.$$

$$\Rightarrow -(1-x) = \cos(2\cos^{-1}x)$$

$$\Rightarrow 2\cos^2\cos^{-1}x - 1 = x-1$$

$$\Rightarrow 2x^2 - 1 = x-1$$

$$\Rightarrow x(2x-1) = 0$$

$$\therefore x = 0,2$$

নিজে চেষ্টা কর: সমাধান কর: (i) $\sin^{-1}x + \sin^{-1}2x = \frac{\pi}{3}$

(ii) $\sin^{-1}\frac{5}{x} + \sin\frac{2}{x} = \pi/2$

ত্রিকোনমিতিক সমীকরণ

সুত্র:

(a) $\sin\theta = 0$ হলে, $\theta = n\pi$

$\cos\theta = 0$ হলে, $\theta = (2n+1)\frac{\pi}{2}$

$\tan\theta = 0$ হলে, $sin\theta = 0, \theta = n\pi.$

$\cot\theta = 0$ হলে, $\cos\theta = 0, \theta = (2n+1)\frac{\pi}{2}.$

(b)

$\sin\theta = \sin\alpha$ হলে, $\theta = n\pi + (-1)^n\alpha$

$\mathrm{cosec}\theta = \mathrm{cosec}\alpha$ হলে , $\sin\theta = \sin\alpha$ হবে,

$\therefore \theta = n\pi + (-1)^n\alpha$

(c)

$\cos\theta = \cos\alpha$ হলে, $\theta = 2n\pi \pm \alpha$

$\sec\theta = \sec\alpha$ হলে, $\theta = 2n\pi \pm \alpha$

(d)

$\tan\theta = \tan\alpha$ হলে, $\theta = n\pi + \alpha$

$\cot\theta = \cot\alpha$ হলে, $\theta = n\pi + \alpha$

(e)

$\sin\theta = 1 = sin\frac{\pi}{2}, \theta = 2n\pi + \frac{\pi}{2} = (4n+1)\frac{\pi}{2}$

$\sin\theta = -1 = \sin(-\frac{\pi}{2}), \theta = 2n\pi - \frac{\pi}{2} = (4n-1)\frac{\pi}{2}$

(f)

$\cos\theta = 1$ হলে, $\theta = 2n\pi$

$\cos\theta = -1$ হলে, $\theta = (2n+1)\frac{\pi}{2}.$

একটি সুত্রের প্রমান: $\sin\theta = \sin\alpha$ হলে , $\theta = n\pi + (-1)^n\alpha$

$\sin\theta - \sin\alpha = 0$

$\Rightarrow 2\sin\frac{\theta-\alpha}{2}.\ \cos\frac{\theta+\alpha}{2} = 0$

$\therefore\ \sin\frac{\theta-\alpha}{2} = 0$ হলে, $\frac{\theta-\alpha}{2} = n\pi$

$\therefore \theta - \alpha = 2n\pi$ এবং $\frac{\theta+\propto}{2} = (2n+1)\ {}^{\pi}\!/_{2}$ $\therefore\ \theta + \alpha = (2n+1)\pi$

$\therefore \theta = 2n\pi + \alpha$ এবং $2n\pi + \pi - \alpha$ $\therefore \theta = \alpha + 2n\pi$ এবং $-\alpha + (2n+1)\pi$

$n = 1$এর জন্য $\theta = \alpha + 2\pi$ এবং $-\alpha + 3\pi$

$n = -1$এর জন্য $\theta = \alpha - 2\pi$ এবং $-\alpha - \pi$

$n = 0$ এর জন্য $\theta = \alpha$ এবং $-\alpha + \pi$

$\therefore \theta = (-1)^{2n}\alpha + 2n\pi$ এবং $(-1)^{2n+1}\alpha + (2n+1)\pi$

$\theta = (-1)^m\alpha + m\pi$ এবং $(-1)^{m+1}\alpha + (m+1)\pi$ [2n =m হলে]

$\therefore\ \theta = n\pi + (-1)^n\alpha$ [n = m হলে]

Type-1: a $\cos\theta$ + b$\sin\theta$ = c
[c $is\ not\ greater\ than\ \sqrt{a^2+b^2}$]

a $\cos\theta$ + b$\sin\theta$ = c

$\Rightarrow \frac{a}{\sqrt{a^2+b^2}}\cos\theta + \frac{b}{\sqrt{a^2+b^2}}\sin\theta = \frac{c}{\sqrt{a^2+b^2}}$ [$\cos\theta$ এবং $\sin\theta$ এর সহগের বর্গের যোগফলের বর্গমূল দ্বারা ভাগ করে]

এখানে, $\cos\alpha = \frac{a}{\sqrt{a^2+b^2}} = \frac{a}{r}$ a = r $\cos\alpha$ এবং $\sin\alpha = \frac{b}{\sqrt{a^2+b^2}} = \frac{b}{r}$ b = r $\sin\alpha$এবং $\tan\alpha = b/a$

$\therefore r = \sqrt{a^2+b^2}$

$\therefore \cos\alpha\cos\theta + \sin\alpha\sin\theta = \frac{c}{r} = \frac{c}{\sqrt{a^2+b^2}}$

$\therefore \cos(\theta-\alpha) = cos\beta$ where $\cos\beta = \frac{c}{r} = \frac{c}{\sqrt{a^2+b^2}}$

$\therefore \theta - \alpha = 2n\pi \pm \beta$

$\therefore \theta = 2n\pi \pm \beta + \alpha$ (n ∈ z)

অংকটিতে অবান্তর মূল (E$x$traneous roots)এর উদ্ভব ঘটে

সাদৃশ্যপূর্ণ একটি সমাধান:

$-\cos x + \sqrt{3}\sin x = \sqrt{2}$.

$= 3\sin^2 x = 2 + 2\sqrt{2}\cos + \cos^2 x$

$= 3 - 3\cos^2 x = 2 + 2\sqrt{2} = \cos x + \cos^2 x$.

$= 4\cos^2 x + 2\sqrt{2}\cos x - 1 = 0$

$\cos x = \frac{1\pm\sqrt{3}}{2\sqrt{2}} = cos\frac{\pi}{12}$

or, $\cos\frac{7\pi}{12}$

$x = 2n\pi \pm \frac{\pi}{12}$

or, $2n\pi \pm \frac{7\pi}{12}$

কিন্তু দেখা যায় যে, $x = 2n\pi - \frac{\pi}{12}$ এবং $2n\pi - \frac{7\pi}{12}$ উক্ত সমীকরণটি সিদ্ধ হয় না।

উদাহরণ-1: সমাধান কর: $\sqrt{3}cosx + sinx = \sqrt{2}$

$$\Rightarrow \frac{\sqrt{3}}{2}cosx + \frac{1}{2}sinx = \frac{\sqrt{2}}{2}$$

$$\Rightarrow cosxcos\frac{\pi}{6} + sinxsin\frac{\pi}{6} = cos\frac{\pi}{4}$$

$$\Rightarrow \cos(x - \frac{\pi}{6}) = cos\frac{\pi}{4}$$

$$\Rightarrow x - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4} \therefore x = 2n\pi + \frac{5\pi}{12}, 2n\pi - \frac{\pi}{12}$$

উদাহরণ-2: সমাধান কর: $a\sec\theta + b\tan\theta = c$

$\Rightarrow \frac{a}{cos\theta} + \frac{bsin\theta}{cos\theta} = c$

$\Rightarrow a + b\sin\theta = c\cos\theta$

$\Rightarrow c\cos\theta - b\sin\theta = a$

$\Rightarrow \frac{c}{\sqrt{b^2+c^2}} cos\theta - \frac{b}{\sqrt{b^2+c^2}} sin\theta = \frac{a}{\sqrt{b^2+c^2}}$

$\Rightarrow \cos\theta . \cos\alpha - \sin\theta . \sin\alpha = \cos\beta$

$\Rightarrow \cos(\theta - \alpha) = \cos\beta$

$\Rightarrow \theta - \alpha = 2n\pi \pm \beta.$

$\therefore \theta = 2n\pi \pm \beta + \alpha$

নিজে চেষ্টা কর: সমাধান কর: (i) $\cos\theta - \sin\theta = \frac{1}{\sqrt{2}}$

(ii) $\cos\theta + \sin\theta = \sqrt{2}$

(iii) $\sqrt{3}\sin\theta - \cos\theta = 2$

Type-2: সমাধান

উদাহরণ-1: সমাধান কর: $\sin\theta + \sin2\theta + \sin3\theta = 1 + \cos\theta + \cos.2\theta$

$\Rightarrow 2\sin2\theta.\cos\theta + \sin2\theta = 2\cos^2\theta + \cos\theta.$

$\Rightarrow \sin2\theta\ (2\cos\theta + 1) = \cos\theta\,(2cos\theta + 1)$

$\Rightarrow (2\cos\theta + 1)\ (\sin2\theta - \cos\theta) = 0$

$\therefore\ 2\cos\theta + 1 = 0$

$\Rightarrow \cos\theta = -\frac{1}{2}$

$\Rightarrow \theta = 2n\pi \pm \frac{2\pi}{3}$

এবং $2\sin\theta.\cos\theta - cos\theta = 0$

$= \cos\theta(2sin\theta - 1) = 0$

$= \cos\theta = 0$

$\theta = (2n + 1)\frac{\pi}{2}$

এবং $2\sin\theta - 1 = 0$

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

সমাধান:

$2n\pi \pm \frac{2\pi}{3}$	$(2n+1)\frac{\pi}{2}$,	$n\pi + (-1)\frac{\pi}{6}$
$n = ০,\quad \pm\frac{2\pi}{3}$ $n = 1,\quad \frac{8\pi}{3}, \frac{4\pi}{3}$	$\pi/2$ $\frac{3\pi}{2}$	$\pi/6$ $\frac{5\pi}{6}$

$n = -1, -\frac{4\pi}{3}, \frac{-8\pi}{3}$	$\frac{-\pi}{2}$	$\frac{-7\pi}{6}$

কোন সাধারণ মান পাওয়া গেলনা
সুতরাং সবগুলো উত্তর হবে।

উদাহরণ-2: সমাধান কর: tanx + tan2x + tan3x = 0

tan3x = tan(2x +x) = $\frac{tan2x+tanx}{1-tan2x,tanx}$.
$\Rightarrow$ tanx + tan2x + tan3x = tanx tan2x tan3x
$\therefore$ tanxtan2xtan3x = 0 হয় tan3x = 0
$\therefore$ x = $\frac{n\pi}{3}$
অথবা,

tanx, tan2x = 0
$\Rightarrow$ tanx. $\frac{2tanx}{1-tan^2x} = 0$
$\Rightarrow$ 2tan2x = 0
$\Rightarrow$ tanx = 0
$\therefore$x $= n\pi(n \in z)$

উদাহরণ-3:

সমাধান কর: $\tan^2\theta - 2\sqrt{3}sec\theta + 4 = 0$
$\Rightarrow \sec^2\theta - 2\sqrt{3}sec\theta + 3 = 0$
$\Rightarrow (\sec\theta - \sqrt{3})^2 = 0$

$\Rightarrow \sec\theta = \sqrt{3} = sec \propto$.
$\therefore \theta = 2n\pi \pm \propto$
$= 2n\pi \pm \sec^{-1}\sqrt{3}$

উদাহরণ-4: সমাধান কর : $\cos^3x \sin3x + \sin^3x \cos3x = \frac{3}{4}$

$\Rightarrow \frac{1}{4}(3\cos x + \cos3x)\sin3x + \frac{1}{4}(3\sin x - \sin3x)\cos3x = \frac{3}{4}$
$\Rightarrow \cos x + \sin x = 1$
$\Rightarrow \cos(x - \frac{\pi}{4}) = \cos\frac{\pi}{4} \Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$ $\therefore x = 2n\pi$ or, $2n\pi + \frac{\pi}{2}$ $(n \in z)$

উদাহরণ-5: সমাধান কর: tanx + tan2x + tan3x = tanx. tan2x. tan3x.

$\Rightarrow$ tanx + tan2x + tan3x − tanx. tan2x. tanx = 0
$\Rightarrow \frac{tanx+tan2x+tan3x-tanx.tan2x.tan3x}{1-tanx.tan2x-tan2x.tan3x-tan3x.tanx} = 0$
$\Rightarrow$ tan(x + 2x + 3x) = 0
$\Rightarrow$ tan6x = 0
$\therefore$ 6x $= n\pi$
x = $\frac{n\pi}{6}$ $(n \in z)$

উদাহরণ-6: সমাধান কর $\sin(\frac{\pi}{4}\tan\theta) = \cos(\frac{\pi}{4}\cot\theta)$ হলে $[\theta = n\pi + \frac{\pi}{4}]$

$\Rightarrow \cos(\pi/2 - \pi/4\, tan\theta) = cos(\frac{\pi}{4}\cot\theta)$

$\Rightarrow \pi/2 - \pi/4\, tan\theta = \frac{\pi}{4}\cot\theta$

$\Rightarrow \tan\theta + \cot\theta = 2$

$\Rightarrow \tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow (\tan\theta - 1)^2 = 0$

$\Rightarrow \tan\theta = 1$

$\theta = n\pi + \frac{\pi}{4}$ Ans:

নিজে চেষ্টা কর: সমাধান কর: (i) $2\sin\theta\tan\theta + 1 = \tan\theta + 2\sin\theta$

(ii) $\cos 6x + \cos 4x = \sin 3x + \sin x$

(iii) $\sin 7\theta - \sqrt{3}\sin 4\theta = \sin\theta$.