যোগজীকরণ-২

ইন্টিগ্র্যাল ক্যালকুলাস

সূত্রাবলী ঃ **1**. মৌলিক ধর্মাবলী(**Fundamental Properties)ঃ**

$(\boldsymbol{i})\ \int\{f_1(x) \pm f_2(x) \pm f_3(x) \pm \cdots \ldots \ldots \ldots .. \pm f_n(x)\}dx$
$=\int f_1(x)dx \pm \int f_2(x)dx \pm \int f_3(x)dx \pm \cdots \ldots \ldots \ldots \ldots .. \pm f_n(x)dx$
$(\boldsymbol{ii})\ \int cf(x)dx = c\int f(x)dx$

2. Standard Integrals :

$(\boldsymbol{i})\ \int x^n dx = \frac{x^{n+1}}{n+1} + c$, $(\boldsymbol{ii})\ \int \frac{dx}{x^n} = -\frac{1}{(n-1)x^{n-1}} + c$, $(\boldsymbol{iii})\ \int dx = x + c$
(iv) $\int \frac{dx}{x} = log|x| + c$, *(v)* $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + c$, $(\boldsymbol{vi})\ \int e^x dx = e^x + c$
(vii) $\int e^{mx}dx = \frac{e^{mx}}{m} + c,$ $(\boldsymbol{viii})\ \int a^{mx}dx = \frac{a^{mx}}{mlog_e a} + c$
(ix) $\int sinmxdx = -\frac{cosmx}{m} + c$, ***(x)*** $\int cosmxdx = \frac{sinmx}{m} + c$
(xi) $\int sec^2xdx = tanx + c$, ***(xii)*** $\int cosec^2xdx = -cotx + c$
(xiii) $\int secxtanxdx = secx + c$, (xiv) $\int cosecxcotxdx = -cosecx + c$

3. Standard Integrals :

(i) $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c$
$(ii)\ \int \tan x dx = \log|\sec x| + c$
$(iii)\ \int cotxdx = \log|\cos x| + c$
(iv) $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c\ , (a \neq 0)$
(v) $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}log\left|\frac{x-a}{x+a}\right| + c, [|x| > |a|]$
(vi) $\int \frac{dx}{a^2-x^2} = \frac{1}{2a}log\left|\frac{x+a}{x-a}\right| + c, [|x| < |a|]$
(vii) $\int \frac{dx}{\sqrt{x^2+a^2}} = log\left|x + \sqrt{x^2 + a^2}\right| + c$
(viii) $\int \frac{dx}{\sqrt{x^2-a^2}} = log\left|x + \sqrt{x^2 - a^2}\right| + c$
(ix) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$
(x) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$
(xi) $\int uvdx = u\int vdx - \int(\frac{du}{dx}\int\ vdx)dx$
(xii) প্রমাণ কর যে , $\int e^{ax}sinbxdx = \frac{e^{ax}(asinbx-bcosbx)}{a^2+b^2} + c$
$= \frac{e^{ax}}{\sqrt{a^2+b^2}}cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

প্রমান ঃ $I = \int e^{ax}sinbxdx = e^{ax}\int sinbxdx - \int(\frac{d}{dx}e^{ax}\int sinbx\,dx\,)dx \quad e^{ax}\left(-\frac{1}{b}cosbx\right) -$
$\int a\,e^{ax}\left(-\frac{1}{b}cosbx\right)dx = -\frac{1}{b}e^{ax}cosbx + \frac{a}{b}\int e^{ax}cosbxdx$
$\int e^{ax}cosbxdx = e^{ax}\int cosbxdx - \int(\frac{d}{dx}e^{ax}\int cosbx\,dx\,)dx$
$= \frac{e^{ax}}{b}sinbx - \frac{a}{b}\int e^{ax}sinbxdx$
$I = -\frac{1}{b}e^{ax}cosbx + \frac{a}{b}\left(\frac{e^{ax}}{b}sinbx - \frac{a}{b}\int e^{ax}sinbxdx\right)$

$= \frac{a}{b}.\frac{e^{ax}}{b} sinbx - \frac{1}{b} e^{ax} cosbx \Rightarrow I + \frac{a^2}{b^2} I = \frac{a}{b}.\frac{e^{ax}}{b} sinbx - \frac{1}{b} e^{ax} cosbx$

$\Rightarrow \dfrac{a^2 + b^2}{b^2} I = \dfrac{a}{b}.\dfrac{e^{ax}}{b} sinbx - \dfrac{1}{b} e^{ax} cosbx$

$\therefore$ I=$\int e^{ax} sinbxdx = \frac{e^{ax}(asinbx - bcosbx)}{a^2+b^2} + c$

ধরি , $a = rcos\theta$, $b = rsin\theta$; $rsinbxcos\theta - rcosbxsin\theta = rsin(bx - \theta)$

$= \sqrt{a^2 + b^2}\, sin\left(bx - \tan^{-1}\frac{b}{a}\right)$

$\therefore$ I=$\int e^{ax} sinbxdx = \frac{e^{ax}(asinbx - bcosbx)}{a^2+b^2} + c = \frac{e^{ax}\, sin\left(bx - \tan^{-1}\frac{b}{a}\right)}{\sqrt{a^2+b^2}} + c$

(xiii) প্রমাণ কর যে , $\int e^{ax} cosbxdx = \frac{e^{ax}(asinbx - bcosbx)}{a^2+b^2} + c$

$= \frac{e^{ax}}{\sqrt{a^2+b^2}} cos\left(bx - \tan^{-1}\frac{b}{a}\right) + c$

(xiv) প্রমাণ কর যে , $\int \sqrt{x^2 + a^2} dx = \frac{x\sqrt{x^2+a^2}}{2} + \frac{a^2}{2} log\left|x + \sqrt{x^2 + a^2}\right| + c$

(xv) প্রমাণ কর যে , $\int \sqrt{x^2 - a^2} dx = \frac{x\sqrt{x^2-a^2}}{2} - \frac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + c$

প্রমাণ ঃ $I = \int \sqrt{x^2 - a^2} dx = \sqrt{x^2 - a^2} \int dx - \int \left[\frac{d\sqrt{x^2-a^2}}{dx} \int dx\right] dx$

$= x\sqrt{x^2 - a^2} - \int \frac{2x}{2\sqrt{x^2-a^2}}.xdx = x\sqrt{x^2 - a^2} - \int \frac{x^2-a^2+a^2}{\sqrt{x^2-a^2}} dx$

$= x\sqrt{x^2 - a^2} - \displaystyle\int \sqrt{x^2 - a^2} dx - a^2 \int \frac{dx}{\sqrt{x^2 - a^2}}$

$= x\sqrt{x^2 - a^2} - I - a^2 log\left|x + \sqrt{x^2 - a^2}\right|$

ধরি,$\sqrt{x^2 - a^2} = z - x \Rightarrow z = x + \sqrt{x^2 - a^2} \Rightarrow \frac{dz}{dx} = 1 + \frac{2x}{2\sqrt{x^2-a^2}}$

$\therefore \dfrac{dz}{dx} = \dfrac{x + \sqrt{x^2 - a^2}}{\sqrt{x^2 - a^2}} = \dfrac{z}{\sqrt{x^2 - a^2}} \Rightarrow \dfrac{dz}{z} = \dfrac{dx}{\sqrt{x^2 - a^2}}$

$\displaystyle\int \frac{dx}{\sqrt{x^2 - a^2}} = \int \frac{dz}{z} = logz = log\left|x + \sqrt{x^2 - a^2}\right|$

$\therefore I = x\sqrt{x^2 - a^2} - I - a^2 log\left|x + \sqrt{x^2 - a^2}\right|$

$\Rightarrow 2I = x\sqrt{x^2 - a^2} - a^2 log\left|x + \sqrt{x^2 - a^2}\right|$

$I = \dfrac{x\sqrt{x^2 - a^2}}{2} + \dfrac{a^2}{2} log\left|x + \sqrt{x^2 - a^2}\right| + c$

(xvi) প্রমাণ কর যে , $\int \sqrt{a^2 - x^2} dx = \frac{x\sqrt{a^2-x^2}}{2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a} + c$

$I = \displaystyle\int \sqrt{a^2 - x^2} dx = \sqrt{a^2 - x^2} \int dx - \int \left[\frac{d\sqrt{a^2 - x^2}}{dx} \int dx\right] dx$

$I = x\sqrt{a^2 - x^2} - \displaystyle\int \frac{-2x}{2\sqrt{a^2 - x^2}}.xdx = x\sqrt{a^2 - x^2} - \int \frac{a^2 - x^2 - a^2}{\sqrt{a^2 - x^2}} dx$

$= x\sqrt{a^2 - x^2} - \int \sqrt{a^2 - x^2} dx + a^2 \int \frac{dx}{\sqrt{a^2-x^2}} = x\sqrt{a^2 - x^2} - I + a^2 \sin^{-1}\frac{x}{a}$

$2I = x\sqrt{a^2 - x^2} + a^2 \sin^{-1}\dfrac{x}{a}$

$\therefore I=\int\sqrt{a^2-x^2}dx=\frac{x\sqrt{a^2-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$

(xvii) প্রমাণ কর যে , $\int cosecxdx=log\left|tan\frac{x}{2}\right|+c=log|cosecx-cotx|+c$

প্রমাণ ঃ $I=\int\frac{cosecx(cosecx-cotx)}{cosecx-cotx}dx=\int\frac{cosec^2x+cosecx.cotx}{cosecx-cotx}dx$

$since,\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+c\quad\therefore\int cosecxdx=log|cosecx-cotx|+c$

$again\ ,cosecx-cotx=\frac{1-cosx}{sinx}=\frac{2sin^2\frac{x}{2}}{2sin\frac{x}{2}cos\frac{x}{2}}=tan\frac{x}{2}$

$$\int cosecxdx=log\left|tan\frac{x}{2}\right|+c=log|cosecx-cotx|+c$$

(xviii) প্রমাণ কর যে , $\int secxdx=log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+c=log|secx+tanx|+c$

প্রমাণ ঃ $I=\int\frac{secx(secx+tanx)}{secx+tanx}dx=\int\frac{secx+secx.tanx}{secx+tanx}dx$

$since,\int\frac{f'(x)}{f(x)}dx=\log|f(x)|+c\quad\therefore\int secxdx=log|secx+tanx|+c$

$again\ ,secx+tanx=\frac{1+sinx}{cosx}=\frac{\left(cos\frac{x}{2}+sin\frac{x}{2}\right)^2}{cos^2\frac{x}{2}-sin^2\frac{x}{2}}=\frac{cos\frac{x}{2}+sin\frac{x}{2}}{cos\frac{x}{2}-sin\frac{x}{2}}=\frac{1+tan\frac{x}{2}}{1-tan\frac{x}{2}}$

$$=\frac{tan\frac{\pi}{4}+tan\frac{x}{2}}{1-tan\frac{\pi}{4}.tan\frac{x}{2}}=tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$\therefore\int secxdx=log\left|\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right|+c=log|secx+tanx|+c$

TYPE-01ঃ মৌলিক সমাকলন সম্পর্কিত সমস্যাবলী ঃ

Example-01 ঃ $\int(x^a+a^{2x})dx=\int x^adx+\int a^{2x}dx=\frac{x^{a+1}}{a+1}+\frac{a^{2x}}{2lna}+c$

Example-02 ঃ$\int\frac{4^{2+x}+4^{2-x}}{2^x}dx=\int 2^{4+x}dx+\int 2^{4-3x}dx=\frac{2^{4+x}}{ln2}+\frac{2^{4-3x}}{-3ln2}+c$

Example-03 ঃ $\int\frac{dx}{\sqrt{x}-\sqrt{x-1}}=\int\frac{\sqrt{x}+\sqrt{x-1}}{(\sqrt{x}-\sqrt{x-1})(\sqrt{x}+\sqrt{x-1})}dx=\int\left(\sqrt{x}+\sqrt{x-1}\right)dx$

$=\frac{2}{3}\left\{x^{\frac{3}{2}}+(x-1)^{\frac{3}{2}}\right\}+c$

Example-04ঃ $\int\frac{1-sinx}{1+sinx}dx=\int\frac{(1-sinx)(1-sinx)}{(1+sinx)(1-sinx)}dx=\int\frac{1-2sinx+sin^2x}{1-sin^2x}dx$

$=\int\frac{1-2sinx+sin^2x}{cos^2x}dx=\int(sec^2x-2secxtanx+tan^2x)\,dx$

$=tanx-2secx+\int(sec^2x-1)dx=tanx-2secx+tanx-x+c$

$=2(tanx-secx)+c$

Example-05៖ $\int \frac{sin2x}{sin5x.sin3x}dx = \int \frac{\sin(5x-3x)}{sin5x.sin3x}dx = \int \frac{\sin 5x \cos 3x - \cos 5x \sin 3x}{sin5x.sin3x}dx$

$= \int \cot 3x dx - \int cot5x dx = \frac{1}{3}ln|sin3x| - \frac{1}{5}ln|sin5x| + c$

Example-06៖ $\int \frac{dx}{\sin(x-a)\sin(x-b)} = \frac{1}{\sin(b-a)}\int \frac{\sin\{(x-a)-(x-b)\}}{\sin(x-a)\sin(x-b)}dx$

$= \frac{1}{\sin(b-a)}\int \frac{\sin(x-a)\cos(x-b)-\cos(x-a)\sin(x-b)}{\sin(x-a)\sin(x-b)}dx$

$= \frac{1}{\sin(b-a)}\left\{\int cot(x-b)dx - \int cot(x-a)\,dx\right\}$

$= \frac{1}{\sin(b-a)}\{ln|\sin(x-b)| - ln|\sin(x-a)|\} + c$

$= \frac{1}{\sin(b-a)} ln\left|\frac{\sin(x-b)}{\sin(x-a)}\right| + c$

Example-07៖ $\int sin^4 x dx = \int \frac{1}{4} \times 4\, sin^4 x dx = \frac{1}{4}\int (2sin^2 x)^2 dx$

$= \frac{1}{4}\int (1 - cos2x)^2 dx = \frac{1}{4}\int (1 - 2cos2x + cos^2 2x)dx$

$= \frac{1}{4}x - \frac{1}{4}\times\frac{2}{2}sin2x + \frac{1}{4}\int \frac{1}{2}.(1 + cos4x)\,dx = \frac{3x}{8} - \frac{sinx}{4} + \frac{sin4x}{32} + c$

Example-08៖ $\int 4\cos x\,.cos2x.cos3x.dx. = \int 2\cos x\,(2\,cos3x.cos2x)dx$

$= \int 2\cos x\,(\cos 5x + \cos x\,)dx = \int 2\cos 5x\,.\mathrm{cosx}\,dx + \int 2cos^2 x.dx.$

$= \int (\cos 6x + cos\,4x)dx + \int (1 + \cos 2x)dx$

$= \frac{1}{6}\sin 6x + \frac{1}{4}sin4x + x + \frac{1}{2}\sin 2x + c$

Example-09 ៖ $\int cos\,^4 x\,.\sin 3x.dx = \int \frac{1}{4}(2cos^2\,x)^2 \sin 3x.dx$

$= \frac{1}{4}\int (1 + cos\,2x)^2 \sin 3x.dx = \frac{1}{4}(1 + 2cos\,2x + cos^2 2x)\sin 3x.dx$

$= \frac{1}{4}\int (\sin 3x + 2\sin 3x.\cos 2x + cos^2\,2x.\sin 3x)\,dx$

$= \frac{1}{4}\times\frac{-1}{3}\cos 3x + \frac{1}{4}\int (\sin 5x + \sin x)dx + \frac{1}{4}\int \frac{1}{2}\;2cos^2\,2x.\sin 3x\,.dx$

$= -\frac{1}{12}\cos 3x + \frac{-1}{20}\cos 5x + \frac{-1}{4}\cos x + \frac{1}{8}\int (1 + cos4\,x)\sin 3x.dx$

$= -\frac{1}{12}\cos 3x - \frac{1}{20}\cos 5x - \frac{1}{4}\cos x + \frac{1}{8}\int \sin 3x.dx. + \frac{1}{8}\int \frac{1}{2}.2\cos 4x.\sin 3x.dx$

$= -\frac{-}{12}\cos 3x - \frac{1}{20}\cos 5x - \frac{1}{4}\cos x - \frac{1}{24}\cos x - \frac{1}{112}\cos 7x. + \frac{1}{16}\cos x + c$

$= c - \frac{1}{112}\cos 7x - \frac{1}{20}\cos 5x - \frac{1}{8}\cos 3x - \frac{3}{16}\cos x.$

TRY YOURLESLF:

1. $\int \frac{e^{3x}+e^{5x}}{e^{x}+e^{-x}}dx = ?\ \left(\boldsymbol{Ans}\text{: } \frac{1}{4}\boldsymbol{e^{4x}}\right)$

2. $\int \frac{8^{1+x}+4^{1-x}}{2x}dx = ?\left(\boldsymbol{Ans}: \frac{4}{10g\,2}\left[2^{2x}-\frac{1}{3}\times 2^{-3x}\right]\right)$

3. $\int \frac{a sin^3 x+b cos^3 x}{sin^2 x.cos^2 x}dx = ?\ (\boldsymbol{Asn}\text{: } \mathbf{asec}\,\boldsymbol{x}-\boldsymbol{bcosec\ x}+\boldsymbol{c})$

4. $\int \frac{\sin x+cosec\ x}{tanx}dx = ?\ (\boldsymbol{Ans}\text{: } \boldsymbol{sinx}-\boldsymbol{cosec\ x}+\boldsymbol{c})$

5. $\int \frac{\cos x-cos2x}{1-\cos x}dx = ?\,(\boldsymbol{Ans}\text{: } \boldsymbol{x}+\mathbf{2}\,\mathbf{sin}\,\boldsymbol{x}+\boldsymbol{c})$

6. $\int \frac{x^3-4x^2+5x-2}{x^2-2x+1}dx = ?\ \left(\boldsymbol{Ans}: \frac{1}{2}\boldsymbol{x}^2-\mathbf{2}\boldsymbol{x}+\boldsymbol{c}\right)$

7. $\int \frac{dx}{1+\sin x} = ?\,(\boldsymbol{Ans}\text{: } \mathbf{tan}\,\boldsymbol{x}-\mathbf{sec}\,\boldsymbol{x}+\boldsymbol{c})$

8. $\int \frac{dx}{1+\cos x} = ?\ (\boldsymbol{Ans}\text{: } \boldsymbol{cosec\ x}-\mathbf{cot}\,\boldsymbol{x}+\boldsymbol{c})$

9. $\int \frac{dx}{sin^2 x.cos^2 x} = ?\ (\boldsymbol{Ans}\text{: } \mathbf{tan}\,\boldsymbol{x}-\boldsymbol{c\ ot\ x}+\boldsymbol{c})$

10. $\int \frac{sin^6 x+cos^6 x}{sin^2 x.cos^2 x}dx = ?\ (\boldsymbol{Ans}\text{: } \mathbf{tan}\,\boldsymbol{x}-\mathbf{cot}\,\boldsymbol{x}-\mathbf{3}\boldsymbol{x}+\boldsymbol{c})$

11 . $\int \frac{sin^5 x - cos^8 x}{1-2sin^2 x cos^2 x}dx = ?\ \left(\boldsymbol{Ans}\text{: } -\frac{1}{2}\mathbf{sin}\,\mathbf{2}\boldsymbol{x}+\boldsymbol{c}\right)$

12. $\int \frac{cos^4 x-sin^4 x}{\sqrt{1+cos4x}}dx = ?\ \left(\boldsymbol{Ans}\text{: } \frac{1}{\sqrt{2}}\boldsymbol{x}+\boldsymbol{c}\right)$

13. $\int \frac{\cos x}{sin^2 x}(1-3cos^3 x)dx = ?\left(\boldsymbol{Ans}\text{: } -\boldsymbol{cosecx}+\mathbf{3}\boldsymbol{cotx}+\frac{9}{2}\ \boldsymbol{x}+\frac{3}{4}\boldsymbol{sin2x}+\boldsymbol{c}\right)$

14. $\int \sin mx.\sin nx.\,dx = ?\ \left(\boldsymbol{Ans}\text{: } \frac{\mathbf{sin}(\boldsymbol{m}+\boldsymbol{n})\boldsymbol{x}}{2(\boldsymbol{m}+\boldsymbol{n})}+\ \frac{\mathbf{sins}(\boldsymbol{m}-\boldsymbol{n})\boldsymbol{x}}{2(\boldsymbol{m}-\boldsymbol{n})}+\boldsymbol{c}\right)$

15. $\int \sin mx.\sin nx.\,dx + ?\ \left[\boldsymbol{Ans}\text{: } \frac{1}{4}\left(\mathbf{sin}\,\mathbf{2}\boldsymbol{x}-\boldsymbol{x}-\frac{1}{4}\mathbf{sin}\,\mathbf{4}\boldsymbol{x}\right)+\boldsymbol{c}\right]$

16. $\int 4\sin x.\sin 2x.\sin 3x.\,dx = ?\ \left(\boldsymbol{Ans}\text{: } -\frac{1}{2}\mathbf{cos}\,\mathbf{2}\boldsymbol{x}-\frac{1}{4}\mathbf{cos}\,\mathbf{4}\boldsymbol{x}+\frac{1}{6}\mathbf{cos}\,\mathbf{6}\boldsymbol{x}\right)+c$

Type - 02 : প্রতিস্থাপন পদ্ধতি

চলকের পরিবর্তন : ধরি, $I = \int f(x)dx$ এবং $x = \emptyset\,(z)$

$\therefore \frac{dI}{dx} = f(x)$ এবং $\frac{dI}{dz} = \frac{dI}{dx}.\frac{dx}{dz} = f(x).\emptyset'(x) \therefore I = \int f\{(\emptyset(z)\}\,\emptyset'(z).dz$

FORM -01 : $\int (a+bx)^n.dx = ?$

ধরি, $a + bx = z \implies bdx = dz \implies dx = \frac{1}{b}dz.$

$$\therefore I = \frac{1}{b}\int z^n.dz = \frac{1}{b}\,\frac{z^{n+1}}{n+1} + c = \frac{1}{b}\frac{(a+bx)^{n+1}}{n+1} + c$$

Example – 01 : $\int \frac{dx}{x\sqrt{x^2-a^2}}$ ধরি, $\mathrm{a + bx = z \implies bdx = dz \implies dx = \frac{1}{b}dz.}$

$\therefore \mathrm{I} = \int \mathrm{z^n.\frac{1}{b}dz = \frac{1}{b}\int z^n.dz = \frac{1}{b}\frac{z^{n+1}}{n+1} + c = \frac{1}{b}\frac{(a+bx)^{n+1}}{n+1} + c}$

Example- 02 : $\int \frac{dx}{x\sqrt{x^2-a^2}} = ?$

ধরি, $\mathrm{x = a\sec\theta \implies dx = a\sec\theta\,.\tan\theta.d\theta.}$

$$\therefore \mathrm{I} = \int \frac{\mathrm{a\sec\theta\,.\tan\theta.d\theta}}{\mathrm{a\sec\theta\,.\sqrt{a^2\sec^2\theta - a^2}}} = \int \frac{\mathrm{a\sec\theta\,.\tan\theta.}}{\mathrm{a\sec\theta\,.a\tan\theta.}}\mathrm{d\theta}$$

$$= \frac{1}{\mathrm{a^2}}\int \mathrm{d\theta} = \frac{1}{\mathrm{a^2}}\theta = \frac{1}{\mathrm{a^2}}\sec^{-1}\frac{\mathrm{x}}{\mathrm{a}} + \mathrm{c.}$$

Example- 03 : $\int \frac{e^{m\tan^{-1}x}}{1+x^2}dx = ?$

ধরি, $\mathrm{m\tan^{-1}x = z \implies \frac{m}{1+x^2}dx = dz \implies \frac{1}{1+x^2}dx = \frac{1}{m}dz.}$

$$\therefore \mathrm{I} = \int \mathrm{e^z.\frac{1}{m}dz = \frac{1}{m}e^z + c = \frac{1}{m}e^{m\tan^{-1}x} + c.}$$

Example- 04 : $\int \frac{\sin 2x.dx}{(a\sin^2 x + b\cos^2 x)^2} = \ ?$

ধরি, $\mathrm{a\sin^2 x + b\cos^2 x = z \implies 2a\sin x\,.\cos x - 2b\sin x.\cos x\ \ dx = dz.}$

$$\implies \mathrm{\sin 2x\,(a-b)dx = dz \implies \sin 2x.dx = \ \frac{dz}{a-b}.}$$

$$\therefore \mathrm{I} = \int \mathrm{\frac{1}{z^2}.\frac{dz}{a-b} = \frac{-1}{a-b}.\frac{1}{z} + c = \frac{1}{b-a}.\frac{1}{a\sin^2 x + b\cos^2 x} + c.}$$

Example- 05 : $\int \frac{\tan x.\sec^2 x.dx}{(a^2+b^2+\tan^2 x)^2} = ?$

ধরি, $\mathrm{a^2 + b^2 + \tan^2 x = z \implies (0 + 2b^2.\tan x.\sec^2 x)dx = dz.}$

$$\implies \mathrm{\tan x\,.\sec^2 x.dx = \frac{dz}{2b^2}.}$$

$$\therefore \mathrm{I} = \int \mathrm{\frac{1}{z^2}.\frac{dz}{2b^2} = -\frac{1}{2b^2 z} + c = -\frac{1}{2b^2(a^2+b^2+\tan^2 x)} + c.}$$

Example- 06: $\int \frac{e^x-1}{e^x+1} dx = ?$

$$I = \int \frac{e^x}{e^x+1} dx - \int \frac{1}{e^x+1} dx = I_1 - I_2.$$

ধরি, $e^x + 1 = z_1 \Longrightarrow e^x dx = dz_1.$ $\quad I_1 = \int \frac{dz_1}{z_1} = \ln z_1 + c_1.$

আবার , $I_2 = \int \frac{e^{-x}}{1+e^{-x}} dx$ ধরি, $e^{-x} + 1 = z_2 \Longrightarrow -e^{-x}.dx = dz_2 \Longrightarrow e^{-x}.dx = -dz_2$

$$I_2 = -\int \frac{dz_2}{z_2} = -\ln z_2 + c_1 = -\ln(1 + e^{-x}) + c_2.$$

$\therefore I = I_1 - I_2 = \ln(e^x + 1) + \ln(1 + e^{-x}) + c = \ln(e^x + 1)(1 + e^{-x}) + c$

$= \ln(e^x + e^{-x} + 2) + c = \ln\left(e^{x/2} + e^{-x/2}\right)^2 + c = 2\ln\left(e^{x/2} + e^{-x/2}\right) + c$

Example- 07 : $\int \frac{dx}{x^2\sqrt{1-x^2}} = ?$ ধরি, $x = \sin\theta \Longrightarrow dx = \cos\theta . d\theta$

$$\therefore I = \int \frac{\cos\theta . d\theta}{\sin\theta^2 . \cos\theta} = \int \operatorname{cosec}^2\theta . d\theta = -\cot\theta + c = \frac{\sqrt{1-x^2}}{x} + c.$$

Example- 08 : $\int \frac{dx}{\sqrt{x}+x} = ?$

ধরি, $\sqrt{x} = z \Longrightarrow \frac{1}{2\sqrt{x}} dx = dz \Longrightarrow dx = 2\sqrt{x}.dz = 2z.dz.$

$$\therefore I = \int \frac{2z.dz.}{z^2+z} = 2\int \frac{dz.}{z+1} = 2\ln|z+1| + c = 2\ln\left|\sqrt{x}+1\right| + c$$

Example-09 : $\int \frac{x.dx}{(2x+1)^3} = ?$ $\quad I = \int \frac{\frac{1}{2}(2x+1)-\frac{1}{2}}{(2x+1)^3} dx = \frac{1}{2}\int \frac{dx}{(2x+1)^2} - \frac{1}{2}\int \frac{dx}{(2x+1)^3}$

ধরি, $2x + 1 = z \Longrightarrow 2dx = dz \Longrightarrow dx = \frac{1}{2}dz.$

$$\therefore I = \frac{1}{4}\int \frac{dz}{z^2} - \frac{1}{4}\int \frac{dz}{z^3} = -\frac{1}{4z} + \frac{1}{8z^2} + c = \frac{1}{8(2x+1)^2} - \frac{1}{4(2x+1)} + c$$

Example-10 : $\int \frac{dx}{\sqrt{x}-1} = ?$

ধরি, $\sqrt{x} = z \Longrightarrow x = z^2 \Longrightarrow dz = 2z.dz$

$$\therefore I = \int \frac{2z.dz}{z-1} = 2\int \frac{z-1+1}{z-1} dz = 2\int dz + 2\int \frac{dz}{z-1}$$

$= 2z + 2\ln|(z-1)| + c = 2\sqrt{x} + 2\ln\left|\sqrt{x}-1\right| + c$

<u>TRY YOURSELF</u>

(i) $\int \sqrt{\frac{a+x}{a-x}} dx = ?$ (put $x = a\cos 2\theta.$) $\left[\textbf{Ans}: -\mathbf{a}\cos^{-1}\left(\frac{\mathbf{x}}{\mathbf{a}}\right) - \sqrt{\mathbf{a}^2 - \mathbf{x}^2}\right] + \mathbf{c}$

(ii) $\int \frac{1+x}{1-x} dx = ?$ $\quad [\textbf{Ans}: -\mathbf{x} - \mathbf{2}\ln|\mathbf{1} - \mathbf{x}| + \mathbf{c}]$

(iii) $\int \frac{x}{\sqrt{x}+1} dx = ?$ $\quad \left[\textbf{Ans}: \frac{1}{6}\mathbf{x}^6 + \frac{1}{5}\mathbf{x}^5 + \frac{1}{4}\mathbf{x}^4 + \frac{1}{3}\mathbf{x}^3 + \frac{1}{2}\mathbf{x}^2 + \mathbf{x} + \log(\mathbf{x} - \mathbf{1})\right]$

(iv) (a) $\int \frac{e^{2x}}{e^x+1} dx = ?$ (b) $\int \frac{\mathbf{dx}}{(e^x-1)^2} = ?$ (c) $\int \frac{dx}{\sqrt{e^x-1}} = ?$

Ans: (**a**)$e^x - \ln|e^x+1|$ (b) $\tan^{-1}(e^x)$ (c) $x - \ln|e^x-1| - (e^x+1)^{-1} + c$

(v) $\int \frac{e^x(x+1)}{\cos^2(xe^x)} dx = ?$ [**Ans**: $\tan(xe^x)$]+**c**

(vi) $\int \frac{xdx}{\sqrt{x}+1} dx = ?$ $\left[\text{Ans: } \frac{2}{3}x^{3/2} - x + 2\sqrt{x} - 2\ln\left|\sqrt{x}+1\right| + c\right]$

Form- 02 : $\int \frac{(a+bx)^m}{(a'+b'x)^n} dx$ কে সমাকলন করা যায়। যেখানে m ধনাত্মক পূর্ণ সংখ্যা ও n যেকোন মূলদ সংখ্যা এক্ষেত্রে $a' + b'x = z$ ধরে অগ্রসর হতে হবে।

Example-01 : $\int \frac{(a+bx)^2}{(a'+b'x)^3} dx$

ধরি, $a' + b'x = z \Rightarrow b'dx = dz \Rightarrow dx = \frac{1}{b'}dx.$

$$I = \int \frac{\left\{a + b\left(\frac{z-a'}{b'}\right)\right\}^2}{z^3} dz = \frac{1}{b'^3}\int \frac{(bz - ab' - a'b)^2}{z^3} dz.$$

$$= \frac{1}{b'^3}\int \frac{b^2z^2 - 2bz(ab'+a'b) + (ab'+a'b)^2}{z^3} dx.$$

$$= \frac{b^2}{b'^3}\int \frac{dz}{z} - \frac{2b(ab'+a'b)}{b'^3}\int \frac{dz}{z^2} + (ab'+a'b)^2\int \frac{dz}{z^3}$$

$$= \frac{b^2}{b'^3}\ln|z| - \frac{2b(ab'+a'b)}{b'^3}\left(\frac{-1}{z}\right) + (ab'+a'b)^2.\left(\frac{-1}{2z^2}\right) + c$$

$$= \frac{b^2}{b'^3}\ln|a' + b'x| + \frac{2b(ab'+a'b)}{b'^3}.\frac{1}{(a'+b'x)} - \frac{ab'+a'b}{2(a'+b'x)} + c$$

Example-02 : $\int \frac{x}{\sqrt[3]{a+bx}} dx = ?$

ধরি, $a + bx = z \Rightarrow b.dx = dz \Rightarrow dx = \frac{1}{b}dz, x = \frac{z-a}{b}$

$$\therefore I = \int \frac{\frac{z-a}{b}}{z^{1/3}} dz = \frac{1}{b}\int z^{2/3}.dz - \frac{a}{b}\int z^{-\frac{1}{3}} dz = \frac{3}{5b}z^{5/3} - \frac{3a}{2b}z^{2/3} + c$$

$$= \frac{3}{5b}(a+bx)^{5/3} - \frac{3a}{2b}(a+bx)^{2/3} + c$$

Example-03 : $\int \frac{x^2}{\sqrt{a^6-x^6}} dx.$

ধরি, $x^3 = a^3\sin\theta \Rightarrow 2x^2.dx = a^3\cos\theta.d\theta \Rightarrow x^2.dx = \frac{a^3}{2}\cos\theta.d\theta$

$$\therefore I = \int \frac{\frac{a^3}{2}\cos\theta.d\theta}{a^3\cos\theta} = \frac{1}{2}\int d\theta = \frac{1}{2}\theta + c = \frac{1}{2}\sin^{-1}\left(\frac{x}{a}\right)^3 + c$$

TRY YOURSELF:

(i) $\int \frac{x}{a+bx} dx = ?$ $\left[\textbf{Ans: } \frac{1}{b^2}[(a+bx) - a\log|a+bx|] + c\right]$

(ii) $\int \frac{2x+1}{\sqrt{3x+2}}dx = ?$ $\left[\textbf{Ans}: \frac{4}{27}(3x+2)^{3/2} - \frac{2}{9}(3x+2)^{1/2} + c\right]$

(iii) $\int \frac{\sqrt{x}}{\sqrt{a^3-x^3}}dx = ?$ $\left[\textbf{Ans}: \frac{2}{3}\sin^{-1}\left(\frac{a}{x}\right)^{3/2} + c.(\text{put } x^3 = a^3\sin^2\theta)\right]$

(iv) $\int \frac{a^3 dx}{\sqrt{1-x^2}} = ?$ $\left[\textbf{Ans}: -\sqrt{1-x^2} + \frac{1}{3}(1-x^2)^{3/2} + c\right]$

Form-03 : $\int \frac{dx}{x^m(a+bx)^n}$ যেখানে (i) m ও n ধনাত্মক পূর্ণ সংখ্যা অথাব জোড় সংখ্যা
(ii) m ও n ভগ্নাংশ হলে যেখানে m + n ধনাত্মক পূর্ণ সংখ্যাও 1 অপেক্ষা ক্ষুদ্রতর হবে।
$[m+n > 1; m, n \epsilon N]$

Example-01 : $\int \frac{dx}{x^3(a+bx)^2}$

ধরি, $a+bx = zx \Longrightarrow (z-x)x = a \Rightarrow z-b = \frac{a}{x} \Rightarrow dz = -\frac{a}{x^2}.dx$

এবং $x = \frac{a}{z-b}$ $\therefore I = \frac{1}{a}\int \frac{-\frac{a}{z^2}.dx}{x(zx)^2} = \frac{1}{a}\int \frac{dz}{z^2}\left(\frac{z-b}{a}\right)^3 = -\frac{1}{a^4}\int\left(z - 3b + \frac{3b^2}{z} - \frac{b^3}{z^2}\right)dz$

$$= -\frac{1}{a^4}\left[\frac{1}{2}\left(\frac{a+bx}{x}\right)^2 - 3b\left(\frac{a+bx}{x}\right) + 3b^2\log\frac{a+bx}{2} + b^3\left(\frac{x}{a+bx}\right) + c\right]$$

Example-02 : $\int \frac{x^{\frac{1}{2}}}{1+x^{3/4}}dx$

ধরি, $x = z^4 \Rightarrow x^{\frac{1}{2}} = z^2 \Longrightarrow dx = 4z^3.dz$

$$\therefore I = \int \frac{z^2}{1+z^3}.4z^3.dz = 4\int \frac{z^5}{1+z^3}dz = 4\int \frac{z^2(z^3+1)-z^2}{1+z^3}dz$$

$$= 4\int z^2dz - 4\int \frac{z^2}{1+z^3}dz = 4.\frac{z^3}{3} - 4.\frac{1}{3}\ln|1+z^3| + c = \frac{4}{3}\left[x^{4/3} - \ln\left|1+x^{3/4}\right| + c\right]$$

ধরি, $z^3 = t \Longrightarrow 3z^2dz = dt. \Longrightarrow z^2.dz = \frac{1}{3}dt$ $\therefore$ $\int \frac{\frac{1}{3}dt}{1+t} = \frac{1}{3}lnt$

Example- 03 : $\int \frac{dx}{x^{1/2}-x^{1/4}}$

এখানে, 2 ও 4 এর ল.স.গু 4 . ধরি , $x = u^4 \Rightarrow dx = 4u^3du$

$I = \int \frac{dx}{x^{1/2}-x^{1/4}} = \int \frac{4u^3du}{u^2-u^2} = 4\int \frac{u^2}{u-1}du = 4\int \frac{u(u-1)+(u-1)+1}{u-1}du$

$= 4\int udu + \int du + \int \frac{du}{u-1} = 2u^2 + u + ln(u-1) + c$

$= 2\sqrt{x} + x^{1/4} + ln\left(x^{1/4} - 1\right) + c$

TRY YOURSELF ঃ

(i) $\int \frac{dx}{x^2(a-bx)^2} = ?$ $\left[\text{Ans: } \frac{2b}{a^2}\log\frac{x}{a-bx} - \frac{(a-bx)}{a^2x((a-bx))}\right]$

(ii) $\int \frac{x^7}{(1-x^4)^2} = ?$ $\left[\text{Ans: } \frac{1}{4}\left\{\log(1-x^4) + \frac{1}{1-x^4}\right\} + c\right]$

(iii) $\int \frac{dx}{x\sqrt{x^4-1}} = ?$ $\left[\text{Ans: } \frac{1}{2}\sec^{-1}x^2 + c \ \ (\text{put}, \ \ x^2 = \sec\theta)\right]$

(iv) $\int \frac{\sqrt{1+x^2}}{x^4}dx = ?$ $\left[\text{Ans: } \frac{-\sqrt{(1-x^2)^3}}{3x^3} + c\right]$

(v) $\int \sqrt{\frac{x}{a-x}}\,dx = ?$ $\left[\text{Ans: } a\sin^{-1}\left(\frac{x}{a}\right)^{1/2} - \sqrt{x(a-x)} + c\right]$

(vi) $\int \frac{(\log\sec x)^2}{\cot x}\,dx = ?$ $\left[\text{Ans: } \frac{1}{3}(\log\sec x)^3\right]$ (vii) $\int \frac{dx}{(a^2-x^2)^{3/2}} = ?$ $\left[\text{Ans: } \frac{x}{a^2\sqrt{a^2-x^2}}\right]$

(viii) $\int \frac{dx}{(1-x)\sqrt{1-x^2}} = ?$ $\left[\text{Ans: } \sqrt{\frac{1+x}{1-x}} + c\right]$ (ix) $\int \frac{x^2+1}{(x^2-1)^2}\,dx = ?$ $\left[\text{Ans: } -\frac{x}{x^2-1}\right]$

(x) $\int \cos\left(2\cot^{-1}\sqrt{\frac{1+x}{1-x}}\right)dx = ?$ $\left[\text{Ans: } -\frac{1}{2}x^2 + c\right]$

(xi) Integrate $\frac{1}{2}f(x)$ with respect to x^4 where .

$f(x) = \tan^{-1}x + \log\sqrt{1+x^2} - \log\sqrt{1-x^2}$. [Ans: $-\log(1-x^4)+c$]

Type – 03 : প্রমিত সমাকলন ঃ

Form – 01 : $\int \frac{dx}{ax^2bx+c} = \frac{1}{a}\int \frac{dz}{z^2 \pm k^2}$ যেখানে, $z = x + \frac{b}{2a}$, $k = \frac{4ac-b^2}{4a^2}$

Example- 01 : $\int \frac{dx}{4x^2+4x+5} = \int \frac{dx}{4\left(x^2+x+\frac{5}{4}\right)} = \int \frac{dx}{4\left\{\left(x+\frac{1}{2}\right)^2+1\right\}}$

ধরি, $x + \frac{1}{2} = z \Rightarrow dx = dz \therefore I = \frac{1}{4}\int \frac{dz}{1+z^2} = \frac{1}{4}\tan^{-1}z + c = \frac{1}{4}\tan^{-1}\left(x+\frac{1}{2}\right)$

TRY YOURSELF:

(i) $\int \frac{dx}{1+x+x^2} = ?$ $\left[\textbf{Ans: } \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)\right]$ (ii) $\int \frac{dx}{1+x-x^2} = ?$ $\left[\textbf{Ans: } \frac{2}{\sqrt{5}}\log\left|\frac{2x+\sqrt{5}-1}{1+\sqrt{5}-2x}\right| + c\right]$

(iii) $\int \frac{dx}{6x^2+7x+2} = ?$ $\left[\textbf{Ans: } \log\left|\frac{2x+1}{3x+2}\right| + c\right]$

Form – 02 : $\int \frac{px+q}{ax^2+bx+c}\,dx$ যেখানে $p \neq 0, a \neq 0$

$$I = \frac{p}{2a}\left\{\int \frac{2ax+b}{ax^2+bx+c}\,dx + \frac{2aq-pb}{p}\int \frac{dx}{ax^2+bx+c}\right\}$$

px + q = l (হরের অন্তরক সহগ) +m যেখানে, l ও m ধ্রুবক।

Example- 01 : $\int \frac{4x+3}{3x^2+3x+1}\,dx = ?$

ধরি, $4x + 3 = l(6x+3) + m = 6lx + 3l + m$, x ও x ধ্রুব পদ সহগ সমীকৃত করে , $6l = 4$, $l = \frac{2}{3}$,

$3l + m = 3 \Rightarrow 3 \times \frac{2}{3} + m = 3 \Rightarrow m = 1$

$$\therefore I = \int \frac{\frac{2}{3}(6x+3)+1}{3x^2+3x+1}\,dx = \frac{2}{3}\int \frac{f'(x)}{f(x)}\,dx + \int \frac{dx}{3\left(x^2+x+\frac{1}{3}\right)}$$

$= \frac{2}{3}\ln|f(x)| + \int \frac{dx}{3\left\{\left(x+\frac{1}{2}\right)^2+\left(\frac{1}{2\sqrt{3}}\right)^2\right\}} = \frac{2}{3}\ln|3x^2+3x+1| + \frac{1}{3}.\tan^{-1}\frac{x+1/2}{1/2\sqrt{3}} + c$

$$= \frac{2}{3}\ln|3x^2+3x+1| + \frac{1}{3}\tan^{-1}\left\{\sqrt{3}(2x-1)\right\} + c$$

TRY YOURSELF:

(i) $\int \frac{x.dx}{x^2+2x+1} = ?$ $\left[\textbf{Ans}: \mathbf{\log(x+1) + \frac{1}{x+1} + c}\right]$

(ii) $\int \frac{2x+3}{4x^2+1} dx = ?$ $\left[\textbf{Ans}: \mathbf{\frac{1}{4}\log\left|4x^2+1\right| + \frac{3}{2}\tan^{-1}(2x) + c}\right]$

(iii) $\int \frac{x+1}{3+2x-x^2} dx = ?$ $[\textbf{Ans}: \mathbf{-\log(x-3) + c}]$

(iv) $\int \frac{x+1}{x^2+4x+5} dx = ?$ $\left[\textbf{Ans}: \mathbf{\frac{1}{2}\log\left|x^2+4x+5\right| - \tan^{-1}(x+2) + c}\right]$

Form – 03 : $\int \frac{dx}{\sqrt{ax^2+bx+c}},\ (a \neq 0) = \int \frac{dx}{\sqrt{a}\left\{\sqrt{\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}}\right\}} = \frac{1}{\sqrt{a}}\int \frac{dz}{z^2 \pm k^2}$

যদি a = - a´,হয় তবে উক্ত সমাকলনের আকার হয়, $\frac{1}{\sqrt{a}} \int \frac{dz}{\sqrt{z^2-k^2}}$

যেখানে, $k = \frac{4a'c+b^2}{4a'^2}, z = \left(x - \frac{a}{2a'}\right)$

Example- 01 : $\int \frac{dx}{\sqrt{2+3x-2x^2}} = \int \frac{dx}{\sqrt{(1+2x)(2-x)}}$

ধরি, $2 - x = z^2 \Longrightarrow -dx = 2z\,.dz, 1 + 2x = 1 + 2(2 - z^2) = 1 + 4 - 2z^2 = 5 - 2z^2$

$\therefore I = \int \frac{-2z.dz}{\sqrt{5-2z^2}} = -\frac{2}{\sqrt{2}}\int \frac{dz}{\sqrt{\left(\frac{\sqrt{5}}{\sqrt{2}}\right)^2 - z^2}} = -\sqrt{2}\sin^{-1}\frac{z}{\sqrt{5/2}} + c = -\sqrt{2}\sin^{-1}\sqrt{\frac{4-2x}{5}} + c$

$= \sqrt{2}\cos^{-1}\sqrt{\frac{4-2x}{5}} + c$

Example- 02 : $\int \frac{\mathbf{dx}}{\sqrt{(x-\alpha)(\beta-x)}}\ (\beta > \alpha)$

ধরি, $x - \alpha = z^2, dx = 2z.dz.\ \beta - x = \beta - \alpha - z^2$

$$\therefore I = \int \frac{-2z.dz}{z.\sqrt{\beta-\alpha-z^2}} = 2\int \frac{dz}{\sqrt{(\beta-\alpha)^2 - z^2}} = 2\sin^{-1}\frac{z}{\sqrt{\beta-x}} + c$$

$$= 2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-x}} + c$$

অথবা $x = \alpha\cos^2\theta + \beta\sin^2\theta$ ধরে অংকটি সমাধান করা যায়।

<u>TRY YOURSELF</u>

(i) $\int \frac{dx}{\sqrt{1-x-x^2}} =?$ $\left[\text{Ans: } \sin^{-1}\left(\frac{2x+1}{\sqrt{5}}\right) + c\right]$

(ii) $\int \frac{dx}{\sqrt{x^2-7x+12}} = ?$ $\left[\text{Ans: } 2\log\left(\sqrt{x-3} + \sqrt{x-4}\right)\right]$

(iii) $\int \frac{\cos x\,.dx}{\sqrt{5\sin^2 - 12\sin x + 4}} = ?$ $\left[\text{Ans: } -\frac{2}{\sqrt{5}}\log\left\{\sqrt{2-5\sin x} + \sqrt{5(2-\sin x)}\right\}\right]$

(iv) $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}}; (\beta > \alpha) = ?$ Ans : $2\log\left(\sqrt{(x-\alpha)} + \sqrt{(\beta-x)}\right) + c$

Form – 04 : (i) $\int \frac{px+q}{\sqrt{ax^2+bx+c}} dx = ?\ (a \neq 0, p \neq 0)$

$= \frac{p}{2a}\int \frac{(2ax+b)+\frac{2aq}{p}-b}{\sqrt{ax^2+bx+c}},$ $ax^2 + bx + c = z$ ধরে অগ্রসর হতে হবে।

(ii) $\int \sqrt{\frac{ax+b}{ca+d}}\,dx = ?$ লবের সার্ড মুক্ত করতে হবে । লব দ্বারা হর ও লবকে গুণ করে হর $= z$ ধরে অগ্রসর হতে হবে।

$\therefore I = \int \frac{(ax+b).dx}{\sqrt{acx^2+(ad+bc)x+bd}}$

Example- 01 : $\int \frac{a+x}{x}\,dx = ?$

$I = \int \frac{a+x}{x}\,dx = \int \frac{\frac{1}{2}(a+2x)+\frac{1}{2}a}{\sqrt{ax+x^2}}\,dx = \frac{1}{2}.\,2\sqrt{ax+x^2} + \frac{1}{2}a\int \frac{\mathbf{dx}}{\sqrt{\left(\mathbf{x}+\frac{1}{2}\mathbf{a}\right)^{\mathbf{2}}-\left(\frac{1}{2}\mathbf{a}\right)^{\mathbf{2}}}}$

$= \sqrt{x(a+x)} + \frac{a}{2}\ln\left|x+\frac{1}{2}a+\sqrt{\left(x+\frac{1}{2}a\right)^{\mathbf{2}}-\left(\frac{1}{2}a\right)^{\mathbf{2}}}\right| + c$

$= \sqrt{x(a+x)} + \frac{a}{2}\ln\left|x+\frac{1}{2}a+\sqrt{x(a+x)}\right| + c$

$= \sqrt{x(a+x)} + \frac{a}{2}\ln\left|2x+a+2\sqrt{x(a+x)}\right| - \frac{a}{2}\ln 2 + c$

$= \sqrt{x(a+x)} + \frac{a}{2}\ln\left|\left(\sqrt{x}+\sqrt{x+a}\right)^2\right| + k = \sqrt{x(a+x)} + a\ln\left|\sqrt{x}+\sqrt{x+a}\right| + k$

Example- 02 : $\int \sqrt{\frac{1+x}{1-x}}\,dx = \int \frac{1+x}{\sqrt{1-x^2}}\,dx = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{x}{\sqrt{1-x^2}}\,dx$

ধরি, $1-x^2 = z^2 \Rightarrow -2x.dx = 2z.dz \Rightarrow \frac{dx}{z} = -\frac{dz}{x} = \frac{-dz}{\sqrt{1-z^2}}$

$\therefore I = \int \frac{dx}{\sqrt{1-x^2}} + \int \frac{\sqrt{1-z^2}}{z}\,dx = \sin^{-1}x + \int \sqrt{1-z^2} \times \frac{-dz}{\sqrt{1-z^2}} = \sin^{-1}x - z + c$

$= \sin^{-1}x - \sqrt{1-x^2} + c$

TRY YOURSELF

(i) $\int \frac{x+b}{\sqrt{x^2+a^2}}\,dx = ?$ $\left[\text{Ans: } \sqrt{x^2+a^2} + b\log\left(x+\sqrt{a^2+x^2}\right)\right]$

(ii) $\int \frac{2x+3}{\sqrt{1+x+x^2}}\,dx = ?$ $\left[\text{Ans: } 2\sqrt{1+x+x^2} + 2\log\left(x+\frac{1}{2}+\sqrt{1+x+x^2}\right)\right]$

(iii) $\int \frac{x-2}{\sqrt{2x^2-8x+5}}\,dx = ?$ $\left[\text{Ans: } \frac{1}{2}\sqrt{2x^2-8x+5}\right]$

(iv) $\int \frac{x+1}{\sqrt{4+8x-5x^2}}\,dx = ?$ $\left[\text{Ans: } \frac{9}{5\sqrt{5}}\sin^{-1}\left(\frac{5x-4}{6}\right) - \frac{1}{5}\sqrt{4+8x-5x^2}\right]$

(v) $\int \sqrt{\frac{x-3}{x-4}}\,dx = ?$ $\left[\text{Ans: } \sqrt{(x-3)(x-4)} + \log\left(\sqrt{x-3}+\sqrt{x-4}\right)\right]$

(vi) $\int \frac{\sqrt{x}dx}{x-1} = ?$ $\left[\text{Ans: } 2\sqrt{x} + \log\frac{\sqrt{x}-1}{\sqrt{x}+1}\right]$

Form – 05 : $\int \frac{dx}{(cx+d)\sqrt{ax^2+bx+c}} = ?$

ধর, $cx + d = z^{-1}$, তারপর অগ্রসর হও।

Example- 01 : $\int \frac{\mathbf{dx}}{\mathbf{(2x+3)\sqrt{x^2+3x+2}}}$

ধরি, $2x + 3 = \frac{1}{z} \Longrightarrow 2dx = -\frac{1}{z^2}dz \Longrightarrow dx = -\frac{1}{2z^2}dz.$ এবং $x = \frac{1}{2}\left(\frac{1}{z} - 3\right)$

$$\therefore z = \frac{1}{2x+3} \quad \therefore I = -\frac{1}{2}\int \frac{dz}{z^2.\frac{1}{z}\sqrt{\frac{1}{z^2}\left(\frac{1}{z}-3\right)^2 + \frac{3}{2}\left(\frac{1}{z}-3\right)+2}}$$

$$= -\int \frac{dz}{\sqrt{1-z^2}} = \cos^{-1} z = \cos^{-1}\left(\frac{1}{2x+3}\right) + c = \sec^{-1}(2x+3) + c$$

অন্যভাবে, $I = \int \frac{2dx}{(2x+3)\sqrt{(2x+3)^2-1}}$ ধরি, $2x + 3 = z \Longrightarrow 2dx = dz.$

$= \int \frac{dz}{z\sqrt{z^2-1}} = \sec^{-1}z + c = \sec^{-1}(2x+3) + c$

TRY YOURSELF:

(i) $\int \frac{dx}{x\sqrt{x^2 \pm a^2}} = ?$ $\left[\textbf{Ans}: \frac{\mathbf{1}}{\mathbf{2a}}\mathbf{log}\left|\frac{\sqrt{\mathbf{x^2+a^2}}-\mathbf{a}}{\sqrt{\mathbf{x^2+a^2}}+\mathbf{a}}\right|; \frac{\mathbf{1}}{\mathbf{a}}\mathbf{sec^{-1}}\frac{\mathbf{x}}{\mathbf{a}}\right]$

(ii) $\int \frac{dx}{(1+x)\sqrt{1-x^2}} = ?$ $\left[\textbf{Ans}: -\sqrt{\frac{\mathbf{1-x}}{\mathbf{1+x}}} + \mathbf{c}\right]$

(iii) $\int \frac{dx}{x\sqrt{9x^2+4x+1}} = ?$ $\left[\textbf{Ans}: \mathbf{logx - log\left(1 + 2x + \sqrt{9x^2+4x+1}\right) + c}\right]$

(iv) $\int \frac{dx}{(1+x)\sqrt{1+2x-x^2}} = ?$ $\left[\textbf{Ans}: \frac{\mathbf{1}}{\sqrt{\mathbf{2}}}\mathbf{sin^{-1}}\left(\frac{\mathbf{x\sqrt{2}}}{\mathbf{1+x}}\right) + \mathbf{c}\right]$

(v) $\int \frac{dx}{x\sqrt{x^2+2x-1}} = ?$ $\left[\textbf{Ans}: \mathbf{sin^{-1}}\left(\frac{\mathbf{x-1}}{\mathbf{x\sqrt{2}}}\right) + \mathbf{c}\right]$

(vi) $\int \frac{dx}{(1+x)\sqrt{1+x-x^2}} = ?$ $\left[\textbf{Ans}: \mathbf{sin^{-1}}\left(\frac{\mathbf{3x+1}}{\mathbf{(1+x)\sqrt{5}}}\right) + \mathbf{c}\right]$

(vii) $\int \frac{dx}{(x-3)\sqrt{x^2-6x+8}} = ?$ $[\textbf{Ans}: \mathbf{sec^{-1}(x-3) + c}]$

(viii) যদি $a < x < b$ হয় তবে দেখাও যে, $\int \frac{dx}{(a-x)\sqrt{(a-x)(b-x)}} = \frac{2}{a-b}\sqrt{\frac{b-x}{x-a}}$

Form – 06 : $\int \frac{dx}{(cx+d)\sqrt{ax+b}} = ?$ $[a \neq 0, \quad c \neq 0]$

ধর, $ax + b = z^2$ তারপর অগ্রসর হও।

$I = \frac{2}{c}\int \frac{z.dz}{\left(a\frac{z^2-d}{c}+b\right)z} = 2\int \frac{dz}{az^2+(bc-ad)}$

Example- 01 : $\int \frac{dx}{(2+x)\sqrt{1+x}} = ?$

ধরি, $1 + x = z^2 \Longrightarrow dx = 2z.dz.$ এবং $x + 1 = z^2 + 1$

$$I = \int \frac{2z.dz}{(z^2+1)z} = \int \frac{dz}{1+z^2} = \tan^{-1} z + c = \tan^{-1}\left(\sqrt{1+x}\right) + c$$

TRY YOURSELF:

(i) $\int \frac{dx}{(2x+1)\sqrt{4x+3}} = ?$ $\left[\text{Ans: } \frac{1}{2}\log\left|\frac{\sqrt{4x+3}-1}{\sqrt{4x+3}+1}\right| + c\right]$

(ii) $\int \frac{dx}{x\sqrt{1+x^3}} = ?$ [ধর, $1 + x^3 = z^2$] $\left[\text{Ans: } \frac{2}{3}\log\left(\sqrt{1+X^3} - 1\right) - \log x + c\right]$

Form – 07 : $\int \frac{dx}{(cx^2+d)\sqrt{ax^2+b}}$, $x = \frac{1}{z}$ ধরে অগ্রসর হতে হবে । অথবা , $\sqrt{ax^2+b} = zx$ ধরে অগ্রসর হতে হবে।

Example- 01 : $\int \frac{\boldsymbol{dx}}{(\boldsymbol{x^2-1})\sqrt{\boldsymbol{x^2+1}}} = ?$

ধরি, $\sqrt{x^2+1} = zx \Rightarrow x^2 + 1 = z^2x^2 \Rightarrow 2xdx = 2zx^2dz + 2z^2.xdx$

$\Rightarrow dx = zx\,dz + z^2.dx \Rightarrow \frac{dx}{zx} = \frac{dz}{1-z^2}$ এবং $x^2(1-z^2) = -1 \Rightarrow x^2 = -\frac{1}{1-z^2}$

$$\Rightarrow x^2 - 1 = -\frac{1}{1-z^2} - 1 = -\frac{-1-1+z^2}{1-z^2} = \frac{z^2-2}{1-z^2}$$

$$\int \frac{dx}{\frac{z^2-2}{1-z}zx} = \int \frac{1-z^2}{z^2-2}.\frac{dz}{(1-z^2)} = \int \frac{dz}{z^2-\left(\sqrt{2}\right)^2} = \frac{1}{2\sqrt{2}}\ln\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right| + c$$

$$= \frac{1}{2\sqrt{2}}\ln\left|\frac{\sqrt{x^2+1}-\sqrt{2}}{\sqrt{x^2+1}+\sqrt{2}}\right| + c$$

$Try\ yourself$: (1) $\int \frac{2xdx}{(1-x^2)\sqrt{x^4-1}} = ?$ $(ii) \int \frac{dx}{(1+x^2)\sqrt{x^2+4}} = ?$

$(iii) \int \frac{dx}{(1+x^2)\sqrt{x^2-1}} = ?$

Form – 08 : $\int \frac{\boldsymbol{dx}}{\boldsymbol{x\sqrt{a+bx^n}}}$ **আকারের জন্য** $\boldsymbol{x^n = \frac{1}{u^2}}$ **ধরে অগ্রসর হতে হবে।**

$\boldsymbol{Example}$: $\int \frac{dx}{x(4+5x^{20})} = ?$ ধরি, $x^{20} = \frac{1}{u^2} \Rightarrow 20x^{19}dx = \frac{-2}{u^3}du$

$$I = \int \frac{dx}{x(4+5x^{20})} = \int \frac{x^{19}}{x^{20}(4+5x^{20})}dx = \frac{-2}{20}\int \frac{du}{u^3.\frac{1}{u^2}\left(4+\frac{5}{u^2}\right)}$$

$$= \frac{-1}{10}\int \frac{udu}{4u^2+5} = \frac{-1}{10}\int \frac{\frac{1}{8}\times 8udu}{4u^2+5} = \frac{-1}{80}ln(4u^2+5) + c = \frac{-1}{80}ln\left(\frac{4+5x^{20}}{x^{20}}\right) + \boldsymbol{c}$$

TRY YOURSELF: (i) $\int \frac{x^{-1/3}}{1+\sqrt{x}}dx = ?\ Ans : 4\tan^{-1}\left(x^{1/4}\right) + c$

(ii) $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}dx = ?\ Ans : \frac{6}{7}x^{7/6} - \frac{6}{5}x^{5/6} + 2x^{1/2} - 6x^{1/6} + 6\tan^{-1}\left(x^{1/6}\right)$

(iii) $\int \frac{1+x^{1/4}}{1+x^{1/2}}dx = ?$

$$Ans : \frac{4}{3}x^{3/4} + 2x^{1/2} - 4x^{1/4} + 2ln\left|x^{1/2}+1\right| + 4\tan^{-1}\left(x^{1/4}\right) + c$$

Type- 04 : অংশক্রমে সমাকলন ঃ

$\frac{d}{dx}(u\,v_1) = \frac{du}{dx}v_1 + u.\frac{dv_1}{dx}$ যেখানে , u = f(x), $v_1 = g(x)$ সমাকলন করে,

$uv_1 = \int\left(\frac{du}{dx}v_1\right)dx + \int\left(u.\frac{dv_1}{dx}\right).dx$ ধরি, $\frac{dv_1}{dx} = v \Rightarrow v_1 = \int vdx$

$$u\int vdx = \int\left(\frac{du}{dx}.\int udx\right)dx + \int uv.dx$$

$$\Longrightarrow \int uv.dx = u\int v.dx - \int\left[\frac{du}{dx}\int vdx\right]dx$$

u ⟶ 1st function , v ⟶ 2nd function , যেভাবে u ও v কে ধরবেঃ LIATE ⟶ ৫ টা function কে তাদের প্রথম অক্ষর দ্বারা চিহ্নিত করা হয়েছে।

L⟶ Logarithmic,I⟶Inverse,A⟶Arithmatic ,

T⟶ Trisometric, E ⟶ Exponential

LI হতে L⟶ u, I⟶ v

IA হতে I⟶ u, A⟶ v

AT হতে A⟶ u, T⟶ v

TE হতে T⟶ u, E⟶ v

LT ⟶? , IE⟶ ?, TI⟶?

Example- 01 : (i) $\int xe^x dx$ এখানে , x ⟶ Arithmatic⟶ u = f(x) = x

e^x ⟶ Exponential ⟶v = g (x) = e^x.

$$\therefore I = x\int e^x dx - \int\left[\frac{d}{dx}(x)\int e^x dx\right]dx = xe^x - \int e^x dx = xe^x - e^x + c$$

Example- 02 : $\int lnx.dx$ এখানে $x^0 = 1 \longrightarrow v, lnx \longrightarrow u$

$$= lnx\int dx - \int\left[\frac{d}{dx}(lnx)\int dx\right]dx = x\,lnx - \int\frac{1}{x}.x.dx = x\,lnx - x + c$$

Example- 03 : $\int \tan^{-1}x.dx. = \tan^{-1}x\int dx - \int\left[\frac{d}{dx}(\tan^{-1}x)\int dx\right]dx$

$$= x\,\tan^{-1}x - \int\frac{x}{1+x^2}dx. = x\,\tan^{-1}x - \frac{1}{2}ln|1+x^2| + c$$

Example- 04 : $\int \log\left(x+\sqrt{x^2+a^2}\right)dx$

$$I = \log\left(x+\sqrt{x^2+a^2}\right).\int dx - \int\left[\frac{d}{dx}\log\left(x+\sqrt{x^2+a^2}\right)\int dx\right]dx.$$

$$= xlog\left(x+\sqrt{x^2+a^2}\right) - \int\frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{1}{2\sqrt{x^2+a^2}}.2x\right)x.dx$$

$$= xlog\left(x+\sqrt{x^2+a^2}\right) - \int\frac{x}{\sqrt{x^2+a^2}}dx.$$

ধরি, $x^2+a^2 = z^2 \Longrightarrow 2xdx = 2z.dz \Longrightarrow \frac{dx}{z}.x = \int\frac{dz}{x}.x = \int dz =$

$z = \sqrt{x^2+a^2}$

$\therefore I = x\log\left(x+\sqrt{x^2+a^2}\right)-\sqrt{x^2+a^2}+c\ (Ans:)$

Form – 01: $\int\sqrt{ax^2+bx+c}.dx\ \ (a\neq 0)=\int\sqrt{a\left(x^2+\frac{a}{b}x+{}^{c}/_{a}\right)}$

$=\sqrt{a}\int\sqrt{\left(x+\frac{b}{2a}\right)^2-\frac{4ac-b^2}{4a^2}}dx=\sqrt{a}\int\sqrt{z^2-k^2}\,.dx$ সূত্র : $\int\sqrt{x^2-a^2}\,dx$ প্রয়োগ করি।

$=\sqrt{a}\int\sqrt{\left(x+\frac{b}{2a}\right)+\frac{b^2-4ac}{4a^2}}dx.=\sqrt{a}\int\sqrt{z^2+a^2}\,dx$ সূত্র : $\int\sqrt{x^2+a^2}\,dx$ প্রয়োগ করি।

Example- 01 : $\int\sqrt{4+8x-5x^2}\,dx\Rightarrow I=\int\sqrt{5\left(\frac{4}{5}\right)+\frac{8}{5}x-x^2}\ dx$

$=\int\sqrt{5}\sqrt{\left\{\frac{4}{5}+\left(\frac{4}{5}\right)^2-\left(\frac{4}{5}\right)^2-2.\frac{4}{5}x+x^2\right\}}dx=\sqrt{5}\int\sqrt{\left(\frac{6}{5}\right)^2-\left(x-\frac{4}{5}\right)^2}\ dx$

$=\sqrt{5}\int\sqrt{\mathrm{a}^2-\mathrm{z}^2}\mathrm{dz}$ ধরি, $z=x-\frac{4}{5}\Rightarrow dx=dz$

$=\sqrt{5}\left[\int\sqrt{a^2-z^2}dz-\int\left\{\frac{1}{2\sqrt{a^2-z^2}}(-2z).\int dz\right\}dz\right]$

ধরি, $\mathrm{I}'=\int\sqrt{\mathrm{a}^2-\mathrm{z}^2}\ \mathrm{dz}=\mathrm{z}\sqrt{\mathrm{a}^2-\mathrm{z}^2}+\int\frac{\mathrm{z}^2}{\sqrt{\mathrm{a}^2-\mathrm{z}^2}}\mathrm{dz}=\mathrm{z}\sqrt{\mathrm{a}^2-\mathrm{z}^2}+\int\frac{-(\mathrm{a}^2-\mathrm{z}^2)+\mathrm{a}^2}{\sqrt{\mathrm{a}^2-\mathrm{z}^2}}\mathrm{dz}$

$=\mathrm{z}\sqrt{\mathrm{a}^2-\mathrm{z}^2}-\int\sqrt{\mathrm{a}^2-\mathrm{z}^2}\ \mathrm{dz}+\mathrm{a}^2\sin^{-1}\frac{\mathrm{z}}{\mathrm{a}}\ \ ,2\mathrm{I}'=\mathrm{z}\sqrt{\mathrm{a}^2-\mathrm{z}^2}+\mathrm{a}^2\sin^{-1}\frac{\mathrm{z}}{\mathrm{a}}$

$\mathrm{I}'=\sqrt{5}\left[\frac{\mathrm{z}\sqrt{\mathrm{a}^2-\mathrm{z}^2}}{2}+\frac{\mathrm{a}^2}{2}\sin^{-1}\frac{\mathrm{z}}{\mathrm{a}}\right]+\mathrm{c}=\sqrt{5}\left[\frac{(5\mathrm{x}-4)\sqrt{4+8\mathrm{x}-5\mathrm{x}^2}}{10\sqrt{5}}+\frac{18}{25}\sin^{-1}\left(\frac{5\mathrm{x}-4}{6}\right)\right]$

$=\frac{1}{10}(5x-4)\sqrt{4+8\mathrm{x}-5\mathrm{x}^2}+\frac{18}{5\sqrt{5}}\sin^{-1}\left(\frac{5\mathrm{x}-4}{6}\right)+\mathrm{c}$

TRY YOURSELF:

(i) $\int\sqrt{25-9x^2}.dx\ \left[Ans:\frac{x}{2}\sqrt{25-9x^2}+\frac{25}{9}\sin^{-1}\frac{3x}{5}\right]$

(ii) $\int\sqrt{5-2x+x^2}dx.\left[Ans:\frac{1}{2}(x-1)\sqrt{5-2x+x^2}+2\log\left(x-1+\sqrt{5-2x+x^2}\right)\right]$

(iii) $\int\sqrt{18x-65-x^2}dx\ \left[Ans:\frac{1}{2}(x-9)\sqrt{18x-65-x^2}+8\sin^{-1}\left(\frac{x-9}{4}\right)\right]$

(iv) $\int\sqrt{4-3x-2x^2}\ \mathrm{dx}\left[Ans:\frac{1}{8}(4x+3)\sqrt{4-3x-2x^2}+\frac{41\sqrt{2}}{32}\sin^{-1}\frac{4x+3}{\sqrt{41}}\right]$

(v) $\int\sqrt{2ax-x^2}\,dx\ \left[Ans:\frac{1}{2}(x-a)\sqrt{2ax-x^2}+\frac{1}{2}\sin^{-1}\left(\frac{x-a}{a}\right)\right]$

(vi) $\int\sqrt{(x-\alpha)(\beta-x)}dx$ ধর, $x=\alpha cos^2\theta+\beta sin^2\theta,$

$\left[Ans:\frac{1}{4}\left[(2x-\alpha-\beta)\sqrt{(x-\alpha)(\beta-x)}+(\beta-\alpha)^2\sin^{-1}\sqrt{\frac{x-\alpha}{\beta-x}}+c\right]\right]$

Form – 02: $\int (px+q)\sqrt{ax^2+bx+c}\, dx \quad [d \neq 0, a \neq 0]$

$ax^2 + bx + cz = z$ ধরে অগ্রসর হতে হবে ।

রূপান্তর ঃ $px + q = \frac{p}{2a}(2ax+b) + \left(q - \frac{bp}{2a}\right)$

Example- 01 : $\int (3x-2)\sqrt{x^2-x+1}\, dx = ?$

ধরি, $x^2 - x + 1 = z \Longrightarrow (2x-1)dx = dz; 3x - 2 = \frac{3}{2}(2x-1) - \frac{1}{2}$

$\therefore \mathrm{I} = \int \left\{\frac{3}{2}(2x-1) - \frac{1}{2}\right\}\sqrt{x^2-x+1}dx = \frac{3}{2}\int (2x-1)\sqrt{x^2-x+1}\,dx - \frac{1}{2}\int \sqrt{x^2-x+1}dx$

$= \frac{3}{2}\int \sqrt{z}.\,dz - \frac{1}{2}\int \sqrt{\left(x-\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)^2}\,dx$

$= z^{3/2} - \frac{1}{2} \times \frac{\left(x-\frac{1}{2}\right)\left(\sqrt{x^2-x+1}\right)}{2} - \frac{1}{2} \times \frac{\frac{3}{4}}{2}\ln\left(x - \frac{1}{2} + \sqrt{x^2-x+1}\right) + c$

$= \left(x^2-x+1\right)^{3/4} - \frac{1}{4} \times \frac{(2x-1)\left(\sqrt{x^2-x+1}\right)}{2} - \frac{3}{16}\ln\left(x - \frac{1}{2} + \sqrt{x^2-x+1}\right) + c$

Form- 03: $\int \frac{px^2+qx+r}{\sqrt{ax^2+bx+c}}\,dx = ?$

$px^2 + qx + r = \frac{p}{a}(ax^2+bx+c) - \frac{pb}{a}x - \frac{pc}{a} = \frac{p}{a}(ax^2+bx+c) - \frac{pb}{2a^2}(2ax+b) - \frac{pa}{a} + \frac{pb^2}{2a^2}$

$= \frac{p}{a}(ax^2+bx+c) - \frac{pb}{2a^2}(2ax+b) - \frac{pb^2 - pac}{2a^2}$

Example – 01: $\int \frac{x^2+x+1}{\sqrt{x^2+2x+3}}\,dx = ?$

$\mathrm{I} = \int \frac{x^2+2x+3-x-2}{\sqrt{x^2+2x+3}}\,dx = \int \sqrt{x^2+2x+3}\,dx - \int \frac{x+2}{\sqrt{x^2+2x+3}}\,dx$

$= \int \sqrt{(x+1)^2 + \sqrt{(2^2)}}\,dx - \int \frac{\frac{1}{2}(2x+2)+1}{\sqrt{x^2+2x+3}}\,dx$

$= \frac{(x+1)\sqrt{x^2+2x+3}}{2} + \log\ \{x+1+\sqrt{x^2+2x+3}\} - \sqrt{x^2+2x+3} - \int \frac{dx}{\sqrt{(x+1)^2+(\sqrt{2})^2}}$

$= \frac{(x+1)\sqrt{x^2+2x+3}}{2} + \log\{x+1+\sqrt{x^2+2x+3}\} - \sqrt{x^2+2x+3} - \log\{x+1+\sqrt{x^2+2x+3}\} + c = \frac{1}{2}(x-1)\sqrt{x^2+2x+3} + c$

Form – 04 : $\int e^{ax}.\cos bx\, dx = ?$ সূত্রে প্রমান করা আছে

Try yourself : (i) $\int e^{ax}.\sin bx\, dx = ?$

Ans: $\frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$, $\frac{e^{ax}}{\sqrt{a^2+b^2}}\sin\left(bx - \tan^{-1}\frac{a}{b}\right)$

alternative method : **P**=$\int e^{ax}$. cos bx.dx , $\boldsymbol{Q} = \int e^{ax}$. sin bx.dx

P+iQ = $\int e^{ax}$(cos bx + sin bx) dx =$\int e^{ax}. e^{ibx}. dx$ = $\int e^{(a+ib)x}$. dx

=$\frac{e^{(a+ib)^x}}{\text{a+ib}} e^{ibx}$ =$\frac{e^{ax}}{a^2+b^2}$ (a-ib)(cos bx +isinbx)

P = $\frac{e^{ax}}{a^2+b^2}$ (a cos bx + b sin bx) [বাস্তব অংশ সমীকৃত করে]

Q = $\frac{e^{ax}}{a^2+b^2}$ (a sin bx – b cos bx) [কাল্পনিক অংশ সমীকৃত করে]

Example -01: $\int 3^x$cos 3x dx =?

I=$\int e^{xln3}$. cos3x.dx=$\frac{e^{xln3}}{(m3)^2+3^2}$ (m3cos 3x + 3 sin3x) = $\frac{3x}{9+(ln3)^2}$ (3sin3x + ln3.cos3x)+ c

Example – 02 : $\int e^{3x}$ cos 4x.dx = ?

I = cos 4x.$\frac{1}{3}e^{3x}$ + $\int \frac{4e^{3x}}{3}$sin 4x.dx =$\frac{1}{3}e^{3x}$. cos4x + 4.$\frac{1}{9}e^{3x}$.sin4x - $\int \frac{4^2}{3^2}$ e^{3x}. cos 4x.dx

=$\frac{1}{3}e^{3x}$. cos4x +$\frac{4}{9}e^{3x}$. sin 4x - $\frac{16}{9}$ I $\Rightarrow$ $\frac{25}{9}$I = $\frac{1}{3}e^{3x}$. cos4x +$\frac{4}{9}e^{3x}$. sin 4x

$\Rightarrow$I = $\frac{e^{3x}}{25}$ (3 cos4x + 4sin4x) + c

Let, 3 = r cosθ, 4=rsinθ; $r^2 = 25 \therefore r = 5$

$\therefore 3cos4x + 4sinx = \cos(4x - \theta) = 5\cos\,(4x - tan^{-1}\frac{4}{3})$

$\therefore I = \frac{e^{3x}}{5}\, cos\,(4x - tan^{-1}\frac{4}{3})$ +c

$Try\ yourself\ : (i) \int 2^2\, sinxdx = ?\, Ans\text{:}\ \frac{2x\sin\{x - cot^{-1}\,(log2)\}}{\sqrt{1+(log2^2)}}$

$(ii) \int e^{3x}\ \ sin4xdx = ?\, Ans\text{:}\ \frac{e^{3x}}{25}(3\sin 4x - 4\cos 4x) + c$

(iii)যদি $u = \int e^{ax}cosbxdx,\quad v = \int e^{ax}\ \ sinbxdx$ হয়, তবে প্রমাণ কর যে,

$(a)\ tan^{-1}\frac{v}{u} + tan^{-1}\frac{b}{a} = bx$ $(b)\ (a^2+b^2)(\,u^2+v^2) = e^{2ax}$

$(v) \int e^x \frac{(1-x)^2}{(1+x^2)^2}dx = ?\ \ Ans\ :\ \frac{e^x}{1+x^2} + c$

$(vi) \int e^x \frac{1+sinx}{1+cosx}\, dx = ?\ \ Ans : e^x tan\frac{x}{2} + c\ \ (vii) \int e^x \frac{1-sinx}{1-cosx}\, dx = ?\, Ans\text{:} -e^x cot\frac{x}{2} + c\ (viii) \int e^x\ \frac{2-sin2x}{1-cos2x} dx = ?\ Ans : -e^x cotx + c\ (ix) \int e^x\ \frac{2+sinx}{1+cos2x} dx = ?\, Ans\text{:}\ e^x\ tanx + c$

Form – 05 : $\int e^x\{f(x) + f(x)\}dx = e^x\{f(x)\} + c$

Example – 01 : $\int \frac{xe^2}{(x+1)^2}dx = ?$ $I = \int \frac{(x+1)e^x - e^x}{(x+1)^2}dx$

$= \int \frac{e^x}{1+x}dx - \int \frac{e^x}{(1+x)^2}dx = \frac{e^x}{1+x} + \int \frac{e^x}{(1+x)^2}dx - \int \frac{e^x}{(1+x)^2}dx = \frac{e^x}{1+x} + c$

সূত্র হতে : $\int \frac{xe^x}{(x+1)^2}\ dx = ?$

$\frac{d}{dx}\left(\frac{1}{1+x}\right) = \frac{-1}{(1+x)^2}$ $\therefore I = \int e^x\,[\frac{1}{1+x} + \{\frac{-1}{(1+x)^2}\}]dx = \frac{e^x}{1+x} + c\quad Ans$ ...

Example- 02 : $= \int\{\frac{1}{lgox} - \frac{1}{(logx)^2}\}dx$

ধরি, $logx = z \Rightarrow x = e^z. \therefore dx = e^z.dx$

$$\therefore I = \int e^z\{\frac{1}{z} - \frac{1}{z^2}\}dx = \int e^z[\frac{1}{z} + (\frac{-1}{z^2})]\,dx = e^z\frac{1}{z} + c = \frac{e^{logx}}{logx} = \frac{x}{logx} + c$$

$try\ yourself\ :(i)\int \frac{e^x}{x}(1 + x\ logx)dx = ?\ \ Ans\ .e^x logx + c$

$(ii)\int e^x\,(tanx - logcosx)dx = ?\quad Ans.e^x logcox + c\ (iii)\int e^x\frac{x^2+1}{(x+1)^2}d = Ans.\ e^x(\frac{x-1}{x+1}) + c$

$(iv)\int e^x\frac{x-1}{(x+1)^3}dx = ?\ \ Ans.\ e^x\frac{1}{(1+x)^2} + c$

ত্রিকোনমিতিক ফাংশনের অংশ্ক্রমে ইন্টিগ্রেশন :

Example -01 : $\int cos2x\log(1 + tanx)dx.$

সমাধান : I = $\int cos2x\log\,(1 + tanx)dx$

$= log(1 + tanx)\frac{1}{2}sin2x - \int \frac{sec^2x}{1+tanx}.\frac{1}{2}sin2x\ dx$

$= \log(1 + tanx)\,sinx\ cosx - \int \frac{sec^2x.sinx.cosx}{1+\frac{sinx}{cosx}}dx$

$= \log(1 + tanx)\,sinx\ cosx - \int \frac{sinx}{sinx+cosx}dx$

ধরি $I, = \int \frac{sinx}{sinx+cosx}dx$

$sinx = l(sinx + cosx) + m(cosx - sinx)$

$sinx = l\ sinx + l\ cosx + mcosx - msinx.$

$= (l - m)sinx + (l + m)cosx$

$l - m = 1,\ l = 1 + m\ \ \therefore\ -m = 1 + m \Rightarrow m = \frac{1}{2}$

$l + m = 0\,,l = -m\quad,\quad l = -\frac{1}{2}$

I, $= \int \frac{\frac{-1}{2}(sinx+cosx)+\frac{1}{2}(cosx-sinx)}{sinx+cosx}dx$

$= -\frac{1}{2}\int dx + \frac{1}{2}\int \frac{\frac{d}{dx}(cosx+sinx)}{sinx+cosx}dx$

$= -\frac{1}{2}x + \frac{1}{2}ln(sinx + cosx)$

$\therefore I = sinx.cosx\log(1 + tanx) - \frac{1}{2}x + \frac{1}{2}ln(sinx + cosx) + c$

Example -2 : $\int \frac{x+sinx}{1+cosx}\ dx.$

$= \int \frac{x+sinx}{2cos^2x/2}dx = \int \frac{x}{2}sec^2\frac{x}{2}.dx + \int \frac{2sin^x/_2.cos^x/_2}{2cos^{2x}/_2}dx$

$= 2.\frac{x}{2}.tan\frac{x}{2} - \int \frac{1}{2}.2tan\frac{x}{2}dx + \int tan\frac{x}{2}dx = xtan\frac{x}{2} + c$

Example - 03: $\int sin^{-1}\sqrt{\frac{x}{x+a}}dx = ?$

ধরি, $x = atan^2\theta$

$\Rightarrow dx = 2a\, tan\theta.sec^2\theta.d\theta$

$I = \int sin^{-1}\sqrt{\frac{a\,tan^2\theta}{a(1+tan^2\theta)}}.\quad 2a\,tan\theta.sec\theta.d\theta$

$= \int sin^{-1}\sqrt{sin^2\theta}.2a\,tan\theta.sec\theta.d\theta$

$= \int \theta.2a\,tan\theta.sec\theta.d\theta$

$= 2a\theta.sec\theta - 2a\int sec\theta.d\theta$

$= 2a\theta.sec\theta - 2a \times \frac{1}{2}log\left|\frac{1+sin\theta}{1-sin\theta}\right| + c$

$\boldsymbol{\frac{x}{a} = tan^2\theta = sec^2\theta - 1}$

$\Rightarrow sec^2\theta = 1 + \frac{x}{a} = \frac{a+x}{a}$

$\Rightarrow sec\theta = \sqrt{\frac{a+x}{a}}$

$\Rightarrow cos\theta = \sqrt{\frac{a}{x+a}}$

$\Rightarrow sin\theta = \sqrt{\frac{x}{x+a}}$

$\boldsymbol{\therefore I = 2a\left[\sqrt{\frac{x+a}{a}}\sec^{-1}\sqrt{\frac{a+x}{a}} - \frac{1}{2}log\left|\frac{1+\sqrt{\frac{x}{x+a}}}{1-\sqrt{\frac{x}{x+a}}}\right|\right] + c}$

$\boldsymbol{= 2a\left[\sqrt{\frac{x+a}{a}} - \frac{1}{2}log\left|\frac{\sqrt{x}+\sqrt{x+a}}{\sqrt{x+a}-\sqrt{x}}\right|\right] + c}$

Phase -01ঃ

Type-06: ত্রিকোনমিতিক ফাংশনের সমাকলন ঃ

Form-01ঃ *R(sinx,cosx)* যেখানে *R* হইল *sinx* এবং *cosx* এর মূলদ(*rational*) ফাংশান।

$$\int\frac{dx}{a+bcosx}, \int\frac{dx}{a+bsinx}, \int\frac{dx}{asinx+bcosx+c}$$

এক্ষেত্রে , $cosx = \frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}$, $sinx = \frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}$, $a = a\frac{1+tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}$

হতে $tan\frac{x}{2} = u$ ধরে অগ্রসর হতে হবে।

Example-01: $I = \int\frac{dx}{a+bcosx} = \int\frac{dx}{a\frac{1+tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}} + b\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}} = \int\frac{sec^2\frac{x}{2}dx}{a+b+(a-b)tan^2\frac{x}{2}}$

ধরি , $tan\frac{x}{2} = u \Rightarrow du = \frac{1}{2}sec^2\frac{x}{2}dx$

$\therefore I = \frac{1}{a-b}\int\frac{2du}{\left(\sqrt{\frac{a+b}{a-b}}\right)^2+u^2} = \frac{2}{a-b}\frac{\sqrt{a-b}}{\sqrt{a+b}}\tan^{-1}\frac{\sqrt{a-b}u}{\sqrt{a+b}} + c$ যখন $a > b$

$= \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\frac{\sqrt{a-b}}{\sqrt{a+b}}\; tan\frac{x}{2} + c = \frac{2}{\sqrt{a^2-b^2}}\cos^{-1}\left(\frac{acosx+b}{a+bcosx}\right)$

যখন $a < b$ $I = \frac{1}{b-a}\int\frac{2du}{\left(\sqrt{\frac{a+b}{a-b}}\right)^2-u^2} = \frac{2}{b-a}\times\frac{1}{2\times\sqrt{\frac{a+b}{a-b}}}ln\frac{\sqrt{\frac{a+b}{b-a}}+u}{\sqrt{\frac{a+b}{b-a}}-u} + c$

$= \frac{2}{\sqrt{b^2-a^2}}ln\left(\frac{\sqrt{a+b}+\sqrt{b-a}\,tan\frac{x}{2}}{\sqrt{a+b}-\sqrt{b-a}\;tan\frac{x}{2}}\right) + c$

বি.দ্র. যদি $a > 0, b > 0$; $a < 0, b > 0$; $a > 0, b < 0$ এবং $a < 0, b < 0$ হয় তবে একই পদ্ধতিতে Solve করতে হবে।

Example-02: $\int \frac{dx}{5+4cosx} = ?$

Solve : $I = \int \frac{dx}{5\frac{1+tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}+4\frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}} = \int \frac{sec^2\frac{x}{2}dx}{9+tan^2\frac{x}{2}}$,

ধরি , $tan\frac{x}{2} = u \Rightarrow du = \frac{1}{2}sec^2\frac{x}{2}dx$

$\therefore I = \int \frac{2du}{3^2+u^2} = \frac{2}{3}.\tan^{-1}\frac{u}{3} + c$

$= \frac{2}{3}\tan^{-1}\left(\frac{tan\frac{x}{2}}{3}\right) + c = \frac{2}{3}\cos^{-1}\left(\frac{5cosx+4}{5+4cosx}\right) + c$

Try yourself : (i) $\int \frac{dx}{4+5cosx} = ?\ Ans: \frac{2}{3}ln\left(\frac{3+\ tan\frac{x}{2}}{3-\ tan\frac{x}{2}}\right) + c$

(ii) $\int \frac{cosxdx}{5-3cosx} = ?\ \ Ans: -\frac{x}{3} + \frac{5}{6}\tan^{-1}\left(2\tan\frac{x}{2}\right) + c$

(iii) $\int \frac{dx}{a^2-b^2\cos^2 x} = ?\ Ans: \frac{1}{a\sqrt{a^2-b^2}}\tan^{-1}\left(\frac{a}{\sqrt{a^2-b^2}}\tan x\right) + c\ ;\ [if\ a > b]$

$Ans: \frac{1}{2a\sqrt{b^2-a}}ln\left(\frac{a\tan x-\sqrt{b^2-a}}{a\tan x+\sqrt{b^2-a}}\right) + c\ ;\ [if\ b > a]$

(iv) $\int \frac{dx}{cos\alpha+cosx} = ?\ \ Ans: \frac{1}{sin\alpha}ln\left(\frac{cos\frac{x-\alpha}{2}}{cos\frac{x+\alpha}{2}}\right) + c$

Example-03 : $I = \int \frac{dx}{a+bsinx} = \int \frac{dx}{a\frac{1+tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}}+b\frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}} = \int \frac{sec^2\frac{x}{2}dx}{a+2btan\frac{x}{2}+atan^2\frac{x}{2}}$

ধরি , $tan\frac{x}{2} = u \Rightarrow du = \frac{1}{2}sec^2\frac{x}{2}dx$

$\therefore I = \int \frac{2du}{a+2bu+au^2} = \frac{2}{a}\int \frac{du}{u^2+2\frac{b}{a}u+1} = \frac{2}{a}\int \frac{du}{\left(u+\frac{b}{a}\right)^2+\left(\sqrt{\frac{a^2-b^2}{a^2}}\right)^2}$

যখন $a > b$, $= \frac{2}{a} \times \frac{a}{\sqrt{a^2-b^2}}\tan^{-1}\frac{u+\frac{b}{a}}{\sqrt{\frac{a^2-b^2}{a^2}}} + c = \frac{2}{\sqrt{a^2-b^2}}\tan^{-1}\left(\frac{atan\frac{x}{2}+b}{\sqrt{a^2-b^2}}\right)$

যখন $a < b$, $I = \frac{2}{a}\int \frac{du}{\left(u+\frac{b}{a}\right)^2-\left(\sqrt{\frac{a^2-b^2}{a^2}}\right)^2} = \frac{2}{a} \times \frac{1}{2\times\sqrt{\frac{a^2-b^2}{a^2}}}ln\frac{u+\frac{b}{a}-\sqrt{\frac{a^2-b^2}{a^2}}}{u+\frac{b}{a}+\sqrt{\frac{a^2-b^2}{a^2}}} + c$

$= \frac{2}{\sqrt{b^2-a^2}}ln\left(\frac{atan\frac{x}{2}+b-\sqrt{b^2-a^2}}{atan\frac{x}{2}+b+\sqrt{b^2-a^2}}\right) + c$

Try yourself : (i) $\int \frac{dx}{5+4sinx} = ?\ Ans : \frac{2}{3}\tan^{-1}\left(\frac{5tan\frac{x}{2}+4}{3}\right)$

(ii) $\int \frac{dx}{4+5sinx} = ?$ $Ans:\ \frac{2}{3} ln\left(\frac{2tan\frac{x}{2}+1}{2tan\frac{x}{2}+4}\right)$

(iii) $\int \frac{dx}{5+4sin2x} = ?$ $Ans:\ \frac{1}{3}\tan^{-1}\left(\frac{5tanx+4}{3}\right)$

Form-02ঃ (i) $\int \frac{acosx+bsinx}{ccosx+dsinx} dx = ?$

এক্ষেত্রে , লব=l×হর +m×হরের অন্তরক সহগ; ধরে অগ্রসর হতে হবে।

(ii) $\int \frac{acosx+bsinx+c}{dcosx+esinx+f} dx = ?$

এক্ষেত্রে , লব=l×হর +m×হরের অন্তরক সহগ+n; ধরে অগ্রসর হতে হবে।

Example-01 : $I = \int \frac{dx}{a+btanx} = \int \frac{cosx\, dx}{acosx + b\, sinx}$

$cosx = l(acosx\ + b\ sinx) + m(bcosx - a\ sinx)$

$= (la + mb)cosx + (lb - ma)sinx$

$cosx$ ও $sinx$ এর সহগ সমীকৃত করে পাই , $la + mb = 1$, $lb - ma = 0$

$l = \frac{a}{a^2+b^2}$, $m = \frac{b}{a^2+b^2}$

$I = \int \frac{cosx\, dx}{acosx + b\, sinx} = \int \frac{\frac{a}{a^2+b^2}(acosx + b\, sinx) + \frac{b}{a^2+b^2}(bcosx - a\, sinx)}{acosx + b\, sinx} dx$

$= \frac{a}{a^2+b^2}\int dx + \frac{b}{a^2+b^2}\int \frac{(bcosx - a\, sinx)}{acosx + b\, sinx} dx = \frac{ax}{a^2+b^2} + \frac{b}{a^2+b^2} ln|acosx\ + b\ sinx|$

Example-02: $I = \int \frac{1-cosx-sinx}{1-cosx+sinx} dx = ?$

$1 - cosx - sinx = l(1 - cosx\ +\ sinx) + m(cosx\ +\ sinx) + n$

$= (l + n) - (l - m)cosx + (l + m)sinx$

ধ্রুব পদ , $cosx$ ও সহগ $sinx$ সমীকৃত করে পাই , $l = 0$, $m = -1$, $n = 1$

$$I = \int \frac{0\times(1 - cosx\ +\ sinx) - 1(cosx\ +\ sinx) + 1}{1 - cosx\ +\ sinx} dx$$

$= -\int \frac{\frac{d}{dx}(1-cosx+sinx)}{1-cosx+sinx} dx + \int \frac{dx}{1-cosx+sinx} = -ln(1 - cosx\ +\ sinx) + I_1$

$I_1 = \int \frac{dx}{1 - \frac{1-tan^2\frac{x}{2}}{1+tan^2\frac{x}{2}} + \frac{2tan\frac{x}{2}}{1+tan^2\frac{x}{2}}} = \frac{1}{2}\int \frac{sec^2\frac{x}{2}\, dx}{tan\frac{x}{2} + tan^2\frac{x}{2}}$ ধরি , $tan\frac{x}{2} = u \Rightarrow du = \frac{1}{2}sec^2\frac{x}{2}dx$

$I_1 = \int \frac{du}{u+u^2} = \int \frac{du}{\left(u+\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2} = \frac{1}{2\times\frac{1}{2}} ln\left|\frac{u-\frac{1}{2}}{u+\frac{1}{2}}\right|$ $\therefore I_1 = ln\left|\frac{u+\frac{1}{2}-\frac{1}{2}}{u+\frac{1}{2}+\frac{1}{2}}\right| = ln\left|\frac{tan\frac{x}{2}}{tan\frac{x}{2}+1}\right|$

$I = \int \frac{1-cosx-sinx}{1-cosx+sinx} dx = -ln(1 - cosx\ +\ sinx) + ln\left|\frac{tan\frac{x}{2}}{tan\frac{x}{2}+1}\right| + c$

Try yourself : (i) $\int \frac{2cosx+3sinx}{3cosx+2\, sinx} dx = ?$ $Ans:\ \frac{12x}{13} - \frac{5}{13} ln|3cosx\ + 2\ sinx|$

(ii) $\int \frac{1+cosx-sinx}{1-cosx+sinx} dx = ?$ $Ans:\ -x + 2ln\left|\frac{tan\frac{x}{2}}{tan\frac{x}{2}+1}\right| + c$

(iii) $\int \frac{dx}{acosx+bsinx} = \int \frac{secxdx}{a+btanx}$ = ? $Ans: \frac{1}{\sqrt{a^2+b^2}}\ ln\left|\frac{\sqrt{a^2+b^2}-\text{b}+\text{a}\ tan\frac{x}{2}}{\sqrt{a^2+b^2}+\text{b}-\text{a}\ tan\frac{x}{2}}\right|+c$

(iv) $\int \frac{tanx}{\sqrt{a+btan^2x}}dx = ?\ Ans: \frac{1}{\sqrt{b-a}}\ cos^{-1}\left|\frac{\sqrt{b-a}}{b}cosx\right|+c$

Hints : tanx কে sinx /cosx এ ভেঙ্গে cosx= u ধরে অগ্রসর হতে হবে ।

Phase -02ঃ

Form-01ঃ $\int sin^n xdx , \int cos^n xdx$

* n বিজোড় পূর্ণ সংখ্যার জন্য $\int sin^n xdx$ এ cosx=u বসাতে হবে এবং $\int cos^n xdx$ এ sinx=u বসাতে হবে ।

Example -01 : $\int sin^7 xdx = ?$

Solve : $I = \int sin^6 xsinx\ dx$

ধরি , $cosx = u \Rightarrow -sinxdx = du$

$I = \int sin^6 xsinx\ dx\ = -\int(1-cos^2x)^3\,(sinxxdx) = -\int(1-u^2)^3 du$

$= -\int(1-3u^2+3u^4-u^6)du = \frac{1}{7}u^7 - \frac{3}{5}u^5 + \frac{3}{3}u^3 - u + c$

$= \frac{1}{7}cosx^7 - \frac{3}{5}cosx^5 + cosx^3 - cosx + c$

Try yourself : $\int cos^7 xdx = ?\ Ans: -\frac{1}{7}sinx^7 + \frac{3}{5}sinx^5 - sinx^3 + sinx + c$

Form-02ঃ $\int sin^n xdx , \int cos^n xdx$

* n জোড় পূর্ণ সংখ্যার জন্য $cos^n x,\ sin^n x$ কে গুনিতক কোনের ফাংশনে পরিণত করে সমাধান কর ।

$Example:$ $\int cos^n xdx = ?$ অথবা, $\int cos^n xdx$ এর লঘূকরন সূত্র নির্ণয় কর ।

$cos^n x = cos^{n-1}xcosx$

ধরি , $cos^{n-1}x = u \Rightarrow du = (n-1)cos^{n-2}xsinxdx$

এবং $v = \int cosx\,dx = sinx$

$I = \int cos^n xd = \int cos^{n-1}xcosxdx = \int udv = uv - \int vdu = cos^{n-1}xsinx + (n-1)\int sin^2xcos^{n-2}xdx = cos^{n-1}xsinx + (n-1)\int(1-cos^2x)cos^{n-2}xdx = cos^{n-1}xsinx + (n-1)\int cos^{n-2}xdx - (n-1)\int cos^n xdx$

পক্ষান্তর করে, $n\int cos^n xdx = cos^{n-1}xsinx + (n-1)\int cos^{n-2}xdx$

$\therefore I = \frac{1}{n}cos^{n-1}xsinx + \frac{(n-1)}{n}\int cos^{n-2}xdx$.......

$Example:$ $\int sin^8 xdx = ?$.

ধরি,$cosx + isinx = y\ \therefore cosx - isinx = \frac{1}{y}$, [De Moivre's theorem]

$cosnx + isinnx = y^{\,n} \therefore cosnx - isinnx = \frac{1}{y^n}$

$\therefore 2cosnx = y^{\,n} + \frac{1}{y^n}$ এবং $2isinnx = y^{\,n} - \frac{1}{y^n}$

$2^8 i^8 sin^8 x = \left(y - \frac{1}{y}\right)^8 = y^8 + 8_{C_1}y^7\left(-\frac{1}{y}\right) + 8_{C_2}y^6\left(-\frac{1}{y}\right)^2 + 8_{C_3}y^5\left(-\frac{1}{y}\right)^3$

$+8_{C_4}y^4\left(-\frac{1}{y}\right)^4+8_{C_5}y^3\left(-\frac{1}{y}\right)^5+8_{C_6}y^2\left(-\frac{1}{y}\right)^6+8_{C_7}y^1\left(-\frac{1}{y}\right)^7+\left(-\frac{1}{y}\right)^8$

$=\left(y^8+\frac{1}{y^8}\right)-8\left(y^6+\frac{1}{y^6}\right)+28\left(y^8+\frac{1}{y^8}\right)-56\left(y^2+\frac{1}{y^2}\right)+70$

$sin^8x=\frac{1}{2^7}(cos8x-8cos6x+28cos4x-56cos2x+35)$

$I=\int sin^8xdx=\frac{1}{128}\left(\frac{sin8x}{8}-\frac{8sin6x}{6}+\frac{28sin4x}{4}-\frac{56}{2}sin2x+35x\right)+c$

n→ অনেক বড় কোন সংখ্যা সেজন্য দ্য’মভারের উপপাদ্য ব্যাবহার করা হয়েছে।

Try yourself : (i) $\int sin^4xdx=?\ Ans:\ \frac{1}{8}\left(3x-2sinx+\frac{1}{4}sin4x\right)+c$

(ii) $\int cos^4xdx=?\ Ans:\ \frac{1}{8}\left(3x+\frac{1}{32}sinx+\frac{1}{4}sin4x\right)+c$

Form-03ঃ $\int sin^mxcos^nxdx=$ **?**

শর্ত ঃ (১) যখন m ও n উভয় বিজোড় ঃ sinx=u অথবা cosx=u ধরে অগ্রসর হবে ।

(২) যখন m জোড় ও n বিজোড় ঃ sinx=u ধরে অগ্রসর হবে ।

(৩) যখন m বিজোড় ও n জোড় ঃ cosx=u ধরে অগ্রসর হবে ।

(৪) যখন m ও n উভয় জোড়ঃ sin^mxcos^nx কে গুনিতক কোনের ত্রিকোণমিতিক ফাংশনে পরিণত করিয়া অগ্রসর হবে।

(৫) যদি m ,n কোন বাস্তব সংখ্যা এবং m+n একটি ঋনাত্বক জোড় সংখ্যা হয়, তবে tanx=u ধরে অগ্রসর হবে ।

$Example-01:\ \int sin^2xcos^5xdx=?$

$I=\int sin^2x(1-sin^2x)^2cosxdx=\int(sin^2x-2sin^4x+sin^6x)d(sinx)$

$=\frac{sin^3x}{3}-\frac{2sin^5x}{5}+\frac{sin^7x}{7}+c$ [∴dsinx= cosxdx]

$Example-02:\ \int sin^4xcos^2xdx=?$

ধরি,$cosx+isinx=y\ \therefore cosx-isinx=\frac{1}{y}$, [De Moivre's theorem]

$2cosnx=\ y+\frac{1}{y}$, $2sinx=y-\frac{1}{y}$

$cosnx+isinnx=y^n\ \therefore cosnx-isinnx=\frac{1}{y^n}$

$\therefore 2cosnx=y^n+\frac{1}{y^n}$ এবং $2isinnx=y^n-\frac{1}{y^n}$

$2^4i^4sin^4xcos^2x=\left(y-\frac{1}{y}\right)^4\left(y+\frac{1}{y}\right)^2=y^6+$

$6_{C_1}y^7\left(-\frac{1}{y}\right)+6_{C_2}y^6\left(-\frac{1}{y}\right)^2+6_{C_3}y^5\left(-\frac{1}{y}\right)^3$

$+6_{C_4}y^4\left(-\frac{1}{y}\right)^4+6_{C_5}y^3\left(-\frac{1}{y}\right)^5+\left(-\frac{1}{y}\right)^6$

$=\left(y^6+\frac{1}{y^6}\right)-2\left(y^4+\frac{1}{y^4}\right)-\left(y^2+\frac{1}{y^2}\right)+c$

$=2cos6x-2.2cos4x-2cos2x+4$

$sin^4xcos^2x=2^{-5}(cos6x-2cos4x-cos2x+2)$

$\int sin^4xcos^2xdx=\frac{1}{2^5}\left[\frac{sin6x}{6}-\frac{sin4x}{2}-\frac{sin2x}{2}\right]+c$

$Example - 03: \int \frac{sin^2x}{cos^6x}dx = ?$

এখানে, m + n = 2- 6 = - 4 ধরি, $tanx = u \Rightarrow sec^2xdx = du = d(tanx)$

$$I = \int tan^2x.sec^4xdx = \int tan^2x(1 + tan^2x)\,sec^2xdx$$

$= \int(tan^2x + tan^4x)d(tanx) = \frac{tan^3x}{3} + \frac{tan^5x}{5} + c$

$Example - 04: \int \frac{dx}{sin^{1/2}cos^{7/2}x} = ?$

এখানে, $m + n = \frac{-1}{2} + \frac{-7}{2} = -4$ ধরি, $tanx = u \Rightarrow sec^2xdx = du = d(tanx)$

$I = \int \frac{sec^4x}{tan^{1/2}}dx = \int \frac{(1+u^2)}{u^{1/2}}du = \int\left(u^{-1/2} + u^{3/2}\right)du = 2u^{1/2} + \frac{2}{5}u^{5/2} + c$

$I = 2tan^{1/2}x + \frac{2}{5}tan^{5/2}x + c$

Form-04ঃ $\int tan^nxdx$ ও $\int cot^nxdx$ এর সমাকলন নির্ণয় সংক্রান্ত সমস্যাবলী ঃ

$Example - 01: \int tan^5xdx = \int tan^3xtan^2xdx = \int tan^3x(sec^2x - 1)dx$

$= \int tan^3x\,sec^2xdx - \int tan^3xdx$

$\int tan^3xdx = \int tanx(sec^2x - 1)dx = \int tanxsec^2xdx - \int tanxdx$

ধরি, $tanx = u \Rightarrow sec^2xdx = du = d(tanx)$

$\int tan^5xdx = \int tan^3x\,sec^2xdx - \int tanxsec^2xdx - \int tanxdx$

$= \int tan^3x\,d(tanx) - \int tanxd(tanx) + ln|secx| + c.$

$= \frac{tan^4x}{4} - \frac{tan^2x}{2} + ln|secx| + c.$

Try yourself : $\int cot^6xdx = ?\ Ans: \frac{-1}{5}cot^5x + \frac{1}{3}cot^3x - cotx + c$

Form-05 ঃ $\int sec^nxdx$ ও $\int cosec^nxdx$ এর সমাকলন নির্ণয় সংক্রান্ত সমস্যাবলী ঃ

শর্ত ঃ (১) যখন n ধনাত্মক জোর পূর্ণ সংখ্যা তখন sec^nx কে $tanx$ এবং $cosec^nx$ কে $cotx$ এর ফাংশনে পরিণত করে অগ্রসর হবে ।

(২) যখন n ধনাত্মক বিজোর পূর্ণ সংখ্যা তখন অংশক্রমে সমাকলন পদ্ধতিতে অগ্রসর হবে ।

$Example - 01: \int sec^5xdx = ?$

$Solve :: \int sec^3xsec^2xdx = sec^3xtanx - 3\int sec^2xsecxtanxtanxdx$

$= sec^3xtanx - 3\int sec^3x(sec^2x - 1)dx$

$= sec^3xtanx - 3\int sec^5xdx + 3\int sec^3xdx$

$= sec^3xtanx - 3I + 3I_1$

$I + 3I = sec^3xtanx + 3I_1 \Rightarrow I = \frac{1}{4}sec^3xtanx + \frac{3}{4}I_1$

$I_1 = \int sec^3xdx = \int secxsec^2xdx = secxtanx - \int secx\,tanxtanxdx$

$= secxtanx - \int secx\,(sec^2x - 1)dx = secxtanx - \int sec^3xdx +$

$\int secx\,dx = secxtanx - I_1 + ln(secx + tanx)$

$2I_1 = secxtanx + ln(secx + tanx)\ \therefore I_1 = \frac{1}{2}\{secxtanx + ln|secx + tanx|\}$

$I = \frac{1}{4}sec^3xtanx + \frac{3}{8}\{secxtanx + ln|secx + tanx|\} + c$

Try yourself : $\int cosec^5xdx = ?$

$Ans : \frac{-1}{4} cosec^3 x cot x - \frac{3}{8}\left\{cosecxcotx - ln\left|tan\frac{x}{2}\right|\right\} + c$

Form-06 : $\int \sqrt{tanx}dx = ?$

$Example - 01: \int \sqrt{tanx}dx = ?$

ধরি, $\sqrt{tanx} = u \Rightarrow tanx = u^2 \Rightarrow sec^2xdx = 2udu$

$\Rightarrow (1 + tan^2x)dx = 2udu \Rightarrow dx = \frac{2udu}{1+u^4}$

$$I = \int \frac{2u^2du}{1+u^4} = 2\int \frac{du}{\frac{1}{u^2}+u^2} = \int \frac{\left(1-\frac{1}{u^2}\right)+\left(1+\frac{1}{u^2}\right)}{\frac{1}{u^2}+u^2}du$$

$$= \int \frac{\left(1-\frac{1}{u^2}\right)}{\left(u+\frac{1}{u}\right)^2-\left(\sqrt{2}\right)^2}du + \int \frac{\left(1+\frac{1}{u^2}\right)}{\left(u-\frac{1}{u}\right)^2+\left(\sqrt{2}\right)^2}du$$

$$= \int \frac{d\left(u+\frac{1}{u}\right)}{\left(u+\frac{1}{u}\right)^2-\left(\sqrt{2}\right)^2} + \int \frac{d\left(u-\frac{1}{u}\right)}{\left(u-\frac{1}{u}\right)^2+\left(\sqrt{2}\right)^2}$$

$$= \frac{1}{2\sqrt{2}} ln\left|\frac{u+\frac{1}{u}-\sqrt{2}}{u+\frac{1}{u}+\sqrt{2}}\right| + \frac{1}{\sqrt{2}}\tan^{-1}\frac{\left(u-\frac{1}{u}\right)}{\sqrt{2}} + c$$

$$= \frac{1}{2\sqrt{2}} ln\left|\frac{tanx-\sqrt{2}tanx+1}{tanx+\sqrt{2}tanx+1}\right| + \frac{1}{\sqrt{2}}\tan^{-1}\frac{(tanx-1)}{\sqrt{2}} + c$$

$Example - 02: \int \sqrt{cosecx}dx = ?$

ধরি, $\sqrt{cosecx} = u \Rightarrow cosecx = u^2 \Rightarrow -cosecx.cotxdx = 2udu \Rightarrow \sqrt{cosecx}dx = -\frac{2du}{\sqrt{u^4-1}}$

$I = \int -\frac{2du}{\sqrt{u^4-1}}$ ধরি, $u^4 - 1 = z^2u^2 \Rightarrow 4u^3du = 2z^2udu + 2u^2zdz$

$\Rightarrow \frac{du}{zx} = \frac{u^2}{u^4-1}, I = \int \frac{1}{z^2}dz = \frac{1}{z} + c = \frac{cosec^2x-1}{\sqrt{cosecx}} + c = cosec^{3/2}x - \sqrt{sinx} + c$

<u>TRY YOURLESLF :</u>

1. (i) $\int sin^3 x\, cos^2 xd\, x.$ (ii) $\int sin^3 x\, cos^4 xd\, x.$

(iii) $\int sin^5 x\, cos^4 xd\, x.$ (iv) $\int sin^3 x\, cos^2 2xd\, x.$

2. (i) $\int \frac{sin^2 x}{cos^6 x}dx$ (ii) $\int \frac{sin^4 x}{cos^8 x}dx$ (iii) $\int \frac{sin^3 x}{cos^9 x}dx$ (iv) $\frac{sec^4 x}{\sqrt{\tan x}}dx$

(v) $\int \frac{dx}{sin^{1/2}xcos^{7/2}x}$ (vi) $\int \frac{dx}{\sqrt[3]{\cos x}\sqrt[3]{sin^5 x}}$ (vii) $\int \frac{dx}{sin^3 x\, cos^5 x}$

3. (i) $\int cos^5 x\, dx$ (ii) $\int tan^6 x\, dx$ (iii) $\int sec^6 x\, dx$
(iv) $\int sec^8 x\, dx$

4. (i) $\int tan^2 x sec^4 x\, dx$ (ii) $\int tan^5 x\, sec^4 x\, dx$ (iii) $\int \sqrt{\tan x}\, sec^4 x\, dx$

(iv) $\int \tan x\, sec^{3/2}\, x\, dx$ (v) $\int tan^3\, 2x\, sec2x\, dx$ (vi) $\int tan^3\, x\, \sqrt{secx}\, dx$

(vii) $\int\left(\sqrt{tanx}+\sqrt{cotx}\right)dx$ (viii) $\int \frac{dx}{\sqrt{x}-\sqrt{x-1}}\,[let, x = sin^2\theta]$

(ix) $\int \frac{sin2x}{\sin x+\cos x}dx$ **(x)** $\int x\sqrt{\frac{(a-x)}{(a+x)}}\,dx$

Ans: 1. (i) $\frac{1}{5}cos^5x-\frac{1}{3}cos^2\,x+c$ (ii) $\frac{1}{7}cos^7\,x-\frac{1}{5}cos^5\,x+c$
(iii) $\frac{2}{7}cos^7\,x-\frac{1}{5}cos^5\,x-\frac{1}{9}cos^9x+c$
(iv) $\frac{4}{7}cos^7x-\frac{8}{5}cos^5x+\frac{5}{3}cos^3x-\cos x+c$

2. (i) $\frac{1}{3}tan\,^3x+\frac{1}{5}tan^5x+c$ (ii) $\frac{1}{5}tan^5x+\frac{1}{7}tan^7x+c$
(iii) $\frac{1}{4}tan^4\,x+\frac{1}{3}tan^6x+\frac{1}{8}tan^8x+c$
(iv) $2\sqrt{(tanx)}+\frac{2}{5}tan^{\frac{5}{2}}x+c$ (v) $2\sqrt{(tanx)}+\frac{2}{5}(tanx)^{\frac{5}{2}}+c$ (vi) $-\frac{3}{2}(\cot x)^{2/3}+c$
(vii) $\frac{-1}{2tan^2x}+3ln\tan x+\frac{3}{2}tan^2x+\frac{1}{4}tan^4x+c$
3. (i) $\frac{-1}{4}cot^4x+\frac{1}{2}cot^2x+ln|sinx|+c$ (ii) $\frac{tan^5x}{5}-\frac{tan^3x}{3}+tanx-x+c$
(iii) $\frac{tan^5x}{5}+\frac{2tan^3x}{3}+tanx+c$
(iv) $\frac{tan^7x}{7}+\frac{3tan^5x}{5}+tan^3x+tanx+c$
4. (i) $\frac{tan^5x}{5}+\frac{tan^3x}{3}+c$ (ii) $\frac{tan^8x}{8}+\frac{tan^6x}{6}+c$ (iii) $\frac{2tan^{3/2}x}{3}+\frac{2tan^{7/2}x}{7}+c$
(iv) $\frac{2sec^{3/2}x}{3}+c$ (v) $\frac{sec^32x}{6}-\frac{sec2x}{2}+c$ (vi) $2\sqrt{(secx)}+\frac{2}{5}(sec)^{\frac{5}{2}}x+c$
(vii) $\sqrt{2}\sin^{-1}(\sin x+\cos x)+c$
(viii) $-\sqrt{x}+\sqrt{x-1}-\frac{1}{2}ln\left|tan\left(\frac{1}{2}\sin^{-1}\sqrt{x}\right)+\frac{\pi}{8}\right|+c$
(ix) $\sin x-\cos x-\frac{1}{2}ln\left|tan\left(\frac{\theta}{2}+\frac{\pi}{8}\right)\right|+c$ (x) $\frac{(x-a)\sqrt{a^2-x^2}}{2}-\frac{1}{2}a^2\sin^{-1}x+c$

Type-07: আংশিক ভগ্নাংশের সহায়্যে সমাকলন ঃ
$\int\frac{N(x)}{D(x)}\,dx$ এ হরের প্রকৃতি অনুযায়ী আংশিক ভগ্নাংশ নির্ণয়ের বিভিন্ন পদ্ধতি ঃ

প্রকৃত ভগ্নাংশের জন্য ঃ

Form-01: $\int \frac{px^2+qx+r}{(ax+\alpha)(bx+\beta)(cx+\gamma)}dx$, $\int \frac{px^2+qx+r}{(ax+b)(cx^2+dx+e)(cx+\gamma)^2}dx$

$$\frac{px^2+qx+r}{(ax+\alpha)(bx+\beta)(cx+\gamma)} = \frac{A}{ax+\alpha} + \frac{B}{bx+\beta} + \frac{C}{cx+\gamma} \quad [A,B,C=?]$$

$$\frac{px^2+qx+r}{(ax+b)(cx^2+dx+e)} = \frac{A}{ax+b} + \frac{Bx+C}{cx^2+dx+e} \quad [A,B,C=?]$$

$Example - 01$: $\int \frac{x^2+x-1}{x^3+x^2-6x}dx$

সমাধান ঃ এখানে $x^3+x^2-6x = x(x^2+x-6) = x(x-2)(x+3)$

ধরি, $\frac{x^2+x-1}{x(x-2)(x+3)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+3}$ উভয়পক্ষকে $x(x-2)(x+3)$দ্বারা গুণ করে পাই,

$x^2+x-1 = A(x-2)(x+3) + Bx(x+3) + Cx(x-2)$....................(i)

(i) অভেদে পার্যয়ক্রমে $x=0,\ x=2, x=-3$ বসিয়ে পাই,

$-1 = A(0-2)(0+3) \Rightarrow -1 = -6A \Rightarrow A\frac{1}{6} \Rightarrow 4+2-1 = B.2(2+3)$

$\Rightarrow 5 = 10B \Rightarrow B = \frac{1}{2}$ এবং $9-3-1 = C(-3)(-3-2) \Rightarrow 5 = 15C \Rightarrow C = \frac{1}{3}$

এখন$\frac{x^2+x-1}{x(x-2)(x+3)} = \frac{1}{6}.\frac{1}{x}.+\frac{1}{2}.\frac{1}{x-2} + \frac{1}{3}.\frac{1}{x-3}$ $\therefore \int \frac{x^2+x-1}{x^3+x^2-6x}dx = \frac{1}{6}\int\frac{dx}{x} + \frac{1}{2}\int\frac{dx}{x-2} + \frac{1}{3}\int\frac{dx}{x+3}$

$= \frac{1}{6}\ln|x| + \frac{1}{2}\ln|x-2| + \frac{1}{3}\ln|x+3| + c$

$$\frac{x^2+x-1}{x(x-2)(x+3)} = \frac{0+0-1}{x(0-2)(0+3)} + \frac{2^2+2-1}{2(x-2)(2+3)} + \frac{(-3)^2+(-3)-1}{(-3)(-3-2)(x+3)}$$

$$= \frac{1}{6x} + \frac{1}{2(x-2)} + \frac{1}{3(x+3)}$$

$Example - 02$: $\int \frac{2x^2+x+17}{(x-1)(x^2+3x-3)}dx = ?$

$Solve$: $\frac{2x^2+x+17}{(x-1)(x^2+3x-3)} = \frac{2x^2+x+17}{(x-1)(x+3)(x-1)} = \frac{2x^2+x+17}{(x+3)(x-1)^2} = \frac{A}{x+3} + \frac{B}{x-1} + \frac{Cx+D}{(x-1)^2}$

$2x^2+x+17 = A(x-1)^2 + B(x+3)(x-1) + (Cx+D)(x+3)$

$= A(x^2-2x+1) + B(x^2+3x-3) + C(x^2+3x) + D(x+3)$

$= (A+B+C)x^2 + (-2A+3B+3C+D)x + A-3B+3D$

$x^2,\ x$ এর সহগ এবং x ধ্রুবপদ সমীকৃত করে , $A+B+C = 2$,

$-2A+3B+3C+D = 1$, এবং $A-3B+3D = 17$

$B=0, A=2, C=0, D=5$

$\int \frac{2x^2+x+17}{(x-1)(x^2+3x-3)}dx = \int\frac{2}{x+3}dx + \int\frac{5}{(x-1)^2}dx = 2ln|x+3| - \frac{5}{x-1} + c$

$Example - 03$: $\int \frac{2x^2+x+17}{(x-1)(x^3+1)}dx = ?$

$\frac{2x^2+x+17}{(x-1)(x^3+1)} = \frac{2x^2+x+17}{(x-1)(x+1)(x^2-x+1)} = \frac{2x^2+x+17}{(x^2-1)(x^2-x+1)}$

$\frac{2x^2+x+17}{(x^2-1)(x^2-x+1)} = \frac{Ax+B}{(x^2-1)} + \frac{Cx+D}{(x^2-x+1)}$

$\Rightarrow 2x^2+x+17 = (Ax+B)(x^2-x+1) + (Cx+D)(x^2-1)$

$= A(x^3-x^2+x) + C(x^3-x) + B(x^2-x+1) + D(x^2-1)$

$= (A+C)x^3 + (-A+B-C+D)x^2 + (A-B-C)x + B-D$

$x^3, x^2,\ x$ এর সহগ এবং x ধ্রুবপদ সমীকৃত করে , $A+C = 0$,

$-A+B-C+D = 2$, $A-B-C = 1, B-D = 17$

$A = \frac{17}{4}$, $B = \frac{19}{2}$, $C = \frac{-17}{4}$, $D = \frac{-15}{2}$,

$$\frac{2x^2+x+17}{(x^2-1)(x^2-x+1)} = \frac{Ax+B}{(x^2-1)} + \frac{Cx+D}{(x^2-x+1)} = \frac{\frac{17}{4}x+\frac{19}{2}}{(x^2-1)} + \frac{\frac{-17}{4}x+\frac{-15}{2}}{(x^2-x+1)}$$

$$= \frac{1}{4} \times \frac{17x+38}{(x^2-1)} - \frac{1}{4} \times \frac{17x-30}{(x^2-x+1)}$$

$$\int \frac{2x^2+x+17}{(x-1)(x^3+1)}dx = \frac{1}{4}\int \frac{17x+38}{(x^2-1)}\, dx - \frac{1}{4}\int \frac{17x-30}{(x^2-x+1)}dx$$

$$= \frac{17}{8}\int \frac{2x}{(x^2-1)}dx + \frac{38}{4}\int \frac{dx}{(x^2-1)} - \frac{17}{8}\int \frac{2x-1}{(x^2-x+1)}dx + \frac{15}{2}\int \frac{dx}{(x^2-x+1)} - \frac{17}{8}\int \frac{1}{(x^2-x+1)}dx$$

$$= \frac{17}{8} ln(x^2-1) + \frac{38}{4} \times \frac{1}{2.1} ln\left|\frac{x-1}{x+1}\right| - \frac{17}{8} ln(x^2-x+1) + \frac{43}{8} \times \frac{1}{\frac{\sqrt{3}}{2}} \tan^{-1}\frac{2x-1}{\sqrt{3}} + c$$

$$= \frac{17}{8} ln\left|\frac{(x^2-1)}{(x^2-x+1)}\right| + \frac{19}{8}\ ln\left|\frac{x-1}{x+1}\right| + \frac{43}{4\sqrt{3}}\tan^{-1}\frac{2x-1}{\sqrt{3}} + c\ ; [x > 1]$$

$if\ x < 1\ then\ ln\left|\frac{x-1}{x+1}\right|\ the\ term\ reduces\ to\ l\,n\left|\frac{1-x}{1+x}\right|$

$Example - 04$: $\int \frac{(1-\cos\theta)d\theta}{\cos\theta\,(1+\cos\theta)}$ ইন্টিগ্রাল নির্ণয় কর।

সমাধানঃ $\frac{1-\cos\theta}{\cos\theta\,(1+\cos\theta)} = \frac{A}{\cos\theta} + \frac{B}{1+\cos\theta}$ উভয় পক্ষকে $\cos\theta\,(1+\cos\theta)$দ্বারা গুণ করে পাই ,

$1 - \cos\theta = A(1+\cos\theta) + B\cos\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(i)$

(i) অভেদে পার্যয়ক্রমে $\cos\theta = 0 , \cos\theta = -1$ বসিয়ে পাই,

$1 - 0 = A(1+0) \Longrightarrow A = 1$ এবং $1 + 1 = B(-1) \Longrightarrow B = -2$ এখন

$\frac{1-\cos\theta}{\cos\theta\,(1+\cos\theta)} = \frac{1}{\cos\theta} + \frac{2}{1+\cos\theta}$ $\quad \therefore \int \frac{1-\cos\theta}{\cos\theta\,(1+\cos\theta)} d\theta = \int \frac{d\theta}{\cos\theta} - 2\int \frac{d\theta}{1+\cos\theta}$

$$= \int \frac{d\theta}{\cos\theta} - 2\int \frac{d\theta}{2\cos^2\frac{\theta}{2}} = \int \sec\theta\, d\theta - \int \sec^2\left(\frac{\theta}{2}\right) d\theta$$

$$= \ln(\sec\theta + \tan\theta) - 2\tan\frac{\theta}{2} + c\ .$$

$Example - 05$: $\int \frac{dx}{x^2(x^2+1)^2}$

সমাধান ঃ$\frac{1}{x^2(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} + \frac{Ex+F}{(x^2+1)^2}$

$\Longrightarrow Ax(x^2+1)^2 + B(x^2+1)^2 + (Cx+D)x^2(x^2+1)^2 + (Ex+F)x^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots (i)$

(i) অভেদে হইতে x^5, x^4, x^3, x^2, x সহগ এবং ধ্রুবকপদ সমকীকৃত করিয়া পাই ,

$A + C = 0, B + D = 0, 2A + C + E = 0,\ \ 2B + D + F = 0, A = 0, B = 1$

উপরোক্ত সমীকরণগুলো সমাধান করে পাই $C = 0, D = -1, E = 0, F = -1$

$\int \frac{dx}{x^2(x^2+1)^2} = \int \frac{dx}{x^2} - \int \frac{dx}{x^2+1} - \int \frac{dx}{(x^2+1)^2}$ এখন $\int \frac{dx}{(x^2+1)^2}$ এর জন্য ধরি, $x = \tan\theta \quad \therefore dx = \sec^2\theta\, d\theta$

$\therefore \int \frac{dx}{(x^2+1)^2} = \int \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2} = \int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^2} = \int \cos^2\theta\, d\theta = \frac{1}{2}\int(1+\cos 2\theta)d\theta$

$$= \frac{1}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + c = \frac{1}{2}\theta + \frac{1}{4}\frac{2\tan\theta}{1+\tan^2\theta} + c = \frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)} + c\ [\because \theta = \tan^{-1}x]$$

$$\int \frac{dx}{x^2(x^2+1)^2} = -\frac{1}{x} - \tan^{-1}x - \frac{1}{2}\tan^{-1}x - \frac{x}{2(1+x^2)} + c = -\frac{1}{x} - \frac{3}{2}\tan^{-1}x - \frac{x}{2(1+x^2)} + c.$$

$Example - 06$: $\int \frac{(x-1)(x-5)}{(x-2)(x-4)}dx$

সমাধান ঃ ধরি,

$\frac{(x-1)(x-5)}{(x-2)(x-4)} = 1 + \frac{A}{x-2} + \frac{B}{x-4} \Longrightarrow (x-1)(x-5) = (x-2)(x-4) + A(x-4) + B(x-2) \ldots (i)$

(i) অভেদে পার্যয়ক্রমে $x = 2,\ x = 4,$ বসিয়ে পাই,

$(2-1)(2-5) = A(2-4) \Longrightarrow A = \frac{3}{2}$ এবং $(4-1)(4-5) = B(4-2) \Rightarrow B = -\frac{3}{2}$

$$\therefore \int \frac{(x-1)(x-5)}{(x-2)(x-4)} dx = \int dx + \frac{3}{2}\int \frac{dx}{x-2} - \frac{3}{2}\int \frac{dx}{x-4} = x + \frac{3}{2}\ln|x-2| - \frac{3}{2}\ln|x-4| + c$$

$$= x + \frac{3}{2}\ln\left|\frac{x-2}{x-4}\right| + c$$

$Example - 07: \int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$

সমাধান ঃ $\int \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)} = \int \left\{1 + \frac{(-a^2)^2}{(x^2+a^2)(-a^2+b^2)} + \frac{(-b^2)^2}{(-b^2+a^2)(x^2+b^2)}\right\} dx$

$$= \int dx - \frac{a^4}{a^2-b^2}\int \frac{dx}{x^2+a^2} + \frac{a^4}{a^2-b^2}\int \frac{dx}{x^2+b^2} = x - \frac{a^4}{a^2-b^2} \cdot \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + \frac{a^4}{a^2-b^2} \cdot \frac{1}{b}\tan^{-1}\left(\frac{x}{b}\right) + c$$

$$= x - \frac{a^3}{a^2-b^2}\tan^{-1}\left(\frac{x}{a}\right) + \frac{b^3}{a^2-b^2}\tan^{-1}\left(\frac{x}{b}\right) + c$$

$Example - 08: \int \frac{dx}{\sin 2x + \sin x}$

ধরি, $I = \int \frac{dx}{\sin 2x+\sin x} = \int \frac{dx}{2\sin x\cos x+\sin x} = \int \frac{dx}{\sin x(2\cos x+1)} = \int \frac{\sin x\, dx}{\sin^2 x(2\cos x+1)}$

$$= \int \frac{\sin x\, dx}{(1-\cos^2 x)(2\cos x+1)} = \int \frac{\sin x\, dx}{(1-\cos x)(1+\cos x)(2\cos x+1)}$$

এবং $\cos x = u \therefore -\sin x\, dx = du \Longrightarrow \sin x\, du = -du \ \therefore I = \int \frac{du}{(1-u)(1+u)(2u+1)}$

ধরি, $\frac{1}{(1-u)(1+u)(2u+1)} = \frac{A}{1-u} + \frac{B}{1+u} + \frac{c}{2u+1}$

$\Longrightarrow 1 = A(1+u)(2u+1) + B(1-u)(2u+1) + C(1-u)(1+u)$

উপরোক্ত অভেদে পর্যায়ক্রমে $u = 1,\ u = -1,\ u = -\frac{1}{2}$ বসিয়ে পাই

$A = \frac{1}{6},\ B = -\frac{1}{2}$ এবং $C = -\frac{4}{3} \therefore \frac{1}{(1-u)(1+u)(2u+1)} = \frac{\frac{1}{6}}{1-u} + \frac{-\frac{1}{2}}{1+u} + \frac{\frac{4}{3}}{2u+1}$

$$\therefore I = -\int \left\{\frac{1}{6} \cdot \frac{1}{1-u} - \frac{1}{2} \cdot \frac{1}{1+u} + \frac{4}{3} \cdot \frac{1}{2u+1}\right\} du = -\frac{1}{6}\int \frac{du}{1-u} + \frac{1}{2}\int \frac{du}{1+u} - \frac{2}{3}\int \frac{2du}{2u+1}$$

$$= \frac{1}{6}\ln|1-u| + \frac{1}{2}\ln|1+u| - \frac{2}{3}\ln|2u+1| + c = \frac{1}{6}\ln|1-u| + \frac{1}{6}\ln(1+u)^3 - \frac{1}{6}\ln(2u+1)^4$$

$$= \frac{1}{6}\ln\frac{(1-u)(1+u)^3}{(2u+1)^4} + c = \frac{1}{6}\ln\frac{(1-u^2)(1-u)^2}{(2u+1)^4} + c = \frac{1}{6}\ln\frac{(1-\cos^2 x)(1+\cos^2 x)}{(2\cos x+1)^4} + c.$$

$$= \frac{1}{6}\ln\frac{\sin^2 x(1+\cos x)^2}{(2\cos x+1)^4} + c = \frac{1}{3}\ln\frac{\sin x(1+\cos x)}{(2\cos x+1)^2} + c$$

TRYYOURSELF:

FIND THE FOLLOWING INDEFINITE INTEGRALS:

1. (i) $\int \frac{dx}{(x-\alpha)(x-\beta)}$ **(ii)** $\int \frac{(x-1)dx}{(x-2)(x-3)}$ **(iii)** $\int \frac{dx}{(x+1)(x+3)(x+5)}$ **(iv)** $\frac{dx}{(x-\alpha)(x-\beta)(x-\gamma)}$

2. (i) $\int \frac{(x+1)dx}{3+2x-x^2}$ **(ii)** $\int \frac{3x}{x^2-x-2}dx$ **(iii)** $\int \frac{dx}{x^3-x^2-9x+9}$ **(iv)** $\int \frac{7x+4}{x^3-4x}dx$

3. (i) $\int \frac{dx}{\sin x\,(1+\sin x)}$ **(ii)** $\int \frac{\cos x\,dx}{(a+\sin x)(b-\sin x)}$ **(iii)** $\int \frac{\cos x\,dx}{(2+\sin x)(1+\sin x)}$

(iv) $\int \frac{\cos x\,dx}{\sin 2x-\sin x}$ **4. (i)** $\int \frac{xdx}{(x+1)(x+2)^2}$ **(ii)** $\int \frac{x^2dx}{(x+1)(x+2)^2}$ **(iii)** $\int \frac{(x+1)dx}{(x-3)(x-1)^2}$

(iv) $\int \frac{(2x^2-1)dx}{(x+1)^2(x-2)}$ **(v)** $\int \frac{x^2+1}{(x^2-1)^2}dx$ **5. (i)** $\int \frac{xdx}{(x-1)(x^2+1)}$ **(ii)** $\int \frac{(x^2+x)dx}{(x-1)(x^2+1)}$

(iii) $\int \frac{(x-1)dx}{(x+1)(x^2+1)}$ **(iv)** $\int \frac{\sin x\,dx}{\cos x(1+\cos^2 x)}$ **6. (i)** $\int \frac{x^2dx}{x^4-x^2-2}$ **(ii)** $\int \frac{dx}{x^3+x^2+x+1}$

(iii) $\int \frac{xdx}{x^3+x^2+x+1}$ **(iv)** $\int \frac{2x\,dx}{(x-1)(x^2+5)}$ **(v)** $\int \frac{x^2+x-2}{3x^3-x^2+3x-1}$

7. (i) $\int \frac{dx}{x^2(a^2+x^2)}$ **(ii)** $\int \frac{dx}{(x-1)^2(x^2+4)}$ **(iii)** $\int \frac{dx}{x(x-1)^2(x^2+1)}$ **(iv)** $\int \frac{(2x^2+x+17)}{(x-1)(x^2+2x+3)}$

8. (i) $\int \frac{dx}{a^4-x^4}$ **(ii)** $\int \frac{x^2dx}{(x^2+a^2)(x^2+b^2)}$

9. (i) $\int \frac{3x^4+3x^3-5x^2+x-1}{x^2+x-2}$ **(ii)** $\int \frac{x^3-3x^2+2x-3}{x^2+1}$ **10. (i)** $\int \frac{x^2+x+1}{(x^2+1)^2}dx$

(ii) $\int \frac{2x^2+3}{(x^2+1)^2}dx$ **11. (i)** $\int \frac{dx}{(x^2+4x+5)^2}$ **(ii)** $\int \frac{dx}{(x^2+2x+3)^2}$

Ans sheet :

1. (i) $\frac{1}{\alpha-\beta}\ln\left|\frac{x-\alpha}{x-\beta}\right|+c$ **(ii)** $\ln\left|\frac{(x-3)^2}{x-2}\right|+c$ **(iii)** $\frac{1}{8}\ln\left|\frac{(x-1)(x+5)}{(x+3)^2}\right|+c$

(iv) $\frac{\alpha^2}{(\alpha-\beta)(\alpha-\gamma)}\ln|x-\alpha|+\frac{\beta^2}{(\beta-\gamma)(\beta-\alpha)}\ln|x-\beta|+\frac{\gamma^2}{(\gamma-\alpha)(\gamma-\beta)}\ln|x-\gamma|+c$

2. (i) $-\ln|3-x|+c$ (ii) $2\ln|x-2|+\ln|x+1|+c$

(iii) $-\frac{1}{8}\ln|x-1|+\frac{1}{12}\ln|x-3|+\frac{1}{24}\ln|x+3|+c$

(iv) $\frac{9}{4}\ln|x-2|-\frac{5}{4}\ln|x+2|-\ln|x|+c$

3. (i) $\ln\left|\tan\frac{x}{2}\right|-\tan x+\sec x+c$ **(ii)** $\frac{1}{a+b}\ln\left|\frac{a+\sin x}{b-\sin x}\right|+c$

(iii) $\ln\left|\frac{1+\sin u}{2+\sin u}\right|+c$ **(iv)** $\frac{1}{3}\ln\frac{\sin x(1-\cos x)}{(2\cos x-1)^2}+c$ **4. (i)** $\ln\left|\frac{x+2}{x+1}\right|-\frac{2}{x+2}+c$

(ii) $\ln(x+1)+\frac{4}{x+2}+c$ **(iii)** $\ln\left|\frac{x-3}{x-1}\right|+\frac{1}{x-1}+c$

(iv) $\frac{11}{9}\ln|x+1|+\frac{1}{3(x+1)}+\frac{7}{9}\ln|x-2|+c$

(v) $-\frac{1}{2(x-1)}-\frac{1}{2(x+1)}+\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right|+c$

5. $(\boldsymbol{i})$ $\frac{1}{2}\ln|x-1|-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\tan^{-1}x+c$ $(\boldsymbol{ii})$ $\ln|x-1|+\tan^{-1}x+c$

$(\boldsymbol{iii})$ $\frac{1}{2}\ln(x^2+1)-\ln|x+1|+c$ $(\boldsymbol{iv})$ $-\ln(\cos x)+\frac{1}{2}\ln(1+\cos^2 x)+c$

6. $(\boldsymbol{i})$ $\frac{1}{3\sqrt{2}}\ln\left|\frac{x-\sqrt{2}}{x+\sqrt{2}}\right|+\frac{1}{3}\tan^{-1}x+c$ $(\boldsymbol{ii})$ $\frac{1}{2}\ln|x+1|-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\tan^{-1}x+c$

(iii) $\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\tan^{-1}x-\frac{1}{2}\ln|x+1|+c$

(iv) $\frac{1}{3}\ln|x-1|-\frac{1}{6}\ln(x^2+5)+\frac{\sqrt{5}}{3}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right)+c$

(v) $-\frac{7}{15}\ln|3x-1|+\frac{2}{5}\ln(x^2+1)+\frac{3}{5}\tan^{-1}x+c$

7.(i) $-\frac{1}{a^2x}-\frac{1}{a^3}tan^{-1}\left(\frac{x}{a}\right)+c$**(ii)** $-\frac{2}{25}ln|x-1|-\frac{1}{5(x-1)}+\frac{1}{25}ln(x^2+4)-\frac{3}{50}tan^{-1}\left(\frac{x}{2}\right)$

(iii) $ln|x|-ln|x-1|-\frac{1}{2(x-1)}+\frac{1}{2}tan^{-1}x+c$ **(iv)** $-\frac{5}{x-1}+2\,ln|x+3|+c$

8. (i) $\frac{1}{4a^3}ln\left|\frac{a+x}{a-x}\right|+\frac{1}{2a^3}tan^{-1}\left|\frac{x}{a}\right|+c$ **(ii)** $\frac{a}{a^2-b^2}tan^{-1}\frac{x}{a}-\frac{b}{a^2-b^2}tan^{-1}\frac{x}{b}+c$

9. (i) $x^3+x+\frac{1}{3}ln\left|\frac{x-1}{x+2}\right|+c$ **(ii)** $\frac{1}{2}x^2-3x+\frac{1}{2}ln(x^2+1)+c$

10. (i) $tan^{-1}x-\frac{1}{2(1+x^2)}+c$ **(ii)** $\frac{5}{2}tan^{-1}x+\frac{x}{2(1+x^2)}+c$

11. (i) $\frac{1}{2}tan^{-1}(x+2)+\frac{(x+2)}{2(x^2+4x+5)}+c$ **(ii)** $\frac{\sqrt{2}}{8}tan^{-1}\left(\frac{x+1}{\sqrt{2}}\right)+\frac{(x+1)}{4(x^2+2x+3)}+c$

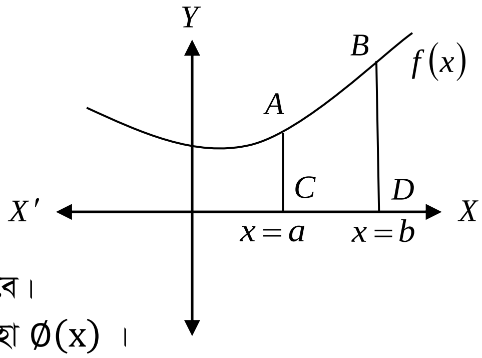
নির্দিষ্ট ইন্টিগ্রাল

➔ নির্দিষ্ট ইন্টিগ্রাল (Difinite Integral)ঃ

জ্যামিতিক, প্রয়োজনে এবং ইন্টিগ্রাল নির্ণয় প্রক্রিয়ার প্রয়োগকালে অনেক সময় স্বাধীন চলকের দুইটি মানের জন্য একটি ফাংশনের ইন্টিগ্রালের পার্থক্য নির্ণয়ের প্রয়োজন হয়। ধরি, স্বাধীন চলক x এর দুইটির মান a ও b এবং f (x) একটি ইন্টিগ্রেশন যোগ্য ফাংশন, যাহার $\int f(x)dx = \emptyset(x)$ অর্থাৎ f (x) এর অনির্দিষ্ট ইন্টিগ্রাল $\emptyset(x)$.এখন $\emptyset(a)$ এবং $\emptyset(b)$ যথাক্রমে x = a এবং x = b বিন্দুতে $\emptyset(x)$ অর্থাৎ $\int f(x)dx$ এর দুইটি মান । এই পার্থক্য $[\emptyset(b)-\emptyset(a)]$ কে [a,b] ব্যবধিতে f (x) এর নির্দিষ্ট ইন্টিগ্রাল বলা হয় । ইহা বুঝাবার সংক্ষিপ্ত প্রতীক নিম্নরূপ ঃ $\int_b^a f(x)dx = [\emptyset(x)]_b^a = \emptyset(a)-\emptyset(b)$, এখানে a নির্দিষ্ট ইন্টিগ্রালের নিম্নমীমা এবং b উহার উর্ধ্বসীমা নামে পরিচিত।

নিদির্ষ্ট ইন্টিগ্রাল বলা হয় কেন ?

যদি f (x) এর অনির্দিষ্ট ইন্টিগ্রাল $\emptyset(x)+c$ হয়, তবে $\int_b^a f(x)dx = [\emptyset(x)+c]_b^a = \emptyset(b)+c-\emptyset(a)-c = \emptyset(b)-\emptyset(a)$ এখানে দেখা যাচ্ছে যে, অনির্দিষ্ট ইন্টিগ্রালের মত নির্দিষ্ট কোন ধ্রুবক পদ যোগ করা হয়নি। ইহাতে c অপসারিত হয়েছে। সুতরাং $\int_b^a f(x)dx = \emptyset(a)-\emptyset(b)$ নির্দিষ্ট বলিয়া ইহাকে নির্দিষ্ট ইন্টিগ্রাল বলা হয়।



$\int_b^a f(x)dx$ এর জ্যামিতিক তাৎপর্য ঃ

y = 0 বা, x – অক্ষ, x = a, x = b সরলরেখ তিনিট এবং

y = f(x) বক্ররেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফলকে

$\int_b^a f(x)dx$ দ্বারা প্রকাশ করা যায়

সুতরাং ABCD ক্ষেত্রের ক্ষেত্রফল = $\int_b^a f(x)dx$

$\int f(x)dx$ এর মান নির্ণয় করতে নিম্নলিখিতভাবে অগ্রসর হইতে হবে।

(i) প্রথমে $\int f(x)dx$ অনির্দিষ্ট ইন্টিগ্রাল নির্ণয় করতে হইবে, ধরি উহা $\emptyset(x)$ ।

(ii) এখন $\emptyset(x)$ এ x এর পরিবর্তে নির্দিষ্ট ইন্টিগ্রালের উর্ধ্বমীমা b বসাইয়া $\emptyset(b)$ এবং x এর পরিবর্তে নিম্নসীমা a বসাইয়া $\emptyset(a)$ নির্ণয় করতে হবে।

(iii) শেষে $\emptyset(a)$ বিয়োগ করলেই $\int_b^a f(x)dx$ নির্ণয় হবে।

TRY YOURSELF:

➔ **Evaluate the following definite integrals :**

1. $\int_0^1 \frac{1-x}{1+x}dx$ **2.** $\int_0^1 \frac{\text{In}(1-x)}{x}$ **3.** $\int \frac{dx}{\sqrt{\{(x-1)(2-x)\}}}$ **4.** $\int_{\sqrt{2}}^2 \frac{dx}{x^2\sqrt{(x^2-1)}}$

5. $\int_0^{\frac{2a}{3}} \sqrt{(4a^2-9x^2)dx}$ **6.** $\int_0^{\sqrt{5}} x^2\sqrt{(5-x^2)}dx$ **7.** $\int_0^{\frac{\pi}{2}} \frac{dx}{a+b\cos x}(a > b > 0)$

8. $\int_0^{\pi} \frac{dx}{3+2\sin x+\cos x}$ **9.** $\int_0^2 xe^{2x}dx$ **10.** $\int_0^1 \tan^{-1}(\sqrt{x})\,dx$ **11.** $\int_0^1 \frac{x^3\sin^{-1}x}{\sqrt{1-x^2}}dx$

সমাধান ঃ ১. $\frac{1-x}{1+x} = \frac{2-(1+x)}{1+x} = \frac{2}{1+x} - 1 \therefore \int \frac{1-x}{1+x}dx = 2\int \frac{dx}{1+x} - \int dx =$
$2\text{In}|1+x| - x + c$ সুতরাং $\int_0^1 \frac{1-x}{1+x}dx = [2\text{In}|1+x| - x]_0^1$
$= \{2\text{In}(1+1)-1\} - \{2\text{In}(1+0)-0\} = 2\text{In} - 1 - 2\text{In}1 = 2\text{In}2 - 1\ [\because \text{In}1 = 0]$

২. $\int_0^1 \frac{\text{In}(1-x)}{x}dx = \int_0^1 \left(\frac{-x-\frac{x^2}{2}-\frac{x^3}{3}-\frac{x^4}{4}\cdots\cdots}{x}\right)dx = \int_0^1 \left(-1 - \frac{x}{2} - \frac{x^2}{3} - \frac{x^3}{4}\cdots\cdots\right)dx = \left[-x - \frac{x^2}{2^2} - \frac{x^3}{3^2} - \frac{x^4}{4^2}\cdots\cdots\cdots\cdots\right]_0^1$
$= -\left(-1 - \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2}\cdots\cdots\cdots\right) = -\frac{\pi^2}{6}$

৩. ধরি, $x-1 = u^2$ বা, $x+1 = u^2 \therefore dx = 2udu$ যখন $x = 1$, তখন $u = 0$ যখন $x = 2$ তখন $u = 1$; $\int_1^2 \frac{dx}{\sqrt{\{(x-1)(2-x)\}}} = \int_0^1 \frac{2udu}{u\sqrt{(2-1-u^2)}}$
$= 2\int_0^1 \frac{du}{\sqrt{(1-u^2)}} = 2[\sin^{-1}u]_0^1 = 2(\sin^{-1}1 - \sin^{-1}0) = 2\left(\frac{\pi}{2} - 0\right) = \pi$

৪. ধরি, $x = \sec\theta \therefore dx = \sec\theta\tan\theta\ d\theta$ যখন $x = \sqrt{2}$ তখন $\theta = \frac{\pi}{4}$; যখন $x = 2$
তখন $\theta = \frac{\pi}{3}$; $\therefore \int_{\sqrt{2}}^2 \frac{dx}{x^2\sqrt{(x^2-1)}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta\,d\theta}{\sec^2\theta\sqrt{\sec^2\theta-1}} = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sec\theta\tan\theta\,d\theta}{\sec^2\theta\tan\theta}$
$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \cos\theta\,d\theta = [\sin\theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \sin\frac{\pi}{3} - \sin\frac{\pi}{4} = \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}$

৫. $\int_0^{\frac{2a}{3}} \sqrt{(4a^2-9x^2)}dx = 3\int_0^{\frac{2a}{3}} \sqrt{\left\{\left(\frac{2a}{3}\right)^2 - x^2\right\}}dx = 3\left[\frac{1}{2}\times\sqrt{\left(\frac{4a^2}{9}-x^2\right)} + \frac{4a^2}{18}\sin^{-1}\left(\frac{3x}{2a}\right)\right]_0^{\frac{2a}{3}}$
$= 3\left\{0 + \frac{4a^2}{10}\sin^{-1}1 - a - 0\right\} = 3\frac{4a^2}{10}.\frac{\pi}{2} - a = \frac{3\pi^2}{5} - a$

৬. ধরি, $x = \sqrt{5}\sin\theta$ $\therefore dx = \sqrt{5}\cos\theta\, d\theta$ যখন $x = 0$ তখন $\theta = 0$ যখন $x = \sqrt{5}$ তখন $\theta = \frac{\pi}{2}$ $\therefore$

$$I = \int_0^{\sqrt{5}} x^2\sqrt{(5 - x^2)}dx = \int_0^{\frac{\pi}{2}} 5\sin^2\theta\sqrt{5 - 5\sin^2\theta}\,\sqrt{5}\cos\theta\, d\theta$$

$$= 25\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\, d\theta = \frac{25}{4}\int_0^{\frac{\pi}{2}} (2\sin\theta\cos\theta)^2 d\theta$$

$$= \frac{25}{4}\int_0^{\frac{\pi}{2}} \sin^2 2\theta\, d\theta = \frac{25}{8}\int_0^{\frac{\pi}{2}} (1 - \cos 4\theta)d\theta = \frac{25}{8}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\frac{\pi}{2}}$$

$$= \frac{25}{8}\left[\left(\frac{\pi}{2} - \frac{1}{2}\sin 2\pi\right) - (0 - 0)\right] = \frac{25\pi}{16}$$

৭. আমরা জানি, $\int \frac{dx}{a+b\cos x} = \frac{2}{\sqrt{(a^2-b^2)}}\tan^{-1}\left\{\left(\sqrt{\frac{a-b}{a+b}}\right)\tan\frac{x}{2}\right\} + c$

$$\therefore \int_0^{\pi} \frac{dx}{a+b\cos x} = \left[\frac{2}{\sqrt{(a^2-b^2)}}\tan^{-1}\left\{\left(\sqrt{\frac{a-b}{a+b}}\right)\tan\frac{x}{2}\right\}\right]_0^{\pi} = \frac{2}{\sqrt{(a^2-b^2)}}\cdot\frac{\pi}{2} = \frac{\pi}{\sqrt{(a^2-b^2)}}$$ ৮.

ধরি, $$l = \int_0^{\pi} \frac{dx}{3+2\sin x+\cos x} = \int_0^{\pi} \frac{dx}{3+2\times\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} = \int_0^{\pi}\frac{\left(1+\tan^2\frac{x}{2}\right)}{3+\left(3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}\right)}$$

$$= \int_0^{\pi} \frac{\sec^2\frac{x}{2}\,dx}{2\left(\tan^2\frac{x}{2}+2\tan\frac{x}{2}+2\right)}$$ ধরি, $\tan\frac{x}{2} = u$ $\therefore \frac{1}{2}\sec^2\frac{x}{2}\,dx = du$ যখন $x = \pi$ তখন $u = \infty$

$$\therefore l = \int_0^{\infty} \frac{du}{u^2+2u+2} = \int_0^{\infty} \frac{du}{(u+1)^2+1^2} = [\tan^{-1}(u + 1)]_0^{\infty} = \tan^{-1}\infty - \tan^{-1}1$$

$$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

৯. ধরি, $u = x$, $dv = e^{2x}dx$ $\therefore du = dx$; $v = \int e^{2x}dx = \frac{1}{2}e^{2x}$ $\therefore \int_0^2 xe^{2x}dx = \int_0^2 u\,dv$

$$= [uv]_0^2 - \int_0^2 u\,dv = \left[\frac{1}{2}\times e^{2x}\right]_0^2 - \int_0^2 \frac{1}{2}e^{2x}dx = \frac{1}{2}(2e^4 - 0) - \frac{1}{4}[e^{2x}]_0^2 = e^4 - \frac{1}{4}e^4 = \frac{3}{4}e^4$$

10. অংশক্রমে ইন্টিগ্রেশন করিয়ে পাই, $\therefore \int \tan^{-1}(\sqrt{x})dx = x\tan^{-1}(\sqrt{x}) - \int \frac{1}{1+x}\cdot\frac{1}{2\sqrt{x}}\cdot x\, dx$

$$= x\tan^{-1}(\sqrt{x}) - \int \frac{1 + x - 1}{1 + x}\cdot\frac{1}{2\sqrt{x}}\, dx = x\tan^{-1}(\sqrt{x}) - \int \frac{1}{2\sqrt{x}}dx + \int \frac{1}{1 + x}\cdot\frac{1}{2\sqrt{x}}\, dx$$

$$= x\tan^{-1}(\sqrt{x}) - \sqrt{x} + \tan^{-1}(\sqrt{x}) + c = (x + 1)\tan^{-1}(\sqrt{x}) - \sqrt{x} + c$$

$$\therefore \int_0^1 \tan^{-1}(\sqrt{x})dx = \left|(x+1)\tan^{-1}(\sqrt{x}) - \sqrt{x}\right|_0^1 = (1+1)\tan^{-1}1 - 1 = 2.\frac{\pi}{2} - 1 = \frac{\pi}{2} - 1$$

১১. ধরি, $I = \int_0^1 \frac{x^3 \sin^{-1} x}{\sqrt{(1-x^2)}} dx$ এবং $x = \sin\theta$, $\therefore dx = \cos\theta\, d\theta$ যখন $x = 0$ তখন $\theta = 0$;

তখন $\theta = \frac{\pi}{2}$; $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ বা, $\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

$$\therefore \int_0^{\frac{\pi}{2}} \frac{\theta \sin^3\theta \cos\theta d\theta}{\cos\theta} = \frac{1}{4}\int_0^{\frac{\pi}{2}} \theta\,(3\sin\theta - \sin 3\theta)d\theta$$

$$= \frac{1}{4}\left[\theta\left(-3\cos\theta + \frac{\cos 3\theta}{3}\right) - 1\left(-3\cos\theta + \frac{\cos 3\theta}{9}\right)\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{36}\left[\theta(3\cos 3\theta - 27\cos\theta) - \sin 3\theta + 27\sin\theta\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{36}\left\{0 - \sin\frac{3\pi}{2} + 27\sin\frac{\pi}{2} - 0 + 0 - 0\right\} = \frac{1}{36}(1+27) = \frac{1}{36}\times 28 = \frac{7}{9}$$

TRY YOURSELF:

1. (i) $\int_{-\ln 3}^{\ln 3} \frac{e^x}{e^x+4}dx$ $\left[\textbf{Ans: In } \frac{21}{13}\right]$ **(ii)** $\int_0^{\ln 3} \frac{e^x}{1+e^x}dx \left[\textbf{Ans: ln } \frac{3}{2}\right]$ **(iii)** $\int_0^1 \frac{dx}{e^x+e^{-x}}\left[\textbf{Ans: } \frac{\pi}{4}\right]$ **(iv)** $\int_0^\infty \frac{dx}{a^2e^x+b^2e^{-x}}\left[\textbf{Ans: } \frac{1}{ab}\tan^{-1}\frac{b}{a}\right]$ **(v)** $\int_0^{\frac{3}{4}} \frac{dx}{1-x}$ $[\textbf{Ans: In4}]$ **(vi)** $\int_0^1 \frac{dx}{\sqrt{(2x-x^2)}}\left[\textbf{Ans: } \frac{\pi}{2}\right]$ **(vii)** $\int_0^{2a} \frac{dx}{\sqrt{(2ax-x^2)}}[\textbf{Ans: } \pi]$ **2. (i)** $\int_0^{\frac{\pi}{4}} \cos^2\theta d\theta.\left[\textbf{Ans: } \frac{1}{8}(\pi+2)\right]$ **(ii)** $\int_0^{\frac{\pi}{4}} \cos^2 2x dx$ $\left[\textbf{Ans: } \frac{1}{3}\right]$ **(iii)** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x}[\textbf{Ans: } 1]$ **(iv)** $\int_0^{\frac{\pi}{4}} \sin^4\theta\, d\theta$ $\left[\textbf{Ans: } \frac{3\pi}{32} - \frac{1}{4}\right]$ **(v)** $\int_0^{\frac{\pi}{4}} \sec^3 x dx$ $\left[\textbf{Ans: } \frac{\sqrt{2}}{2} + \frac{1}{2}\textbf{In}(1+\sqrt{2})\right]$ **(vi)** $\int_0^{\frac{\pi}{4}} \sec^4 x dx\left[\textbf{Ans: } \frac{4}{3}\right]$
3.(i) $\int_a^b \frac{\ln x}{x}$. $\left[\textbf{Ans: } \frac{1}{2}\textbf{In (ab)ln}\left(\frac{a}{b}\right)\right]$ **(ii)** $\int_1^3 (2x+1)\sqrt{(x^2+x+1)}dx\left[\textbf{Ans: } \frac{2}{3}\left\{13\sqrt{(13)} - 3\sqrt{3}\right\}\right]$ **(iii)** $\int_0^1 x^3\sqrt{(1+3x^4)}dx\left[\textbf{Ans: } \frac{7}{18}\right]$ **(iv)** $\int_0^{\frac{\pi}{2}} \cos^3 x^4\sqrt{(\sin x)}dx\left[\textbf{Ans: } \frac{32}{65}\right]$**(v)** $\int_0^{\frac{\pi}{3}} \frac{\cos x\, dx}{3+4\sin x}\left[\textbf{Ans: } \frac{1}{4}\textbf{In}\left(\frac{3+2\sqrt{3}}{2}\right)\right]$ **(vi)** $\int_0^\infty \frac{(\tan^{-1}x)^2}{1+x^2}dx\left[\textbf{Ans: } \frac{\pi^3}{24}\right]$ **(vii)** $\int_2^4 \frac{(x-3)dx}{\sqrt{(5-x)(x-1)}}$ [Ans: 0] **(viii)** $\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^3\theta d\theta.\left[\textbf{Ans: } \frac{1}{15}\right]$ **4. (i)** $\int_a^b (x-a)^3(b-x)^2\, dx.\left[\textbf{Ans: } \frac{(b-a)^6}{60}\right]$ **(ii)** $\int_0^1 \frac{xdx}{1+\sqrt{x}}\left[\textbf{Ans: } \frac{5}{3} - 2\textbf{In}2\right]$ **(iii)** $\int_0^a \frac{x^3dx}{\sqrt{x^2+9}}.\left[\textbf{Ans: } \frac{44}{3}\right]$ **(iv)** $\int_0^{16} \frac{x^{\frac{1}{4}}dx}{1+\sqrt{x}}$ $\left[\textbf{Ans: } \frac{8}{3} + 4\tan^{-1}2\right]$ **5. (i)** $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\sin^2 x+b^2\cos^2 x}$. $\left[\textbf{Ans: } \frac{\pi}{2ab}\right]$ **(ii)** $\int_0^{\frac{\pi}{2}} \frac{dx}{a^2\cos^2 x+b^2\sin^2 x}$. $\left[\textbf{Ans: } \frac{\pi}{2ab}\right]$ **(iii)** $\int_0^3 \frac{dx}{(16+x^2)^{\frac{3}{2}}}\left[\textbf{Ans: } \frac{3}{80}\right]$ **(iv)** $\int_0^a \frac{a^2-x^2}{(a^2+x^2)^2}dx.\left[\textbf{Ans: } \frac{1}{2a}\right]$

(v) $\int_0^{\frac{\pi}{4}} \frac{\cos 2x-1}{\cos 2x+1} dx.$ $\left[\textbf{Ans: } \frac{\pi}{4}-1\right]$ **6.** **(i)** $\int_0^{\frac{\pi}{2}} \frac{\cos x\, dx}{(1+\sin x)\,(2+\sin x)}$. [**Ans: In** $(4/3)$]

(ii) $\int_1^3 \frac{x-3}{x^3+x^2} dx$ $\left[\textbf{Ans: } 4\ \text{In}\frac{3}{2}-2\right]$ **(iii)** $\int_0^{\frac{1}{2}} \frac{dx}{(1-x^2)^2}$. $\left[\textbf{Ans: } \frac{1}{3}+\frac{1}{4}\text{In}3\right]$ **(iv)** $\int_0^{\infty} \frac{xdx}{(1+x)(1+x^2)}$ $\left[\textbf{Ans: } \frac{\pi}{4}\right]$ **(v)** $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$ $\left[\textbf{Ans: } \frac{\pi}{2ab\,(a+b)}\right]$ **(vi)** $\int_0^{\infty} \frac{xdx}{(x^2+a^2)(x^2+b^2)}$ $\left[\textbf{Ans: } \frac{1}{a^2-b^2}\text{In}\left(\frac{a}{b}\right)\right]$ **(vii)** $\int_0^1 \frac{x}{x^4+1} dx$ $\left[\textbf{Ans: } \frac{\pi}{8}\right]$ **7. (i)** $\int_{\sqrt{e}}^{e} \frac{\text{In}\, x}{x^2} dx$ $\left[\textbf{Ans: } \frac{3\sqrt{e}-4}{2e}\right]$ **(ii)** $\int_1^3 \text{In}\left(x+\sqrt{x^2}-1\right)dx$[**Ans: 3In** $\left(3+2\sqrt{2}\right)-2\sqrt{2}$]

(iii) $\int_1^6 e^{2x}\cos 3x\, dx$ $\left[\textbf{Ans: } \frac{3}{13}e^{\frac{\pi}{3}}-\frac{2}{13}\right]$ **(iv)** $\int_1^{\infty} e^{-ax}\sin bx\, dx$ $\left[\textbf{Ans: } \frac{b}{a^2+b^2}\right]$

8. (i) $\int_0^1 \sqrt{(1-x^2)}dx$ $\left[\textbf{Ans: } \frac{\pi}{4}\right]$ **(ii)** $\int_0^{\frac{\pi}{2}} \frac{\cos^2\theta\,\sin\theta}{\sqrt{1+a^2\,\text{COS}^2\,\theta}} d\theta$ $\left[\textbf{Ans: } \frac{1}{a^2}(1_1-1_2)\right]$

9. (i) $\int_2^3 \frac{dx}{\sqrt{\{(x-1)(5-x)\}}}$ $\left[\textbf{Ans: } \frac{\pi}{6}\right]$ **(ii)** $\int_{\alpha}^{\beta} \frac{dx}{\sqrt{\{(x-\alpha)(\beta-x)\}}}$ [**Ans:** π]

(iii) $\int_3^4 \frac{dx}{\sqrt{\{(x-1)(x+1)\}}}$ [**Ans:** π] **(iv)** $\int_a^b \frac{dx}{x\sqrt{\{(x-a)(b-x)\}}}$ $\left[\textbf{Ans: } \frac{\pi}{\sqrt{(ab)}}\right]$

(v) $\int_8^{15} \frac{dx}{(x-3)\sqrt{(x+1)}}$ $\left[\textbf{Ans: } \frac{1}{3}\text{In}\frac{5}{3}\right]$ **(vi)** $\int_1^{\infty} \frac{dx}{x\sqrt{(x^2-1)}}$ $\left[\textbf{Ans: } \frac{\pi}{2}\right]$ **(vii)** $\int_1^2 \frac{dx}{x\sqrt{(x^2+5x+1)}}$ $\left[\textbf{Ans: In}\left(\frac{7+2\sqrt{7}}{6+\sqrt{15}}\right)\right]$ **(viii)** $\int_0^1 \frac{dx}{(1+x)\sqrt{1+2x-x^2}}$ $\left[\textbf{Ans: } \frac{\pi}{4\sqrt{2}}\right]$

(ix) $\int_0^1 \frac{dx}{(1+x^2)\sqrt{(1-x^2)}}$ $\left[\textbf{Ans: } \frac{\pi}{2\sqrt{2}}\right]$ **(x)** $\int_0^1 x^2\sqrt{(1-x^2)}\, dx$ $\left[\textbf{Ans: } \frac{\pi}{16}\right]$

10. (i) $\int_0^{\pi} \frac{dx}{5+3\cos x}$ $\left[\textbf{Ans: } \frac{\pi}{4}\right]$

11. (i) $\int_0^{\pi} \frac{dx}{1-2a\cos x+a^2}$ $\left[\textbf{Ans: } \frac{\pi}{1-a^2}\right]$ **(ii)** $\int_0^{\pi} \frac{dx}{a^2-2a\cos x+b^2}$ $\left[\textbf{Ans: } \frac{\pi}{a^2-b^2}\right]$

12. (i) $\int_1^2 \sqrt{\{(x-1)(2-x)\}}\, dx$ $\left[\textbf{Ans: } \frac{\pi}{8}\right]$ **(ii)** $\int_{\alpha}^{\beta} \sqrt{\{(x-\alpha)(2-x)\}}\, dx$ $\left[\textbf{Ans: } \frac{\pi}{8}-(\beta-\alpha)^2\right]$

<u>নির্দিষ্ট ইন্টিগ্রালের সাধারণ ধর্ম</u>

নির্দিষ্ট ইন্টিগ্রালের গুরূত্বপূর্ণ ধর্মগুলো নিম্নে আলোচনা করা হলঃ (i) $\int_b^a f(x)dx = \int_b^a f(u)du$

প্রমাণ ঃ ধরি, $\int f(x)dx = F(x)$

এবং $\int_b^a f(u)du = F(u)$

$\int_b^a f(x)dx = [F(x)]_a^b = F(b) - F(a) \ldots\ldots\ldots\ldots..(i)$ এবং

$\int_b^a f(u)du = [F(u)]_a^b = F(b) - F(a) \ldots\ldots\ldots\ldots..(ii)$

সুতরাং (i) ও (ii) হতে পাই , $\int_b^a f(x)dx = \int_b^a f(u)du$ (প্রমাণিত)

(ii) $\int_b^a f(x)dx = -\int_b^a f(x)dx$

প্রমাণ ঃ ধরি, $\int f(x)dx = F(x)$, $\int_b^a f(x)dx = [F(x)]_a^b = F(b) - F(a) \ldots\ldots\ldots\ldots..(i)$ এবং

$$\int_b^a f(x)dx = [F(x)]_a^b = -[F(a) - F(b)] = F(b) - F(a) \ldots\ldots\ldots\ldots..(ii)$$

সুতরাং (i) ও (ii) হতে পাই, $\int_b^a f(x)dx = \int_b^a f(x)dx$ (প্রামণিত)

(iii) $\int_b^a f(x)dx = \int_b^a f(a-x)dx$

প্রমাণঃ ধরি, $a - x = u$ বা, $x = a - u$ $\therefore dx = -du$ যখন $x = 0$ তখন $u = 0$ সুতরাং

$\int_b^a f(x)dx = \int_b^a f(a-x)(-du) = -\int_a^b f(a-u)du = \int_a^b f(a-u)du$ [(ii) এর সাহায্য]

$= \int_0^a f(a-x)dx$[(i) এর সাহায্য]

(iv) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ যখন, $a < c < b$ হয়।

প্রমাণ ঃ ধরি, $\int f(x)dx = F(x)$

$$\therefore \int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a) \ldots\ldots\ldots\ldots\ldots\ldots(i)$$

$$\int_a^c f(x)dx = [F(x)]_a^c = F(c) - F(a) \dots\dots\dots\dots\dots\dots (ii)$$

$$\int_c^b f(x)dx = [F(x)]_c^b = F(b) - F(c) \dots\dots\dots\dots\dots\dots (iii)$$

এখন (ii) ও (iii) যোগ করে পাই,

$$\int_a^c f(x)dx + \int_c^b f(x)dx = F(b) - F(a) \dots\dots\dots\dots\dots\dots (iv)$$

সুতরাং (i) ও (iv) হতে পাই,

$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx,$ (প্রমাণিত)

সাধারণভাবে যদি $a < c_1 < c_2 \dots\dots\dots\dots\dots < c_n < b$ হয়, তবে

$\int_a^b f(x)dx = \int_a^{c_1} f(x)dx + \int_{c_1}^{c_2} f(x)dx + \cdots \dots\dots\dots\dots.. + \int_{c_n}^b f(x)dx$

(v) $\int_0^{na} f(x)dx = n\int_0^a f(x)dx$ যদি $f(a + x) = f(x)$ হয়।

প্রমাণ ঃ

$\int_0^{na} f(x)dx = \int_0^{na} f(x)dx + \int_a^{2a} f(x)dx + \int_a^{3a} f(x)dx + \cdots \dots\dots\dots\dots + \int_{(n-1)a}^{na} f(x)dx$

এখন $\int_a^{2a} f(x)dx$ ইন্টিগ্রাল নির্ণয়ের জন্য ধরি, $x = u + a$, $\therefore dx = du$;

x	a	2a
u	0	a

$$\therefore \int_a^{2a} f(x)dx = \int_0^a f(u+a)du = \int_0^a f(x+a)dx = \int_0^a f(x)dx \, [\because f(a+x) = f(x)]$$

অনুরূপভাবে আমরা দেখতে পারি যে, $\int_{2a}^{3a} f(x)dx = \int_a^{2a} f(x)dx = \int_0^a f(x)dx$ এবং

$\int_{(n-1)a}^{na} f(x)dx = \int_0^a f(x)dx$ সুতরাং (i) হতে পাই, $\int_0^{na} f(x)dx$

$= \int_0^a f(x)dx + \int_0^a f(x)dx + \int_0^a f(x)dx + \cdots \ldots \ldots \ldots . n$ সংখ্যক পদ পর্যন্ত । $= n\int_0^a f(x)dx$ **(প্রমাণিত)**

(vi) $\int_a^{2a} f(x)dx = 2\int_0^a f(x)dx$ যদি f (2a – x) = f(x) হয়, এবং $\int_a^{2a} f(x)dx = 0$ যদি f (2a – x) = – f(x) হয়।

প্রমাণ ঃ

$\int_a^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(x)dx = \int_0^a f(x)dx - \int_a^0 f(2a-u)du$ [x = 2a – u ধরিয়া]

$\int_0^a f(x)dx + \int_0^a f(2a-x)dx = \int_0^a f(x)dx + \int_0^a f(x)dx = 2\int_0^a f(x)dx$ **(প্রমাণিত)**

এবং $\int_a^{2a} f(x)dx = \int_0^a f(x)dx + \int_a^{2a} f(2a-x)dx = \int_0^a f(x)dx - \int_a^0 f(2a-u)du$ [x = 2a – u ধরিয়া]

$\int_0^a f(x)dx + \int_0^a f(2a-x)dx = \int_0^a f(x)dx - \int_0^a f(x)dx = 0$ **(প্রমাণিত)**

প্রমাণ ঃ $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ এর সাহায্য পাই,

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_{0}^{a} f(x)dx \ldots \ldots \ldots \ldots \ldots (i)$$

এখন, $\int_{-a}^0 f(x)dx$ এর মান নির্ণয়ের জন্য x = $-$ u বসাই, তাহা হইলে dx = –du;

x	-a	0
u	a	0

$\therefore \int_{-a}^0 f(x)dx = -\int_a^0 f(-u)du = \int_0^a f(-x)dx$ সুতরাবং (i) নং হতে পাই,

$\int_{-a}^a f(x)dx = \int_0^a f(-x)dx + \int_0^a f(x)dx = \int_0^a [f(x) + f(-x)]dx$ **(প্রমাণিত)**

(viii) $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx; & \text{যদি } f(-x) = f(x) \text{ হয়।} \\ 0 & \text{যদি } f(-x) =- f(x) \text{ হয়।} \end{cases}$

প্রমাণ ঃ আমরা জানি f(x) যুগ্ম ফাংশন (even function) হইলে f(- x) = f(x) সুতরাং (viii) হতে পাই,

$\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(x)dx + \int_{0}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$, যদি f(- x) = f(x) হয়।

আবার, f(x) অযুগ্ম ফাংশন হলে f(- x) = - f(x) সুতরাং (viii) হতে পাই,

$\int_{-a}^{a} f(x)dx = \int_{0}^{a} f(x)dx - \int_{0}^{a} f(x)dx = 0$ যদি f(- x) = - f(x) হয়।

নিম্নলিখিত নির্দিষ্ট ইন্টিগ্রালের মান নির্ণয় কর ঃ

(i) $\int_{0}^{\frac{\pi}{2}} \frac{\cos x\, dx}{\sin x + \cos x}$ (ii) $\int_{0}^{\frac{\pi}{2}} \frac{\sqrt{(\tan x)}dx}{1+\sqrt{(\tan x)}}$ (iii) $\int_{0}^{\frac{\pi}{2}} \frac{\cos^2 x\, dx}{\sin x + \cos x}$ (iv) $\int_{0}^{1} \operatorname{In}\left(\frac{1}{x} - 1\right) dx$

(v) $\int_{0}^{\pi} \frac{x\, dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$ (vi) $\int_{0}^{\frac{\pi}{2}} \operatorname{Incos} x\, dx$ (vii) $\int_{0}^{\pi} \sin x \operatorname{In}(\sin x)dx$

(viii) $\int_{0}^{1} \cot^{-1}(1 - x + x^2)dx$ (ix) $\int_{0}^{\frac{\pi}{4}} \sqrt{(\tan x)}dx$ (x) $\int_{0}^{\frac{\pi}{2}} \frac{x^2 \sin 2x \sin(\pi 2 \cos x)}{2x-\pi}$

(xi) $\int_{0}^{\frac{\pi}{4}} \sqrt[3]{(\tan x)}dx$ (xii) $\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \sin^{7} x\, dx$

সমাধান ঃ (i) ধরি,

$l = \int_{0}^{\frac{\pi}{2}} \frac{\cos x\, dx}{\sin x + \cos x} \ldots\ldots\ldots\ldots.(i) = \int_{0}^{\frac{\pi}{2}} \frac{\cos\left(\frac{\pi}{2}-x\right)dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)} = \int_{0}^{\frac{\pi}{2}} \frac{\sin x\, dx}{\cos x + \sin x}$

এখন (i) ও (ii) যোগ করে পাই, $l+l = \int_{0}^{\frac{\pi}{2}} \frac{\cos x\, dx}{\sin x + \cos x} dx + \int_{0}^{\frac{\pi}{2}} \frac{\sin x\, dx}{\cos x + \sin x}$

বা, $2l = \int_{0}^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$ বা, $l = \int_{0}^{\frac{\pi}{2}} [x]_{0}^{\frac{\pi}{2}} = \frac{\pi}{2}$ বা, $l = {}^{\pi}/_{4}$ অর্থাৎ $\int_{0}^{\frac{\pi}{2}} \frac{\cos x\, dx}{\sin x + \cos x} = {}^{\pi}/_{4}$

(ii) ধরি, (ii) $l = \int_{0}^{\frac{\pi}{2}} \frac{\sqrt{(\sin x)}dx}{1+\sqrt{(\tan x)}} = \int_{0}^{\pi/2} \frac{\sqrt{(\sin x)}dx}{\sqrt{(\cos x)}+\sqrt{(\sin x)}} \ldots\ldots\ldots\ldots (i)$

$$= \int_0^{\pi/2} \frac{\sqrt{\{\sin(\pi/_2 - x)dx\}}}{\sqrt{\{\cos(\pi/_2 - x)\}} + \sqrt{\{\sin(\pi/_2 - x)\}}} = \int_0^{\pi/2} \frac{\sqrt{(\cos x)dx}}{\sqrt{(\sin x)} + \sqrt{(\cos x)}} \ldots\ldots\ldots..(2)$$

এখন (i) ও (ii) যোগ করে পাই, $l+l = \int_0^{\frac{\pi}{2}} \frac{\sqrt{(\sin x)dx}}{\sqrt{(\cos x)}+\sqrt{(\sin x)}} + \int_0^{\frac{\pi}{2}} \frac{\sqrt{(\cos x)dx}}{\sqrt{(\sin x)}+\sqrt{(\cos x)}}$

বা, $2l = \int_0^{\pi/2} \frac{\sqrt{(\sin x)}dx}{\sqrt{(\cos x)}+\sqrt{(\sin x)}} dx$ বা, $l = \pi/_4$ অর্থাৎ $\int_0^{\pi/2} \frac{\sqrt{(\tan x)}dx}{1+\sqrt{(\tan x)}dx} = \pi/_4$

ধরি, (iii)

$$l = \int_0^{\pi/2} \frac{\cos^2 x\, dx}{\sin x + \cos x} = \frac{\cos^2(\pi/_2 - x)dx}{\sin(\pi/_2 - x) + \cos(\pi/_2 - x)} = \int_0^{\pi/2} \frac{\sin^2 x\, dx}{\cos x + \sin x}$$

$$\therefore 2l = \int_0^{\pi/2} \left[\frac{\cos^2 x}{\sin x+\cos x} + \frac{\sin^2 x}{\sin x+\cos x}\right] = \int_0^{\pi/2} \frac{\sin^2 x+\cos^2 x}{\sin x+\cos x} = \int_0^{\pi/2} \frac{dx}{\sin x+\cos x}$$

$$= \int_0^{\pi/2} \frac{dx}{\frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} + \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}} = \int_0^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{2\tan\frac{x}{2} + 1 - \tan^2\frac{x}{2}} = \int_0^{\pi/2} \frac{sce^2\frac{x}{2}dx}{1+2\tan\frac{x}{2} - \tan^2\frac{x}{2}}$$

ধরি, $\tan^2\frac{x}{2} = u$, $\therefore \frac{1}{2}\sec^2 dx = du$;

x	0	π_2
u	0	1

$$\therefore l = \int_0^{\pi/2} \frac{du}{1+2u-u^2} = \int_0^1 \frac{du}{\left(\sqrt{2}\right)^2 - (u-1)^2} = \frac{1}{2\sqrt{2}}\left[\text{In}\frac{\sqrt{2}+u-1}{\sqrt{2}-u+1}\right]_0^1$$

$$= \frac{1}{2\sqrt{2}}\text{In}\left(\frac{\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}-1}{\sqrt{2}+1}\right) = \frac{1}{2\sqrt{2}}\text{In}\frac{\left(\sqrt{2}-1\right)^2}{2-1} = \frac{1}{\sqrt{2}}\text{In}\left(\sqrt{2}-1\right)$$

(iv) ধরি, $l = \int_0^1 \text{In}\left(\frac{1}{x} - 1\right)dx = \int_0^1 \text{In}\left(\frac{1}{x-1} - 1\right)dx = \int_0^1 \text{In}\left(\frac{1-1+x}{1-x}\right)d\,x$

$$= \int_0^1 \text{In}\left(\frac{x}{1-x}\right)d\,x = -\int_0^1 \text{In}\left(\frac{1-x}{x}\right)d\,x = \int_0^1 \text{In}\left(\frac{1}{x} - 1\right)dx = -1$$

বা, $2l = 0$ বা, $l= 0$ অর্থাৎ $\int_0^1 \text{In}\left(\frac{1}{x} - 1\right)dx = 0$

(v) ধরি, $l = \int_0^{\pi} \frac{x\,dx}{(a^2\cos^2 x + b^2\sin^2 x)} = \int_0^{\pi} \frac{(\pi - x)dx}{\{a^2\cos^2(\pi - x) + b^2\sin^2(\pi - x)\}^2}$

$= \int_0^{\pi} \frac{(\pi - x)dx}{(a^2\cos^2 + b^2\sin^2)^2} \quad \therefore 2l = \int_0^{\pi} \frac{\pi\,dx}{(a^2\cos^2 x + b^2\sin^2 x)^2} = \pi \int_0^{\pi} \frac{\pi\,dx}{(a^2 + b^2\tan^2 x)^2}$

বা, $l = \frac{\pi}{2} \int_0^{\pi} \frac{(1+\tan^2 x)\sec^2 x\,dx}{(a^2+b^2\tan^2 x)^2} = \pi \int_0^{\pi} \frac{(1+\tan^2 x)\sec^2 x\,dx}{(a^2+b^2\tan^2 x)^2}$

ধরি, $b\tan x = a\tan\theta \quad \therefore b\sec^2 x\,dx = a\sec^2\theta\,d\theta$

$\sec^2 x\,dx = \frac{a}{b}\sec^2\theta\,d\theta$

X	0	$\pi/2$
θ	0	$\pi/2$

সুতরাং $l = \pi \int_0^{\pi/2} \frac{\left(1+\frac{a^2}{b^2}\tan^2\theta\right)\frac{a}{b}\sec^2\theta\,d\theta}{(a^2+b^2\tan^2\theta)^2} = \frac{\pi}{a^3b^3} \int_0^{\frac{\pi}{2}} \frac{(a^2+b^2\tan^2\theta)}{\sec^2\theta}d\theta$

$\frac{\pi}{a^3b^3} \int_0^{\frac{\pi}{2}} (b^2\cos^2\theta + a^2\sin^2\theta)d\theta = \frac{\pi}{a^3b^3}\left(b^2.\frac{1}{2}.\frac{\pi}{2} + a^2.\frac{1}{2}.\frac{\pi}{2}\right) = \frac{\pi^2}{4a^2b^2}(a^2 + b^2)$

(vi) ধরি, $l = \int_0^{\pi/2} \text{In}\cos x\,dx = \int_0^{\pi/2} \text{In}\cos(\pi/2 - x)dx = \int_0^{\pi/2} \text{Insin}\,x\,dx$

$\therefore 2l = \int_0^{\pi/2} (\text{Insin}\,x + \text{In}\cos x)dx = \int_0^{\pi/2} \text{In}\frac{2\sin x\cos x}{2}dx = \int_0^{\pi/2} \text{In}\frac{(\sin 2x)}{2}dx$

$\int_0^{\pi/2} \text{In}\sin 2x\,dx - \text{in}2 \int_0^{\pi/2} dx = l_1 - \text{In}2[x]_0^{\pi/2} = l_1(\pi/2)$ যেখানে, $\int_0^{\pi/2} \text{In}\sin 2x\,dx$

ধরি, $2x = u; 2dx = du;$

X	0	$\pi/2$
u	0	π

$\therefore l_1 = \frac{1}{2}\int_0^{\pi} \text{In}\sin u\,du = \frac{1}{2}\int_0^{\pi} \text{In}\sin u\,du = l_1 = \frac{1}{2}\int_0^{\pi} \text{In}\sin x\,du = \frac{1}{2}\int_0^{\pi} \text{In}\cos x\,du = l$

সুতরাং $2l = l - (\pi/2)\text{In}2$ বা, $l = (\pi/2)\text{In}\frac{1}{2}$

(vii) ধরি, $l = \int_0^{\pi/2} \text{In}\sin x\,\text{In}\,(\sin x)dx = \frac{1}{2}\int_0^{\pi/2} \text{In}\sin x\,\text{In}\,(\sin^2 x)dx = \frac{1}{2}\int_0^{\pi/2} \sin x\,\text{In}(1 - \cos^2 x)dx$

X	0	$\pi/2$
u	1	0

ধরি, cos x = u; sin x dx = du;

সুতরাং $l = \frac{1}{2}\int_1^0 \text{In}\,(1 - u^2)du = \frac{1}{2}\int_1^0 \left(-u^2 - \frac{u^4}{2} - \frac{u^6}{3} - \cdots \ldots \ldots \ldots . .\right)du$

$$\frac{1}{2}\int_{1}^{0}\left(u^2+\frac{u^4}{2}+\frac{u^6}{3}+\cdots\ldots\ldots\ldots..\right)du=\frac{1}{2}\left[\frac{u^3}{3}+\frac{u^5}{2.5}+\frac{u^7}{3.7}\right]_1^0$$
$$=\frac{1}{2}\left(-\frac{1}{3}-\frac{1}{2.5}-\frac{1}{3.7}-\cdots\ldots\ldots\ldots\ldots..\right)$$

$$=-\left\{\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\cdots\ldots\ldots\right\}=-\left\{\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\left(\frac{1}{6}-\frac{1}{7}\right)+\cdots\ldots\ldots\ldots.\right\}$$

$$=\left(1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+\frac{1}{7}-\cdots\ldots\ldots\ldots.\right)-1=\text{In}\ (1+1)-1=\text{In}2-\text{Ine}=\text{In}\left(\frac{2}{e}\right)$$

(viii) ধরি, $l=\int_0^1\cot^{-1}(1-x+x^2)dx=\int_0^1\tan^{-1}\frac{1}{1-x+x^2}dx=\int_0^1\tan^{-1}\frac{1}{1+x(x-1)}dx$

$=\int_0^1\tan^{-1}\frac{x-(x-1)}{1+x(x-1)}dx=\int_0^1\{\tan^{-1}x-\tan^{-1}(x-1)\}dx=\int_0^1\tan^{-1}xdx-\int_0^1\tan^{-1}(x-1)dx$

$\int_0^1\tan^{-1}xdx-\int_0^1\tan^{-1}\{(1-x)-1\}dx=\int_0^1\tan^{-1}dx+\int_0^1\tan^{-1}xdx=2\int_0^1\tan^{-1}dx$

$$=2\left[x\tan^{-1}x-\frac{1}{2}\text{In}(x+x^2)\right]_0^1=2\left\{\tan^{-1}1-\frac{1}{2}\text{In}\ (1+1)-0+0\right\}=2\left({}^{\pi}/_{4}-\frac{1}{2}\text{In}2\right)$$

$={}^{\pi}/_{2}-\text{In}2$ (ix) ধরি, $l=\int_0^{\pi/2}\sqrt{(\tan x)}dx$ এবং $\tan x=u^2\ \therefore\sec^2xdx=2udu$

বা, $dx=\frac{2udu}{\sec^2 x}=\frac{2udu}{1+\tan^2 x}=\frac{2udu}{1+u^4}$ $\therefore l=\int_0^1\frac{2u^2du}{1+u^4}=\int_0^1\frac{(u^2+1)+(u^2-1)}{u^4}du$

$$=\int_0^1\frac{u^2+1}{u^4+1}du+\int_0^1\frac{u^2-1}{u^4+1}du=\int_0^1\frac{1+\frac{1}{u^2}}{u^2+\frac{1}{u^2}}du+\int_0^1\frac{1-\frac{1}{u^2}}{u^2+\frac{1}{u^2}}du$$

$=\int_0^1\frac{\left(1+\frac{1}{u^2}\right)du}{\left(u-\frac{1}{u}\right)^2+2}+\int_0^1\frac{\left(1-\frac{1}{u^2}\right)du}{\left(u+\frac{1}{u}\right)^2-2}$ ধরি, $u-\frac{1}{u}=v$ এবং $u+\frac{1}{u}=\omega$

$\therefore\left(1+\frac{1}{u^2}\right)du=dv$ এবং $\left(1-\frac{1}{u^2}\right)du=d\omega$, ং

u	0	1
-v	$-\infty$	0

u	0	1
ω	∞	2

সুতরাং $l=\int_{-\infty}^0\frac{dv}{v^2+(\sqrt{2})^2}+\int_{\infty}^0\frac{d\omega}{\omega^2-(\sqrt{2})^2}=\frac{1}{\sqrt{2}}\left[\tan^{-1}\frac{v}{\sqrt{2}}\right]_{-\infty}^0+\frac{1}{\sqrt{2}}\left[\text{In}\frac{\omega-\sqrt{2}}{\omega+\sqrt{2}}\right]_{\infty}^2$

$$=\frac{1}{\sqrt{2}}\{\tan^{-1}0-\tan^{-1}(-\infty)\}+\frac{1}{2\sqrt{2}}\left\{\text{In}\frac{2-\sqrt{2}}{2+\sqrt{2}}-\text{In}1\right\}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\infty + \frac{1}{2\sqrt{2}}\text{In}\left\{\frac{\sqrt{2}(\sqrt{2}-1)}{\sqrt{2}(\sqrt{2}+1)}\right\} = \frac{1}{\sqrt{2}}.\frac{\pi}{2} + \frac{1}{2\sqrt{2}}\text{In}\left(\frac{2-\sqrt{2}}{2+\sqrt{2}}\right)$$

$= \frac{\pi}{2\sqrt{2}} + \frac{1}{\sqrt{2}}\text{In}\left(\sqrt{2}-1\right)$ (x) ধরি, $l = \int_0^{\pi}\frac{x^2\sin 2x\sin({}^{\pi}/_{2}\cos x)}{2x-\pi}dx$

$$= \int_0^{\pi}\frac{(\pi-x)^2\sin(2\pi-2x)\sin\{({}^{\pi}/_{2}\ \cos x(\pi-x)\}}{2(\pi-x)-\pi}dx$$

$$= \int_0^{\pi}\frac{(\pi^2-2\pi x+x^2)(-\sin 2x)\sin({}^{\pi}/_{2}\ \cos x)}{\pi-2x}dx \ \therefore 2l$$

$$= \int_0^{\pi}\frac{\pi(2x-\pi)\sin 2x\sin({}^{\pi}/_{2}\ \cos x)}{\pi-2x}dx$$

$= 2\pi\int_0^{\pi}\sin x\cos x\sin({}^{\pi}/_{2}\ \cos x)dx$ বা, $l = \pi\int_0^{\pi}\sin x\cos x\sin({}^{\pi}/_{2}\ \cos x)dx$

$= 2\pi\int_0^{\pi/2}\sin x\cos x\sin({}^{\pi}/_{2}\ \cos x)dx$ ধরি, ${}^{\pi}/_{2}\cos x = u \ \therefore {}^{\pi}/_{2}\sin x\,dx = du$

বা, $\sin x\,dx = -\frac{2}{\pi}u\sin u\,du$;

x	0	$\pi/2$
u	$\pi/2$	0

$\therefore -\frac{2}{\pi}.2\pi\int_{\pi/2}^{0}\frac{2}{\pi}u\sin u\,du$

$= -{}^{8}/_{\pi}[-u\cos u+\sin u]_{\pi/2}^{0}$ $0-\sin{}^{\pi}/_{2}\} = -{}^{8}/_{\pi}(-1)$

$= {}^{8}/_{\pi}$

ধরি, $l = \int_0^{\pi/4}\sqrt[3]{\tan x\,dx}$ এবং $\tan x = u^3 \quad \therefore \sec^2 x\,dx = 3u^2du$

বা, $dx = \frac{3u^2du}{\sec^2 x} = \frac{3u^2du}{1+\tan^2 x} = \frac{3u^2du}{1+u^6}$ $\quad \therefore l' = \int_0^1\frac{u.3u^2du}{1+u^6}$

$= 3\int_0^1\frac{u.u^2du}{(u^2+1)(u^4-u^2+1)}dx$ পুনরায় ধরি, $u^2 = v : 2udu = dv$ পুনরায় ধরি, $u^2 = v \ \therefore 2udu = dv$

$udu = \frac{1}{2}dv$;

u	0	1
v	0	1

$\therefore l = \frac{3}{2}\int_0^1\frac{vdv}{(v+1)(v^2-v+1)}$ ধরি, $\frac{v}{(v+1)(v^2-v+1)} = \frac{A}{v+1}+\frac{Bv+C}{v^2-v+1}$,

$v = A(v^2-v+1)+(Bv+C)(v+1)$......................(i)

(i) অভেদে v = -1 বসিয়ে পাই $A = -\frac{1}{3}$ (i) অভেদে হতে v^2 এবং v এর সহগ সমীকৃত করে পাই, A + B = 0 এবং -A + B + C = 1

সমাধান করে পাই, $B = \frac{1}{3}, C = \frac{1}{3}$ $\quad \therefore \frac{3}{2}\left\{-\frac{1}{3}\int_0^1\frac{dv}{v+1}+\frac{1}{3}\int_0^1\frac{(v+1)}{(v^2-v+1)}dv\right\}$

$$= -\frac{1}{2}\int_0^1\frac{dv}{(v+1)}+\frac{1}{2}\int_0^1\frac{\frac{1}{2}(2v-1)+\frac{3}{2}}{(v^2-v+1)}dv$$

$$= -\frac{1}{2}\int_0^1\frac{dv}{v+1}+\frac{1}{4}\int_0^1\frac{(2v-1)dv}{(v^2-v+1)}+\frac{4}{3}\int_0^1\frac{dv}{\left(v-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= -\frac{1}{2}[\text{In}(v+1)]_0^1+\frac{1}{4}[(v^2-v+1)]_0^1+\frac{3}{4}.\frac{2}{\sqrt{3}}\left[\tan^{-1}\frac{v-\frac{1}{2}}{\frac{2}{\sqrt{3}}}\right]_0^1$$

$$= -\frac{1}{2}\text{In}2+\frac{1}{4}(\text{In}1-\text{In}1)+\frac{\sqrt{3}}{2}\tan^{-1}\frac{2v-1}{\sqrt{3}}$$

$= -\frac{1}{2}\ln 2 + \frac{1}{4}.0 + \frac{\sqrt{3}}{2}\left\{\tan^{-1}\frac{1}{\sqrt{3}} - \left(-\frac{1}{\sqrt{3}}\right)\right\}_0^1 = \frac{1}{2}\ln\frac{1}{2} + \frac{\sqrt{3}}{2}2\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{2}\ln\frac{1}{2} + \frac{\pi\sqrt{3}}{6}$

(xii) ধরি, $f(x) = \sin^7 x$, $f(-x) = \sin^7(-x) = -\sin^7 x = -f(x)$ অতএব, $f(x) = \sin^7 x$ একটি অযুগ্ম ফাংশন। আমরা জানি , $f(x)$ অযুগ্ম হইলে $\int_{-a}^{a} f(x)dx = 0$ সুতরাং $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^7 dx = 0.$

TRY YOURSELF:

Evalute the following definite intergrals

1. (i) $\int_0^{\frac{\pi}{2}} \frac{\sin x\, dx}{\sin x + \cos\, dx}$ **(ii)** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cot x}$ **(iii)** $\int_0^{\frac{\pi}{2}} \frac{\sin^3 x dx}{\sin^3 x + \cos^3 x}$ **(iv)** $\int_0^{\frac{\pi}{2}} \frac{\sin x dx}{\sin x - \cos x}$ **(v)** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\tan x}$ **(vi)** $\int_0^{\frac{\pi}{2}} \frac{\sin^n x dx}{\sin^n x + \cos^n x}$ **2. (i)** $\int_0^{\frac{\pi}{2}} \frac{\sqrt{(\sin x)}dx}{\sqrt{(\sin x)} + \sqrt{(\cos x)}}$ **(ii)** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{(\cot x)}}$ **(iii)** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sqrt{(\tan x)}}$ **(iv)** $\int_0^{\frac{\pi}{2}} \frac{\sqrt{(\cot x)}\, dx}{1+\sqrt{(\cot x)}}$ **(v)** $\int_0^{\frac{\pi}{2}} \frac{(\sin x)^{3/2} dx}{(\sin x)^{3/2} + (\cos x)^{3/2}}$ **(vi)** $\int_0^{\pi} \frac{\sqrt{(\tan x)}\, dx}{\sqrt{(\tan x)} + \sqrt{(\cot x)}}$ **3. (i)** $\int_0^{\pi} \frac{x\, dx}{1+\sin x}$ **(ii)** $\int_0^{\pi} \frac{x\, dx}{1+\cos^2 x}$ **(iii)** $\int_0^{\pi} \frac{x\, dx}{a^2 - \cos^2 x}$ **(iv)** $\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x + b^2 \sin^2 x}$ **4. (i)** $\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}$ **(ii)** $\int_0^{\pi} \frac{x \tan x\, xdx}{\sec x + \tan x}$ **5. (i)** $\int_0^{\frac{\pi}{2}} \frac{\cos x - \sin x}{1+\sin x\cos x}dx$ **(ii)** $\int_0^{\frac{\pi}{2}} \frac{\sin x - \cos x}{1+\sin x\cos x}dx$ **6. (i)** $\int_0^{\frac{\pi}{2}} \frac{xdx}{\sin x + \cos x}$ **(ii)** $\int_0^{\frac{\pi}{2}} \frac{\sin^2 xdx}{\sin x + \cos xs}$ **7. (i)** $\int_0^{\frac{\pi}{2}} \ln(\tan x)dx$ **(ii)** $\int_0^{\infty} \frac{\ln x}{1+x^2}dx$ **8. (i)** $\int_0^{\frac{\pi}{4}} \ln(1+\tan\theta)d\theta$ **(ii)** $\int_0^{1} \frac{\ln(1+x)}{1+x^2}$ **9. (i)** $\int_0^{\frac{\pi}{2}} \ln(\sin x)dx$ **(ii)** $\int_0^{1} \frac{\ln xdx}{\sqrt{1-x^2}}$ **(iii)** $\int_0^{\frac{\pi}{2}} x\ln(\sin x)dx$ **(iv)** $\int_0^{\frac{\pi}{2}} \ln(\tan x + \cot x)dx$ **(v)** $\int_0^{1} \ln(1+\cos x)dx$ **(vi)** $\int_0^{1} \ln \sin\left(\frac{\pi\theta}{2}\right)dx$ 10. **(i)** $\int_{-a}^{a} x\sqrt{a^2 - x^2 dx}$ **(ii)** $\int_{-2}^{2} x^9(1-x^2)^7 dx$ **11. (i)** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^3\frac{x}{2}\cos^5\frac{x}{2}dx$ **(ii)** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^{15} xdx$

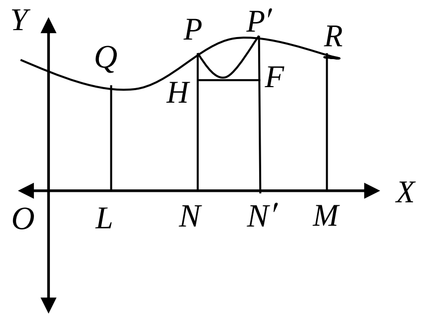
Ans:

1. (i) $\pi/4$ (ii) ইহা |(i)| (iii) $\frac{\pi}{4}$ (iv) $\frac{\pi}{4}$ (v) $\frac{\pi}{4}$ (vi) $\frac{\pi}{4}$ 2. (i) $\frac{\pi}{4}$ (ii) ইহা 2(i)l (iii) $\frac{\pi}{4}$ (iv) $\frac{\pi}{4}$ (v) $\frac{\pi}{2}$ (vi) $\frac{\pi}{2}$ 3. (i) π (ii) $\frac{\pi^2}{2\sqrt{2}}$ (iii) $\frac{\pi^2}{2a\sqrt{a^2-1}}$ (iv) $\frac{\pi^2}{2ab}$ 4. (i) $\frac{\pi^2}{4}$ (ii) $\pi\left(\frac{\pi}{2} - 1\right)$ (iii) ইহা 2(ii). (iv) $\frac{\pi}{3\sqrt{3}}$ 5. (i) 0. (ii) 0 6. (i) $\frac{\pi}{2\sqrt{2}}\ln(\sqrt{2}+1)$ (ii) $\frac{\sqrt{2}}{2}\ln(\sqrt{2}+1)$ 7. (i) 0 (ii) 0 8. (i) $\frac{\pi}{8}\ln 2$ (ii) $\frac{\pi}{8}\ln 2$ 9. (i) $\frac{\pi}{2}\ln\frac{1}{2}$ (ii) $\frac{1}{2}\ln\frac{1}{2}$ (iii) $\frac{\pi^2}{2}\ln\frac{1}{2}$ (iv) $\pi\ln 2$ (v) $\pi\ln\frac{1}{2}$ (vi) $\ln\frac{1}{2}$ 10. (i) 0 (ii) 0 11. (i) 0 (ii) 0

নির্দিষ্ট যোগজ এর প্রয়োগঃ

কার্তেসীয় স্থানাঙ্কে সামতলিক ক্ষেত্রের ক্ষেত্রফল ঃ

মনে করি, $y = f(x), x$ —অক্ষ এবং $x = a$ ও $x = b$ কোটি দ্বারা আবদ্ধ সামতলিক ক্ষেত্রের ক্ষেত্রফল $A_1; A_1$এর মান নির্ণয় করতে হবে। $f(x)$ফাংশনটি (a, b)ব্যবধিতে সীমিত মানের একটি অবিচ্ছিন্ন ফাংশন। $y = f(x)$বক্ররেখা xঅক্ষ, কোটি QLএবং PN-দ্বারা আবদ্ধ QLNP=A-এর ক্ষেত্রফল বিবেচনা করি। OL= a একটি নির্দিষ্ট রাশি এবং ON = x রাশিটি পরিবর্তনশীল । যেহেতু, x একটি পরিবর্তনশীল রাশি, A_1 সামতলিক ক্ষেত্রের ক্ষেত্রফলও পরিবর্তনশীল এবং এর মান x এর মানের উপর নির্ভরশীল।



যখন x -এর মান $\delta x (= NN')$পরিমাণ বৃদ্ধি পায়
তখন A- এর মানের আনুষঙ্গিক বৃদ্ধি $\delta A = PNN'P'$
যদি δx ব্যবধিতে $f(x_1)$ এবং $f(x_2)$যথাক্রমে
বৃহত্তম ও ক্ষুদ্রতম কোটি হয় তবে
$x \le x_1 \le x + \delta x$এবং $\le x_2 \le x + \delta x$;
স্পষ্টতই,δA এর ক্ষেত্রফল আয়তক্ষেত্র HN´অপেক্ষা বৃহত্তর ও আয়তক্ষেত্র FN´অপেক্ষা ক্ষুদ্রতর।অর্থাৎ $f(x_2)\delta x < \delta A < f(x_1).\delta x \Rightarrow f(x_2) < \frac{\delta A}{\delta x} < f(x_1)$ যখন, $\delta x \longrightarrow 0, f(x_1) \longrightarrow f(x)$ ও $f(x_2) \longrightarrow f(x)$ এবং $\underset{\delta x \to 0}{\mathrm{Lt}} \frac{\delta A}{\delta x} = \frac{dA}{dx}$

অতএব, $\frac{dA}{dx} = f(x)$, যোগজীকরণ করে পাই, $A = \int f(x)dx = F(x) + C$

এখন, $x = a$ হলে, A = 0 এবং x = b হলে, A = A_1 $\therefore\ A_1 = F(b) + C$

এবং $0 = F(a) + C \ \therefore\ A_1 = F(b) - F(a) = \int_a^b f(x)dx = \int_a^b ydx$

অতএব, নির্দিষ্ট যোগজ $\int_a^b f(x)dx = \int_a^b ydx$, $y = f(x)$ বক্ররেখা,
x- অক্ষ ও দুইটি নির্দিষ্ট কোটি $x = a$ এবং x = b এর দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্দেশ করে ।

অনুসি ঃ একই প্রকারে দেখানো যায় যে, $\int_a^b ydx$ নির্দিষ্ট যোগজটি, যে কোনো বক্ররেখা, y- অক্ষ এবং দুইটি প্রদত্ত ভুজ y = c, y = d দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্দেশ করে।

উদাহরণ-০১ ঃ $4x^2 + 9y^2 = 36$ অথবা $\frac{x^2}{9} + \frac{y^2}{4}$ উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রেফল নির্ণয় কর।

$= 4\int_0^3 ydx = 4\int_0^3 \frac{2}{3}\sqrt{9-x^2}\,dx = \frac{8}{3}\int_0^3 \sqrt{9-x^2}dx$ এখন, $x = 3\sin\theta$ হলে,

$dx = 3\cos\theta\, d\theta$ এবং $\sqrt{9-x^2} = \sqrt{9 - 9sin^2\theta} = 3\cos\theta$, $x = 0$ হলে,

$\theta = 0$ এবং $\theta = \frac{\pi}{2}$

উপবৃত্তের ক্ষেত্রফল = $\frac{8}{3}\int_0^{\pi/2} 3\cos\theta.3\cos\theta\, d\theta = 12\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$$= 12\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right) = 6\pi\ (Ans:)$$

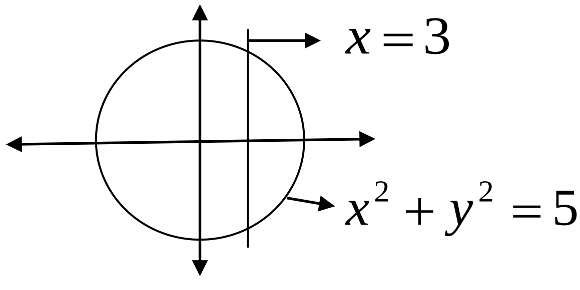
উদাহরণ-০২ ঃ $x^2 + y^2 = 25$ বৃত্ত এবং x = 3 সরলরেখা দ্বারা আবদ্ধ ক্ষুদ্রতর ক্ষেত্রটির ক্ষেত্রফল নির্ণয় কর।

নির্ণেয় ক্ষেত্রফল = ক্ষেত্রফল PQR= $2\int_3^5 ydx$

$= 2\int_3^5 \sqrt{25-x^2}dx$, $x = 5\sin\theta$ হলে,

$dx = 5\cos\theta\, d\theta$

এবং $\sqrt{25-x^2} = \sqrt{25 - 25\sin\theta}$



$= 5\cos\theta$: আবার , x = 3 হলে, $\sin\theta = \frac{3}{5} = \theta = \frac{\pi}{2}$

$\therefore$ নির্ণেয় ক্ষেত্রফল $= 2\int_{\sin^{-1}\frac{3}{5}}^{\pi/2} cos^2\,\theta d\theta = 25\int_{\sin^{-1}\frac{3}{5}}^{\pi/2} 2cos^2\theta d\theta$

$= 25\int_{\sin^{-1}\frac{3}{5}}^{\pi/2}(1+\cos 2\theta)d\theta = 25\left[\theta + \frac{1}{2}\right]_{\sin^{-1}\frac{3}{5}}^{\pi/2} = 25[\theta + \sin\theta\cos\theta]_{\sin^{-1}\frac{3}{5}}^{\pi/2}$

$= 25\left[\frac{\pi}{2} - \sin^{-1}\frac{3}{5} - \sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\sin^{-1}\frac{3}{5}\right)\right] = 25\left[\frac{\pi}{2} - \sin^{-1}\frac{3}{5} - \frac{3}{5}\cos\left(\cos^{-1}\frac{4}{5}\right)\right]$

$= \dfrac{25\pi}{2} - 25\sin^{-1}\dfrac{3}{5} - 12\ (Ans:)$

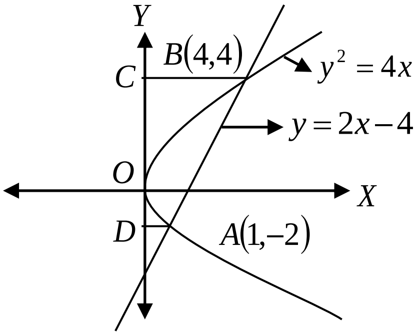
উদাহরণ -০৩ ঃ দেখাও যে, $y^2 = 4x$ পরাবৃত্ত এবং $y = 2x - 4$ সরলরেখার অর্ন্তগত অঞ্চলের ক্ষেত্রফল 9 বর্গ একক।

সমাধান ঃ, $y^2 = 4x$ সমীকরণে $y = 2x - 4$ বসিয়ে পাই,

$(2x-4)^2 = 4x = 4x^2 - 16x + 16 = 4x = x^2 - 5x + 4 = 0\ \therefore x = 1,4$

কাজেই $y = -2, 4$ অর্থাৎ পরাবৃত্ত এবং সরলরেখা পরস্পর (1, -2) ও (4, 4) বিন্দুদ্বয়ে ছেদ করে।

এখন AOB অঞ্চলের ক্ষেত্রফল নির্ণয় করতে হবে।



$\therefore$ নির্ণেয় ক্ষেত্রফল = ABCD ট্রাপিজিয়মের ক্ষেত্রফল

- (OBC ক্ষেত্র + OAD ক্ষেত্র) এর ক্ষেত্রফল।

$= \left(\frac{1}{2}\right)(BC + AD) \times CD - \int_{-2}^{4} xdy$

$= \left(\frac{1}{2}\right)(4+1) \times 6 - \int_{-2}^{4}\frac{y^2}{4}dy = \left(\frac{1}{2}\right) \times 30 - \left[\frac{y^3}{12}\right]_{-2}^{4}$

$= 154 - \left(\dfrac{64}{12}\right) + \left(\dfrac{8}{12}\right) = 15 - 6 = 9$

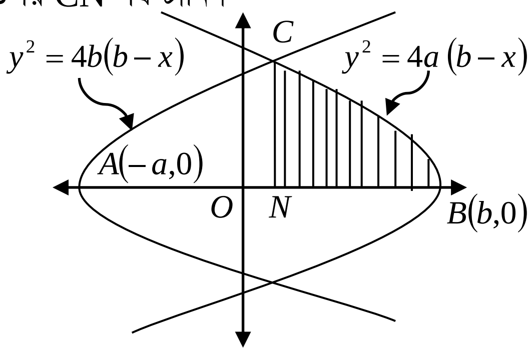
বিকল্প পদ্ধতি ঃ

নির্ণেয় ক্ষেত্রফল $= \int_{c}^{d}\big(f(y) - g(y)\big)dy = \int_{-2}^{4}\left[\frac{y+4}{2} - \frac{y^2}{4}\right]dy.$

$= \left[\frac{y^2}{4} + 2y - \frac{y^3}{12}\right]_{-2}^{4} = \left(4 + 8 + \frac{16}{3}\right) - \left(1 - 4 + \frac{2}{3}\right) = 9$

উদাহরণ -০৪ ঃ $y^2 = 4a(x+a)$ এবং $y^2 = 4b(b-x).(a > 0\,, b > 0)$ পরাবৃত্ত দুইটি দ্বারা সীমাবদ্ধ এলকার ক্ষেত্রফল নির্ণয় কর।

সমাধাণ ঃ $y^2 = 4b(b-x)$ পরাবৃত্তের শীর্ষবিন্দু B(b, 0) এবং উপকেন্দ্রিক লম্বের দৈর্ঘ্য $=4b$ অনুরূপভাবে $y^2 = 4a(x+a)$পরাবৃত্তের শীর্ষবিন্দুর A(- a, 0) এবং উপকেন্দ্রিক লম্বের দৈর্ঘ্য $= 4a$ পরাবৃত্ত দুইটি থেকে পাই $4a(x+a) = 4b(b-x) \Rightarrow x = b - a$ অতএব, পরাবৃত্ত দুইটি C বিন্দুতে ছেদ করে, যাহার ভূজ $b - a$ এবং C বিন্দু হইতে OX এর উপর CN লম্ব টানি।



সুতরাং নির্ণেয় ক্ষেত্রফল = $2 \times ABC$ অঞ্চলের ক্ষেত্রফল

= 2[NBC অঞ্চলের ক্ষেত্রফল + ACN অঞ্চলের ক্ষেত্রফল]

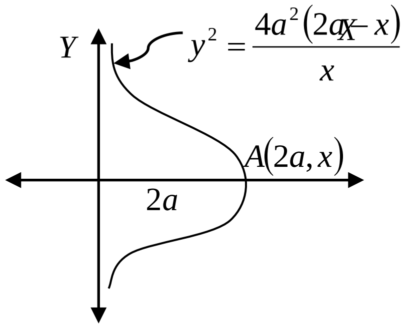
$= 2\left[\int_{b-a}^{b}\sqrt{\{4b(b-x)\}}dx + \int_{-a}^{b-a}\sqrt{\{4a(a+x)\}}dx\right]$

$= 2\{2\sqrt{b}\}\left[-\left(\dfrac{2}{3}\right)(b-x)^{\frac{3}{2}}\right]_{b-a}^{b} + 2\sqrt{a}\left[\left(\dfrac{2}{3}\right)(a+x)^{\frac{3}{2}}\right]_{-a}^{b-a}$

$= 4\sqrt{b}\left\{-\left(\frac{2}{3}\right)\left(0-a^{\frac{3}{2}}\right)\right\} + 4\sqrt{a}\left\{\left(\frac{2}{3}\right)\left(b^{\frac{3}{2}}-0\right)\right\} = \left(\frac{8}{3}\right)\sqrt{ba^2}^{\,3} + \left(\frac{8}{3}\right)\sqrt{(ab)}(a+b)$ বর্গ একক

উদাহরণ -০৫ ঃ $xy^2 = 4a^2(2a-x)$ বক্ররেখা এবং ইহার অসীমতটের অন্তবর্তী অংশের ক্ষেত্রফল নির্ণয় কর।

সমাধান ঃ বক্ররেখাটি অংকন করিলে দেখা যায় ইহা $x = 0$ এবং $x = 2a$ এর মধ্যে থাকিবে $x = 0$ রেখাটি এখানে অসীমতট হবে।



সুতরাং নির্ণেয় ক্ষেত্রফল $= \int_0^{2a} y\,dx = 2\int_0^{2a} \frac{2a\sqrt{(2a-x)}}{\sqrt{x}}\,dx$

$\therefore dx = 4a\sin\theta\cos\theta\, d\theta$ যখন $x = 0, \theta = 0$ যখন

$x = 2a, \theta = \frac{\pi}{2}$ $\quad = 4a\int_0^{\frac{\pi}{2}} \frac{\sqrt{(2a)}\cos\theta 4a\sin\theta.\cos\theta\, d\theta}{\sqrt{(2a)}\sin\theta}$

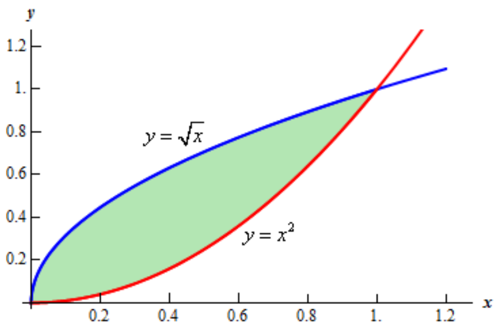
$= 16a^2 \int_0^{\frac{\pi}{2}} (1+\cos 2\theta)d\theta$

$= 8a^2\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = 8a^2\left[\left(\frac{\pi}{2}\right) + \left(\frac{1}{2}\right) + \sin\pi - \theta - \left(\frac{1}{2}\right)\sin\theta\right]$

$= 8a^2\left(\frac{\pi}{2}\right) = 4\pi a^2$

চিত্র থেকে ক্ষেত্রফল নির্ণয়ের কিছু গুরুত্বপূর্ণ সমাধান দেখানো হল ঃ

1.



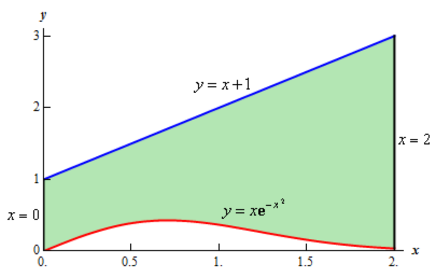
$$A = \int_a^b \left(\begin{matrix}\text{upper}\\ \text{function}\end{matrix}\right) - \left(\begin{matrix}\text{lower}\\ \text{function}\end{matrix}\right) dx$$

$$= \int_0^1 \sqrt{x} - x^2\, dx$$

$$= \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Bigg|_0^1$$

$$= \frac{1}{3}$$

2.



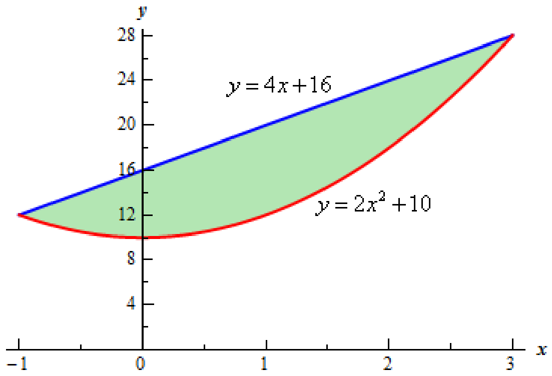
$$A = \int_a^b \left(\begin{matrix}\text{upper}\\ \text{function}\end{matrix}\right) - \left(\begin{matrix}\text{lower}\\ \text{function}\end{matrix}\right) dx$$

$$= \int_0^2 x + 1 - xe^{-x^2}\, dx$$

$$= \left(\frac{1}{2}x^2 + x + \frac{1}{2}e^{-x^2}\right)\Bigg|_0^2$$

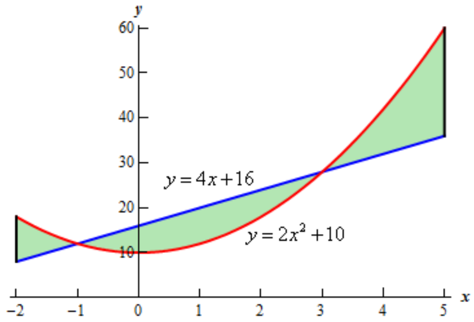
$$= \frac{7}{2} + \frac{e^{-4}}{2} = 3.5092$$

3.



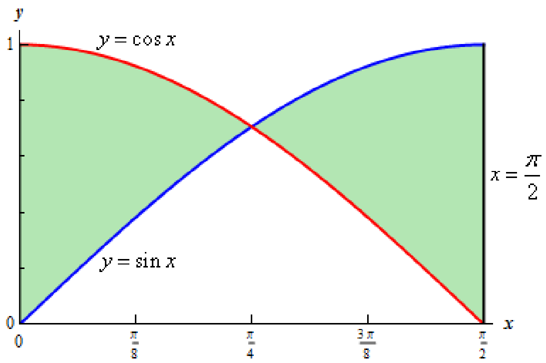
$$A = \int_a^b \begin{pmatrix} \text{upper} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{lower} \\ \text{function} \end{pmatrix} dx$$
$$= \int_{-1}^{3} 4x + 16 - \left(2x^2 + 10\right) dx$$
$$= \int_{-1}^{3} -2x^2 + 4x + 6\, dx$$
$$= \left(-\frac{2}{3}x^3 + 2x^2 + 6x\right)\Big|_{-1}^{3}$$
$$= \frac{64}{3}$$

4.



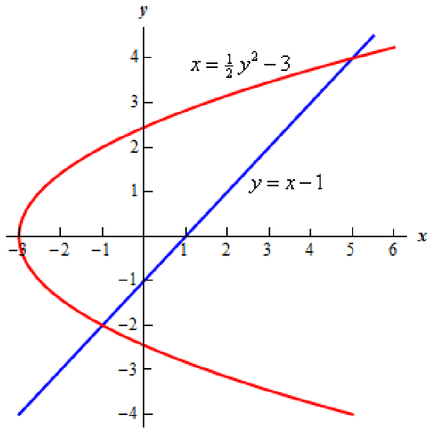
$$A = \int_{-2}^{-1} 2x^2 + 10 - (4x + 16)\, dx + \int_{-1}^{3} 4x + 16 - \left(2x^2 + 10\right) dx + \int_{3}^{5} 2x^2 + 10 - (4x + 16)\, dx$$
$$= \int_{-2}^{-1} 2x^2 - 4x - 6\, dx + \int_{-1}^{3} -2x^2 + 4x + 6\, dx + \int_{3}^{5} 2x^2 - 4x - 6\, dx$$
$$= \left(\frac{2}{3}x^3 - 2x^2 - 6x\right)\Big|_{-2}^{-1} + \left(-\frac{2}{3}x^3 + 2x^2 + 6x\right)\Big|_{-1}^{3} + \left(\frac{2}{3}x^3 - 2x^2 - 6x\right)\Big|_{3}^{5}$$
$$= \frac{14}{3} + \frac{64}{3} + \frac{64}{3}$$
$$= \frac{142}{3}$$

5.



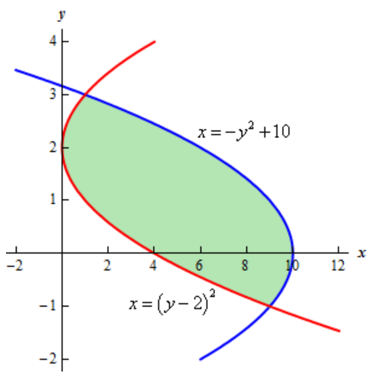
$$A = \int_0^{\frac{\pi}{4}} \cos x - \sin x\, dx + \int_{\pi/4}^{\pi/2} \sin x - \cos x\, dx$$
$$= \left(\sin x + \cos x\right)\Big|_0^{\frac{\pi}{4}} + \left(-\cos x - \sin x\right)\Big|_{\pi/4}^{\pi/2}$$
$$= \sqrt{2} - 1 + \left(\sqrt{2} - 1\right)$$
$$= 2\sqrt{2} - 2 = 0.828427$$

6.



$$A=\int_{-3}^{-1}\sqrt{2x+6}-\left(-\sqrt{2x+6}\right)dx+\int_{-1}^{5}\sqrt{2x+6}-(x-1)dx$$
$$=\int_{-3}^{-1}2\sqrt{2x+6}\,dx+\int_{-1}^{5}\sqrt{2x+6}-x+1dx$$
$$=\int_{-3}^{-1}2\sqrt{2x+6}\,dx+\int_{-1}^{5}\sqrt{2x+6}\,dx+\int_{-1}^{5}-x+1dx$$
$$=\frac{2}{3}u^{\frac{3}{2}}\Big|_0^4+\frac{1}{3}u^{\frac{3}{2}}\Big|_4^{16}+\left(-\frac{1}{2}x^2+x\right)\Big|_{-1}^{5}$$
$$=18$$

7.



$$A=\int_c^d\left(\begin{array}{c}\text{right}\\\text{function}\end{array}\right)-\left(\begin{array}{c}\text{left}\\\text{function}\end{array}\right)dy$$
$$=\int_{-1}^{3}-y^2+10-(y-2)^2\,dy$$
$$=\int_{-1}^{3}-2y^2+4y+6\,dy$$
$$=\left(-\frac{2}{3}y^3+2y^2+6y\right)\Big|_{-1}^{3}=\frac{64}{3}$$

TRY YOURSELF

FIND THE FOLLOWING INTEGRALS CALCULATIONS

1. **$xy=c^2$** পরাবৃত্ত x- অক্ষ এবং $x = a$ ও $x = b$ রেখাদ্বয় দ্বারা সীমাবদ্ধ অঞ্চলের ক্ষেত্রফল নির্ণয় কর।

2. প্রমাণ কর যে, $\sqrt{x}+\sqrt{y}=\sqrt{a}$ রেখা দ্বয় এবং অক্ষ রেখাগুলি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $\frac{a^2}{6}$ ।

3. $y^2=2x$ পরাবৃত্ত হইতে $y = 4x - 1$ সরলরেখার দ্বারা কর্তিত অংশের ক্ষেত্রফল নির্ণয় কর।

4. $y^2=4ax$ এবং $x^2=4ay$ পরাবৃত্ত দুইটি দ্বারা সীমাবদ্ধ এলাকার ক্ষেত্রফল নির্ণয় কর।

5. $y^2=4a(x+a)$ এবং $y^2=-4a(x-a)$ পরাবৃত্ত দুইটি দ্বারা সীমাবদ্ধ এলাকার ক্ষেত্রফল নির্ণয় কর।

6. $x^2+y^2=1$ বৃত্তটির যে অংশটুকু $y^2=1-x$ পরাবৃত্তের মধ্যে অবস্থিত উহার ক্ষেত্রফল নির্ণয় কর।

7 . $y^2=ax$ পরাবৃত্ত এবং $x^2+y^2=2ax$ বৃত্ত দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

8. $y^2(2a-x)=x^3$ বক্ররেখা এবং ইহার অসীমতটরেখা দ্বারা সীমাবদ্ধ ক্ষেত্রের ক্ষেত্রফল নির্ণয় কর।

Ans:

1. $c^2ln\left(\frac{b}{a}\right)$ বর্গ একক। **2.** $\frac{a^2}{6}$ বর্গ একক। **3.** $\frac{9}{32}$ বর্গ একক। **4.**$\frac{16}{3}a^2$ বর্গ একক।
5. $\frac{16}{3}a^2$ বর্গ একক।**6.** $\left(\frac{4}{3}+\frac{\pi}{2}\right)$ বর্গ একক।7.$a^2\left(\frac{\pi}{2}-\frac{4}{3}\right)$ বর্গ একক।**8.** $3\pi a^2$ বর্গ একক।