5. $\int \frac{5x^4+4x^2+3}{\sqrt[3]{x}}dx = \int \frac{5x^4+4x^2+3}{x^{\frac{1}{3}}}dx$

$= \int \left(5.x^{4-\frac{1}{3}} + 4x^{2-\frac{1}{3}} + 3x^{-\frac{1}{3}}\right)dx$

$= \int \left(5x^{\frac{11}{3}} + 4x^{\frac{5}{3}} + 3x^{-\frac{1}{3}}\right)dx$

$= 5.\frac{x^{\frac{11}{3}+1}}{\frac{11}{3}+1} + 4.\frac{x^{\frac{5}{3}+1}}{\frac{5}{3}+1} + 3.\frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c$

$= 5.\frac{x^{\frac{14}{3}}}{\frac{14}{3}} + 4.\frac{x^{\frac{8}{3}}}{\frac{8}{3}} + 3.\frac{x^{\frac{2}{3}}}{\frac{2}{3}} + c$

$= \frac{15}{14}x^{\frac{14}{3}} + \frac{3}{2}x^{\frac{8}{3}} + \frac{9}{2}x^{\frac{2}{3}} + c$ **(Ans.)**

6. $\int \sin^2\frac{\theta}{2}d\theta = \int \frac{1}{2}.2\sin^2\frac{\theta}{2}d\theta$

$= \int \frac{1}{2}(1-\cos\theta)\,d\theta$

$= \frac{1}{2}(\theta - \sin\theta) + c$ **(Ans.)**

7. $\int (3^x + e^x)\,dx = \int 3^x dx + \int e^x dx$

$= \frac{3^x}{\ln 3} + e^x + c$ **(Ans.)**

8. $\int \sec x(\sec x + \tan x)\,dx$

$= \int (\sec^2x + \sec x\tan x)\,dx$

$= \int \sec^2x\,dx + \int \sec x\tan x\,dx$

$= \tan x + \sec x + c$ **(Ans.)**

9.(i) $\int \frac{dx}{1+\cos 2x} = \int \frac{dx}{2\cos^2x}$

$= \frac{1}{2}\int \sec^2x dx = \frac{1}{2}\tan x + c$ **(Ans.)**

(ii) $\int \sqrt{1-\cos 2x}\,dx = \int \sqrt{2\sin^2x}\,dx$

$= \sqrt{2}\int \sin x\,dx = -\sqrt{2}\cos x + c$ **(Ans.)**

10. $\int \sqrt{1+\sin 2x}\,dx$

$= \int \sqrt{\sin^2x + \cos^2x + 2\sin x\cos x}\,dx$

$= \int \sqrt{(\sin x + \cos x)^2}\,dx$

$= \int (\sin x + \cos x)\,dx = -\cos x + \sin x + c$

$= \sin x - \cos x + c$ **(Ans.)**

11. $\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}dx$

$= \int \frac{\sin x + \cos x}{\sqrt{\sin^2x + \cos^2x + 2\sin x\cos x}}dx$

$= \int \frac{\sin x + \cos x}{\sqrt{(\sin x + \cos x)^2}}dx$

$= \int \frac{\sin x + \cos x}{\sin x + \cos x}dx$

$= \int 1.\,dx = x + c$ **(Ans.)**

12.(i) $\int \tan^2x\,dx$

$= \int (\sec^2x - 1)\,dx \quad [\because \sec^2x = 1 + \tan^2x]$

$= \int \sec^2x\,dx - \int dx$

$= \tan x - x + c$ **(Ans.)**

(ii) $\int \frac{1}{1+\sin x}dx = \int \frac{(1-\sin x)}{(1+\sin x)(1-\sin x)}dx$

$= \int \frac{1-\sin x}{(1-\sin^2x)}dx$

$= \int \frac{1-\sin x}{\cos^2x}dx$

$= \int \left(\frac{1}{\cos^2x} - \frac{\sin x}{\cos^2x}\right)dx$

$= \int \sec^2x\,dx - \int \sec x\tan x\,dx$

$= \tan x - \sec x + c$ **(Ans.)**

13. $\int \frac{dx}{1-\sin x} = \int \frac{(1+\sin x)\,dx}{(1-\sin x)(1+\sin x)}$

$= \int \frac{1+\sin x}{1-\sin^2x}dx$

$= \int \frac{1+\sin x}{\cos^2x}dx$

$= \int \left(\frac{1}{\cos^2x} + \frac{\sin x}{\cos^2x}\right)dx$

$= \int \sec^2x\,dx + \int \sec x\tan x\,dx$

$= \tan x + \sec x + c$ **(Ans.)**

14. $\int \frac{\cos\theta}{\sin^2\theta}d\theta = \int \frac{\cos\theta}{\sin\theta}.\frac{1}{\sin\theta}d\theta$

$= \int \cot\theta.\mathrm{cosec}\theta\,d\theta$

$= -\mathrm{cosec}\theta + c$ **(Ans.)**

15. $\int \frac{a\sin^3\theta + b\cos^3\theta}{\sin^2\theta\cos^2\theta}d\theta$

$= \int \left(\frac{a\sin\theta}{\cos^2\theta} + \frac{b\cos\theta}{\sin^2\theta}\right)d\theta$

$= \int (a\sec\theta\tan\theta + b\,\mathrm{cosec}\theta\cot\theta)\,d\theta$

$= a\sec\theta - b\,\mathrm{cosec}\theta + c$ **(Ans.)**

অনুশীলনী-10(B) এর সমাধান

1.(i) $\int \frac{dx}{\sqrt{16-9x}} = \int (16-9x)^{-\frac{1}{2}} dx$

$= \frac{(16-9x)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \cdot \frac{-1}{9} + c$

$= \frac{(16-9x)^{\frac{1}{2}}}{\frac{1}{2}} \cdot \frac{-1}{9} + c$

$= -\frac{2}{9}\sqrt{16-9x} + c$ **(Ans.)**

(ii) $\int \frac{dx}{\sqrt[3]{(2x-3)^2}} = \int (2x-3)^{-\frac{2}{3}} dx$

$= \frac{(2x-3)^{-\frac{2}{3}+1}}{\left(-\frac{2}{3}+1\right).2} + c$

$= \frac{1}{2} \cdot \frac{(2x-3)^{\frac{1}{3}}}{\frac{1}{3}} + c$

$= \frac{3}{2}\sqrt[3]{2x-3} + c$ **(Ans.)**

(iii) $\int \frac{dx}{\sqrt[3]{3x-5}} = \int (3x-5)^{-\frac{1}{3}} dx$

$= \frac{(3x-5)^{-\frac{1}{3}+1}}{\left(-\frac{1}{3}+1\right).3} + c$

$= \frac{(3x-5)^{\frac{2}{3}}}{\frac{2}{3}.3} + c$

$= \frac{1}{2}(3x-5)^{\frac{2}{3}} + c$ **(Ans.)**

2. $\int \left(\frac{2}{x-3} + \frac{3}{5-x}\right) dx = \int \frac{2\,dx}{x-3} + \int \frac{3\,dx}{5-x}$

$= 2\ln(x-3) + 3\ln(5-x)(-1) + c$

$= \ln(x-3)^2 - \ln(5-x)^3 + c$

$= \ln\left|\frac{(x-3)^2}{(5-x)^3}\right| + c$ **(Ans.)**

3. $\int e^{5-\frac{x}{2}} dx = \frac{e^{5-\frac{x}{2}}}{1} \cdot \frac{1}{-\frac{1}{2}} + c = -2e^{5-\frac{x}{2}} + c$ **(Ans.)**

4. $\int \frac{(e^x+1)^2}{\sqrt{e^x}} dx = \int \frac{e^{2x}+2e^x+1}{e^{\frac{x}{2}}} dx$

$= \int \left(e^{\frac{3x}{2}} + 2e^{\frac{x}{2}} + e^{-\frac{x}{2}}\right) dx$

$= \frac{2}{3}e^{\frac{3x}{2}} + 4e^{\frac{x}{2}} - 2e^{-\frac{x}{2}} + c$ **(Ans.)**

5. $\int \frac{dx}{\sqrt{x+2}-\sqrt{x}}$

$= \int \frac{(\sqrt{x+2}+\sqrt{x})}{(\sqrt{x+2}-\sqrt{x})(\sqrt{x+2}+\sqrt{x})} dx$

$= \int \frac{\sqrt{x+2}+\sqrt{x}}{x+2-x} dx$

$= \frac{1}{2}\left[\int (x+2)^{\frac{1}{2}} dx + \int x^{\frac{1}{2}} dx\right]$

$= \frac{1}{2} \cdot \frac{(x+2)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{1}{2} \cdot \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$

$= \frac{1}{3} \cdot \left[\left((x+2)^{\frac{3}{2}} + x^{\frac{3}{2}}\right)\right] + c$ **(Ans.)**

6.(i) $\int \frac{x}{\sqrt{x+3}} dx = \int \frac{x+3-3}{\sqrt{x+3}} dx$

$= \int \left(\frac{x+3}{\sqrt{x+3}} - \frac{3}{\sqrt{x+3}}\right) dx$

$= \int \left(\sqrt{x+3} - \frac{3}{\sqrt{x+3}}\right) dx$

$= \frac{(x+3)^{\frac{3}{2}}}{\frac{3}{2}} - 3\,\frac{(x+3)^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= \frac{2}{3}(x+3)^{\frac{3}{2}} - 6\sqrt{x+3} + c$ **(Ans.)**

(ii) $\int \frac{x}{\sqrt{1-x}} dx = \int \frac{1-(1-x)}{\sqrt{1-x}} dx$

$= \int \left\{\frac{1}{\sqrt{1-x}} - \sqrt{1-x}\right\} dx = \int \frac{dx}{\sqrt{1-x}} - \int \sqrt{1-x}\, dx$

$= 2\sqrt{1-x}\,(-1) - \frac{2}{3}(1-x)^{\frac{3}{2}}(-1) + c$

$= \frac{2}{3}(1-x)^{\frac{3}{2}} - 2\sqrt{1-x} + c$ **(Ans.)**

7. $\int\sqrt{1-\cos 4x}\,dx = \int\sqrt{2\sin^2 2x}\,dx$
$= \int\sqrt{2}\sin 2x\,dx$
$= \sqrt{2}\left(\frac{-\cos 2x}{2}\right) + c$
$= -\frac{\cos 2x}{\sqrt{2}} + c$ **(Ans.)**

8.(i) $\int\frac{dx}{1+\cos x} = \int\frac{dx}{2\cos^2\frac{x}{2}}$
$= \frac{1}{2}\int\sec^2\frac{x}{2}\,dx$
$= \frac{1}{2}\,\frac{\tan\frac{x}{2}}{\frac{1}{2}} + c$
$= \tan\frac{x}{2} + c$ **(Ans.)**

(ii) $\int\frac{dx}{1-\cos x} = \int\frac{dx}{2\sin^2\frac{x}{2}}$
$= \frac{1}{2}\int\operatorname{cosec}^2\frac{x}{2}\,dx = -\frac{1}{2}\,\frac{\cot\frac{x}{2}}{\frac{1}{2}} + c$
$= -\cot\frac{x}{2} + c$ **(Ans.)**

9. $\int\cos^3 3\theta\,d\theta = \int\frac{1}{4}(\cos 9\theta + 3\cos 3\theta)\,d\theta$
$= \frac{1}{4}\left(\frac{\sin 9\theta}{9} + 3.\frac{1}{3}\sin 3\theta\right) + c$
$= \frac{1}{36}\sin 9\theta + \frac{1}{4}\sin 3\theta + c$ **(Ans.)**

10. $\int\sin^2 x\cos^2 x\,dx = \frac{1}{4}\int 4\sin^2 x\cos^2 x\,dx$
$= \frac{1}{4}\int(2\sin x\cos x)^2\,dx$
$= \frac{1}{4}\int(\sin 2x)^2 dx$
$= \frac{1}{4}\int\sin^2 2x\,dx$
$= \frac{1}{4}\cdot\frac{1}{2}\int 2\sin^2 2x\,dx$
$= \frac{1}{4}\cdot\frac{1}{2}\int(1-\cos 4x)\,dx$
$= \frac{1}{8}[x - \frac{1}{4}\sin 4x] + c$ **(Ans.)**

11. $\int\cos^4 x\,dx = \frac{1}{4}\int(2\cos^2 x)^2\,dx$
$= \frac{1}{4}\int(1+\cos 2x)^2\,dx$
$= \frac{1}{4}\int(1 + 2\cos 2x + \cos^2 2x)\,dx$
$= \frac{1}{4}\left[x + 2.\frac{1}{2}\sin 2x + \frac{1}{2}\int 2\cos^2 2x\,dx\right]$
$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\int(1+\cos 4x)\,dx\right]$
$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{4}\sin 4x\right] + c$
$= \frac{1}{4}\left[\frac{3x}{2} + \sin 2x + \frac{1}{8}\sin 4x\right] + c$ **(Ans.)**

12. $\int\sin x^0\,dx$
$= \int\sin\left(\frac{x\pi}{180}\right)dx \qquad \left|\because 1^\circ = \frac{\pi}{180}\right.$ রেডিয়ান
$= -\frac{180}{\pi}\cos\frac{\pi x}{180} + c$ **(Ans.)**

13.(i) $\int 5\cos 4x\sin 3x\,dx = \frac{5}{2}\int 2\sin 3x\cos 4x\,dx$
$= \frac{5}{2}\int(\sin 7x - \sin x)\,dx$
$= \frac{5}{2}\left(-\frac{1}{7}\cos 7x + \cos x\right) + c$
$= \frac{5}{14}(7\cos x - \cos 7x) + c$ (Ans.)

(ii) $\int\sin 2x\sin 4x\,dx = \frac{1}{2}\int 2\sin 2x\sin 4x\,dx$
$= \frac{1}{2}\int\{\cos(2x-4x) - \cos(2x+4x)\}\,dx$
$= \frac{1}{2}\int(\cos 2x - \cos 6x)\,dx$
$= \frac{1}{2}\left[\frac{1}{2}\sin 2x - \frac{1}{6}\sin 6x\right] + c$ **(Ans.)**

(iii) $\int\sin 5x\sin 3x\,dx = \int\frac{1}{2}.2\sin 5x\sin 3x\,dx$
$= \frac{1}{2}(\cos 2x - \cos 8x)\,dx$
$= \frac{1}{2}\left(\frac{\sin 2x}{2} - \frac{\sin 8x}{8}\right) + c$
$= \frac{1}{4}\sin 2x - \frac{1}{16}\sin 8x + c$ (Ans.)

(iv) $\int\cos 2\theta\cos\theta\,d\theta = \int(2\cos^2\theta - 1)\cos\theta\,d\theta$
$= \int(2\cos^3\theta - \cos\theta)\,d\theta$
$= \int\left\{\frac{1}{2}(\cos 3\theta + 3\cos\theta) - \cos\theta\right\}d\theta$

$= \int \left(\frac{1}{2}\cos 3\theta + \frac{3}{2}\cos\theta - \cos\theta\right) d\theta$

$= \int \left(\frac{1}{2}\cos 3\theta + \frac{1}{2}\cos\theta\right) d\theta$

$= \frac{1}{2}\left(\frac{\sin 3\theta}{3} + \sin\theta\right) + c$

$= \frac{1}{6}\sin 3\theta + \frac{1}{2}\sin\theta + c$ **(Ans.)**

14.(i) $\int \sin^2 x \cos 2x\, dx = \frac{1}{2}\int (2\sin^2 x)\cos 2x\, dx$

$= \frac{1}{2}\int (1 - \cos 2x)\cos 2x\, dx$

$= \frac{1}{2}\int (\cos 2x - \cos^2 2x)\, dx$

$= \frac{1}{2}\int \cos 2x\, dx - \frac{1}{4}\int 2\cos^2 2x\, dx$

$= \frac{1}{2}\int \cos 2x\, dx - \frac{1}{4}\int (1 + \cos 4x)\, dx$

$= \frac{1}{2}\int \cos 2x\, dx - \frac{1}{4}\int dx - \frac{1}{4}\int \cos 4x\, dx$

$= \frac{\sin 2x}{4} - \frac{1}{4}x - \frac{\sin 4x}{16} + c$ **(Ans.)**

(ii) $\int (2\cos x + \sin x)\cos x\, dx$

$= \int 2\cos^2 x\, dx + \int \sin x \cos x\, dx$

$= \int (1 + \cos 2x)\, dx + \frac{1}{2}\int 2\sin x\cos x\, dx$

$= \int dx + \int \cos 2x\, dx + \frac{1}{2}\int \sin 2x\, dx$

$= x + \frac{1}{2}\sin 2x - \frac{1}{4}\cos 2x + c$ **(Ans.)**

15.(i) $\int x e^{x^2} dx$

ধরি, $x^2 = z$

বা, $2x dx = dz$

$\therefore x dx = \frac{1}{2} dz$

$= \frac{1}{2}\int e^z dz$

$= \frac{1}{2} e^z + c$

$= \frac{1}{2} e^{x^2} + c$ **(Ans.)**

(ii) $\int (2x + 3)\sqrt{x^2 + 3x}\, dx$

ধরি, $x^2 + 3x = z$, $\therefore (2x + 3)\, dx = dz$

$= \int \sqrt{z}\, dz$

$= \int z^{\frac{1}{2}} dz = \frac{z^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \frac{2}{3}(x^2 + 3x)^{\frac{3}{2}} + c$

$= \frac{2}{3}\sqrt{(x^2 + 3x)^3} + c$ **(Ans.)**

(iii) $\int \left(e^x + \frac{1}{x}\right)(e^x + \ln x)\, dx$

ধরি, $e^x + \ln x = z$

$\therefore \left(e^x + \frac{1}{x}\right) dx = dz$

$= \int z\, dz$

$= \frac{z^2}{2} + c$

$= \frac{1}{2}(e^x + \ln x)^2 + c$ **(Ans.)**

(iv) $\int \sec^2 x\, e^{\tan x} dx$

ধরি, $\tan x = z$

$\therefore \sec^2 x\, dx = dz$

$= \int e^z dz$

$= e^z + c$

$= e^{\tan x} + c$ **(Ans.)**

(v) $\int \frac{e^{a\sin^{-1}x}}{\sqrt{1 - x^2}} dx$

ধরি, $\sin^{-1}x = z$

$\therefore \frac{dx}{\sqrt{1 - x^2}} = dz$

$= \int e^{az} dz$

$= \frac{1}{a} e^{az} + c$

$= \frac{1}{a} e^{a\sin^{-1}x} + c$ **(Ans.)**

(vi) $\int \frac{e^{a\tan^{-1}x}}{1 + x^2} dx$

ধরি, $\tan^{-1}x = z$

$\therefore \frac{1}{1 + x^2} dx = dz$

$= \int e^{az} dz$

$= \frac{1}{a} e^{az} + c$

$= \frac{1}{a} e^{a\tan^{-1}x} + c$ **(Ans.)**

(vii) $\int \cos x\, e^{\sin x} dx$

ধরি, $\sin x = y$

$\therefore \cos x\, dx = dy$

$= \int e^y dy$

$= e^y + c$

$= e^{\sin x} + c$ **(Ans.)**

(viii) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx$

ধরি, $\sqrt{x} = z$

$\therefore \frac{1}{\sqrt{x}} dx = 2dz$

$= \int 2e^z dz$

$= 2e^z + c$

$= 2e^{\sqrt{x}} + c$ **(Ans.)**

(ix) $\int \frac{dx}{x(1 + \ln x)}$

ধরি, $1 + \ln x = z$

$\therefore \frac{1}{x} dx = dz$

$= \int \frac{dz}{z}$

$= \ln z + c$

$= \ln|1 + \ln x| + c$ **(Ans.)**

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(x) $\int \frac{dx}{x(\ln x)^2}$

$= \int \frac{dz}{z^2}$ | ধরি, $\ln x = z$ ∴ $\frac{1}{x}dx = dz$

$= \frac{z^{-2+1}}{-2+1} + c$

$= -\frac{1}{z} + c$

$= \frac{-1}{\ln|x|} + c$ **(Ans.)**

(xi) $\int \frac{dx}{x(1+\ln x)^2}$ | ধরি, $1 + \ln x = z$ ∴ $\frac{1}{x}dx = dz$

$= \int \frac{dz}{z^2}$

$= \int z^{-2}\, dz$

$= \frac{z^{-2+1}}{-2+1} + c$

$= -z^{-1} + c$

$= -\frac{1}{z} + c$

$= -\frac{1}{1+\ell n\, x} + c$ **(Ans.)**

(xii) $\int (1+\cos\theta)^3.\sin\theta\, d\theta$

$= -\int z^3 dz$ | ধরি, $1 + \cos\theta = z$ ∴ $\sin\theta d\theta = -dz$

$= -\frac{z^4}{4} + c$

$= -\frac{1}{4}(1+\cos\theta)^4 + c$ **(Ans.)**

(xiii) $\int \frac{\cos x}{\sqrt{\sin x}} dx$ | ধরি, $z = \sin x$ ∴ $dz = \cos x\, dx$

$= \int \frac{dz}{\sqrt{z}}$

$= \int z^{-\frac{1}{2}} dz$

$= \frac{z^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2\sqrt{z} + c$

$= 2\sqrt{\sin x} + c$ **(Ans.)**

(xiv) $\int \frac{1+\cos x}{x+\sin x} dx$ | ধরি, $x + \sin x = z$ ∴ $(1+\cos x)\, dx = dz$

$= \int \frac{dz}{z} = \ell n\, |z| + c$

$= \ell n\, |x + \sin x| + c$ **(Ans.)**

(xv) $\int \frac{1+\cos x}{\sqrt[3]{x+\sin x}} dx$ | ধরি, $x + \sin x = z$ ∴ $(1+\cos x)\, dx = dz$

$= \int \frac{dz}{\sqrt[3]{z}}$

$= \frac{z^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c = \frac{z^{\frac{2}{3}}}{\frac{2}{3}} + c$

$= \frac{3}{2} z^{\frac{2}{3}} + c$

$= \frac{3}{2}(x+\sin x)^{\frac{2}{3}} + c$ **(Ans.)**

(xvi) $\int \frac{\sin x\, dx}{3+4\cos x}$ | ধরি, $3 + 4\cos x = z$ ∴ $4(-\sin x)\, dx = dz$ ∴ $\sin x\, dx = \frac{-dz}{4}$

$= \int \frac{\frac{-dz}{4}}{z}$

$= -\frac{1}{4}\int \frac{1}{z}\, dz$

$= -\frac{1}{4}\ln z + c$

$= -\frac{1}{4}\ln|3+4\cos x| + c$ **(Ans.)**

(xvii) $\int \frac{\cos x}{(1-\sin x)^2} dx$ | ধরি, $1 - \sin x = y$ ∴ $-\cos x\, dx = dy$ ∴ $\cos x\, dx = -dy$

$= \int \frac{-dy}{y^2}$

$= \frac{1}{y} + c$

$= \frac{1}{1-\sin x} + c$ **(Ans.)**

(xviii) $\int \frac{\sin(2+5\ln x)}{x} dx$ | ধরি, $2 + 5\ln x = z$ ∴ $5.\frac{1}{x}\, dx = dz$ ∴ $\frac{dx}{x} = \frac{dz}{5}$

$= \int \sin z \frac{dz}{5}$

$= -\frac{1}{5}\cos z + c$

$= -\frac{1}{5}\cos(2+5\ln x) + c$ **(Ans.)**

(xix) $\int \frac{\cos\sqrt{x}}{\sqrt{x}} dx$ | ধরি, $\sqrt{x} = z$ ∴ $\frac{1}{\sqrt{x}}\, dx = 2dz$

$= \int 2\cos z\, dz$

$= 2\sin z + c$

$= 2\sin\sqrt{x} + c$ **(Ans.)**

(xx) $\int \frac{\sec^2\theta}{2+4\tan\theta} d\theta$

$= \int \frac{1}{4} \cdot \frac{1}{z} dz$

$= \frac{1}{4} \ln z + c$

$= \frac{1}{4} \ln|2+4\tan\theta| + c$ **(Ans.)**

ধরি, $2 + 4\tan\theta = z$

$\therefore 4\sec^2\theta \, d\theta = dz$

$\therefore \sec^2\theta \, d\theta = \frac{1}{4} dz$

(xxi) $\int \frac{\tan x}{\ln(\cos x)} dx$

$= -\int \frac{dz}{z}$

$= -\ln z + c$

$= -\ln[\ln|\cos x|] + c$ **(Ans.)**

ধরি, $\ell n(\cos x) = z$

$\therefore \frac{1}{\cos x}(-\sin x)dx = dz$

$\therefore \tan x \, dx = -dz$

(xxii) $\int e^{2x} \tan e^{2x} \sec e^{2x} dx$

$= \int \tan z \sec z . \frac{dz}{2}$

$= \frac{1}{2} \int \sec z \tan z \, dz$

$= \frac{1}{2} \sec z + c$

$= \frac{1}{2} \sec e^{2x} + c$ **(Ans.)**

ধরি, $e^{2x} = z$

$\therefore 2e^{2x} dx = dz$

$\therefore e^{2x} dx = \frac{dz}{2}$

(xxiii) $\int \frac{\sin\left(\frac{1}{x}\right)}{x^2} dx$

$= \int \sin z(-dz)$

$= -\int \sin z \, dz$

$= -(-\cos z) + c$

$= \cos z + c$

$= \cos\left(\frac{1}{x}\right) + c$ **(Ans.)**

ধরি, $\frac{1}{x} = z$

$\therefore -\frac{1}{x^2} dx = dz$

$\therefore \frac{dx}{x^2} = -dz$

(xxiv) $\int \frac{x dx}{\sqrt{16-x^2}}$

$= -\frac{1}{2} \int \frac{dz}{\sqrt{z}}$

$= -\sqrt{z} + c$

$= -\sqrt{16-x^2} + c$ **(Ans.)**

ধরি, $16 - x^2 = z$

$\therefore -2x \, dx = dz$

বা, $x dx = -\frac{1}{2} dz$

(xxv) $\int \frac{2x \tan^{-1}x^2}{1+x^4} dx$

$= \int z \, dz$

$= \frac{z^2}{2} + c$ [যেখানে, c যোগজীকরণ ধ্রুবক]

$= \frac{1}{2}(\tan^{-1}x^2)^2 + c$ **(Ans.)**

ধরি, $\tan^{-1}x^2 = z$

$\therefore \frac{2x}{1+x^4} dx = dz$

16.(i) $\int \frac{dx}{e^x+1} = \int \frac{e^{-x}}{1+e^{-x}} dx$

[লব ও হরকে e^{-x} দ্বারা গুণ করে]

মনে করি, $1 + e^{-x} = z$

$\therefore -e^{-x}dx = dz$ বা, $e^{-x} dx = -dz.$

$\therefore \int \frac{dx}{e^x+1} = -\int \frac{dz}{z}$

$= -\ln|z| + c$

$= -\ln|1+e^{-x}| + c$

(ii) $\int \frac{e^x}{1+e^{2x}} dx$

$= \int \frac{dz}{1+z^2}$

$= \tan^{-1}z + c$

$= \tan^{-1}(e^x) + c$ **(Ans.)**

ধরি, $z = e^x$

$\therefore dz = e^x dx$

(iii) $\int \frac{e^{5x}+e^{3x}}{e^x+e^{-x}} dx$

$= \int \frac{e^{4x}(e^x+e^{-x})}{(e^x+e^{-x})} dx$

$= \int e^{4x} dx$

$= \frac{e^{4x}}{4} + c$ **(Ans.)**

(iv) $\int \frac{e^x-1}{e^x+1} dx$

$= \int \frac{(e^x+1)-2}{e^x+1} dx$

$= \int \frac{e^x+1}{e^x+1} dx - \int \frac{2}{e^x+1} dx$

$= \int dx - 2\int \frac{e^{-x}}{1+e^{-x}} dx$

$= x + 2\int \frac{dz}{z}$

$= x + 2\ln z + c$

$= x + 2\ln(1+e^{-x}) + c$ **(Ans)**

ধরি, $1 + e^{-x} = z$

বা, $-e^{-x} dx = dz$

$\therefore e^{-x} dx = -dz$

17.(i) $\int \frac{dx}{(1+x^2)\tan^{-1}x}$

$= \int \frac{1}{z} dz$

$= \ln z + c$

$= \ln|\tan^{-1}x| + c$ **(Ans.)**

ধরি, $\tan^{-1}x = z$

$\therefore \frac{1}{1+x^2} dx = dz$

(ii) $\int \frac{dx}{(1+x^2)\sqrt{\tan^{-1}x+3}}$

$= \int \frac{dz}{\sqrt{z}}$

$= 2\sqrt{z} + c$

$= 2\sqrt{\tan^{-1}x+3} + c$ **(Ans.)**

ধরি, $\tan^{-1}x + 3 = z$

$\therefore \frac{1}{1+x^2} dx = dz$

(iii) $\int \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$

ধরি, $\sin^{-1}x = z$

$\therefore \frac{1}{\sqrt{1+x^2}}dx = dz$

$= \int z\,dz = \frac{1}{2}z^2 + c$

$= \frac{1}{2}(\sin^{-1}x)^2 + c$ **(Ans.)**

(iv) $\int \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx$

$= \int z^2\,dz$

ধরি, $\sin^{-1}x = z$

$\therefore \frac{1}{\sqrt{1+x^2}}dx = dz$

$= \frac{1}{3}z^3 + c$

$= \frac{1}{3}(\sin^{-1}x)^3 + c$ **(Ans.)**

(v) $\int \frac{2t\sin^{-1}t^2}{\sqrt{1-t^4}}dt$

ধরি, $\sin^{-1}t^2 = z$

$\therefore \frac{2t}{\sqrt{1-t^4}}dt = dz$

$= \int z\,dz$

$= \frac{1}{2}z^2 + c$

$= \frac{1}{2}(\sin^{-1}t^2)^2 + c$ **(Ans.)**

(vi) মনে করি, $\sec^{-1}x = z$

$\therefore \frac{dx}{x\sqrt{x^2-1}} = dz.$

$\therefore \int \frac{(\sec^{-1}x)^4}{x\sqrt{x^2-1}}dx = \int z^4\,dz = \frac{1}{5}z^5 + c$

$= \frac{1}{5}(\sec^{-1}x)^5 + c.$

18. $\int \frac{1-\tan\theta}{1+\tan\theta}d\theta = \int \frac{1-\frac{\sin\theta}{\cos\theta}}{1+\frac{\sin\theta}{\cos\theta}}d\theta$

$= \int \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}d\theta$

ধরি, $\cos\theta + \sin\theta = z$

$\therefore (-\sin\theta + \cos\theta)\,d\theta = dz$

বা, $(\cos\theta - \sin\theta)\,d\theta = dz$

$= \int \frac{dz}{z}$

$= \ln z + c$

$= \ln|\cos\theta + \sin\theta| + c$ **(Ans.)**

19. $\int \sqrt{1+\sin\theta}\,d\theta$

$= \int \sqrt{\sin^2{}^{\theta}/_2 + \cos^2{}^{\theta}/_2 + 2\sin{}^{\theta}/_2\cos{}^{\theta}/_2}\,d\theta.$

$= \int \sqrt{(\sin{}^{\theta}/_2 + \cos{}^{\theta}/_2)^2}\,d\theta$

$= \int (\sin{}^{\theta}/_2 + \cos{}^{\theta}/_2)\,d\theta$

$= \int \sin{}^{\theta}/_2\,d\theta + \int \cos{}^{\theta}/_2\,d\theta$

$= -2\cos\frac{\theta}{2} + 2\sin\frac{\theta}{2} + c$

$= 2(\sin\frac{\theta}{2} - \cos\frac{\theta}{2}) + c$ **(Ans.)**

20. $\int \frac{1}{1+\tan x}dx = \int \frac{1}{1+\frac{\sin x}{\cos x}}dx$

$= \int \frac{\cos x\,dx}{\cos x + \sin x}$

$= \frac{1}{2}\int \frac{(\cos x + \sin x) + (\cos x - \sin x)}{\cos x + \sin x}dx$

$= \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{\cos x - \sin x}{\cos x + \sin x}dx$

$= \frac{1}{2}x + \frac{1}{2}\ln|\cos x + \sin x| + c$ **(Ans.)**

$[\because \int \frac{f'(x)}{f(x)}dx = \ln|f(x)| + c]$

21. $\int \frac{\tan(\sin^{-1}x)}{\sqrt{1-x^2}}dx$

ধরি, $\sin^{-1}x = z$

$\therefore \frac{1}{\sqrt{1-x^2}}dx = dz$

$= \int \tan z\,dz$

$= \ln|\sec z| + c$

$= \ln|\sec(\sin^{-1}x)| + c$ **(Ans.)**

22. $\int \frac{\sqrt{\tan x}}{\sin x\cos x}dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x}\cos^2 x}dx$

$= \int \frac{\sqrt{\tan x}}{\tan x\cos^2 x}dx$

$= \int \frac{\sec^2 x}{\sqrt{\tan x}}dx$

ধরি, $\tan x = z$

$\therefore \sec^2 x\,dx = dz$

$= \int \frac{dz}{\sqrt{z}} = 2\sqrt{z} + c$

$= 2\sqrt{\tan x} + c$ **(Ans.)**

23. $\int \cos^3 x\sqrt{\sin x}\,dx = \int \cos^2 x.\cos x\sqrt{\sin x}\,dx$

$= \int (1-\sin^2 x)\cos x\sqrt{\sin x}\,dx$

$= \int (1-z^2)\,z^{\frac{1}{2}}\,dz$

ধরি, $\sin x = z$

$\therefore \cos x\,dx = dz$

$= \int z^{1/2}\,dz - \int z^{\frac{5}{2}}\,dz$

$= \frac{z^{\frac{3}{2}}}{\frac{3}{2}} - \frac{z^{\frac{7}{2}}}{\frac{7}{2}} + c = \frac{2}{3}z^{\frac{3}{2}} - \frac{2}{7}z^{\frac{7}{2}} + c$

$= \frac{2}{3}(\sin x)^{\frac{3}{2}} - \frac{2}{7}(\sin x)^{\frac{7}{2}} + c$

$= \frac{2}{3}(\sqrt{\sin x})^3 - \frac{2}{7}(\sqrt{\sin x})^7 + c$ **(Ans.)**

24.(i) $\int \frac{\sec^4 x}{\sqrt{\tan x}}\,dx$

$= \int \frac{(1+\tan^2 x)\sec^2 x\,dx}{\sqrt{\tan x}}$

ধরি, $\tan x = z$
$\therefore \sec^2 x\,dx = dz$

$= \int \frac{(1+z^2)}{\sqrt{z}}\,dz$

$= \int \left(z^{-\frac{1}{2}} + z^{2-\frac{1}{2}}\right) dz$

$= \int \left(z^{-\frac{1}{2}} + z^{\frac{3}{2}}\right) dz$

$= \frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + \frac{z^{\frac{3}{2}+1}}{\frac{3}{2}+1} + c$

$= 2z^{\frac{1}{2}} + \frac{2}{5}z^{\frac{5}{2}} + c$

$= 2\sqrt{\tan x} + \frac{2}{5}\sqrt{(\tan x)^5} + c$ **(Ans.)**

(ii) $\int \sin^3 x \cos x\,dx$

ধরি, $\sin x = z$
$\therefore \cos x\,dx = dz$

$= \int z^3\,dz = \frac{1}{4}z^4 + c$

$= \frac{1}{4}\sin^4 x + c$ **(Ans.)**

25. $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$

ধরি, $x = z^6$
$\therefore dx = 6z^5 dz$
$\therefore x^{\frac{1}{3}} = z^2$

$= \int \frac{z^3.6z^5}{1+z^2}\,dz$

$= 6\int \frac{z^8}{1+z^2}\,dz$

$= 6\int \left(z^6 - z^4 + z^2 - z + \frac{1}{1+z^2}\right)dz$

$= 6\left[\int z^6 dz - \int z^4 dz + \int z^2 dz - \int dz + \tan^{-1} z\right]$

$= 6\left[\frac{z^7}{7} - \frac{z^5}{5} + \frac{z^3}{3} - z + \tan^{-1} z\right] + c$

$= 6\left[\frac{1}{7}x^{\frac{7}{6}} - \frac{1}{5}x^{\frac{5}{6}} + \frac{1}{3}\sqrt{x} - x^{\frac{1}{6}} + \tan^{-1} x^{\frac{1}{6}}\right] + c$ **(Ans.)**

26. $\int \frac{dx}{\sqrt{x}+x}$

$= \int \frac{dx}{\sqrt{x}(1+\sqrt{x})}$

ধরি, $1+\sqrt{x} = z$
$\therefore \frac{1}{\sqrt{x}}dx = 2dz$

$= \int \frac{2dz}{z}$

$= 2\ln z + c$

$= 2\ln|1+\sqrt{x}| + c$ **(Ans.)**

27. $\int \frac{dx}{x\sqrt{x^4-1}}$

$= \int \frac{dx}{x.x^2\sqrt{1-\frac{1}{x^4}}}$

ধরি, $\frac{1}{x^2} = z$
বা, $-2.\frac{1}{x^3}dx = dz$
$\therefore \frac{1}{x^3}dx = -\frac{1}{2}dz$

$= \int \frac{dx}{x^3\sqrt{1-\left(\frac{1}{x^2}\right)^2}}$

$= \int \frac{-dz}{2\sqrt{1-z^2}}$

$= \frac{1}{2}\cos^{-1} z + c = \frac{1}{2}\cos^{-1}\left(\frac{1}{x^2}\right) + c$

$= \frac{1}{2}\sec^{-1} x^2 + c$ **(Ans.)**

28. $\int \frac{dx}{(1+x^2)^{\frac{3}{2}}}$

$= \int \frac{\sec^2\theta\,d\theta}{(1+\tan^2\theta)^{\frac{3}{2}}}$

ধরি, $x = \tan\theta$
$\therefore dx = \sec^2\theta\,d\theta$

$= \int \frac{\sec^2\theta}{\sec^3\theta}\,d\theta$

$= \int \frac{1}{\sec\theta}\,d\theta$

$= \int \cos\theta\,d\theta = \sin\theta + c$

$= \frac{1}{\mathrm{cosec}\,\theta} + c = \frac{1}{\sqrt{1+\cot^2\theta}} + c$

$= \frac{\tan\theta}{\sqrt{\tan^2\theta+1}} + c$

$= \frac{x}{\sqrt{x^2+1}} + c$ **(Ans.)**

29.(i) $\int \frac{x\,dx}{x^4+1}$

ধরি, $x^2 = z$
$\therefore 2x\,dx = dz$
বা, $x\,dx = \frac{1}{2}dz$

$= \frac{1}{2}\int \frac{dz}{z^2+1}$

$= \frac{1}{2}\tan^{-1} z + c$

$= \frac{1}{2}\tan^{-1} x^2 + c$ **(Ans.)**

(ii) $\int \frac{x^2}{\sqrt{1-x^6}}\,dx$

ধরি, $x^3 = z$
বা, $3x^2\,dx = dz$
$\therefore x^2\,dx = \frac{1}{3}dz$

$= \frac{1}{3}\int \frac{dz}{\sqrt{1-z^2}}$

$= \frac{1}{3}\sin^{-1} z + c$

$= \frac{1}{3}\sin^{-1}(x^3) + c$ **(Ans.)**

▶ অনুচ্ছেদ-10.5 | পৃষ্ঠা-৪১৩

(i) $\int \sqrt{36-x^2}\,dx$

$= \int \sqrt{6^2-x^2}\;dx$

$= \frac{x}{2}\sqrt{6^2-x^2} + \frac{6^2}{2}\sin^{-1}\frac{x}{6} + c$

$= \frac{x}{2}\sqrt{36-x^2} + 18\sin^{-1}\frac{x}{6} + c$ **(Ans.)**

(ii) $\int \sqrt{25-9x^2}dx$

| ধরি, $3x = z$
| $\therefore 3\;dx = dz$
| বা, $dx = \frac{dz}{3}\;dz$

$= \int \sqrt{25-(3x)^2}\;dx$

$= \int \sqrt{25-z^2}\,\frac{dz}{3}$

$= \frac{1}{3}\int \sqrt{25-z^2}\;dz$

$= \frac{1}{3}\left(\frac{z}{2}\sqrt{5^2-z^2} + \frac{5^2}{2}\sin^{-1}\frac{z}{5}\right) + c$

$= \frac{1}{3}\,\frac{3x}{2}\sqrt{25-(3x)^2} + \frac{25}{6}\sin^{-1}\left(\frac{3x}{5}\right) + c$

$= \frac{1}{2}x\sqrt{25-9x^2} + \frac{25}{6}\sin^{-1}\left(\frac{3x}{5}\right) + c$ **(Ans.)**

অনুশীলনী-10(C) এর সমাধান

1.(i) $\int\frac{dx}{25+4x^2} = \int\frac{dx}{4\left(\frac{25}{4}+x^2\right)}$

$= \frac{1}{4}\int\frac{dx}{\left(\frac{5}{2}\right)^2+x^2}$

$= \frac{1}{4}\cdot\frac{1}{\frac{5}{2}}\tan^{-1}\frac{x}{\frac{5}{2}} + c$

$= \frac{1}{10}\tan^{-1}\left(\frac{2x}{5}\right) + c$ **(Ans.)**

(ii) $\int\frac{dx}{4x^2+5}$

| ধরি, $3x = z$
| $\therefore 3dx = dz$
| বা, $dx = \frac{dz}{3}$

$= \frac{1}{4}\int\frac{dx}{x^2+\frac{5}{4}}$

$= \frac{1}{4}\int\frac{dx}{x^2+\left(\frac{\sqrt{5}}{2}\right)^2}$

$= \frac{1}{4}\cdot\frac{1}{\frac{\sqrt{5}}{2}}\tan^{-1}\frac{x}{\frac{\sqrt{5}}{2}} + c$

$= \frac{1}{2\sqrt{5}}\tan^{-1}\frac{2x}{\sqrt{5}} + c$ **(Ans.)**

2. (i) $\int\frac{dx}{9x^2-16} = \int\frac{dx}{9\left(x^2-\frac{16}{9}\right)}$

$= \frac{1}{9}\int\frac{dx}{x^2-\left(\frac{4}{3}\right)^2}$

$= \frac{1}{9}\cdot\frac{1}{2.\frac{4}{3}}\ln\left|\frac{x-\frac{4}{3}}{x+\frac{4}{3}}\right| + c$

$= \frac{1}{9}\cdot\frac{3}{8}\ln\left|\frac{3x-4}{3x+4}\right| + c$

$= \frac{1}{24}\ln\left|\frac{3x-4}{3x+4}\right| + c$ **(Ans.)**

(ii) $\int\frac{dx}{9-4x^2} = \frac{1}{4}\int\frac{dx}{\frac{9}{4}-x^2}$

$= \frac{1}{4}\int\frac{dx}{\left(\frac{3}{2}\right)^2-x^2} = \frac{1}{4}\,\frac{1}{2.\frac{3}{2}}\ln\left|\frac{\frac{3}{2}+x}{\frac{3}{2}-x}\right| + c$

[যেখানে, c যোগজীকরণ ধ্রুবক]

$= \frac{1}{12}\ln\left|\frac{3+2x}{3-2x}\right| + c$ **(Ans.)**

(iii) $\int\frac{dx}{25a^2-49x^2} = \int\frac{dx}{49\left(\frac{25}{49}a^2-x^2\right)}$

$= \frac{1}{49}\int\frac{dx}{\left(\frac{5a}{7}\right)^2-x^2}$

$= \frac{1}{49}\cdot\frac{1}{2.\frac{5}{7}a}\ln\left|\frac{\frac{5a}{7}+x}{\frac{5a}{7}-x}\right| + c$

$= \frac{1}{49}\cdot\frac{7}{10a}\ln\left|\frac{5a+7x}{5a-7x}\right| + c$

$= \frac{1}{70a}\ln\left|\frac{5a+7x}{5a-7x}\right| + c$ **(Ans.)**

(iv) $\int\frac{dx}{16-25x^2} = \frac{1}{25}\int\frac{dx}{\frac{16}{25}-x^2}$

$= \frac{1}{25}\int\frac{dx}{\left(\frac{4}{5}\right)^2-x^2} = \frac{1}{25}\frac{1}{2.\frac{4}{5}}\ln\left|\frac{\frac{4}{5}+x}{\frac{4}{5}-x}\right| + c$

$= \frac{1}{40}\ln\left|\frac{4+5x}{4-5x}\right| + c$ **(Ans.)**

(v) $\int\sqrt{\frac{1-x}{1+x}}dx = \int\sqrt{\frac{(1-x)(1-x)}{(1+x)(1-x)}}dx$

$= \int\sqrt{\frac{(1-x)^2}{1-x^2}}dx = \int\frac{1-x}{\sqrt{1-x^2}}dx$

$= \int\frac{dx}{\sqrt{1-x^2}} - \int\frac{x\,dx}{\sqrt{1-x^2}}$

$= \sin^{-1}x - \int\frac{x\,dx}{\sqrt{1-x^2}}$

$= \sin^{-1}x + \int\frac{z\,dz}{z}$

$= \sin^{-1}x + \int dz$

$= \sin^{-1}x + z + c$

$= \sin^{-1}x + \sqrt{1-x^2} + c$ **(Ans.)**

ধরি, $\sqrt{1-x^2} = z$
বা, $1 - x^2 = z^2$
$\therefore\ -2x\,dx = 2z\,dz$
বা, $x dx = -z\,dz$

3.(i) $\int\frac{dx}{\sqrt{25-x^2}}$

$= \int\frac{5\cos\theta\,d\theta}{\sqrt{25(1-\sin^2\theta)}}$

$= \int\frac{5\cos\theta\,d\theta}{5\cos\theta}$

$= \int d\theta = \theta + c$

$= \sin^{-1}\frac{x}{5} + c$ **(Ans.)**

ধরি, $x = 5\sin\theta$
$\therefore dx = 5\cos\theta\,d\theta$
$\therefore\ \theta = \sin^{-1}\frac{x}{5}$

(ii) $\int\frac{dx}{\sqrt{2-3x^2}} = \int\frac{dx}{\sqrt{3\left(\frac{2}{3}-x^2\right)}}$

$= \frac{1}{\sqrt{3}}\int\frac{dx}{\sqrt{\left(\sqrt{\frac{2}{3}}\right)^2 - x^2}}$

$= \frac{1}{\sqrt{3}}\sin^{-1}\frac{x}{\sqrt{\frac{2}{3}}} + c$

$= \frac{1}{\sqrt{3}}\sin^{-1}\left(\sqrt{\frac{3}{2}}x\right) + c$ **(Ans.)**

(iii) $\int\frac{dx}{\sqrt{9-16x^2}} = \int\frac{dx}{\sqrt{16\left(\frac{9}{16}-x^2\right)}}$

$= \frac{1}{4}\int\frac{dx}{\sqrt{\left(\frac{3}{4}\right)^2 - x^2}}$

$= \frac{1}{4}\sin^{-1}\left(\frac{x}{\frac{3}{4}}\right) + c$

$= \frac{1}{4}\sin^{-1}\frac{4x}{3} + c$ **(Ans.)**

(iv) $\int\frac{dx}{\sqrt{5-4x^2}} = \int\frac{dx}{\sqrt{4\left(\frac{5}{4}-x^2\right)}}$

$= \frac{1}{2}\int\frac{dx}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2 - (x)^2}}$

$= \frac{1}{2}\sin^{-1}\frac{x}{\frac{\sqrt{5}}{2}} + c$

$= \frac{1}{2}\sin^{-1}\frac{2x}{\sqrt{5}} + c$ **(Ans.)**

(v) $\int\frac{dx}{\sqrt{9-25x^2}} = \frac{1}{5}\int\frac{dx}{\sqrt{\left(\frac{3}{5}\right)^2 - x^2}}$

$= \frac{1}{5}\sin^{-1}\frac{x}{\frac{3}{5}} + c = \frac{1}{5}\sin^{-1}\frac{5x}{3} + c$ **(Ans.)**

4.(i) $\int\frac{x^2dx}{\sqrt{1-x^6}} = \int\frac{x^2dx}{\sqrt{1-(x^3)^2}}$

$= \frac{1}{3}\int\frac{dz}{\sqrt{1-z^2}} = \frac{1}{3}\sin^{-1}z + c$

$= \frac{1}{3}\sin^{-1}(x^3) + c$ **(Ans.)**

ধরি, $x^3 = z$
$\therefore\ x^2\,dx = \frac{1}{3}\,dz$

(ii) $\int\frac{x^3dx}{\sqrt{a^8-x^8}}$

$= \int\frac{x^3dx}{\sqrt{a^8-(x^4)^2}}$

$= \frac{1}{4}\int\frac{dz}{\sqrt{(a^4)^2-(z)^2}}$

$= \frac{1}{4}\cdot\sin^{-1}\frac{z}{a^4} + c$

$= \frac{1}{4}\sin^{-1}\left(\frac{x^4}{a^4}\right) + c$ **(Ans.)**

ধরি, $x^4 = z$
$\therefore\ x^3\,dx = \frac{1}{4}\,dz$

(iii) $\int\frac{5e^{2x}dx}{1+e^{4x}}$

$= \frac{5}{2}\int\frac{2e^{2x}dx}{1+e^{4x}}$

$= \frac{5}{2}\int\frac{2e^{2x}dx}{1+(e^{2x})^2}$

$= \frac{5}{2}\int\frac{dz}{1+z^2}$

$= \frac{5}{2}\tan^{-1}z + c$ [যেখানে, c যোগজীকরণ ধ্রুবক।]

$= \frac{5}{2}\tan^{-1}(e^{2x}) + c$ **(Ans.)**

ধরি, $e^{2x} = z$
$\therefore\ 2e^{2x}dx = dz$

5.(i) $\int \frac{3dx}{x^2-8x+25} = \int \frac{3dx}{(x-4)^2+(3)^2}$

$= 3.\frac{1}{3}\tan^{-1}\left(\frac{x-4}{3}\right) + c$

$= \tan^{-1}\left(\frac{x-4}{3}\right) + c$ **(Ans.)**

(ii) $\int \frac{dx}{x^2+6x+25} = \int \frac{dx}{(x^2+6x+9)+16}$

$= \int \frac{dx}{(x+3)^2+4^2}$

$= \frac{1}{4}\tan^{-1}\left(\frac{x+3}{4}\right) + c$ **(Ans.)**

(iii) $\int \frac{dx}{x^2-x+1} = \int \frac{dx}{x^2-2.\frac{1}{2}.x+\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+1}$

$= \int \frac{dx}{\left(x-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{\frac{\sqrt{3}}{2}}\tan^{-1}\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + c$

$= \frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{2x-1}{\sqrt{3}}\right) + c$ **(Ans.)**

6.(i) $\int \frac{dx}{\sqrt{3-2x-x^2}} = \int \frac{dx}{\sqrt{-(x^2+2x+1-4)}}$

$= \int \frac{dx}{\sqrt{-(x+1)^2+(2)^2}}$

$= \int \frac{dx}{\sqrt{(2)^2-(x+1)^2}}$

$= \sin^{-1}\left(\frac{x+1}{2}\right) + c$ **(Ans.)**

(ii) $\int \frac{dx}{\sqrt{4x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-4x+4)+4}}$

$= \int \frac{dx}{\sqrt{(2)^2-(x-2)^2}}$

$= \sin^{-1}\left(\frac{x-2}{2}\right) + c$ **(Ans.)**

7(i) $\int \frac{dx}{\sqrt{x^2+4x+13}}$

ধরি, $x + 2 = 3\tan\theta$

$\therefore dx = 3\sec^2\theta \, d\theta$

$= \int \frac{dx}{\sqrt{x^2+2.x.2+2^2+9}}$

$= \int \frac{dx}{\sqrt{(x+2)^2+3^2}}$

$= \ln\left[(x+2)+\sqrt{(x+2)^2+3^2}\right] + c$

[যেখানে, c যোগজীকরণ ধ্রুবক]

$= \ln\left|(x+2)+\sqrt{(x^2+4x+13)}\right| + c$ **(Ans.)**

(ii) $\int \frac{e^x}{\sqrt{e^{2x}+1}}dx = \int \frac{dz}{\sqrt{z^2+1}}$

ধরি, $e^x = z$

$\therefore e^x dx = dz$

$= \ln\left(\sqrt{1+z^2}+z\right) + c$

$= \ln\left|\sqrt{1+e^{2x}}+e^x\right| + c$ **(Ans.)**

(iii) $\int \frac{dx}{\sqrt{2x+x^2}} = \int \frac{dx}{\sqrt{x^2+2x+1-1}}$

$= \int \frac{dx}{\sqrt{(x+1)^2-1}}$

ধরি, $x + 1 = \sec\theta$

$\therefore dx = \sec\theta\tan\theta \, d\theta$

$= \int \frac{\sec\theta\tan\theta \, d\theta}{\sqrt{\sec^2\theta-1}}$

$= \int \frac{\sec\theta\tan\theta}{\tan\theta} d\theta$

$= \int \sec\theta \, d\theta$

$= \ln(\sec\theta+\tan\theta) + c$

$= \ln\left\{\sec\theta+\sqrt{\sec^2\theta-1}\right\} + c$

$= \ln\left[(x+1)+\sqrt{(x+1)^2-1}\right] + c$

$= \ln\left|(x+1)+\sqrt{2x+x^2}\right| + c$ **(Ans.)**

8. $\int \frac{dx}{5+4x-x^2} = \int \frac{dx}{-(x^2-4x+4)+9}$

$= \int \frac{dx}{(3)^2-(x-2)^2}$

$= \frac{1}{2.3}\ln\left|\frac{3+x-2}{3-x+2}\right| + c$

$= \frac{1}{6}\ln\left|\frac{x+1}{5-x}\right| + c$ **(Ans.)**

9.(i) $\int \sqrt{1-9x^2}\,dx$

$= \frac{1}{3}\int \sqrt{1-9.\frac{1}{9}\sin^2\theta}\,\cos\theta \, d\theta$

$= \frac{1}{3}\int \sqrt{1-\sin^2\theta}\,\cos\theta \, d\theta$

ধরি, $x = \frac{1}{3}\sin\theta$

$\therefore dx = \frac{1}{3}\cos\theta \, d\theta$

$\therefore \theta = \sin^{-1}3x$

$= \frac{1}{3}\int \cos^2\theta \, d\theta$

$= \frac{1}{6}\int 2\cos^2\theta \, d\theta$

$= \frac{1}{6}\int (1+\cos2\theta) \, d\theta$

$= \frac{1}{6}\left[\theta+\frac{\sin2\theta}{2}\right] + c$

$= \frac{1}{6}\left[\sin^{-1}3x+\frac{2\sin\theta\cos\theta}{2}\right] + c$

$= \frac{1}{6}\sin^{-1}3x + \frac{1}{6}\sin\theta\sqrt{1-\sin^2\theta} + c$

$= \frac{1}{6}\cdot 3x\sqrt{1-9x^2} + \frac{1}{6}\sin^{-1}3x + c$

$= \frac{x\sqrt{1-9x^2}}{2} + \frac{1}{6}\sin^{-1}3x + c$ **(Ans.)**

(ii) $\int\sqrt{2ax-x^2}\,dx$

$= \int\sqrt{-(x^2-2ax+a^2)+a^2}\;dx$

$= \int\sqrt{a^2-(x-a)^2}\;dx$

$= \int\sqrt{a^2-a^2\sin^2\theta}\;a\cos\theta\,d\theta$

$= \int a\sqrt{1-\sin^2\theta}\;a\cos\theta\,d\theta$

$= a^2\int\cos^2\theta\,d\theta$

$= \frac{a^2}{2}\int 2\cos^2\theta\,d\theta$

$= \frac{a^2}{2}\int(1+\cos2\theta)\,d\theta$ | ধরি, $x-a=a\sin\theta$ $\therefore dx=a\cos\theta\,d\theta$

$= \frac{a^2}{2}\left(\theta+\frac{\sin2\theta}{2}\right)+c$

$= \frac{a^2}{2}\left[\sin^{-1}\left(\frac{x-a}{a}\right)+\frac{2\sin\theta\cos\theta}{2}\right]+c$

$= \frac{a^2}{2}\left[\sin^{-1}\left(\frac{x-a}{a}\right)+\sin\theta\sqrt{1-\sin^2\theta}\right]+c$

$= \frac{a^2}{2}\left[\sin^{-1}\left(\frac{x-a}{a}\right)+\frac{x-a}{a}\sqrt{1-\left(\frac{x-a}{a}\right)^2}\right]+c$

$= \frac{a^2}{2}\left[\sin^{-1}\left(\frac{x-a}{a}\right)+\frac{x-a}{a}\sqrt{\frac{a^2-x^2+2ax-a^2}{a^2}}\right]+c$

$= \frac{a^2}{2}\left[\sin^{-1}\frac{x-a}{a}+\frac{x-a}{a^2}\sqrt{2ax-x^2}\right]+c$

$= \frac{(x-a)\sqrt{2ax-x^2}}{2}+\frac{a^2}{2}\sin^{-1}\left(\frac{x-a}{a}\right)+c$ **(Ans.)**

10. $\int\frac{dx}{1+\cos^2x}$

$= \int\frac{\sec^2x}{\sec^2x+1}\,dx$ [লব ও হরকে $\cos^2x$ দ্বারা ভাগ করে]

$= \int\frac{\sec^2x}{1+\tan^2x+1}\,dx$

$= \int\frac{\sec^2x}{2+\tan^2x}\,dx$

$= \int\frac{dz}{(\sqrt{2})^2+z^2}$ | ধরি, $\tan x=z$ $\therefore \sec^2x\,dx=dz$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}+c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+c$ **(Ans.)**

11. $\int\frac{\sec^2x}{4+9\tan^2x}\,dx$

$= \int\frac{dz}{4+9z^2}$ | ধরি, $\tan x=z$ $\therefore \sec^2x\,dx=dz$

$= \int\frac{dz}{9\left(\frac{4}{9}+z^2\right)}$

$= \frac{1}{9}\int\frac{dz}{\left(\frac{2}{3}\right)^2+(z)^2}$

$= \frac{1}{9}\cdot\frac{1}{\frac{2}{3}}\tan^{-1}\frac{z}{\frac{2}{3}}+c$

$= \frac{1}{9}\cdot\frac{3}{2}\tan^{-1}\frac{3z}{2}+c$

$= \frac{1}{6}\tan^{-1}\frac{3z}{2}+c$

$= \frac{1}{6}\tan^{-1}\left(\frac{3\tan x}{2}\right)+c$ **(Ans.)**

12. $\int\frac{\sec^2x}{\sqrt{1-\tan^2x}}\,dx$

$= \int\frac{dz}{\sqrt{1-z^2}}$ | ধরি, $\tan x=z$ $\therefore \sec^2x\,dx=dz$

$= \sin^{-1}z+c$

$= \sin^{-1}(\tan x)+c$ **(Ans.)**

13. $\int\frac{3\tan^2x\sec^2x}{1+\tan^6x}\,dx$

$= \int\frac{3\tan^2x\sec^2x}{1+(\tan^3x)^2}\,dx$ | ধরি, $\tan^3x=z$ বা, $3\tan^2x.\frac{d}{dx}(\tan x)=\frac{dz}{dx}$ $\therefore 3\tan^2x.\sec^2x\,dx=dz$

$= \int\frac{dz}{1+z^2}$

$= \tan^{-1}z+c$

$= \tan^{-1}(\tan^3x)+c$ **(Ans.)**

14. $\int\frac{\cos x}{\sqrt{3+\cos^2x}}\,dx$ | ধরি, $\sin x=z$ $\therefore \cos x\,dx=dz$

$= \int\frac{\cos x}{\sqrt{3+1-\sin^2x}}\,dx$

$= \int\frac{\cos x}{\sqrt{4-\sin^2x}}\,dx$

$= \int\frac{dz}{\sqrt{(2)^2-z^2}}=\sin^{-1}\frac{z}{2}+c$

$= \sin^{-1}\left(\frac{\sin x}{2}\right)+c$ **(Ans.)**

15.(i) $\int\frac{x^2-1}{x^4+1}\,dx=\int\frac{x^2\left(1-\frac{1}{x^2}\right)}{x^2\left(x^2+\frac{1}{x^2}\right)}\,dx$

$= \int\frac{1-\frac{1}{x^2}}{x^2+\frac{1}{x^2}}\,dx$ | ধরি, $x+\frac{1}{x}=z$ $\therefore \left(1-\frac{1}{x^2}\right)dx=dz$

$= \int\frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}\right)^2-2}$

$$= \int \frac{dz}{z^2-(\sqrt{2})^2} = \frac{1}{2\sqrt{2}} \ln\left|\frac{z-\sqrt{2}}{z+\sqrt{2}}\right| + c$$

$$= \frac{1}{2\sqrt{2}} \ln\left|\frac{x+\frac{1}{x}-\sqrt{2}}{x+\frac{1}{x}+\sqrt{2}}\right| + c$$

$$= \frac{1}{2\sqrt{2}} \ln\left|\frac{x^2+1-\sqrt{2}x}{x^2+1+\sqrt{2}x}\right| + c \textbf{ (Ans.)}$$

(ii) ধরি, $I = \int \frac{x^2+1}{x^4+1}dx = \int \frac{1+\frac{1}{x^2}}{x^2+\frac{1}{x^2}}dx$

[লব ও হরকে x^2 দ্বারা ভাগ করে]

$$= \int \frac{1+\frac{1}{x^2}}{\left(x-\frac{1}{x}\right)^2+2}dx$$

মনে করি, $x-\frac{1}{x} = z$ বা, $\left(1+\frac{1}{x^2}\right)dx = dz$

$$\therefore I = \int \frac{dz}{z^2+(\sqrt{2})^2} = \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{z}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x-\frac{1}{x}}{\sqrt{2}}\right) + c$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{\sqrt{2}x}\right) + c$$

16(i). $\int \frac{dx}{(x-3)\sqrt{x+1}}$

| ধরি, $x+1=z^2$; $\therefore dx = 2z\,dz$; $x = z^2-1$; এবং $z=\sqrt{x+1}$

$$= \int \frac{2z\,dz}{(z^2-1-3)z} = 2\int\frac{dz}{z^2-4}$$

$$= 2.\frac{1}{2.2}\ln\left|\frac{z-2}{z+2}\right| + c$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{x+1}-2}{\sqrt{x+1}+2}\right| + c \quad \textbf{(Ans.)}$$

(ii) $\int \frac{dx}{(x+2)\sqrt{x+3}}$

| ধরি, $x+3=z^2$; বা, $x = z^2-3$; $\therefore dx = 2zdz$

$$= \int \frac{2zdz}{(z^2-3+2).z} = \int\frac{2dz}{z^2-1}$$

$$= \frac{2}{2.1}\ln\left|\frac{z-1}{z+1}\right| + c$$

$$= \ln\left|\frac{\sqrt{x+3}-1}{\sqrt{x+3}+1}\right| + c \textbf{ (Ans.)}$$

(iii) $\int \frac{dx}{(2x+1)\sqrt{4x+3}}$

| ধরি, $4x+3=z^2$; বা, $4x = z^2-3$; বা, $4dx = 2z\,dz$; $\therefore dx = \frac{zdz}{2}$

$$= 2\int\frac{dx}{(4x+2)\sqrt{4x+3}}$$

$$= 2\int \frac{\frac{z.dz}{2}}{(z^2-3+2).z}$$

$$= \int\frac{dz}{z^2-1}$$

$$= \frac{1}{2}\ln\left|\frac{z-1}{z+1}\right| + c$$

$$= \frac{1}{2}\ln\left|\frac{\sqrt{4x+3}-1}{\sqrt{4x+3}+1}\right| + c \textbf{ (Ans.)}$$

অনুশীলনী-10(D) এর সমাধান

1.(i) $\frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)}$

$$= \frac{1}{(x+3)(-3-2)} + \frac{1}{(x-2)(2+3)}$$

[By cover-up rule]

$$= \frac{1}{-5(x+3)} + \frac{1}{5(x-2)}$$

$$\therefore \int\frac{dx}{x^2+x-6} = \int\left\{\frac{-1}{5(x+3)} + \frac{1}{5(x-2)}\right\}dx$$

$$= -\frac{1}{5}\ln(x+3) + \frac{1}{5}\ln(x-2) + c$$

$$= \frac{1}{5}\ln\left|\frac{x-2}{x+3}\right| + c \textbf{ (Ans.)}$$

(ii) $\frac{x+1}{3x^2-x-2} = \frac{x+1}{(3x+2)(x-1)}$

$$= \frac{-\frac{2}{3}+1}{(3x+2)\left(-\frac{2}{3}-1\right)} + \frac{1+1}{(3\times1+2)(x-1)}$$

[By cover-up rule]

$$= \frac{\frac{1}{3}}{-\frac{5}{3}(3x+2)} + \frac{2}{5(x-1)}$$

$$= \frac{2}{5(x-1)} - \frac{1}{5(3x+2)}$$

$$\therefore \int\frac{x+1}{3x^2-x-2}dx = \frac{2}{5}\int\frac{1}{x-1}dx - \frac{1}{5}\int\frac{dx}{3x+2}$$

$$= \frac{2}{5}\ln|x-1| - \frac{1}{5}.\frac{1}{3}\ln|3x+2| + c$$

$$= \frac{2}{5}\ln|x-1| - \frac{1}{15}\ln|3x+2| + c$$

(Ans.)

(iii) $\frac{1}{x(x-1)(x-3)}$

$= \frac{1}{x(0-1)(0-3)} + \frac{1}{1.(x-1).(1-3)} + \frac{1}{3(3-1)(x-3)}$ [By cover-up rule]

$= \frac{1}{3x} - \frac{1}{2(x-1)} + \frac{1}{6(x-3)}$

$\therefore \int \frac{1}{x(x-1)(x-3)} dx$

$= \frac{1}{3}\int \frac{dx}{x} - \frac{1}{2}\int \frac{1}{x-1} dx + \frac{1}{6}\int \frac{1}{x-3} dx$

$= \frac{1}{3}\ln|x| - \frac{1}{2}\ln|x-1| + \frac{1}{6}\ln|x-3| + c$ **(Ans.)**

(iv) $\frac{x-3}{(1-2x)(1+x)}$

$= \frac{\frac{1}{2}-3}{(1-2x)\left(1+\frac{1}{2}\right)} + \frac{-1-3}{(1+x)\{1-2(-1)\}}$ [By cover-up rule]

$= \frac{-\frac{5}{2}}{\frac{3}{2}(1-2x)} - \frac{4}{3(1+x)} = \frac{-5}{3(1-2x)} - \frac{4}{3(1+x)}$

$\therefore \int \frac{(x-3)dx}{(1-2x)(1+x)} = -\frac{5}{3}\int \frac{dx}{1-2x} - \frac{4}{3}\int \frac{dx}{1+x}$

$= -\frac{5}{3}\int \frac{-\frac{1}{2}du}{u} - \frac{4}{3}\int \frac{dx}{1+x}$ | ধরি, $u = 1-2x$ $\therefore du = -2dx$

$= \frac{5}{3}\cdot\frac{1}{2}\ln u - \frac{4}{3}\ln(1+x) + c$

$= \frac{5}{6}\ln|1-2x| - \frac{4}{3}\ln|1+x| + c$ **(Ans.)**

(v) ধরি, $\frac{2x+3}{x^3+x^2-2x} = \frac{2x+3}{x(x^2+x-2)}$

$= \frac{2x+3}{x(x-1)(x+2)} \equiv \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+2}$

যেখানে, $A, B, C \in \mathbb{R}$

উভয় পক্ষকে $x(x-1)(x+2)$ দ্বারা গুণ করে,

$\therefore A(x-1)(x+2) + Bx(x+2) + Cx(x-1) \equiv 2x+3$

$x=0$ হলে, $\therefore A = -\frac{3}{2}$

$x=1$ হলে, $\therefore B = \frac{5}{3}$

$x=-2$ হলে, $\therefore C = -\frac{1}{6}$

$\therefore \int \frac{(2x+3)dx}{x^3+x^2-2x} = \int\left\{\frac{\frac{-3}{2}}{x} + \frac{\frac{5}{3}}{x-1} + \frac{\frac{-1}{6}}{x+2}\right\} dx$

$= -\frac{3}{2}\ln|x| + \frac{5}{3}\ln|x-1| - \frac{1}{6}\ln|x+2| + c$ **(Ans.)**

2.(i) $\frac{1}{x(x+1)^2}$

$= \frac{1}{x+1}\left[\frac{1}{x(x+1)}\right]$

$= \frac{1}{x+1}\left[\frac{1}{x(0+1)} + \frac{1}{(-1)(x+1)}\right]$ [By cover-up rule]

$= \frac{1}{x+1}\left[\frac{1}{x} - \frac{1}{x+1}\right]$

$= \frac{1}{x(x+1)} - \frac{1}{(x+1)^2}$

$= \frac{1}{x(0+1)} + \frac{1}{(-1)(x+1)} - \frac{1}{(x+1)^2}$ [By cover-up rule]

$= \frac{1}{x} - \frac{1}{x+1} - \frac{1}{(x+1)^2}$

[বি.দ্র. উক্ত নিয়ম kelley's cover-up rule নামে পরিচিত।]

$\therefore \int \frac{1}{x(x+1)^2} dx = \int \frac{1}{x} dx - \int \frac{1}{x+1} dx - \int \frac{1}{(x+1)^2} dx$

$= \ln(x) - \ln(x+1) - \frac{(x+1)^{-1}}{-1} + c$

$= \ln\left|\frac{x}{x+1}\right| + \frac{1}{x+1} + c$ **(Ans.)**

(ii) $\frac{1}{x^2(x-1)} = \frac{1}{x}\left[\frac{1}{x(x-1)}\right]$

$= \frac{1}{x}\left[\frac{1}{x(0-1)} + \frac{1}{1(x-1)}\right]$ [By cover-up rule]

$= -\frac{1}{x^2} + \frac{1}{x(x-1)}$

$= -\frac{1}{x^2} + \frac{1}{x(0-1)} + \frac{1}{1(x-1)}$ [By cover-up rule]

$= \frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}$

$\therefore \int \frac{dx}{x^2(x-1)} = \int\left\{\frac{1}{x-1} - \frac{1}{x} - \frac{1}{x^2}\right\} dx$

$= \int \frac{dx}{x-1} - \int \frac{dx}{x} - \int x^{-2} dx$

$= \ln|x-1| - \ln x + x^{-1} + c$

$= \ln\left|\frac{x-1}{x}\right| + \frac{1}{x} + c$ **(Ans.)**

3.(i) $\int \frac{x+35}{x^2-25}dx$

$= \int \frac{x+35}{(x+5)(x-5)}dx$

$= \frac{-5+35}{(x+5)(-5-5)} + \frac{5+35}{(5+5)(x-5)}$ [By cover-up rule]

$= \frac{30}{-10(x+5)} + \frac{40}{10(x-5)}$

$= \frac{4}{x-5} - \frac{3}{x+5}$

$\therefore \int \frac{x+35}{x^2-25}dx = 4\int \frac{1}{x-5}dx - 3\int \frac{1}{x+5}dx$

$= 4\ln|x-5| - 3\ln|x+5| + c$ **(Ans.)**

(ii) মনে করি, $x+2 = y$ তাহলে $x = y-2$

$x^2 = y^2 - 4y + 4$ বা, $x^2+1 = y^2-4y+5$

$\therefore \frac{x^2+1}{(x+2)^2} = \frac{5-4y+y^2}{y^2} = \frac{5}{y^2} - \frac{4}{y} + 1$

$= \frac{5}{(x+2)^2} - \frac{4}{x+2} + 1$

$\therefore \int \frac{x^2+1}{(x+2)^2}dx = 5\int \frac{1}{(x+2)^2}dx - 4\int \frac{1}{x+2}dx + \int dx$

$= 5\frac{(x+2)^{-1}}{(-1)} - 4\ln(x+2) + x + c$

$= -\frac{5}{x+2} - 4\ln|x+2| + x + c$ **(Ans.)**

(iii) $\int \frac{x^3-2x+3}{x^2+x-2}dx$

ধরি $\frac{x^3-2x+3}{x^2+x-2} = \frac{x^3-2x+3}{(x-1)(x+2)} \equiv Ax + B + \frac{C}{x+2} + \frac{D}{x-1}$

$\therefore$ $x^3 - 2x + 3 \equiv Ax(x-1)(x+2) + B(x-1)(x+2) + C(x-1) + D(x+2)$

যখন, $x = 1$ তখন $D = \frac{2}{3}$

যখন, $x = -2$ তখন $C = \frac{1}{3}$

এখন x^3-এর সহগ সমীকৃত করে পাই, $A = 1$

এবং x^2-এর সহগ সমীকৃত করে পাই, $A + B = 0$

$\therefore B = -1$

$\therefore$ $\int \frac{x^3-2x+3}{x^2+x-2}dx$

$= \int x\,dx - \int dx + \frac{1}{3}\int \frac{1}{x+2}dx + \frac{2}{3}\int \frac{1}{x-1}dx$

$= \frac{1}{2}x^2 - x + \frac{1}{3}\ln(x+2) + \frac{2}{3}\ln(x-1) + c$ **(Ans.)**

(iv) $\int \frac{2x+1}{(x+2)(x-3)^2}dx$

ধরি, $\frac{2x+1}{(x+2)(x-3)^2} \equiv \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

উভয় পক্ষকে $(x+2)(x-3)^2$ দ্বারা গুণ করে,

$2x+1 = A(x-3)^2 + B(x+2)(x-3) + C(x+2)$

$x = -2$ হলে, $-2 \times 2 + 1 = A(-2-3)^2 + 0 + 0$

বা, $25A = -3$ $\therefore A = -\frac{3}{25}$

$x = 3$ হলে, $5C = 7$ $\therefore C = \frac{7}{5}$

উভয়পক্ষ থেকে x^2 এর সহগ সমীকৃত করে,

$A + B = 0$ বা, $B = -A$ $\therefore B = \frac{3}{25}$

অতএব, $\int \frac{(2x+1)}{(x+2)(x-3)^2}dx$

$= -\frac{3}{25}\int \frac{1}{x+2}dx + \frac{3}{25}\int \frac{1}{x-3}dx + \frac{7}{5}\int \frac{1}{(x-3)^2}dx$

$= -\frac{3}{25}\ln|x+2| + \frac{3}{25}\ln|x-3| + \frac{7}{5}\cdot\frac{(x-3)^{-1}}{(-1)} + c$

$= -\frac{3}{25}\ln|x+2| + \frac{3}{25}\ln|x-3| - \frac{7}{5}\cdot\frac{1}{x-3} + c$ **(Ans.)**

(v) $\frac{x}{(x-1)^2(x+2)} \equiv \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+2}$

যেখানে, $A, B, C \in R$

উভয় পক্ষকে $(x-1)^2(x+2)$ দ্বারা গুণ করে,

সুতরাং, $A(x-1)(x+2) + B(x+2) + C(x-1)^2 \equiv x$

$x = 1$ হলে, $3B = 1$ $\therefore B = \frac{1}{3}$

$x = -2$ হলে, $9C = -2$ $\therefore C = -\frac{2}{9}$

x^2 এর সহগ সমীকৃত করে, $A + C = 0$

$\therefore A = -C = \frac{2}{9}$

$\therefore \int \frac{x\,dx}{(x-1)^2(x+2)}$

$= \int \left\{ \frac{\frac{2}{9}}{x-1} + \frac{\frac{1}{3}}{(x-1)^2} + \frac{\frac{-2}{9}}{x+2} \right\} dx$

$= \frac{2}{9}\int \frac{dx}{x-1} - \frac{2}{9}\int \frac{dx}{x+2} + \frac{1}{3}\int \frac{dx}{(x-1)^2}$

$= \frac{2}{9}\ln(x-1) - \frac{2}{9}\ln(x+2) - \frac{1}{3}\frac{1}{x-1} + c$

$= \frac{2}{9}\ln\left|\frac{x-1}{x+2}\right| - \frac{1}{3}\cdot\frac{1}{x-1} + c$ **(Ans.)**

(vi) $\int \frac{2x^2-1}{(x+1)^2(x-2)}dx$

ধরি, $\frac{2x^2-1}{(x+1)^2(x-2)} \equiv \frac{A}{x-2}+\frac{B}{x+1}+\frac{C}{(x+1)^2}$

যেখানে, $A, B, C \in \mathbb{R}$

উভয়পক্ষকে $(x+1)^2(x-2)$ দ্বারা গুণ করে,

$2x^2-1 = A(x+1)^2+B(x+1)(x-2)+C(x-2)$

$x=-1$ হলে,

$2(-1)^2-1 = A\times 0 + B\times 0 + C\times(-3)$

$\therefore C=-\frac{1}{3}$ $x=2$ হলে, $7=A.9$ $\therefore A=\frac{7}{9}$

এখন, উভয় পক্ষ হতে x^2 এর সহগ সমীকৃত করে,

$2=A+B$ বা, $B=2-\frac{7}{9}$ $\therefore B=\frac{11}{9}$

$\therefore \int \frac{(2x^2-1)}{(x+1)^2(x-2)}dx$

$=\frac{7}{9}\int\frac{1}{x-2}dx+\frac{11}{9}\int\frac{1}{x+1}dx-\frac{1}{3}\int\frac{dx}{(x+1)^2}$

$=\frac{7}{9}\ln|x-2|+\frac{11}{9}\ln|x+1|+\frac{1}{3}\frac{1}{(x+1)}+c$ **(Ans.)**

(vii) $\int\frac{dx}{x^2(x+1)^2}$

মনে করি, $\frac{1}{x^2(x+1)^2}\equiv\frac{A}{x}+\frac{B}{x^2}+\frac{C}{(x+1)}+\frac{D}{(x+1)^2}$

উভয় পক্ষকে $x^2(x+1)^2$ দ্বারা গুণ করে,

$\therefore 1=Ax(x+1)^2+B(x+1)^2+Cx^2(x+1)+Dx^2$

বা, $1=Ax(x^2+2x+1)+B(x^2+2x+1)+Cx^3+Cx^2+Dx^2$

$=Ax^3+2Ax^2+Ax+Bx^2+2Bx+B+Cx^3+Cx^2+Dx^2$

$x=0$ হলে, $1=B.1$ $\therefore B=1$

$x=-1$ হলে, $D=1$

উভয় পক্ষ হতে x এর সহগ সমীকৃত করে,

$\therefore A+2B=0$ বা, $A=-2$ $\therefore A=-2$

আবার উভয় পক্ষ হতে x^3 এর সহগ সমীকৃত করে,

$A+C=0$ বা, $C=2$

$\therefore\int\frac{dx}{x^2(1+x)^2}=-2.\int\frac{1}{x}dx+\int\frac{1}{x^2}dx+2\int\frac{1}{x+1}dx+\int\frac{1}{(1+x)^2}dx$

$=-2.\ln|x|-\frac{1}{x}+2\ln|x+1|-\frac{1}{(1+x)}+c$

$\therefore\int\frac{dx}{x^2(1+x)^2}=-\frac{1}{x}-2\ln|x|+2\ln|x+1|-\frac{1}{(1+x)}+c$

(Ans.)

4.(i) $\int\frac{x^2-1}{x^2-4}dx$

$\frac{x^2-1}{x^2-4}=1+\frac{3}{x^2-4}=1+\frac{3}{(x+2)(x-2)}$

ধরি, $\frac{3}{(x+2)(x-2)}\equiv\frac{A}{x+2}+\frac{B}{x-2}$ যেখানে, $A, B\in\mathbb{R}$

উভয় পক্ষকে (x^2-4) দ্বারা গুণ করে,

$\therefore\quad 3\equiv A(x-2)+B(x+2)$

$x=2$ হলে, $B=\frac{3}{4}$; $x=-2$ হলে, $A=-\frac{3}{4}$

$\therefore\int\frac{x^2-1}{x^2+4}dx=\int dx-\frac{3}{4}\int\frac{1}{x+2}dx+\frac{3}{4}\int\frac{1}{x-2}dx$

$=x-\frac{3}{4}\ln(x+2)+\frac{3}{4}\ln(x-2)+c$

$=x+\frac{3}{4}\ln\left|\frac{x-2}{x+2}\right|+c$ **(Ans.)**

(ii) $\int\frac{x^2}{x^2-4}dx$

ধরি, $\frac{x^2}{x^2-4}=\frac{x^2}{(x-2)(x+2)}\equiv A+\frac{B}{x-2}+\frac{C}{x+2}$

$\therefore x^2=A(x-2)(x+2)+B(x+2)+C(x-2)$

যখন, $x=2$ তখন $B=1$

যখন, $x=-2$ তখন $C=-1$

উভয়পক্ষে থেকে x^2-এর সহগ সমীকৃত করে পাই,

$A=1$

$\therefore\int\frac{x^2}{x^2-4}dx=\int dx+\int\frac{1}{x-2}dx-\int\frac{1}{x+2}dx$

$=x+\ln(x-2)-\ln(x+2)+c$

$=x+\ln\left|\frac{x-2}{x+2}\right|+c$ **(Ans.)**

5.(i) $\int\frac{x\,dx}{(x-1)(x^2+4)}$

মনে করি, $\frac{x}{(x+1)(x^2+4)}\equiv\frac{A}{x-1}+\frac{Bx+C}{x^2+4}$

$\therefore x=A(x^2+4)+(Bx+C)(x-1)$

বা, $x=Ax^2+4A+Bx^2-Bx+Cx-C$

x^2 এর সহগ সমীকৃত করে, $A+B=0$

এবং $4A-C=0$

আবার, $x=1$ হলে, $A=\frac{1}{5}$ $\therefore B=-\frac{1}{5}, C=\frac{4}{5}$

$\therefore\int\frac{x\,dx}{(x-1)(x^2+4)}=\int\frac{dx}{5(x-1)}+\int\frac{4-x}{5(x^2+4)}dx$

$=\frac{1}{5}\ln|x-1|+\frac{4}{5}\int\frac{1}{x^2+4}dx-\frac{1}{5}\int\frac{x}{x^2+4}dx$

$=\frac{1}{5}\ln|x-1|+\frac{2}{5}\tan^{-1}\frac{x}{2}-\frac{1}{10}\ln|x^2+4|+c$

(Ans.)

(ii) $\int \frac{dx}{x(x^2+1)}$

মনে করি, $\frac{1}{x(x^2+1)} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+1}$

$\therefore 1 = A(x^2+1) + (Bx+C)x$

$x = 0$ হলে, $A = 1$

উভয় পক্ষ হতে x^2 এর সহগ সমীকৃত করে,

$A + B = 0$ বা, $B = -A \quad \therefore B = -1$

উভয় পক্ষ থেকে x এর সহগ সমীকৃত করে, $C = 0$

$$\therefore \int \frac{dx}{x(x^2+1)} = \int \frac{1}{x}dx - \int \frac{x}{x^2+1}dx$$
$$= \int \frac{1}{x}dx - \frac{1}{2}\int \frac{2x}{x^2+1}dx$$
$$= \ln|x| - \frac{1}{2}\ln|x^2+1| + c \text{ (Ans.)}$$

(iii) $\int \frac{x^2 dx}{x^4-1}$

ধরি, $\frac{x^2}{x^4-1} = \frac{x^2}{(x+1)(x-1)(x^2+1)} \equiv \frac{A}{x+1} + \frac{B}{x-1} + \frac{Cx+D}{x^2+1}$

যেখানে, $A, B, C, D \in \mathbb{R}$

উভয় পক্ষকে $(x+1)(x-1)(x^2+1)$ দ্বারা গুণ করে,

$$\therefore x^2 \equiv A(x-1)(x^2+1) + B(x+1)(x^2+1) + (Cx+D)(x^2-1) \text{ ... ... (i)}$$

আমরা পাই,

$$x^2 \equiv (A+B+C)x^3 + (-A+B+D)x^2 + (A+B-C)x + (-A+B-D) \text{ ... ... (ii)}$$

$x = 1$ হলে, (i) নং হতে পাই,

$4B = 1 \quad \therefore B = \frac{1}{4}$

$x = -1$ হলে, (i) নং হতে পাই,

$-4A = 1 \quad \therefore A = -\frac{1}{4}$

এখন, x^3 ও x^2 এর সহগ উপরোক্ত (ii) নং অভেদের উভয় পক্ষ থেকে সমীকৃত করে, $A + B + C = 0$

বা, $-\frac{1}{4} + \frac{1}{4} + C = 0 \quad \therefore C = 0$

এবং $-A + B + D = 1$ বা, $\frac{1}{4} + \frac{1}{4} + D = 1 \therefore D = \frac{1}{2}$

তাহলে, $\frac{x^2}{x^4-1} = -\frac{1}{4}\cdot\frac{1}{x+1} + \frac{1}{4}\cdot\frac{1}{x-1} + \frac{1}{2}\cdot\frac{1}{x^2+1}$

$$\therefore \int \frac{x^2}{x^4-1}dx = -\frac{1}{4}\int\frac{1}{x+1}dx + \frac{1}{4}\int\frac{1}{x-1}dx + \frac{1}{2}\int\frac{dx}{1+x^2}$$
$$= -\frac{1}{4}\ln|x+1| + \frac{1}{4}\ln|x-1| + \frac{1}{2}\tan^{-1}x + c$$
$$= \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| + \frac{1}{2}\tan^{-1}x + c \text{ (Ans.)}$$

(iv) $\int \frac{2x^2}{(x^2+1)(x^2+3)}dx$

ধরি, $y = x^2$

এবং $\frac{2y}{(y+1)(y+3)} \equiv \frac{A}{y+1} + \frac{B}{y+3}$

যেখানে, $A, B \in \mathbb{R}$

উভয় পক্ষকে $(y+1)(y+3)$ দ্বারা গুণ করে,

$2y = (y+3)A + (y+1)B$

$y = -1$ হলে, $-2 = 2A$

$\therefore A = -1$

এবং $y = -3$ হলে, $-6 = -2B$

$\therefore B = 3$

$$\therefore \int \frac{2x^2}{(x^2+1)(x^2+3)}dx$$
$$= \int \frac{-1}{x^2+1}dx + \int \frac{3}{x^2+3}dx$$
$$= -\tan^{-1}(x) + 3.\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c$$
$$= -\tan^{-1}x + \sqrt{3}\tan^{-1}\left(\frac{x}{\sqrt{3}}\right) + c \text{ (Ans.)}$$

6. $\int \frac{dx}{(e^x-1)(e^x+3)}$

মনে করি, $\frac{1}{(e^x-1)(e^x+3)} \equiv \frac{A}{e^x-1} + \frac{B}{e^x+3}$

উভয় পক্ষকে $(e^x-1)(e^x+3)$ দ্বারা গুণ করে,

$1 = A(e^x+3) + B(e^x-1)$

$x = 0$ হলে, $A = \frac{1}{4}$

$x = -\ln 3$ হলে, $1 = A(e^{-\ln 3}+3) + B(e^{-\ln 3}-1)$

বা, $1 = A(-3+3) + B(-3-1)$

বা, $1 = -4B$

$\therefore B = -\frac{1}{4}$

$$\therefore \int \frac{dx}{(e^x-1)(e^x+3)} = \int\left(\frac{1}{4(e^x-1)} - \frac{1}{4(e^x+3)}\right)dx$$
$$= \frac{1}{4}\int\left(\frac{e^{-x}}{1-e^{-x}} + \frac{e^{-x}}{1+3e^{-x}}\right)dx$$
$$= \frac{1}{4}\left[\ln(1-e^{-x}) + \frac{1}{3}\ln(1+3e^{-x})\right] + c$$
$$= \frac{1}{4}\left[\ln\left(\frac{e^x-1}{e^x}\right) + \frac{1}{3}\ln\left(\frac{e^x+3}{e^x}\right)\right] + c$$
$$= \frac{1}{4}\left[\ln|e^x-1| - \ln e^x + \frac{1}{3}(\ln|e^x+3| - \ln|e^x|)\right] + c$$
$$= \frac{1}{4}\left[-\frac{4}{3}\ln|e^x| + \ln|e^x-1| + \frac{1}{3}\ln(e^x+3)\right] + c$$
$$= -\frac{1}{3}\ln|e^x| + \frac{1}{4}\ln|e^x-1| + \frac{1}{12}\ln|e^x+3| + c \text{ (Ans.)}$$

অনুশীলনী-10(E) এর সমাধান

1.(i) $\int x^2e^{-3x}dx$

$= x^2\int e^{-3x}dx - \int\left\{\frac{d}{dx}(x^2)\int e^{-3x}dx\right\}dx$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\int xe^{-3x}dx$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left[x\int e^{-3x}dx - \int\left\{\frac{d}{dx}(x)\int e^{-3x}dx\right\}dx\right]$

$= -\frac{1}{3}x^2e^{-3x} + \frac{2}{3}\left[-\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx\right]$

$= -\frac{1}{3}x^2e^{-3x} - \frac{2}{9}xe^{-3x} - \frac{2}{27}e^{-3x} + c$

$= -\frac{1}{3}\left(x^2 + \frac{2}{3}x + \frac{2}{9}\right)e^{-3x} + c$ **(Ans.)**

(ii) $\int x^3e^{x^2}dx$

ধরি, $x^2 = z$

$\therefore 2x\,dx = dz$

$\therefore x\,dx = \frac{dz}{2}$

$= \int x^2e^{x^2}x\,dx$

$= \int ze^z\frac{dz}{2}$

$= \frac{1}{2}\int ze^z dz$

$= \frac{1}{2}\left[z\int e^z dz - \int\left\{\frac{d}{dz}(z)\int e^z dz\right\}dz\right]$

$= \frac{1}{2}[ze^z - \int e^z dz]$

$= \frac{1}{2}[ze^z - e^z] + c$

$= \frac{1}{2}e^z(z-1) + c$

$= \frac{1}{2}(x^2-1)e^{x^2} + c$ **(Ans.)**

2.(i) $\int x^2\sin x\,dx$

$= x^2\int\sin x\,dx - \int\left\{\frac{d}{dx}(x^2)\int\sin x\,dx\right\}dx$

$= -x^2\cos x + 2\int x\cos x\,dx$

$= -x^2\cos x + 2\left[x\int\cos x\,dx - \int\left\{\frac{d}{dx}(x)\int\cos x\,dx\right\}dx\right]$

$= -x^2\cos x + 2x\sin x - 2\int\sin x\,dx$

$= -x^2\cos x + 2x\sin x + 2\cos x + c$ **(Ans.)**

(ii) $\int x^2\cos x\,dx$

$= x^2\int\cos x\,dx - \int\left\{\frac{d}{dx}(x^2)\int\cos x\,dx\right\}dx$

$= x^2\sin x - 2\int x\sin x\,dx$

$= x^2\sin x - 2\left[x\int\sin x\,dx - \int\left\{\frac{d}{dx}(x)\int\sin x\,dx\right\}dx\right]$

$= x^2\sin x - 2[-x\cos x + \int\cos x\,dx]$

$= x^2\sin x + 2x\cos x - 2\sin x + c$ **(Ans.)**

3.(i) $\int x\cos x\,dx$

$= x\int\cos x\,dx - \int\left\{\frac{d}{dx}(x)\int\cos x\,dx\right\}dx$

$= x\sin x - \int\sin x\,dx$

$= x\sin x + \cos x + c$ **(Ans.)**

(ii) $\int e^{2x}\cos e^x dx$

ধরি, $e^x = z$

$\therefore e^x dx = dz$

$= \int e^x.e^x\cos e^x dx$

$= \int z\cos z\,dz$

$= z\int\cos z\,dz - \int\left\{\frac{d}{dz}(z)\int\cos z\,dz\right\}dz$

$= z\sin z - \int\sin z\,dz$

$= z\sin z + \cos z + c$

$= e^x\sin e^x + \cos e^x + c$ **(Ans.)**

4.(i) $\int x\sin x\cos x\,dx$

$= \frac{1}{2}\int x(2\sin x\cos x)dx$

$= \frac{1}{2}\int x\sin 2x\,dx$

$= \frac{1}{2}\left[x\int\sin 2x\,dx - \left\{\frac{d}{dx}(x)\int\sin 2x\,dx\right\}dx\right]$

$= \frac{1}{2}\left[-\frac{x\cos 2x}{2} + \frac{1}{2}\int\cos 2x\,dx\right]$

$= -\frac{1}{4}x\cos 2x + \frac{1}{4}\int\cos 2x\,dx$

$= -\frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + c$ **(Ans.)**

(ii) $\int x\sin x\sin 2x\,dx$

$= \frac{1}{2}\int x\,2\sin x\sin 2x\,dx$

$= \frac{1}{2}\int x(\cos x - \cos 3x)dx$

$= \frac{1}{2}\int x\cos x\,dx - \frac{1}{2}\int x\cos 3x\,dx$

$= \frac{1}{2}\left[x\int\cos x\,dx - \int\left\{\frac{d}{dx}(x)\int\cos x\,dx\right\}dx\right] - \frac{1}{2}\left[x\int\cos 3x - \int\left\{\frac{d}{dx}(x)\int\cos 3x\,dx\right\}dx\right]$

$= \frac{1}{2}(x\sin x - \int\sin x\,dx) - \frac{1}{2}\left(\frac{1}{3}x\sin 3x - \frac{1}{3}\int\sin 3x\,dx\right)$

$= \frac{1}{2}x\sin x + \frac{1}{2}\cos x - \frac{1}{6}x\sin 3x - \frac{1}{18}\cos 3x + c$

$= \frac{1}{2}\left(x\sin x + \cos x - \frac{1}{3}x\sin 3x - \frac{1}{9}\cos 3x\right) + c$ **(Ans.)**

5.(i) $\int x\sec^2x\,dx$

$= x\int\sec^2x\,dx - \int\left\{\frac{d}{dx}(x)\int\sec^2x\,dx\right\}dx$

$= x\tan x - \int\tan x\,dx$

$= x\tan x - \ln|\sec x| + c$

$= x\tan x - \ln|\sec x| + c$ **(Ans.)**

(ii) $\int\frac{x}{\cos^2x}dx = \int x\sec^2x\,dx$

$= x\int\sec^2x - \int\left\{\frac{d}{dx}(x)\int\sec^2x\,dx\right\}dx$

$= x\tan x - \int\tan x\,dx$

$= x\tan x - \ln|\sec x| + c$ **(Ans.)**

6.(i) $\int x\cos^2x\,dx$

$= \frac{1}{2}\int x(2\cos^2x)dx$

$= \frac{1}{2}\int x(1+\cos 2x)\,dx$

$= \frac{1}{2}\int x\,dx + \frac{1}{2}\int x\cos 2x\,dx$

$= \frac{1}{4}x^2 + \frac{1}{2}\left[x\int\cos 2x\,dx - \int\{\frac{d}{dx}(x)\int\cos 2x\,dx\}\,dx\right]$

$= \frac{1}{4}x^2 + \frac{1}{2}[\frac{1}{2}x\sin 2x - \frac{1}{2}\int\sin 2x\,dx]$

$= \frac{1}{4}x^2 + \frac{1}{4}x\sin 2x - \frac{1}{4}\int\sin 2x\,dx$

$= \frac{1}{4}x^2 + \frac{1}{4}x\sin 2x + \frac{1}{8}\cos 2x + c$

$= \frac{1}{4}(x^2 + x\sin 2x) + \frac{1}{8}\cos 2x + c$ **(Ans.)**

(ii) $\int x\sin^2\frac{x}{2}dx = \frac{1}{2}\int x.\,2\sin^2\frac{x}{2}dx$

$= \frac{1}{2}\int x(1-\cos x)\,dx$

$= \frac{1}{2}\int x\,dx - \frac{1}{2}\int x\cos x\,dx$

$= \frac{1}{4}x^2 - \frac{1}{2}[x\int\cos x\,dx - \int\{\frac{d}{dx}(x)\int\cos x\,dx\}\,dx]$

$= \frac{1}{4}x^2 - \frac{1}{2}[x\sin x - \int\sin x\,dx]$

$= \frac{1}{4}x^2 - \frac{1}{2}[x\sin x + \cos x] + c$

$= \frac{1}{4}x^2 - \frac{1}{2}x\sin x - \frac{1}{2}\cos x + c$ **(Ans.)**

7. $\int x\tan^2x\,dx$

$= \int x\sec^2x\,dx - \int x\,dx$

$= \left[x\int\sec^2x\,dx - \int\left\{\frac{d}{dx}(x)\int\sec^2x\,dx\right\}\right] - \int x\,dx$

$= \left(x\tan x - \int\tan x\,dx\right) - \frac{1}{2}x^2$

$= x\tan x + \ln|\cos x| - \frac{1}{2}x^2 + c$ **(Ans.)**

8. $\int x\cos^3x\,dx$

$= \int x.\frac{1}{4}(\cos 3x + 3\cos x)\,dx$

$= \frac{1}{4}\int x\cos 3x\,dx + \frac{3}{4}\int x\cos x\,dx$

$= \frac{1}{4}\left[x\int\cos 3x\,dx - \int\left\{\frac{d}{dx}(x)\int\cos 3x\,dx\right\}dx\right] + \frac{3}{4}\left[x\int\cos x\,dx - \int\left\{\frac{d}{dx}(x)\int\cos x\,dx\right\}\right]$

$= \frac{1}{4}.\frac{1}{3}x\sin 3x - \frac{1}{4}.\frac{1}{3}\int\sin 3x\,dx + \frac{3}{4}x\sin x - \frac{3}{4}\int\sin x\,dx$

$= \frac{1}{12}x\sin 3x + \frac{1}{12}.\frac{1}{3}\cos 3x + \frac{3}{4}x\sin x + \frac{3}{4}\cos x + c$

$= \frac{3}{4}(x\sin x + \cos x) + \frac{1}{36}(3x\sin 3x + \cos 3x) + c$

(Ans.)

9.(i) $\int\sin^{-1}x\,dx$

$= \sin^{-1}x\int 1\,dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int 1.dx\right\}dx$

$= \sin^{-1}x.\,x - \int\frac{1}{\sqrt{1-x^2}}.x\,dx$

$= x\sin^{-1}x + \frac{1}{2}\int\frac{-2x}{\sqrt{1-x^2}}dx$

$= x\sin^{-1}x + \frac{1}{2}\int\frac{dz}{\sqrt{z}}$ | ধরি, $z = 1-x^2$ ∴ $dz = -2x\,dx$

$= x\sin^{-1}x + \frac{1}{2}.2\sqrt{z} + c$

$= x\sin^{-1}x + \sqrt{1-x^2} + c$ **(Ans.)**

(ii) $\int\tan^{-1}x\,dx$

$= \tan^{-1}x\int dx - \int\{\frac{d}{dx}(\tan^{-1}x)\int dx\}\,dx$

$= x\tan^{-1}x - \int\frac{x}{1+x^2}dx$

$= x\tan^{-1}x - \frac{1}{2}\int\frac{2x\,dx}{1+x^2}$

$= x\tan^{-1}x - \frac{1}{2}\ln|1+x^2| + c$ **(Ans.)**

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(iii) $\int \sec^{-1} x\, dx$

$= \sec^{-1}x \int dx - \int\left\{\frac{d}{dx}(\sec^{-1}x)\int dx\right\} dx$

$= \sec^{-1}x.x - \int x.\frac{1}{x\sqrt{x^2-1}}\, dx$

$= x\sec^{-1}x - \int\frac{1}{\sqrt{x^2-1}}\, dx$

$= x\sec^{-1}x - \ln\left|x+\sqrt{x^2-1}\right| + c$ **(Ans.)**

10.(i) $\int x \ln x\, dx$

$= \ln x \int x\, dx - \int\{\frac{d}{dx}(\ln x)\int x\, dx\} dx$

$= \ln x.\frac{x^2}{2} - \int\frac{1}{x}.\frac{x^2}{2} dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}\int x\, dx$

$= \frac{x^2}{2}\ln x - \frac{1}{2}.\frac{x^2}{2} + c$

$= \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + c$ **(Ans.)**

(ii) $\int (\ln x)^2 dx$

$= (\ln x)^2\int dx - \int\left[\frac{d}{dx}\{(\ln x)^2\}\int dx\right] dx$

$= x(\ln x)^2 - \int\frac{2\ln x}{x}\cdot x\, dx$

$= x(\ln x)^2 - 2\int \ln x\, dx$

$= x(\ln x)^2 - 2\left[\ln x\int dx - \int\left\{\frac{d}{dx}(\ln x)\int dx\right\} dx\right]$

$= x(\ln x)^2 - 2\left[x\ln x - \int\frac{1}{x}\cdot x\, dx\right]$

$= x(\ln x)^2 - 2\left[x\ln x - \int dx\right]$

$= x(\ln x)^2 - 2x\ln x + 2x + c$ **(Ans.)**

(iii) $\int x^2(\ln x)^2 dx$

$= (\ln x)^2\int x^2 dx - \int\left[\frac{d}{dx}\{(\ln x)^2\}\int x^2 dx\right] dx$

$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\int\frac{\ln(x)}{x}.x^3 dx$

$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\int\ln(x).x^2 dx$

$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\left[\ln(x)\int x^2 dx - \int\left\{\frac{d}{dx}(\ln x)\int x^2 dx\right\}dx\right]$

$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{3}\left[\frac{1}{3}x^3\ln x - \frac{1}{3}\int\frac{1}{x}.x^3 dx\right]$

$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x + \frac{2}{9}\int x^2 dx$

$= \frac{1}{3}x^3(\ln x)^2 - \frac{2}{9}x^3\ln x + \frac{2}{27}x^3 + c$ **(Ans.)**

(iv) $\int x^n \ln x\, dx = \ln x$

$\int x^n dx - \int\left\{\frac{d}{dx}(\ln x)\int x^n dx\right\} dx$

$= \frac{x^{n+1}\ln x}{(n+1)} - \int\frac{1}{x}.\frac{x^{n+1}}{(n+1)} dx$

$= \frac{x^{n+1}\ln x}{(n+1)} - \frac{1}{(n+1)}\int x^n dx$

$= \frac{x^{n+1}\ln x}{(n+1)} - \frac{x^{n+1}}{(n+1)^2} + c$ **(Ans.)**

11. $\int x\sin^{-1}x\, dx$

$= \sin^{-1}x\int x\, dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int x\, dx\right\} dx$

$= \sin^{-1}x.\frac{x^2}{2} - \int\frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2} dx$

$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}} dx$

$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}I$ $\quad [I = \int\frac{x^2}{\sqrt{1-x^2}} dx$ ধরে$]$

এখন, $I = \int\frac{x^2}{\sqrt{1-x^2}} dx$

$= \int\frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}.\cos\theta\, d\theta$

$= \int\sin^2\theta\, d\theta$

$= \int\frac{1}{2}(1-\cos 2\theta)\, d\theta$

$= \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$

$= \frac{1}{2}(\sin^{-1}x - \frac{1}{2}.2\sin\theta\cos\theta)$

ধরি, $x = \sin\theta$
বা, $\theta = \sin^{-1}x$
$\therefore dx = \cos\theta\, d\theta$

$\therefore I = \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}$

$\therefore \int x\sin^{-1}x\, dx = \frac{1}{2}x^2\sin^{-1}x - \frac{1}{2}\left(\frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}\right) + c$

$= \frac{1}{2}x^2\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + c$ **(Ans.)**

12.(i) ধরি, $I = \int x\cos^{-1}x\, dx$

$= \cos^{-1}x\int x\, dx - \int\left\{\frac{d}{dx}(\cos^{-1}x)\int x\, dx\right\} dx$

$= \cos^{-1}x.\frac{x^2}{2} - \frac{1}{2}\int\frac{-1}{\sqrt{1-x^2}}.x^2 dx$

$= \frac{x^2}{2}\cos^{-1}x + \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}} dx$ ... ... (i)

এখন, $\int \frac{x^2}{\sqrt{1-x^2}} dx$

ধরি, $x = \sin\theta$

বা, $\theta = \sin^{-1}x$

$\therefore dx = \cos\theta \, d\theta$

$= \int \frac{\sin^2\theta}{\cos\theta} . \cos\theta \, d\theta$

$= \int \sin^2\theta . d\theta$

$= \frac{1}{2}\int (1-\cos 2\theta) \, d\theta = \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$

$= \frac{1}{2}\left(\sin^{-1}x - \frac{1}{2}. 2\sin\theta\cos\theta\right)$

$= \frac{1}{2}\left(\sin^{-1}x - x\sqrt{1-x^2}\right)$

$= \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}$

$\therefore I = \frac{x^2}{2}\cos^{-1}x + \frac{1}{2}\left(\frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}\right)$

$= \frac{x^2}{2}\cos^{-1}x + \frac{1}{4}\sin^{-1}x - \frac{1}{4}x\sqrt{1-x^2} + c$ **(Ans.)**

(ii) ধরি,

ধরি, $x^2 = z$

বা, $2x \, dx = dz$

$\therefore x \, dx = \frac{1}{2}dz$

$I = \int x \sin^{-1}x^2 \, dx$

$= \frac{1}{2}\int \sin^{-1}z \, dz$

$= \frac{1}{2}\left[\sin^{-1}z \int dz - \int \left\{\frac{d}{dz}(\sin^{-1}z)\int dz\right\} dz\right]$

$= \frac{1}{2}\left[(\sin^{-1}z) z - \int \frac{1}{\sqrt{1-z^2}} z \, dz\right]$

$= \frac{1}{2}\left[z(\sin^{-1}z) + \frac{1}{2}\int \frac{-2z}{\sqrt{1-z^2}} dz\right]$

$= \frac{1}{2}\left[z(\sin^{-1}z) + \frac{1}{2}2\sqrt{1-z^2}\right] + c$

$= \frac{1}{2}\left[z\sin^{-1}z + \sqrt{1-z^2}\right] + c$

$= \frac{1}{2}\left[x^2\sin^{-1}(x^2) + \sqrt{1-x^4}\right] + c$ **(Ans.)**

(iii) ধরি, $I = \int x\cos^{-1}x^2 \, dx$

ধরি, $x^2 = z$

বা, $2x \, dx = dz$

$\therefore x \, dx = \frac{1}{2}dz$

$= \frac{1}{2}\int \cos^{-1}z \, dz$

$= \frac{1}{2}\left[\cos^{-1}z \int dz - \int\left\{\frac{d}{dz}(\cos^{-1}z)\int dz\right\} dz\right]$

$= \frac{1}{2}\left[\cos^{-1}z. z - \int \frac{-1}{\sqrt{1-z^2}} z.dz\right]$

$= \frac{1}{2}\left[z\cos^{-1}z + \int \frac{z}{\sqrt{1-z^2}} dz\right]$

$= \frac{1}{2}\left[z\cos^{-1}z - \sqrt{1-z^2}\right] + c$

$= \frac{1}{2}\left[x^2\cos^{-1}(x^2) - \sqrt{1-x^4}\right] + c$ **(Ans.)**

13.(i) $\int \tan^{-1}\frac{2x}{1-x^2} dx$

$= \int 2\tan^{-1}x \, dx$

$= 2\tan^{-1}x \int dx - 2\int\left\{\frac{d}{dx}(x)\int \tan^{-1}x \, dx\right\} dx$

$= 2\tan^{-1}x . x - \int \frac{2x}{1+x^2} dx$

$= 2x\tan^{-1}x - \ln(1+x^2) + c$ **(Ans.)**

(ii) $\int \sin^{-1}\frac{2x}{1+x^2} dx$

$= \int 2\tan^{-1}x \, dx \quad \left[\because 2\tan^{-1}x = \sin^{-1}\frac{2x}{1+x^2}\right]$

$= 2\tan^{-1}x \int dx - 2\int\left\{\frac{d}{dx}(x)\int \tan^{-1}x \, dx\right\} dx$

$= 2\tan^{-1}x . x - \int \frac{2x}{1+x^2} dx$

$= 2x\tan^{-1}x - \ln(1+x^2) + c$ **(Ans.)**

(iii) $\int \cos^{-1}\frac{1-x^2}{1+x^2} dx$

$= \int 2\tan^{-1}x \, dx \quad \left[\because 2\tan^{-1}x = \cos^{-1}\frac{1-x^2}{1+x^2}\right]$

$= 2\tan^{-1}x . x - \int \frac{2x}{1+x^2} dx$

$= 2x\tan^{-1}x - \ln(1+x^2) + c$ **(Ans.)**

14. $\int \frac{\ln(\ln x)}{x} dx$

$= \ln(\ln x)\int \frac{1}{x} dx - \int [\frac{d}{dx}\{\ln(\ln x)\}\int \frac{1}{x} dx] dx$

$= \ln x \ln(\ln x) - \int (\frac{1}{\ln x} \times \frac{1}{x} \times \ln x) dx$

$= \ln x \ln(\ln x) - \int \frac{dx}{x}$

$= \ln x \ln(\ln x) - \ln x + c$

$= \ln x \{\ln(\ln x) - 1\} + c$ **(Ans.)**

15. ধরি, $I = \int \frac{\ln(\sec^{-1}x)}{x\sqrt{x^2-1}} dx$

ধরি, $\sec^{-1}x = z$

$\frac{1}{x\sqrt{x^2-1}} dx = dz$

$\therefore I = \int \ln(z) \, dz$

$= \ln z . \int dz - \int\left\{\frac{d}{dz}\ln(z)\int dz\right\} dz$

$= \ln z.z - \int \frac{1}{z}.z.dz$

$= z\ln(z) - \int dz$

$= z\ln(z) - z + c$

$= \sec^{-1}x. \ln(\sec^{-1}x) - \sec^{-1}x + c$

$= \sec^{-1}x[\ln|\sec^{-1}x| - 1] + c$ **(Ans.)**

16.(i) ধরি, $I = \int e^x \cos x\, dx$

$= \cos x \int e^x dx - \int\left\{\frac{d}{dx}(\cos x)\int e^x dx\right\}dx$

$= e^x \cos x + \int \sin x.\, e^x dx$

$= e^x \cos x + \sin x \int e^x dx - \int\left\{\frac{d}{dx}(\sin x)\int e^x dx\right\}dx$

$= e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$

$= e^x \cos x + e^x \sin x - I$

$\therefore 2I = e^x(\cos x + \sin x)$

$\therefore I = \frac{e^x}{2}(\cos x + \sin x) + c$ **(Ans.)**

(ii) ধরি, $I = \int e^x \sin x\, dx$

$= e^x.\int \sin x\, dx - \int\{\frac{d}{dx}(e^x)\int \sin x\, dx\}dx$

$= -e^x \cos x + \int e^x \cos x\, dx$

$= -e^x \cos x + \left[e^x \sin x - \int e^x \sin x\, dx\right]$

$= e^x \sin x - e^x \cos x - I$

বা, $2I = e^x \sin x - e^x \cos x$

$\therefore I = \frac{1}{2}e^x(\sin x - \cos x) + c$ **(Ans.)**

(iii) ধরি, $I = \int e^x \sin 2x\, dx$

$= e^x \int \sin 2x\, dx - \int\{\frac{d}{dx}(e^x)\int \sin 2x\, dx\}dx$

$= \frac{e^x(-\cos 2x)}{2} - \int \frac{\{e^x(-\cos 2x)\}}{2}dx$

$= -\frac{1}{2}e^x \cos 2x + \frac{1}{2}\int e^x \cos 2x\, dx$

$= -\frac{1}{2}e^x \cos 2x + \frac{1}{2}[e^x.\frac{\sin 2x}{2} - \frac{1}{2}\int e^x \sin 2x\, dx]$

$\therefore I = -\frac{1}{2}e^x \cos 2x + \frac{1}{4}e^x \sin 2x - \frac{1}{4}I$

বা, $I + \frac{1}{4}I = \frac{1}{4}e^x(\sin 2x - 2\cos 2x)$

বা, $I = \frac{4}{5}\cdot\frac{1}{4}.\, e^x(\sin 2x - 2\cos 2x)$

$= \frac{1}{5}e^x(\sin 2x - 2\cos 2x) + c$ **(Ans.)**

(iv) মনে করি, $I = \int e^{-3x}\cos 4x\, dx$

তাহলে, $I = e^{-3x}\left(\frac{1}{4}\sin 4x\right) - \int(-3)e^{-3x}\left(\frac{1}{4}\sin 4x\right)dx$

$= \frac{1}{4}e^{-3x}\sin 4x + \frac{3}{4}\int e^{-3x}\sin 4x\, dx$

$= \frac{1}{4}e^{-3x}\sin 4x + \frac{3}{4}$

$\left[e^{-3x}\left(\frac{-1}{4}\cos 4x\right) - \int -3e^{-3x}\left(\frac{-1}{4}\cos 4x\right)\right]dx$

$= \frac{1}{4}e^{-3x}\sin 4x - \frac{3}{16}e^{-3x}\cos 4x - \frac{9}{16}I$

$\therefore \left(1 + \frac{9}{16}\right)I = \frac{e^{-3x}}{16}(4\sin 4x - 3\cos 4x)$

$\therefore I = \frac{1}{25}e^{-3x}(4\sin 4x - 3\cos 4x) + c$ **(Ans.)**

17.(i) $\int e^x(\sin x + \cos x)dx$

$= \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c$

$= e^x \sin x + c$ **(Ans.)**

ধরি, $f(x) = \sin x$, $f'(x) = \cos x$

(ii) $\int e^x(\tan x - \ln\cos x)dx$

$= \int e^x(-\ln\cos x + \tan x)dx$

$= \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c$

$= e^x(-\ln\cos x) + c$

$= e^x \ln|\sec x| + c$ **(Ans.)**

ধরি, $f(x) = -\ln\cos x$, $f'(x) = \tan x$

(iii) $\int e^x \sec x(1 + \tan x)dx$

$= \int e^x(\sec x + \sec x\tan x)dx$

$= \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c$

$= e^x \sec x + c$ **(Ans.)**

ধরি, $f(x) = \sec x$, $f'(x) = \sec x\tan x$

18. $\int e^x\left(\ln x + \frac{1}{x}\right)dx$

$= \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c$

$= e^x \ln|x| + c$ **(Ans.)**

ধরি, $f(x) = \ln x$, $f'(x) = \frac{1}{x}$

19.(i) $\int e^x\left(\frac{1}{x} - \frac{1}{x^2}\right)dx$

$= \int e^x\{f(x) + f'(x)\}dx$

$= e^x f(x) + c$

$= \frac{e^x}{x} + c$ **(Ans.)**

ধরি, $f(x) = \frac{1}{x}$, $f'(x) = -\frac{1}{x^2}$

(ii) $\int e^{-x}\left(\frac{1}{x} + \frac{1}{x^2}\right)dx$

$= \int e^{-x}.\frac{1}{x}dx + \int e^{-x}.\frac{1}{x^2}dx$

$= \frac{1}{x}\int e^{-x}dx - \int\left(\frac{d}{dx}.\frac{1}{x}\int e^{-x}dx\right)dx + \int e^{-x}.\frac{1}{x^2}dx$

$= \frac{1}{x}e^{-x} - \int\left(-\frac{1}{x^2}.\frac{e^{-x}}{-1}\right) + \int e^{-x}.\frac{1}{x^2}dx$

$= -\frac{e^{-x}}{x} - \int e^{-x}.\frac{1}{x^2}dx + \int e^{-x}.\frac{1}{x^2}dx$

$= -\frac{e^{-x}}{x} + c$ **(Ans.)**

20. $\int e^{5x}\left[5\ln x+\frac{1}{x}\right]dx$

$= 5\int e^{5x}\ln x\,dx+\int e^{5x}.\frac{1}{x}dx$

$= 5\left[\ln x\int e^{5x}dx-\int\frac{d}{dx}(\ln x)\int e^{5x}dx\right]+\int e^{5x}.\frac{1}{x}dx$

$= 5\left[\ln x.\frac{e^{5x}}{5}-\frac{1}{5}\int\frac{1}{x}e^{5x}dx\right]+\int e^{5x}.\frac{1}{x}dx$

$= e^{5x}\ln x-\int\frac{1}{x}e^{5x}dx+\int e^{5x}.\frac{1}{x}dx$

$= e^{5x}\ln|x|+c$ **(Ans.)**

21. ধরি, $I=\int\sin^{-1}\sqrt{\frac{x}{a+x}}\,dx$

$= \int\sin^{-1}\left(\sqrt{\frac{a\tan^2\theta}{a(1+\tan^2\theta)}}\right)2a\tan\theta\sec^2\theta\,d\theta$

$= \int\sin^{-1}\left(\sqrt{\frac{\tan^2\theta}{\sec^2\theta}}\right)2a\tan\theta\sec^2\theta\,d\theta$

$= \int\sin^{-1}(\sin\theta)\,2a\tan\theta\sec^2\theta\,d\theta$

$\therefore I=\int 2a\theta\tan\theta\sec^2\theta\,d\theta$

ধরি, $x=a\tan^2\theta$
$\therefore dx=2a\tan\theta\sec^2\theta d\theta$
এবং $\theta=\tan^{-1}\sqrt{\frac{x}{a}}$

$= 2a\left[\theta\tan\theta\int\sec^2\theta\,d\theta-\int\left\{\frac{d}{d\theta}(\theta\tan\theta)\int\sec^2\theta\,d\theta\right\}d\theta\right]$

$= 2a\,[\theta\tan^2\theta-\int(\theta\sec^2\theta+\tan\theta)\tan\theta\,d\theta]$

$= 2a\theta\tan^2\theta-\int 2a\,\theta\tan\theta\sec^2\theta\,d\theta-2a\int\tan^2\theta\,d\theta$

$= 2a\theta\tan^2\theta-I-2a\int(\sec^2\theta-1)\,d\theta$

$\therefore I+I=2a\theta\tan^2\theta-2a(\tan\theta-\theta)$

বা, $2I=2a\theta\tan^2\theta-2a(\tan\theta-\theta)$

$\therefore I=a\tan^{-1}\sqrt{\frac{x}{a}}\left(\tan\tan^{-1}\sqrt{\frac{x}{a}}\right)^2-a\left(\tan\tan^{-1}\sqrt{\frac{x}{a}}-\tan^{-1}\sqrt{\frac{x}{a}}\right)+c$

$= a\tan^{-1}\left(\sqrt{\frac{x}{a}}\right)\frac{x}{a}-a\left(\sqrt{\frac{x}{a}}-\tan^{-1}\sqrt{\frac{x}{a}}\right)+c$

$= x\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+a\tan^{-1}\sqrt{\frac{x}{a}}+c$

$= (x+a)\tan^{-1}\sqrt{\frac{x}{a}}-\sqrt{ax}+c$ **(Ans.)**

অনুশীলনী-10(F) এর সমাধান

1.(i) $\int_0^{\pi/2}(\sin\theta+\cos\theta)\,d\theta=[-\cos\theta+\sin\theta]_0^{\pi/2}$

$= [(-\cos\frac{\pi}{2}+\sin\frac{\pi}{2})-(-\cos 0+\sin 0)]$

$= (0+1)-(-1+0)=1+1=2$ **(Ans.)**

(ii) $\int_0^{\pi/2}\cos 4x\,dx$

$= \frac{1}{4}[\sin 4x]_0^{\frac{\pi}{2}}=\frac{1}{4}[\sin 2\pi-\sin 0]$

$= \frac{1}{4}[0-0]=0$ **(Ans.)**

2.(i) $\int_0^4\frac{dx}{\sqrt{2x+1}}$

ধরি, $z=2x+1$
$\therefore dz=2dx$

x	4	0
z	9	1

$= \frac{1}{2}\int_1^9\frac{dz}{\sqrt{z}}$

$= \frac{1}{2}\int_1^9 z^{-\frac{1}{2}}dz=\frac{1}{2}\left[\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^9=[9^{\frac{1}{2}}-1]=3-1=2$ **(Ans.)**

(ii) $\int_1^4\frac{dx}{(2+3x)^2}$

ধরি, $2+3x=z$
বা, $3\,dx=dz$
$\therefore dx=\frac{dz}{3}$
যখন $x=4$ তখন $z=14$
যখন $x=1$ তখন $z=5$

$= \int_1^{14}\frac{\frac{dz}{3}}{z^2}$

$= \frac{1}{3}\int_5^{14}\frac{dz}{z^2}$

$= \frac{1}{3}.\left[-\frac{1}{z}\right]_5^{14}$

$= \frac{1}{3}\left(-\frac{1}{14}+\frac{1}{5}\right)=\frac{1}{3}\times\frac{9}{70}=\frac{3}{70}$ **(Ans.)**

(iii) $\int_0^1 x(1-\sqrt{x})^2dx=\int_0^1 x(1-2\sqrt{x}+x)\,dx$

$= \int_0^1(x-2x^{\frac{1}{2}}.x+x^2)dx$

$= \int_0^1(x-2x^{\frac{3}{2}}+x^2)\,dx$

$= \left[\frac{x^2}{2}-2.\frac{2}{5}x^{\frac{5}{2}}+\frac{x^3}{3}\right]_0^1$

$= \frac{1}{2}-\frac{4}{5}+\frac{1}{3}-0$

$= \frac{15-24+10}{30}=\frac{1}{30}$ **(Ans.)**

(iv) $\int_0^4(\sqrt{x}+\frac{1}{\sqrt{x}})\,dx$

$= \int_0^4\left(x^{\frac{1}{2}}+x^{-\frac{1}{2}}\right)dx=\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}+\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_0^4$

$= \left[\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}\right]_0^4=\frac{2}{3}(4)^{\frac{3}{2}}+2(4)^{\frac{1}{2}}-0$

$= \frac{2}{3}.8+2.2=\frac{16}{3}+4=\frac{28}{3}$ **(Ans.)**

(v) $\int_2^3 \frac{2x}{1+x^2}dx$

$$=[\ln(1+x^2)]_2^3 \quad \left[\because \int \frac{f'(x)}{f(x)}dx = \ln|f(x)|\right]$$

$$= \ln 10 - \ln 5 = \ln\left(\frac{10}{5}\right) = \ln 2 \textbf{ (Ans.)}$$

(vi) $\int_4^8 \frac{x\,dx}{\sqrt{x^2-15}}$

ধরি, $z = x^2 - 15$
$\therefore dz = 2x\,dx$

x	4	8
z	1	49

$$= \frac{1}{2}\int_4^8 \frac{2x\,dx}{\sqrt{x^2-15}}$$

$$= \frac{1}{2}\int_1^{49}\frac{dz}{\sqrt{z}} = \frac{1}{2}\int_1^{49} z^{\frac{-1}{2}}\,dz$$

$$= \frac{1}{2}\left[\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^{49} = \frac{1}{2}\cdot 2\left[(49)^{\frac{1}{2}} - 1\right]$$

$$= 7 - 1 = 6 \textbf{ (Ans.)}$$

(vii) $\int_0^1 \frac{x\,dx}{\sqrt{1-x^2}}dx$

$$= -\frac{1}{2}\int_0^1 \frac{-2x}{\sqrt{1-x^2}}dx$$

$$= -\frac{1}{2}\int_1^0 \frac{dz}{\sqrt{z}}$$

ধরি, $z = 1 - x^2$
$\therefore dz = -2x\,dx$

x	0	1
z	1	0

$$= -\frac{1}{2}[2\sqrt{z}]_1^0$$

$$= -\frac{1}{2}[0-2] = 1 \textbf{ (Ans.)}$$

(viii) $\int_0^2 \frac{x\,dx}{\sqrt{9-2x^2}}$

$$= -\frac{1}{4}\int_0^2 \frac{-4x}{\sqrt{9-2x^2}}dx$$

$$= -\frac{1}{4}\int_9^1 \frac{dz}{\sqrt{z}}$$

ধরি, $z = 9 - 2x^2$
$\therefore dz = -4x\,dx$

x	0	1
z	9	1

$$= -\frac{1}{4}[2\sqrt{z}]_9^1$$

$$= -\frac{1}{4}[2 - 2\sqrt{9}] = 1 \textbf{ (Ans.)}$$

(ix) $\int_0^1 \frac{x\,dx}{\sqrt{4-x^2}}$

ধরি, $z = 4 - x^2$
$\therefore dz = -2x\,dx$
$\therefore x\,dx = -\frac{dz}{2}$
সীমা : $x = 0$ হলে, $z = 4$
$x = 1$ হলে, $z = 3$

$$= -\frac{1}{2}\int_4^3 \frac{dz}{\sqrt{z}}$$

$$= \frac{1}{2}\int_3^4 \frac{dz}{\sqrt{z}}$$

$$= \frac{1}{2}\left[\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right]_3^4 = [\sqrt{4} - \sqrt{3}] = 2 - \sqrt{3} \textbf{ (Ans.)}$$

(x) $\int_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}dx$

$$= 2\int_0^1 e^z\,dz$$

$$= 2[e^z]_0^1$$

$$= 2(e^1 - e^0)$$

$$= 2(e-1) \textbf{ (Ans.)}$$

ধরি, $z = \sqrt{x}$
$\therefore dz = \frac{1}{2\sqrt{x}}dx$

x	0	1
z	0	1

3.(i) $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x]_0^1$

$$= [\tan^{-1}1 - \tan^{-1}0] = \frac{\pi}{4} - 0 = \frac{\pi}{4} \textbf{ (Ans.)}$$

(ii) $\int_0^1 \frac{dx}{\sqrt{1-x^2}} = [\sin^{-1}x]_0^1$

$$= \sin^{-1}(1) - \sin^{-1}(0) = \frac{\pi}{2} - 0 = \frac{\pi}{2} \textbf{ (Ans.)}$$

(iii) $\int_3^4 \frac{dx}{25-x^2}$

$$= \int_3^4 \frac{dx}{(5)^2 - (x)^2} = \left[\frac{1}{2.5}\ln\left|\frac{5+x}{5-x}\right|\right]_3^4$$

$$= \frac{1}{10}\left[\ln\frac{5+4}{5-4} - \ln\frac{5+3}{5-3}\right] = \frac{1}{10}(\ln 9 - \ln 4)$$

$$= \frac{1}{10}(2\ln 3 - 2\ln 2)$$

$$= \frac{2}{10}\cdot\ln\frac{3}{2} = \frac{1}{5}\ln\frac{3}{2} \textbf{ (Ans.)}$$

(iv) $\int_0^1 \frac{dx}{\sqrt{4-3x^2}} = \frac{1}{\sqrt{3}}\int_0^1 \frac{dx}{\sqrt{\left(\frac{2}{\sqrt{3}}\right)^2 - x^2}}$

$$= \frac{1}{\sqrt{3}}\left[\sin^{-1}\frac{\sqrt{3}\,x}{2}\right]_0^1 = \frac{1}{\sqrt{3}}\sin^{-1}\frac{\sqrt{3}}{2}$$

$$= \frac{1}{\sqrt{3}} \times \frac{\pi}{3} = \frac{\pi}{3\sqrt{3}} \textbf{ (Ans.)}$$

(v) $\int_2^3 \frac{dx}{9x^2-16} = \frac{1}{9}\int_2^3 \frac{dx}{x^2 - \frac{16}{9}}$

$$= \frac{1}{9}\int_2^3 \frac{dx}{x^2 - \left(\frac{4}{3}\right)^2} = \left[\frac{1}{9}\cdot\frac{1}{2\cdot\frac{4}{3}}\ln\left|\frac{x-\frac{4}{3}}{x+\frac{4}{3}}\right|\right]_2^3$$

$$= \frac{1}{24}\left[\ln\frac{3-\frac{4}{3}}{3+\frac{4}{3}} - \ln\frac{2-\frac{4}{3}}{2+\frac{4}{3}}\right] = \frac{1}{24}\left[\ln\frac{5}{13} - \ln\frac{2}{10}\right]$$

$$= \frac{1}{24}\left[\ln\frac{5}{13} - \ln\frac{1}{5}\right] = \frac{1}{24}\ln\left[\frac{5}{13}\times\frac{5}{1}\right] = \frac{1}{24}\ln\frac{25}{13} \textbf{ (Ans.)}$$

4. $\int_0^{\pi/4} \frac{1-\cos 2\theta}{1+\cos 2\theta}\, d\theta = \int_0^{\pi/4} \frac{2\sin^2\theta}{2\cos^2\theta}\, d\theta$

$= \int_0^{\pi/4} \tan^2\theta\, d\theta = \int_0^{\pi/4} (\sec^2\theta - 1)\, d\theta$

$= [\tan\theta - \theta]_0^{\pi/4} = \tan\frac{\pi}{4} - \frac{\pi}{4} - (\tan 0 - 0)$

$= 1 - \frac{\pi}{4}$ **(Ans.)**

5.(i) $\int_0^{\pi/2} \frac{dx}{1+\cos x} = \int_0^{\pi/2} \frac{dx}{2\cos^2\frac{x}{2}}$

$= \int_0^{\pi/2} \frac{1}{2}\, \text{sce}^2\frac{x}{2}\, dx = \left[\tan\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$= \tan\frac{\pi}{4} - \tan 0 = 1$ **(Ans.)**

(ii) $\int_0^{\pi/4} \frac{1}{1+\cos 2x}\, dx$

$= \int_0^{\pi/4} \frac{1}{2\cos^2 x}\, dx = \frac{1}{2}\int_0^{\pi/4} \sec^2 x\, dx$

$= \frac{1}{2}[\tan x]_0^{\frac{\pi}{4}} = \frac{1}{2}[\tan\frac{\pi}{4} - 0]$

$= \frac{1}{2}.1 = \frac{1}{2}$ **(Ans.)**

(iii) $\int_0^{\pi/4} \frac{1}{1-\cos 2x}\, dx$

$= \int_0^{\pi/4} \frac{1}{2\sin^2 x}\, dx = \frac{1}{2}\int_0^{\pi/4} \text{cosec}^2 x\, dx$

$= \frac{1}{2}[-\cot x]_0^{\pi/4} = -\frac{1}{2}[\cot\frac{\pi}{4} - \cot 0]$

$= -\frac{1}{2}[1-\infty] = \infty$

6.(i) $\int_0^{\pi/3} \frac{dx}{1-\sin x}$

$= \int_0^{\pi/3} \frac{(1+\sin x)dx}{1-\sin^2 x}$

$= \int_0^{\pi/3} \frac{1+\sin x}{\cos^2 x}\, dx$

$= \int_0^{\pi/3} (\sec^2 x + \sec x \tan x)\, dx$

$= [\tan x + \sec x]_0^{\pi/3}$

$= \tan\frac{\pi}{3} + \sec\frac{\pi}{3} - (\tan 0 + \sec 0)$

$= (\sqrt{3} + 2) - (0+1)$

$= \sqrt{3} + 1$ **(Ans.)**

(ii) $\int_0^{\pi/4} \frac{dx}{1+\sin x} = \int_0^{\pi/4} \frac{(1-\sin x)dx}{(1+\sin x)(1-\sin x)}$

$= \int_0^{\pi/4} \frac{1-\sin x}{1-\sin^2 x}\, dx = \int_0^{\pi/4} \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x}\right) dx$

$= \int_0^{\pi/4} (\sec^2 x - \tan x.\sec x)\, dx$

$= [\tan x - \sec x]_0^{\pi/4}$

$= 1 - \sqrt{2} - 0 + 1 = 2 - \sqrt{2}$ **(Ans.)**

7.(i) $\int_0^{\pi/2} \sqrt{1+\cos x}\, dx$

$= \int_0^{\pi/2} \sqrt{2\cos^2\frac{x}{2}}\, dx = \sqrt{2}\int_0^{\pi/2} \cos\frac{x}{2}\, dx$

$= \sqrt{2}\left[\frac{\sin\frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}} = 2\sqrt{2}\left[\sin\frac{\pi}{4} - \sin 0\right]$

$= 2\sqrt{2}.\frac{1}{\sqrt{2}} = 2$ **(Ans.)**

(ii) $\int_0^{\pi} 3\sqrt{1-\cos x}\, dx$

$= \int_0^{\pi} 3\sqrt{1-\cos\left(2\times\frac{x}{2}\right)}\, dx$

$= 3\int_0^{\pi} \sqrt{2\sin^2\frac{x}{2}}\, dx = 3\sqrt{2}\int_0^{\pi} \sin\frac{x}{2}\, dx$

$= 3\sqrt{2}\left[\frac{-\cos\frac{x}{2}}{\frac{1}{2}}\right]_2^{\pi} = -6\sqrt{2}\left(\cos\frac{\pi}{2} - \cos 0\right)$

$= -6\sqrt{2}(0-1) = 6\sqrt{2}$ **(Ans.)**

(iii) $\int_0^{\pi/2} \sqrt{1+\sin x}\, dx$

$= \int_0^{\pi/2} \sqrt{\left(\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}\right)}\, dx$

$= \int_0^{\pi/2} \sqrt{\left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2}\, dx$

$= \int_0^{\pi/2} \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right) dx$

$= \left[\frac{-\cos\frac{x}{2}}{\frac{1}{2}} + \frac{\sin\frac{x}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}} = \left[-2\cos\frac{x}{2} + 2\sin\frac{x}{2}\right]_0^{\frac{\pi}{2}}$

$= 2\left[-\cos\frac{\pi}{4} + \cos 0 + \sin\frac{\pi}{4} - \sin 0\right]$

$= 2\left[-\frac{1}{\sqrt{2}} + 1 + \frac{1}{\sqrt{2}} - 0\right] = 2.1 = 2$ **(Ans.)**

(iv) $\int_{-\pi/2}^{\pi/2} \frac{\sec x + 1}{\sec x}\, dx$

$= \int_{-\pi/2}^{\pi/2} (1 + \cos x)\, dx = [x + \sin x]_{-\pi/2}^{\pi/2}$

$= \left(\frac{\pi}{2} + \sin\frac{\pi}{2}\right) - \left(-\frac{\pi}{2} - \sin\frac{\pi}{2}\right) = \frac{\pi}{2} + 1 + \frac{\pi}{2} + 1$

$= \pi + 2$ **(Ans.)**

(v) $\int_0^{\pi/4} \frac{\cos 2x}{\cos^2 x}\, dx$

$= \int_0^{\pi/4} \left(\frac{2\cos^2 x - 1}{\cos^2 x}\right) dx = \int_0^{\pi/4} (2 - \sec^2 x)\, dx$

$= [2x - \tan x]_0^{\frac{\pi}{4}} = \frac{\pi}{2} - \tan\frac{\pi}{4} - 0 + \tan 0$

$= \frac{\pi}{2} - 1$ **(Ans.)**

8.(i) $\int_0^{\pi/2} \cos^2 x\, dx = \frac{1}{2}\int_0^{\pi/2} 2\cos^2 x\, dx$

$= \frac{1}{2}\int_0^{\pi/2} (1 + \cos 2x)\, dx$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2} + \frac{\sin \pi}{2}\right) - 0\right]$

$= \frac{1}{2}\left(\frac{\pi}{2} + 0\right) = \frac{\pi}{4}$ **(Ans.)**

(ii) $\int_0^{\pi/2} \sin^2 x\, dx$

$= \int_0^{\pi/2} \frac{1}{2} . 2\sin^2 x\, dx = \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x)\, dx$

$= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\frac{x}{2}} = \frac{1}{2}\left[\left(\frac{\pi}{2} - \frac{1}{2}\sin 2 . \frac{\pi}{2}\right) - 0\right]$

$= \frac{1}{2}\left(\frac{\pi}{2} - 0\right) = \frac{\pi}{4}$ **(Ans.)**

(iii) $\int_0^{\pi/4} \tan^2\theta\, d\theta$

$= \int_0^{\pi/4} (\sec^2\theta - 1)\, d\theta = [\tan\theta - \theta]_0^{\pi/4}$

$= \tan\frac{\pi}{4} - \frac{\pi}{4} - \tan 0 + 0 = 1 - \frac{\pi}{4}$ **(Ans.)**

9.(i) $\int_0^{\pi/2} \cos^3 x\, dx = \int_0^{\pi/2} \frac{1}{4}\ (\cos 3x + 3\cos x)\, dx$

$= \frac{1}{4}\left[\frac{1}{3}\sin 3x + 3\sin x\right]_0^{\frac{\pi}{2}}$

$= \frac{1}{4}\left[\frac{1}{3}\sin\frac{3\pi}{2} + 3\sin\frac{\pi}{2}\right] - \frac{1}{4}\left[\frac{1}{3}\sin 3.0 + 3\sin 0\right]$

$= \frac{1}{4}[-\frac{1}{3} + 3] - \frac{1}{4}\ [0 + 0] = \frac{1}{4}\cdot\frac{8}{3} = \frac{2}{3}$ **(Ans.)**

(ii) $\int_0^{\pi/2} \sin^3\theta\, d\theta$

$= \frac{1}{4}\int_0^{\pi/2} (3\sin\theta - \sin 3\theta)\, d\theta$

$= \frac{1}{4}\left[-3\cos\theta + \frac{1}{3}\cos 3\theta\right]_0^{\pi/2}$

$= \frac{1}{4}[(-3\cos\frac{\pi}{2} + \frac{1}{3}\cos\frac{3\pi}{2}) - (-3\cos 0 + \frac{1}{3}\cos 0)]$

$= -\frac{1}{4}(-3 + \frac{1}{3})$

$= -\frac{1}{4} \times \left(-\frac{8}{3}\right) = \frac{2}{3}$ **(Ans.)**

10. $\int_0^{\pi/4} \sin^4 x\, dx = \int_0^{\pi/4} (\sin^2 x)^2\, dx$

$= \frac{1}{4}\int_0^{\pi/4} (2\sin^2 x)^2\, dx$

$= \frac{1}{4}\int_0^{\pi/4} (1 - \cos 2x)^2\, dx$

$= \frac{1}{4}\int_0^{\pi/4} (1 - 2\cos 2x + \cos^2 2x)\, dx$

$= \frac{1}{4}\int_0^{\pi/4} \left\{(1 - 2\cos 2x) + \frac{1}{2}(2\cos^2 2x)\right\} dx$

$= \frac{1}{4}\int_0^{\pi/4} \left\{1 - 2\cos 2x + \frac{1}{2}(1 + \cos 4x)\right\} dx$

$= \frac{1}{4}\int_0^{\pi/4} \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$

$= \frac{1}{4}\left[\frac{3x}{2} - \frac{2\sin 2x}{2} + \frac{\sin 4x}{2.4}\right]_0^{\frac{\pi}{4}}$

$= \frac{1}{4}\left(\frac{3.\pi}{2.4} - \sin\frac{\pi}{2} + \frac{1}{8}\sin\pi - 0\right)$

$= \frac{1}{4}\left(\frac{3\pi}{8} - 1 + 0\right) = \frac{1}{4}\left(\frac{3\pi - 8}{8}\right) = \frac{3\pi - 8}{32}$ **(Ans.)**

11. $\int_0^1 \frac{dx}{\sqrt{x+1} + \sqrt{x}}$

$= \int_0^1 \frac{(\sqrt{x+1} - \sqrt{x})\, dx}{(\sqrt{x+1} + \sqrt{x})(\sqrt{x+1} - \sqrt{x})}$

$= \int_0^1 \left(\frac{\sqrt{x+1} - \sqrt{x}}{x + 1 - x}\right) dx$

$= \int_0^1 (\sqrt{x+1} - \sqrt{x})\, dx$

$= \left[\frac{(x+1)^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \left[\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{2}{3}x^{\frac{3}{2}}\right]_0^1$

$= \frac{2}{3}(2)^{\frac{3}{2}} - \frac{2}{3} - \frac{2}{3} = \frac{2}{3}(2\sqrt{2} - 2)$ **(Ans.)**

12.(i) $\int_0^1 \frac{1+x}{1+x^2}dx = \int_0^1 \frac{1}{1+x^2}dx + \int_0^1 \frac{x}{1+x^2}dx$

$= [\tan^{-1}x]_0^1 + \frac{1}{2}\int_0^1 \frac{2x}{1+x^2}dx$

$= (\tan^{-1}1 - \tan^{-1}0) + \frac{1}{2}[\ln(1+x^2)]_0^1$

$= \frac{\pi}{4} + \frac{1}{2}(\ln 2 - \ln 1)$

$= \frac{\pi}{4} + \frac{1}{2}\ln 2$ **(Ans.)**

(ii) $\int_0^1 \frac{1-x}{1+x}dx = \int_0^1 \frac{-(1+x)+2}{1+x}dx$

$= \int_0^1\left(-1+\frac{2}{1+x}\right)dx$

$= [-x+2\ln(1+x)]_0^1$

$= (-1+2\ln 2) - (0+2\ln 1)$

$= 2\ln 2 - 1$

$= \ln 2^2 - \ln e \quad [\because \ln e = 1]$

$= \ln\left(\frac{4}{e}\right)$ **(Ans.)**

13.(i) $\int_0^1 xe^{x^2}dx$

$= \int_0^1 e^z \cdot \frac{1}{2}dz$

$= \frac{1}{2}\int_0^1 e^z dz$

$= \frac{1}{2}[e^z]_0^1 = \frac{1}{2}(e^1 - e^0)$

$= \frac{1}{2}(e-1)$ **(Ans.)**

ধরি, $x^2 = z$
বা, $2xdx = dz$
$\therefore xdx = \frac{1}{2}dz$

x	1	0
z	1	0

(ii) মনে করি, $x^3 = z$
তাহলে, $3x^2dx = dz$
$\therefore x^2dx = \frac{1}{3}dz.$
যখন, $x = 1$ তখন $z = 1$ এবং যখন $x = 2$ তখন $z = 8$.
এখন, $\int_1^2 x^2e^{x^3}dx$

$= \int_1^8 \frac{1}{3}e^z dz = \frac{1}{3}[e^z]_1^8 = \frac{1}{3}[e^8 - e^1]$

$= \frac{1}{3}e(e^7-1)$ **(Ans.)**

14(i). $\int_0^{\ln 2}\frac{e^x}{1+e^x}dx$

$= \int_2^3 \frac{1}{z}dz = [\ln z]_2^3$

$= \ln 3 - \ln 2 = \ln\frac{3}{2}$ **(Ans.)**

ধরি, $z = 1 + e^x$
$\therefore dz = e^x dx$

x	ln2	0
z	3	2

(ii) $\int_0^1 \frac{dx}{e^x+e^{-x}}$

$= \int_0^1 \frac{e^x dx}{e^{2x}+1}$

$= \int_0^1 \frac{e^x dx}{1+(e^x)^2}$

$= \int_1^e \frac{dz}{1+z^2} = [\tan^{-1}z]_1^e = \tan^{-1}e - \tan^{-1}1$

$= \tan^{-1}e - \frac{\pi}{4}$ **(Ans.)**

ধরি, $e^x = z$
$\therefore e^xdx = dz$
সীমা : $x = 0$ হলে, $z = 1$
$x = 1$ হলে, $z = e$

(iii) $\int_a^b \frac{\ln x}{x}dx$

$= \int_{\ln a}^{\ln b} z\,dz$

$= \left[\frac{z^2}{2}\right]_{\ln a}^{\ln b}$

$= \frac{(\ln b)^2}{2} - \frac{(\ln a)^2}{2}$

$= \frac{1}{2}\{(\ln b)^2 - (\ln a)^2\} = \frac{1}{2}(\ln b + \ln a)(\ln b - \ln a)$

$= \frac{1}{2}\ln(ab)\ln\left(\frac{b}{a}\right)$ **(Ans.)**

ধরি, $z = \ln x$
$\therefore dz = \frac{dx}{x}$

x	a	b
z	ln a	ln b

15(i). $\int_1^3 \frac{\cos(\ln x)dx}{x}$

$= \int_0^{\ln 3}\cos u\,du$

$= [\sin u]_0^{\ln 3}$

$= \sin(\ln 3)$ **(Ans.)**

ধরি, $u = \ln x$
$\therefore du = \frac{1}{x}dx$
সীমা : $x = 1$ হলে, $u = \ln 1 = 0$
$x = 3$ হলে, $u = \ln 3$

(ii) $\int_0^1 x^3\sqrt{1+3x^4}\,dx$

$= \int_1^4 \frac{1}{12}\sqrt{z}\,dz$

$= \left[\frac{1}{12}\times\frac{2}{3}z^{3/2}\right]_1^4$

$= \frac{1}{18}[4^{3/2} - 1^{3/2}]$

$= \frac{1}{18}(8-1)$

$= \frac{7}{18}$ **(Ans.)**

মনে করি, $1 + 3x^4 = z$
$\therefore 12x^3dx = dz$
$\therefore x^3dx = \frac{1}{12}dz$
যখন, $x = 1$ তখন $z = 4$
যখন, $x = 0$ তখন $z = 1$

(iii) $\int_1^{e^2}\frac{dx}{x(1+\ln x)}$

$= \int_1^3 \frac{du}{u}$

$= [\ln u]_1^3$

$= \ln 3 - \ln 1$

$= \ln 3$ **(Ans.)**

ধরি, $u = 1 + \ln x$
$\therefore du = \frac{1}{x}dx$
সীমা : $x = 1$ হলে, $u = 1$
$x = e^2$ হলে, $u = 1 + \ln e^2$
$= 1 + 2\ln e$
$= 1 + 2 = 3$

(iv) $\int_0^{e^2} \frac{dx}{x(1+\ln x)^2}$

$= \int_1^3 z^{-2}\, dz$

$= \left[-\frac{1}{z}\right]_1^3$

$= \left(-\frac{1}{3}\right) - (-1)$

$= 1 - \frac{1}{3} = \frac{2}{3}$ **(Ans.)**

মনে করি, $1 + \ln x = z$

$\therefore \frac{1}{x} dx = dz$

সীমা: যখন, $x = e^2$

তখন, $z = 3$

যখন, $x = 1$ তখন, $z = 1$

(v) $\int_0^{\pi} 3\sqrt{1-\cos x}\, \sin x\, dx$

$= 3\int_0^2 \sqrt{z}\, dz$

$= 3\left[\frac{z^{3/2}}{\frac{3}{2}}\right]_1^2$

$= 3 \cdot \frac{2}{3}(2^{3/2} - 0) = 2.2.2^{1/2} = 4\sqrt{2}$ **(Ans.)**

ধরি, $z = 1 - \cos x$

$\therefore dz = \sin x\, dx$

x	0	π
z	0	2

(vi) $\int_0^{\pi/2} \frac{\cos\theta}{(1+\sin\theta)^3} d\theta$

$= \int_1^2 \frac{dz}{z^3}$

$= \left[\frac{z^{-2}}{-2}\right]_1^2$

$= -\frac{1}{2} \cdot (2^{-2} - 1^{-2})$

$= -\frac{1}{2}\left(\frac{1}{4} - 1\right) = \left(-\frac{1}{2}\right) \cdot \frac{1-4}{4} = \frac{3}{8}$ **(Ans.)**

ধরি, $z = 1 + \sin\theta$

$\therefore dz = \cos\theta\, d\theta$

θ	0	$\frac{\pi}{2}$
z	1	2

(vii) $\int_0^1 \frac{(\tan^{-1}x)^2}{1+x^2} dx$

$= \int_0^{\pi/4} u^2\, du$

$= \left[\frac{u^3}{3}\right]_0^{\pi/4}$

$= \frac{1}{3}\left\{\left(\frac{\pi}{4}\right)^3 - 0\right\}$

$= \frac{\pi^3}{192}$ **(Ans.)**

ধরি, $u = \tan^{-1}x$

$\therefore du = \frac{1}{1+x^2} dx$

যখন, $x = 0$ তখন, $u = 0$

যখন, $x = 1$

তখন, $u = \tan^{-1}(1) = \frac{\pi}{4}$

(viii) $\int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}} dx$

$= \int_0^{\pi/2} u\, du = \frac{1}{2}[u^2]_0^{\frac{\pi}{2}}$

$= \frac{\pi^2}{8}$ **(Ans.)**

ধরি, $\sin^{-1}x = u$

$\therefore \frac{dx}{\sqrt{1-x^2}} = du$

সীমা:

যখন, $x = 0$ তখন, $u = 0$

যখন, $x = 1$ তখন, $u = \frac{\pi}{2}$

(ix) $\int_0^{\pi/2} \frac{\cos x\, dx}{9 - \sin^2 x}$

$= \int_0^1 \frac{dz}{9 - z^2}$

$= \frac{1}{6}\left[\ln \frac{3+z}{3-z}\right]_0^1$

$= \frac{1}{6}\left[\ln \frac{3+1}{3-1} - \ln \frac{3+0}{3-0}\right]$

$= \frac{1}{6}(\ln 2 - \ln 1)$

$= \frac{1}{6} \ln 2$ **(Ans.)**

ধরি, $\sin x = z$

$\therefore \cos x\, dx = dz$

সীমা : যখন, $x = 0$ তখন $z = 0$

যখন, $x = \frac{\pi}{2}$ তখন $z = 1$

(x) $\int_0^{\pi/4} \tan^2 x \sec^2 x\, dx$

$= \int_0^1 z^2\, dz$

$= \frac{1}{3}[z^3]_0^1$

$= \frac{1}{3}$ **(Ans.)**

মনে করি, $\tan x = z$

$\therefore \sec^2 x\, dx = dz$

যখন $x = 0$ তখন $z = 0$

যখন $x = \frac{\pi}{4}$ তখন $z = 1$

(xi) $\int_0^{\pi/4} 4\tan^3 x \sec^2 x\, dx$

$= \int_0^1 4z^3 dz$

$= 4\left[\frac{z^4}{4}\right]_0^1$

$= 1$ **(Ans.)**

মনে করি, $\tan x = z$

$\therefore \sec^2 x\, dx = dz$

যখন $x = 0$ তখন $z = 0$

যখন $x = \frac{\pi}{4}$ তখন $z = 1$

(xii) $\int_0^{\pi/4} \sin^3\theta \cos\theta\, d\theta$

$= \int_0^{\frac{1}{\sqrt{2}}} z^3\, dz$

$= \frac{1}{4}[z^4]_0^{\frac{1}{\sqrt{2}}}$

$= \frac{1}{4} \cdot \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} \cdot \frac{1}{4}$

$= \frac{1}{16}$ **(Ans.)**

ধরি, $z = \sin\theta$

$\therefore dz = \cos\theta\, d\theta$

θ	0	$\frac{\pi}{4}$
z	0	$\frac{1}{\sqrt{2}}$

(xiii) $\int_0^{\pi/2} \cos^5\theta \sin\theta\, d\theta$

$= -\int_1^0 (z^5) dz$

$= -\left[\frac{z^6}{6}\right]_1^0$

$= \frac{1}{6}$ **(Ans.)**

ধরি, $z = \cos\theta$

$\therefore dz = -\sin\theta\, d\theta$

θ	0	$\frac{\pi}{2}$
z	1	0

(xiv) $\int_0^1 \frac{(\cos^{-1}x)^3}{\sqrt{1-x^2}}dx$

$= -\int_{\pi/2}^{0} z^3\,dz$

$= \int_0^{\pi/2} z^3\,dz = \frac{1}{4}[z^4]_0^{\pi/2}$

$= \frac{1}{4}\cdot\frac{\pi^4}{2^4} = \frac{\pi^4}{64}$ **(Ans.)**

ধরি, $z = \cos^{-1}x$

$\therefore dz = \frac{-dx}{\sqrt{1-x^2}}$

x	0	1
z	$\frac{\pi}{2}$	0

(xv) $\int_0^1 \frac{(\sin^{-1}x)^2}{\sqrt{1-x^2}}dx)$

$\therefore I = \int_0^{\frac{\pi}{2}} z^2\,dz$

$= \left[\frac{z^3}{3}\right]_0^{\pi/2}$

$= \frac{1}{3}\left[\left(\frac{\pi}{2}\right)^3 - 0\right]$

$= \frac{1}{3}\cdot\frac{\pi^3}{8} = \frac{\pi^3}{24}$ **(Ans.)**

মনে করি, $\sin^{-1}x = z$

$\therefore \frac{1}{\sqrt{1-x^2}}dx = dz$

সীমা : $x = 0$ হলে, $z = 0$

$x = 1$ হলে, $z = \frac{\pi}{2}$

(xvi) $\int_0^{\pi/2} (1+\cos x)^2 \sin x\,dx$

$= \int_2^1 z^2(-dz)$

$= \int_1^2 z^2\,dz$

$= \frac{1}{3}[z^3]_1^2$

$= \frac{1}{3}[2^3 - 1^3] = \frac{1}{3}(8-1) = \frac{7}{3}$ **(Ans.)**

ধরি, $1 + \cos x = z$

বা, $-\sin x\,dx = dz$

$\therefore \sin x\,dx = -dz$

এখন,

$x = 0$ হলে, $z = 2$

$x = \frac{\pi}{2}$ হলে, $z = 1$

(xvii) $\int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx$

$= \int_1^{-1}\frac{(-dz)}{1+z^2}$

$= -\int_1^{-1}\frac{dz}{1+z^2}$

$= -[\tan^{-1}z]_1^{-1}$

$= -[\tan^{-1}(-1) - \tan^{-1}(1)] = -\left[-\frac{\pi}{4} - \frac{\pi}{4}\right]$

$= -\left(-\frac{\pi}{2}\right) = \frac{\pi}{2}$ **(Ans.)**

ধরি, $\cos x = z$

$\therefore -\sin x\,dx = dz$

x	0	π
z	1	−1

(xviii) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{1+\sin^2 x}dx$

$= \int_0^1 \frac{1}{1+z^2}dz$

$= [\tan^{-1}z]_0^1$

$= \tan^{-1}1 - \tan^{-1}0 = \frac{\pi}{4}$ **(Ans.)**

ধরি, $\sin x = z$

$\therefore \cos x\,dx = dz$

সীমা: যখন $x = 0$ তখন $z = 0$

যখন $x = \frac{\pi}{2}$ তখন $z = 1$

(xix) $\int_0^{\pi/2} \frac{\cos\theta}{\sqrt{4-\sin^2\theta}}d\theta$

$= \int_0^1 \frac{dz}{\sqrt{2^2 - z^2}}$

$= \left[\sin^{-1}\frac{z}{2}\right]_0^1 = \sin^{-1}\frac{1}{2} - 0$

$= \frac{\pi}{6}$ **(Ans.)**

ধরি, $z = \sin\theta$

$\therefore dz = \cos\theta\,d\theta$

θ	0	$\frac{\pi}{2}$
z	0	1

(xx) $\int_0^1 \frac{2x(\tan^{-1}x^2)^3}{1+x^4}dx$

$= \int_0^{\pi/4} z^3\,dz$

$= \left[\frac{z^4}{4}\right]_0^{\pi/4}$

$= \frac{1}{4}\left(\frac{\pi}{4}\right)^4$

$= \frac{1}{4}\cdot\frac{\pi^4}{256}$

$= \frac{\pi^4}{1024}$ **(Ans.)**

ধরি, $z = \tan^{-1}x^2$

$\therefore dz = \frac{2x\,dx}{1+x^4}$

যখন $x = 0$ তখন $z = 0$

$x = 1$ তখন $z = \frac{\pi}{4}$

16.(i) $\int_0^{\pi/2} \frac{\cos^3\theta\,d\theta}{\sqrt{\sin\theta}}$

$= \int_0^{\pi/2} \frac{\cos^2\theta\cos\theta\,d\theta}{\sqrt{\sin\theta}}$

$= \int_0^{\pi/2} \frac{(1-\sin^2\theta)\cos\theta\,d\theta}{\sqrt{\sin\theta}}$

$= \int_0^1 \frac{(1-z^2)\,dz}{\sqrt{z}}$

$= \int_0^1 \left(z^{-\frac{1}{2}} - z^{\frac{3}{2}}\right)dz$

$= \left[\frac{z^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} - \frac{z^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^1$

$= \left[2z^{\frac{1}{2}} - \frac{2}{5}z^{\frac{5}{2}}\right]_0^1$

$= 2 - \frac{2}{5} = \frac{8}{5}$ **(Ans.)**

ধরি, $\sin\theta = z$

$\therefore \cos\theta\,d\theta = dz$

যখন $\theta = 0$, তখন $z = 0$

যখন $\theta = \frac{\pi}{2}$, তখন $z = 1$

(ii) ধরি, $I = \int_0^{\pi/2} \sin^3 x\sqrt{\cos x}\,dx$

$= \int_0^{\pi/2} \sqrt{\cos x}\,\sin^2 x\sin x\,dx$

$= \int_0^{\pi/2} \sqrt{\cos x}\,(1-\cos^2 x)\sin x\,dx$

আবার ধরি, $\cos x = z$

$\therefore \sin x\,dx = -dz$

যখন, $x = \frac{\pi}{2}$ তখন, $z = 0$

যখন, $x = 0$ তখন, $z = 1$

$$\therefore I = \int_1^0 \sqrt{z}(1-z^2)(-dz)$$

$$= -\int_1^0 \left(z^{\frac{1}{2}} - z^{\frac{5}{2}}\right) dz$$

$$= -\left[\frac{2}{3}z^{\frac{3}{2}} - \frac{2}{7}z^{\frac{7}{2}}\right]_0^1 = \frac{2}{3} - \frac{2}{7} = \frac{8}{21} \text{ (Ans.)}$$

(iii) $\int_0^{\pi/2} \cos^3 x \sqrt{\sin x}\, dx$

ধরি, $z = \sin x$

$\therefore dz = \cos x\, dx$

x	0	$\frac{\pi}{2}$
z	0	1

$$= \int_0^{\pi/2} \cos^2 x \sqrt{\sin x}\,.\cos x\, dx$$

$$= \int_0^{\pi/2} (1-\sin^2 x)\sqrt{\sin x} \cos x\, dx$$

$$= \int_0^1 (1-z^2) z^{\frac{1}{2}}\, dz$$

$$= \int_0^1 (z^{\frac{1}{2}} - z^{\frac{5}{2}})\, dz = \left[\frac{2}{3}z^{\frac{3}{2}} - \frac{2}{7}z^{\frac{7}{2}}\right]_0^1$$

$$= \left(\frac{2}{3} - \frac{2}{7}\right) = \frac{8}{21} \text{ (Ans.)}$$

17. $\int_0^{\pi/2} \sin^5\theta \cos^4\theta\, d\theta$

$$= -\int_0^{\pi/2} \sin^4\theta \cos^4\theta (-\sin\theta\, d\theta)$$

$$= -\int_0^{\pi/2} (1-\cos^2\theta)^2 \cos^4\theta(-\sin\theta\, d\theta)$$

$$= -\int_1^0 (1-z^2)^2 z^4\, dz$$

ধরি, $z = \cos\theta$

$\therefore dz = -\sin\theta\, d\theta$

θ	0	π/2
z	1	0

$$= \int_0^1 (z^4 - 2z^6 + z^8)\, dz$$

$$= \left[\frac{z^5}{5} - \frac{2}{7}z^7 + \frac{z^9}{9}\right]_0^1$$

$$= \frac{1}{5} - \frac{2}{7} + \frac{1}{9} - 0 = \frac{8}{315} \text{ (Ans.)}$$

18. মনে করি, $I = \int_{\pi/3}^{\pi/2} \frac{\cos^5 x}{\sin^7 x} dx = \int_{\pi/3}^{\pi/2} \cot^5 x \operatorname{cosec}^2 x\, dx$

মনে করি, $z = \cot x$

$\therefore \quad dz = -\operatorname{cosec}^2 x\, dx$

$x = \frac{\pi}{3}$ হলে, $z = \cot\frac{\pi}{3} = \frac{1}{\sqrt{3}}$

$x = \frac{\pi}{2}$ হলে, $z = \cot\frac{\pi}{2} = 0$

$$\therefore I = \int_{\frac{1}{\sqrt{3}}}^{0} z^5(-dz) = \int_0^{\frac{1}{\sqrt{3}}} z^5\, dz$$

$$= \frac{1}{6}\left[z^6\right]_0^{\frac{1}{\sqrt{3}}} = \frac{1}{6}\left[\left(\frac{1}{\sqrt{3}}\right)^6\right] = \frac{1}{6}\cdot\frac{1}{3^3} = \frac{1}{162} \text{ (Ans.)}$$

19. $\int_0^{\pi/4} (\tan^3 x + \tan x)\, dx$

$$= \int_0^{\pi/4} \tan x\,(1 + \tan^2 x)\, dx$$

$$= \int_0^{\pi/4} \tan x.\sec^2 x\, dx$$

মনে করি, $\tan x = z$

$\therefore \sec^2 x\, dx = dz$

যখন $x = 0$ তখন $z = 0$

এবং যখন $x = \frac{\pi}{4}$ তখন $z = 1$

$$= \int_0^1 z\, dz = \left[\frac{1}{2}z^2\right]_0^1$$

$$= \frac{1}{2} \text{ (Ans.)}$$

20. (i) $\int_0^{\pi/2} \sin 2x \cos x\, dx$

$$= \frac{1}{2}\int_0^{\pi/2} 2\sin 2x \cos x\, dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (\sin 3x + \sin x)\, dx$$

$$= \frac{1}{2}\left[-\frac{1}{3}\cos 3x - \cos x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}[(-\frac{1}{3}\cos\frac{3\pi}{2} - \cos\frac{\pi}{2}) - (-\frac{1}{3}\cos 3.0 - \cos 0)]$$

$$= \frac{1}{2}\left[(0-0) - \left(-\frac{1}{3}.1 - 1\right)\right] = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3} \text{ (Ans.)}$$

(ii) $\int_0^{\pi/2} \cos 3\theta \cos 2\theta\, d\theta$

$$= \frac{1}{2}\int_0^{\pi/2} 2\cos 3\theta \cos 2\theta\, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/2} (\cos 5\theta + \cos\theta)\, d\theta$$

$$= \frac{1}{2}\left[\frac{1}{5}\sin 5\theta + \sin\theta\right]_0^{\pi/2} = \frac{1}{2}[\frac{1}{5}\sin\frac{5\pi}{2} + \sin\frac{\pi}{2}]$$

$$= \frac{1}{2}\left(\frac{1}{5} + 1\right) = \frac{3}{5} \text{ (Ans.)}$$

(iii) $\int_0^{\pi/2} \sin^2 x \sin 3x\, dx$

$$= \int_0^{\pi/2} \frac{1}{2}(1 - \cos 2x)\sin 3x\, dx$$

$$= \frac{1}{2}\int_0^{\pi/2} (\sin 3x - \sin 3x \cos 2x)\, dx$$

$$= \frac{1}{2}\int_0^{\pi/2} [\sin 3x - \frac{1}{2}(\sin 5x + \sin x)]\, dx$$

$$= -\left[\frac{1}{6}\cos 3x - \frac{1}{20}\cos 5x - \frac{1}{4}\cos x\right]_0^{\pi/2}$$

$$= -\left[\left(\frac{1}{6}\cos\frac{3\pi}{2} - \frac{1}{20}\cos\frac{5\pi}{2} - \frac{1}{4}\cos\frac{\pi}{2}\right) - \left(\frac{1}{6} - \frac{1}{20} - \frac{1}{4}\right)\right]$$

$$= -[0 - 0 - 0) - \left(\frac{10 - 3 - 15}{60}\right)]$$

$$= -\frac{8}{60} = -\frac{2}{15} \text{ (Ans.)}$$

21(i) $\int_8^{27} \frac{dx}{x - x^{\frac{1}{3}}}$

$= \int_2^3 \frac{3z^2\, dz}{z^3 - z}$

$= \int_2^3 \frac{2z\, dz}{z^2 - 1}$

$= \frac{3}{2} \int_2^3 \frac{2z\, dz}{z^2 - 1}$

ধরি, $x = z^3$

$\therefore dx = - 3z^2\, dz$

x	8	27
z	2	3

$= \frac{3}{2} \int_3^8 \frac{dy}{y}$

$= \frac{3}{2} [\ln y]_3^8$

$= \frac{3}{2} [\ln 8 - \ln 3]$

$= \frac{3}{2} \ln \left(\frac{8}{3}\right)$ **(Ans.)**

ধরি, $y = z^2 - 1$

$\therefore dy = 2z\, dz$

z	2	3
y	3	8

(ii) $\int_0^{16} \frac{x^{\frac{1}{4}}}{1 + x^{\frac{1}{2}}} dx$

ধরি, $x = z^4$

$\therefore dx = 4z^3\, dz$

x	0	16
z	0	2

$= \int_0^2 \frac{z.4z^3 dz}{1 + z^2}$

$= 4\int_0^2 \frac{z^2(z^2 + 1) - z^2}{1 + z^2} dz$

$= 4\int_0^2 \left(z^2 - \frac{z^2}{z^2 + 1}\right) dz = 4\int_0^2 \left(z^2 - \frac{z^2 + 1 - 1}{z^2 + 1}\right) dz$

$= 4\int_0^2 \left(z^2 - 1 + \frac{1}{z^2 + 1}\right) dz = 4\left[\frac{z^3}{3} - z + \tan^{-1} z\right]_0^2$

$= 4\left(\frac{8}{3} - 2 + \tan^{-1}2\right) = 4\left(\frac{2}{3} + \tan^{-1}2\right)$ **(Ans.)**

(iii) $\int_1^3 \frac{x - 3}{x^3 + x^2} dx = \int_1^3 \frac{x - 3}{x^2(x + 1)} dx$

ধরি, $\frac{x - 3}{x^2(x + 1)} = \frac{A}{x + 1} + \frac{Bx + C}{x^2}$ ... ... (i)

বা, $\frac{x - 3}{x^2(x + 1)} = \frac{Ax^2 + (Bx + C)(x + 1)}{x^2(x + 1)}$

বা, $x - 3 = Ax^2 + (Bx + C)(x + 1)$ ... ... (ii)

(ii) নং এ $x = - 1$ বসিয়ে পাই, $A = -4$

(ii) নং হতে x^2 এর সহগ সমীকৃত করে পাই,

$A + B = 0$ বা, $B = - A = 4$

(ii) নং হতে ধ্রুবক পদ সমীকৃত করে পাই, $C = - 3$

A, B ও C এর মান (i) নং এ বসিয়ে পাই,

$\frac{x - 3}{x^2(x + 1)} = -\frac{4}{x + 1} + \frac{4x - 3}{x^2}$

$\therefore \int_1^3 \frac{x - 3}{x^2(x + 1)} dx = \int_1^3 \left(-\frac{4}{x + 1} + \frac{4x - 3}{x^2}\right) dx$

$= -4\int_1^3 \frac{dx}{x + 1} + 4\int_1^3 \frac{dx}{x} - 3\int_1^3 \frac{dx}{x^2}$

$= - 4[\ln(x + 1)]_1^3 + 4[\ln x]_1^3 - 3\left[\frac{x^{-1}}{-1}\right]_1^3$

$= - 4(\ln 4 - \ln 2) + 4(\ln 3 - \ln 1) + 3\left(\frac{1}{3} - 1\right)$

$= - 4\ln\left(\frac{4}{2}\right) + 4 \ln 3 - 2$

$= 4(\ln 3 - \ln 2) - 2$

$= 4\ln\left(\frac{3}{2}\right) - 2$ **(Ans.)**

(iv) $\int_0^2 \frac{x^4 + 1}{x^2 + 1} dx$

$= \int_0^2 \frac{x^2(x^2 + 1) - x^2 + 1}{x^2 + 1} dx$

$= \int_0^2 \left(x^2 - \frac{x^2}{x^2 + 1} + \frac{1}{1 + x^2}\right) dx$

$= \int_0^2 \left(x^2 - \frac{x^2 + 1 - 1}{x^2 + 1} + \frac{1}{1 + x^2}\right) dx$

$= \int_0^2 \left(x^2 - 1 + \frac{1}{x^2 + 1} + \frac{1}{1 + x^2}\right) dx$

$= \left[\frac{x^3}{3} - x + 2\tan^{-1}x\right]_0^2$

$= \frac{8}{3} - 2 + 2\tan^{-1}2$

$= \frac{2}{3} + 2\tan^{-1}2$ **(Ans.)**

(v) $\int_0^3 \frac{dx}{(2 + x^2)^{\frac{3}{2}}}$

$= \int_0^{\tan^{-1}\frac{3}{\sqrt{2}}} \frac{\sqrt{2}\sec^2\theta\, d\theta}{(2 + 2\tan^2\theta)^{3/2}}$

ধরি, $x = \sqrt{2}\tan\theta$

$\therefore dx = \sqrt{2}\sec^2\theta\, d\theta$

x	0	3
θ	0	$\tan^{-1}\frac{3}{\sqrt{2}}$

$= \sqrt{2} \int_0^{\tan^{-1}\frac{3}{\sqrt{2}}} \frac{\sec^2\theta\, d\theta}{2^{3/2}(1 + \tan^2\theta)^{3/2}}$

$= \frac{\sqrt{2}}{2\sqrt{2}} \int_0^{\tan^{-1}\frac{3}{\sqrt{2}}} \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^{3/2}}$

$= \frac{1}{2} \int_0^{\tan^{-1}\frac{3}{\sqrt{2}}} \frac{\sec^2\theta\, d\theta}{\sec^3\theta}$

$= \frac{1}{2} \int_0^{\tan^{-1}\frac{3}{\sqrt{2}}} \frac{d\theta}{\sec\theta} = \frac{1}{2} \int_0^{\tan^{-1}\frac{3}{\sqrt{2}}} \cos\theta\, d\theta$

$= \frac{1}{2} [\sin\theta]_0^{\tan^{-1}\frac{3}{\sqrt{2}}}$

$= \frac{1}{2}\left(\sin \tan^{-1}\frac{3}{\sqrt{2}} - 0\right)$

$= \frac{1}{2} \sin \sin^{-1}\left(\frac{3}{\sqrt{11}}\right)$ [চিত্র হতে]

$= \frac{1}{2} \cdot \frac{3}{\sqrt{11}} = \frac{3}{2\sqrt{11}} = \frac{3\sqrt{11}}{22}$ **(Ans.)**

22.(i) মনে করি, $x = 4\sin\theta$

$\therefore dx = 4\cos\theta\, d\theta$.

এবং $\sqrt{16 - x^2} = \sqrt{4^2 - 4^2\sin^2\theta} = 4\cos\theta$

যখন, $x = 0$, তখন, $\theta = 0$ এবং যখন, $x = 4$ তখন, $\theta = \frac{\pi}{2}$

এখন, $\int_0^4 \sqrt{16 - x^2}\, dx = \int_0^{\pi/2} 4\cos\theta .\, 4\cos\theta\, d\theta$

$= 8\int_0^{\pi/2} 2\cos^2\theta\, d\theta = 8\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$

$= 8\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}} = 8\left[\frac{\pi}{2} + 0 - 0 - 0\right]$

$= 4\pi$ **(Ans.)**

(ii) মনে করি, $I = \int_0^5 \sqrt{25 - x^2}\, dx$

ধরি, $x = 5\sin\theta$

$\therefore dx = 5\cos\theta d\theta$

x	0	5
θ	0	$\frac{\pi}{2}$

$\therefore\ I = \int_0^{\frac{\pi}{2}} \sqrt{25 - 25\sin^2\theta} .5\cos\theta\, d\theta$

$= 25\int_0^{\frac{\pi}{2}} \cos\theta . \cos\theta\, d\theta$

$= \frac{25}{2}\int_0^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$

$= \frac{25}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$

$= \frac{25}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = \frac{25}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right]$

$= \frac{25\pi}{4}$ **(Ans.)**

(iii) মনে করি, $I = \int_{-1}^{1} x^2\sqrt{4 - x^2}\, dx$

ধরি, $x = 2\sin\theta$

$\therefore dx = 2\cos\theta\, d\theta$

x	−1	1
θ	$-\frac{\pi}{6}$	$\frac{\pi}{6}$

$\therefore I = \int_{-\pi/6}^{\pi/6} 4\sin^2\theta\sqrt{4 - 4\sin^2\theta}\ .2\cos\theta\, d\theta$

$= \int_{-\pi/6}^{\pi/6} 8\sin^2\theta\, .2\sqrt{1 - \sin^2\theta}\ .\cos\theta\, d\theta$

$= \int_{-\pi/6}^{\pi/6} 16\sin^2\theta\cos^2\theta\, d\theta$

$= 4\int_{-\pi/6}^{\pi/6} (2\sin\theta\cos\theta)^2\, d\theta$

$= 4\int_{-\pi/6}^{\pi/6} \sin^2 2\theta\, d\theta$

$= 2\int_{-\pi/6}^{\pi/6} (1 - \cos 4\theta)\, d\theta$

$= 2\left[\theta - \frac{\sin 4\theta}{4}\right]_{-\pi/6}^{\pi/6}$

$= 2\left[\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3} + \frac{\pi}{6} + \frac{1}{4}\sin\left(\frac{-2\pi}{3}\right)\right]$

$= 2\left[\frac{\pi}{3} - \frac{1}{4}\cdot\frac{\sqrt{3}}{2} - \frac{1}{4}\cdot\frac{\sqrt{3}}{2}\right]$

$= 2\left[\frac{\pi}{3} - \frac{2\sqrt{3}}{8}\right]$

$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ **(Ans.)**

23. মনে করি, $I = \int_0^4 y\sqrt{4 - y}\, dy$

ধরি, $4 - y = z^2$ বা, $y = 4 - z^2$ $\therefore dy = -2zdz$

$y = 0$ হলে, $z = 2$

$y = 4$ হলে, $z = 0$

$\therefore I = \int_2^0 (4 - z^2) . z . (-2zdz)$

$= -2\int_2^0 (4 - z^2)\, z^2 dz$

$= 2\int_0^2 (4z^2 - z^4)\, dz$

$= 2\left[\frac{4}{3}z^3 - \frac{1}{5}z^5\right]_0^2$

$= 2\left[\frac{4}{3}.8 - \frac{1}{5}.32 - 0 + 0\right]$

$= 2\left[\frac{32}{3} - \frac{32}{5}\right] = 2\left[\frac{160 - 96}{15}\right]$

$= 2 \times \frac{64}{15} = \frac{128}{15}$ **(Ans.)**

বিকল্প সমাধান:

$\int_0^4 y\sqrt{4 - y}\, dy$

$= \int_0^4 (4 - y)\sqrt{y}\, dy$

[সূত্র $\int_0^a f(x)dx = \int_0^a f(a-x)dx$]

$= 4\int_0^4 y^{\frac{1}{2}}\, dy - \int_0^4 y^{\frac{3}{2}}\, dy$

$$= 4\left[\frac{y^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_0^4 - \left[\frac{y^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right]_0^4$$

$$= 4\times\frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^4 - \frac{2}{5}\left[y^{\frac{5}{2}}\right]_0^4$$

$$= \frac{8}{3}\left[4^{\frac{3}{2}}-0\right] - \frac{2}{5}\left[4^{\frac{5}{2}}-0\right]$$

$$= \frac{8}{3}\times 8 - \frac{2}{5}\times 32 = 64\left(\frac{1}{3}-\frac{1}{5}\right)$$

$$= 64\times\frac{2}{15} = \frac{128}{15} \text{ (Ans.)}$$

24.(i) $\int_0^{2a}\sqrt{2ax-x^2}\,dx$

ধরি, $x = 2a\sin^2\theta$

$\therefore dx = 4a\sin\theta\cos\theta\, d\theta$

$= 2a\sin 2\theta\, d\theta$

x	0	$2a$
θ	0	$\frac{\pi}{2}$

$$= \int_0^{\pi/2}\sqrt{(4a^2\sin^2\theta - 4a^2\sin^4\theta)}\; 2a\sin 2\theta\, d\theta$$

$$= \int_0^{\pi/2}\sqrt{4a^2\sin^2\theta\,(1-\sin^2\theta)}\; 2a\sin 2\theta\, d\theta$$

$$= \int_0^{\pi/2}\sqrt{4a^2\sin^2\theta\cos^2\theta}\; 2a\sin 2\theta\, d\theta$$

$$= \int_0^{\pi/2} 2a\sin\theta\cos\theta .\, 2a\sin 2\theta\, d\theta$$

$$= a^2\int_0^{\pi/2}\sin 2\theta .\, 2\sin 2\theta\, d\theta$$

$$= a^2\int_0^{\pi/2} 2\sin^2 2\theta\, d\theta$$

$$= a^2\int_0^{\pi/2}(1-\cos 4\theta)\, d\theta$$

$$= a^2\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$$

$$= a^2\left(\frac{\pi}{2} - \frac{1}{4}\sin 2\pi\right)$$

$$= \frac{\pi a^2}{2} \text{ (Ans.)}$$

(ii) $\int_0^3\frac{dx}{\sqrt{3x-x^2}} = \int_0^3\frac{dx}{\sqrt{\frac{9}{4}-(x^2-2.\frac{3}{2}x+\frac{9}{4})}}$

$$= \int_0^3\frac{dx}{\sqrt{\left(\frac{3}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}} = \left[\sin^{-1}\frac{x-\frac{3}{2}}{\frac{3}{2}}\right]_0^3$$

$$= \sin^{-1}\left(\frac{3}{2}\times\frac{2}{3}\right) - \sin^{-1}(-1)$$

$$= \sin^{-1}1 + \sin^{-1}1 = 2\sin^{-1}\sin\frac{\pi}{2}$$

$$= 2.\frac{\pi}{2} = \pi \text{ (Ans.)}$$

25.(i) $\int_{-1}^1\sqrt{\frac{1-x}{1+x}}\,dx = \int_{-1}^1\sqrt{\frac{(1-x)(1-x)}{(1+x)(1-x)}}\,dx$

$$= \int_{-1}^1\frac{1-x}{\sqrt{1-x^2}}dx = \int_{-1}^1\frac{dx}{\sqrt{1-x^2}} - \int_{-1}^1\frac{x\,dx}{\sqrt{1-x^2}}$$

$$= [\sin^{-1}x]_{-1}^1 + \left[\sqrt{1-x^2}\right]_{-1}^1 = \frac{\pi}{2}+\frac{\pi}{2}+0 = \pi \text{ (Ans.)}$$

(ii) $\int_0^a\sqrt{\frac{a+x}{a-x}}\,dx$

$$= \int_0^a\sqrt{\frac{(a+x)(a+x)}{(a-x)(a+x)}}\,dx$$

$$= \int_0^a\frac{a+x}{\sqrt{a^2-x^2}}\,dx$$

$$= a\int_0^a\frac{dx}{\sqrt{a^2-x^2}} + \int_0^a\frac{x}{\sqrt{a^2-x^2}}\,dx$$

$$= a\left[\sin^{-1}\frac{x}{a}\right]_0^a - \left[\sqrt{a^2-x^2}\right]_0^a$$

$$= a\left(\sin^{-1}\frac{a}{a}-0\right) - [0-a]$$

$$= a\frac{\pi}{2} + a = \frac{a}{2}(\pi+2) \text{ (Ans.)}$$

26. $\int_0^4\frac{x^3}{\sqrt{x^2+9}}\,dx$

$$= \int_0^4\frac{x^2.x}{\sqrt{x^2+9}}\,dx$$

ধরি, $z = x^2+9$

$\therefore dz = 2x\,dx$

x	0	4
z	9	25

$$= \frac{1}{2}\int_9^{25}\frac{(z-9)\,dz}{\sqrt{z}}$$

$$= \frac{1}{2}\int_9^{25}\left(\frac{z}{\sqrt{z}}-\frac{9}{\sqrt{z}}\right)dz$$

$$= \frac{1}{2}\int_9^{25}\left(\sqrt{z}-9z^{-1/2}\right)dz$$

$$= \frac{1}{2}\left[\frac{2}{3}z^{3/2} - 9.2\,z^{1/2}\right]_9^{25}$$

$$= \frac{1}{2}\left[\frac{2}{3}(25)^{3/2} - 18\,(25)^{1/2} - \frac{2}{3}(9)^{3/2} + 18\,(9)^{1/2}\right]$$

$$= \frac{1}{2}\left(\frac{2}{3}5^3 - 18.5 - \frac{2}{3}.3^3 + 18.3\right)$$

$$= \frac{125}{3} - 45 - 9 + 27$$

$$= \frac{125-135-27+81}{3}$$

$$= \frac{44}{3} \text{ (Ans.)}$$

27.(i) ধরি, $I = \int_1^4 \ln x\,dx$

এখন, $\int \ln x\,dx = \ln x \int 1dx - \int\left\{\frac{d}{dx}(\ln x)\int 1.dx\right\}dx$

$= x\ln x - \int \frac{1}{x}.x\,dx$

$= x\ln x - \int 1dx$

$= x\ln x - x + c$

$\therefore\ I = [x\ln x\ \ x]_1^4$

$= 4\ln 4 - 4 - 1.\ln 1 + 1$

$= 4\ln 4 - 3$ **(Ans.)**

(ii) ধরি, $I = \int_0^1 \ln(x^2+1)\,dx$

এখন, $\int \ln(x^2+1)\,dx$

$= \ln(x^2+1)\int dx - \int\left[\frac{d}{dx}\{\ln(x^2+1)\}\int dx\right]dx$

$= \ln(x^2+1).\,x - \int\frac{2x^2}{x^2+1}dx$

$= x\ln(x^2+1) - 2\int\left(1 - \frac{1}{1+x^2}\right)dx$

$= x\ln(x^2+1) - 2(x - \tan^{-1}x)$

$= x\ln(x^2+1) - 2x + 2\tan^{-1}x + c$

[যেখানে, c যোগজীকরণ ধ্রুবক]

$\therefore I = \int_0^1 \ln(x^2+1)\,dx$

$= [x\ln(x^2+1) - 2x + 2\tan^{-1}x]_0^1$

$= \ln 2 - 2 + 2\tan^{-1}1$

$= \ln 2 - 2 + 2.\frac{\pi}{4}$

$= \ln 2 - 2 + \frac{\pi}{2}$ **(Ans.)**

28. $\int_1^4 \frac{\ln x}{\sqrt{x}}dx = I$ (ধরি)

এখন, $\int\frac{\ln x}{\sqrt{x}}dx$

$= \ln x\int x^{-1/2}dx - \int\left\{\frac{d}{dx}(\ln x)\int x^{-1/2}dx\right\}dx$

$= 2\ln x\sqrt{x} - \int\left(\frac{1}{x}.2\sqrt{x}\right)dx$

$= 2\ln x\sqrt{x} - 2\int x^{-1/2}dx$

$= 2\ln x\sqrt{x} - 4\sqrt{x}$

$\therefore I = [2\ln x\sqrt{x} - 4\sqrt{x}]_1^4$

$= 2\ln 4.\sqrt{4} - 4\sqrt{4} - 2.0 + 4.1$

$= 4\ln 4 - 4 = 8\ln 2 - 4$ **(Ans.)**

29.(i) $\int_0^1 2x^3e^{-x^2}dx$

$= \int_0^1 2x.x^2e^{-x^2}dx$

$= \int_0^1 ze^{-z}dz$

মনে করি, $x^2 = z$
$\therefore 2x\,dx = dz$
যখন, $x = 0$ তখন, $z = 0$
যখন, $x = 1$ তখন, $z = 1$

এখন, $\int ze^{-z}dz$

$= z\int e^{-z}dz - \int\left[\frac{d}{dz}(z)\int e^{-z}dz\right]dz$

$= -ze^{-z} + \int e^{-z}dz$

$= -ze^{-z} - e^{-z} + c$

$\therefore \int_0^1 ze^{-z}dz = [-ze^{-z} - e^{-z}]_0^1$

$= (-e^{-1} - e^{-1}) - (-e^0)$

$= -\frac{2}{e} + 1 = 1 - \frac{2}{e}$ **(Ans.)**

(ii) $\int xe^{-3x}dx$

$= x\int e^{-3x}dx - \int\left\{\frac{d}{dx}(x)\int e^{-3x}dx\right\}dx$

$= x\left(\frac{e^{-3x}}{-3}\right) - \int\frac{e^{-3x}}{-3}dx$

$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}dx$

$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\left(\frac{e^{-3x}}{-3}\right)$

$= -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x}$

$= -\frac{1}{3}e^{-3x}\left(x + \frac{1}{3}\right) + c$

$\therefore \int_0^1 xe^{-3x}dx = \left[-\frac{1}{3}e^{-3x}\left(x+\frac{1}{3}\right)\right]_0^1$

$= -\frac{1}{3}e^{-3}\left(1 + \frac{1}{3}\right) + \frac{1}{3}e^{-3\times 0}\left(0 + \frac{1}{3}\right)$

$= -\frac{4}{9}e^{-3} + \frac{1}{9}$

$= \frac{1}{9}(1 - 4e^{-3})$ **(Ans.)**

30. ধরি, $I = \int_0^{\pi/2} x^2\cos x\,dx$

এখন, $\int x^2\cos x\,dx$

$= x^2\int\cos x\,dx\ \int\left\{\frac{d}{dx}(x^2)\int\cos x\,dx\right\}dx$

$= x^2\sin x - \int 2x\sin x\,dx$

$= x^2\sin x - 2\left[x\int\sin x\,dx - \int\left\{\frac{d}{dx}(x)\int\sin x\,dx\right\}dx\right]$

$= x^2 \sin x - 2\left[x(-\cos x) - \int(-\cos x)\,dx\right]$

$= x^2 \sin x - 2\,[-x\cos x + \sin x]$

$= x^2 \sin x + 2x\cos x - 2\sin x + c$

$\therefore\ I = [x^2 \sin x + 2x\cos x - 2\sin x]_0^{\frac{\pi}{2}}$

$= \left(\frac{\pi^2}{4}\sin\frac{\pi}{2} + \pi\cos\frac{\pi}{2} - 2\sin\frac{\pi}{2}\right) - (0 + 0 - 2\sin 0)$

$= \frac{\pi^2}{4} + \pi \times 0 - 2 + 0 = \frac{\pi^2}{4} - 2$ **(Ans.)**

31. ধরি, $I = \int_0^1 \sin^{-1}x\,dx$

এখন, $\int \sin^{-1}x\,dx = \sin^{-1}x \int dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\right\}dx$

$= x\sin^{-1}x - \int\frac{x}{\sqrt{1-x^2}}\,dx$

$= x\sin^{-1}x + \sqrt{1-x^2}$

$\therefore I = \left[x\sin^{-1}x + \sqrt{1-x^2}\right]_0^1$

$= \sin^{-1}1 + 0 - 0 - 1 = \frac{\pi}{2} - 1$ **(Ans.)**

32. এখানে, $\int x\tan^{-1}x\,dx$

$= \tan^{-1}x \int x\,dx - \int\left\{\frac{d}{dx}(\tan^{-1}x)\int x\,dx\right\}dx$

$= \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int\frac{x^2}{1+x^2}\,dx$

$= \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int\left(\frac{1+x^2-1}{1+x^2}\right)dx$

$= \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{dx}{1+x^2}$

$= \frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x + c$

$\therefore \int_1^{\sqrt{3}} x\tan^{-1}x\,dx$

$= \left[\frac{1}{2}x^2\tan^{-1}x - \frac{1}{2}x + \frac{1}{2}\tan^{-1}x\right]_1^{\sqrt{3}}$

$= \left\{\frac{1}{2}(\sqrt{3})^2\tan^{-1}\sqrt{3} - \frac{1}{2}\sqrt{3} + \frac{1}{2}\tan^{-1}\sqrt{3}\right\}$

$\quad - \left(\frac{1}{2}\tan^{-1}1 - \frac{1}{2} + \frac{1}{2}\tan^{-1}1\right)$

$= \left(\frac{3}{2}\cdot\frac{\pi}{3} - \frac{\sqrt{3}}{2} + \frac{1}{2}\cdot\frac{\pi}{3}\right) - \left(\frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2} + \frac{1}{2}\cdot\frac{\pi}{4}\right)$

$= \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right) - \left(\frac{\pi}{4} - \frac{1}{2}\right)$

$= \frac{5\pi}{12} - \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{1}{12}(5\pi - 6\sqrt{3} + 6)$ **(Ans.)**

33. মনে করি, $I = \int e^x(\sin x + \cos x)\,dx$

$= \int e^x\sin x\,dx + \int e^x\cos x\,dx$

$= \sin x\int e^x dx - \int\left\{\frac{d}{dx}(\sin x)\int e^x dx\right\}dx + \int e^x\cos x\,dx$

$= e^x\sin x - \int e^x\cos x\,dx + \int e^x\cos x\,dx$

$\therefore\ I = e^x\sin x + c$

অতএব, $\int_0^{\pi/2} e^x(\sin x + \cos x)\,dx = [e^x\sin x]_0^{\frac{\pi}{2}}$

$= \left[e^{\frac{\pi}{2}}\sin\frac{\pi}{2} - 0\right] = e^{\frac{\pi}{2}}$ **(Ans.)**

34. মনে করি, $I = \int_0^{\pi/2}(a\cos^2x + b\sin^2x)\,dx$

$= \frac{1}{2}\int_0^{\pi/2} a.2\cos^2x\,dx + \frac{1}{2}\int_0^{\pi/2} b.2\sin^2x\,dx$

$= \frac{a}{2}\int_0^{\pi/2}(1 + \cos 2x)\,dx + \frac{b}{2}\int_0^{\pi/2}(1 - \cos 2x)\,dx$

$= \frac{a}{2}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi/2} + \frac{b}{2}\left[x - \frac{\sin 2x}{2}\right]_0^{\pi/2}$

$= \frac{a}{2}\left[\frac{\pi}{2} + 0 - 0 - 0\right] + \frac{b}{2}\left[\frac{\pi}{2} - 0 - 0 + 0\right]$

$= \frac{\pi a}{4} + \frac{\pi b}{4} = \frac{1}{4}(a + b)\pi$ **(Ans.)**

35.(i) $\int_0^{\pi/2}\frac{dx}{a^2\cos^2x + b^2\sin^2x}$

ধরি, $b\tan x = a\tan\theta$

$\therefore\ b\sec^2x = a\sec^2\theta\,d\theta$

x	0	$\frac{\pi}{2}$
θ	0	$\frac{\pi}{2}$

$= \int_0^{\pi/2}\frac{\sec^2x\,dx}{a^2 + b^2\tan^2x}$

$= \frac{1}{b}\int_0^{\pi/2}\frac{a\sec^2\theta\,d\theta}{a^2 + a^2\tan^2\theta}$

$= \frac{1}{b}\int_0^{\pi/2}\frac{a\sec^2\theta\,d\theta}{a^2\sec^2\theta}$

$= \frac{1}{ab}\int_0^{\pi/2}d\theta = \frac{1}{ab}[\theta]_0^{\pi/2} = \frac{\pi}{2ab}$ **(Ans.)**

(ii) $\int_0^{\pi/2}\frac{d\theta}{a\sin^2\theta + b\cos^2\theta}$

$= \int_0^{\pi/2}\frac{\sec^2\theta\,d\theta}{b + a\tan^2\theta}$

$= \frac{1}{\sqrt{a}}\int_0^{\infty}\frac{dz}{(\sqrt{b})^2 + z^2}$

ধরি, $\sqrt{a}\tan\theta = z$

$\therefore\ \sqrt{a}\sec^2\theta\,d\theta = dz$

θ	0	$\frac{\pi}{2}$
z	0	∞

$= \frac{1}{\sqrt{a}}\left[\frac{1}{\sqrt{b}}\tan^{-1}\frac{z}{\sqrt{b}}\right]_0^{\infty}$

$= \frac{1}{\sqrt{a}}\cdot\frac{1}{\sqrt{b}}\cdot\frac{\pi}{2}$

$= \frac{\pi}{2\sqrt{ab}}$ **(Ans.)**

36(i) $\int_0^{\pi/2} \frac{dx}{1+\cot x} = \int_0^{\pi/2} \frac{\sin x}{\cos x + \sin x} dx$

$= \frac{1}{2}\int_0^{\pi/2} \frac{\sin x + \cos x - \cos x + \sin x}{\cos x + \sin x} dx$

$= \frac{1}{2}\int_0^{\pi/2} \left(\frac{\sin x + \cos x}{\cos x + \sin x} + \frac{\sin x - \cos x}{\cos x + \sin x}\right) dx$

$= \frac{1}{2}\int_0^{\pi/2} dx - \frac{1}{2}\int_0^{\pi/2} \frac{-\sin x + \cos x}{\cos x + \sin x} dx$

$= \frac{1}{2}[x]_0^{\pi/2} - \frac{1}{2}[\ln(\cos x + \sin x)]_0^{\pi/2}$

$= \frac{\pi}{4} - \frac{1}{2}(\ln 1 - \ln 1) = \frac{\pi}{4}$ **(Ans.)**

(ii) ধরি, $I = \int_0^{\pi} \frac{x\,dx}{1+\sin x}$

$= \int_0^{\pi} \frac{x(1-\sin x)\,dx}{1-\sin^2 x}$

$= \int_0^{\pi} \frac{x - x\sin x}{\cos^2 x} dx$

$= \int_0^{\pi} (x\sec^2 x - x\sec x \tan x)\, dx$

এখন, $\int (x\sec^2 x - x\sec x\tan x)\, dx$

$= \int x\sec^2 x\, dx - \int x\sec x \tan x\, dx$

$= x\int \sec^2 x\, dx - \int\left\{\frac{dx}{dx}\int \sec^2 x\, dx\right\} dx$

$-\left[x\int \sec x\tan x\, dx - \int\left\{\frac{dx}{dx}\int \sec x \tan x\, dx\right\}\right] dx$

$= x\tan x - \int \tan x\, dx - x\sec x - \int \sec x\, dx$

$= x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x|$

$\therefore I = [x\tan x - \ln|\sec x| - x\sec x + \ln|\sec x + \tan x|]_0^{\pi}$

$= 0 - \ln|\sec\pi| + \pi + \ln|\sec\pi| - 0 + 0 + 0 + 0$

$= \pi$ **(Ans.)**

37. $\int_8^{15} \frac{dx}{(x-3)\sqrt{x+1}}$

ধরি, $x + 1 = z^2$

$\therefore dx = 2z\,dz$

$x = z^2 - 1,\ x - 3 = z^2 - 4$

x	8	15
z	3	4

$= \int_3^4 \frac{2z\,dz}{(z^2-4)z}$

$= 2\int_3^4 \frac{dz}{z^2 - 2^2}$

$= 2.\frac{1}{4}\left[\ln\left(\frac{z-2}{z+2}\right)\right]_3^4$

$= \frac{1}{2}\left[\ln\frac{4-2}{4+2} - \ln\frac{3-2}{3+2}\right]$

$= \frac{1}{2}\ln\frac{1}{3} - \ln\frac{1}{5}$

$= \frac{1}{2}\ln\left(\frac{5}{3}\right)$ **(Ans.)**

38. $\int_0^1 \frac{x\,dx}{1+x^4}$

ধরি, $x^2 = z$

$\therefore 2x\,dx = dz$

বা, $x\,dx = \frac{dz}{2}$

x	1	0
z	1	0

$= \frac{1}{2}\int_0^1 \frac{dz}{1+z^2}$

$= \frac{1}{2}[\tan^{-1} z]_0^1$

$= \frac{1}{2}[\tan^{-1}(1) - \tan^{-1}(0)]$

$= \frac{1}{2}[\tan^{-1}(\tan\frac{\pi}{4}) - 0]$

$= \frac{1}{2}.\frac{\pi}{4}$

$= \frac{\pi}{8}$ **(Ans.)**

39. ধরি, $I = \int \ln x\, dx$

$= \ln x\int dx - \int\left\{\frac{d}{dx}(\ln x)\int dx\right\} dx$

$= x\ln x - \int \frac{1}{x}.x\, dx$

$= x\ln x - x + c$

$\therefore \int_0^1 \ln x\, dx = [x\ln x - x]_0^1$

$= 0 - 1 - 0 + 0$

$= -1$ **(Ans.)**

40. ধরি, $I = \int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ... ... (i)

$\therefore I = \int_0^{\pi/2} \frac{\sqrt{\cos\left(\frac{\pi}{2} - x\right)}}{\sqrt{\sin\left(\frac{\pi}{2} - x\right)} + \sqrt{\cos\left(\frac{\pi}{2} - x\right)}} dx$

$[\because \int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx]$

$= \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx$

বা, $I = \int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$ ... ... (ii)

(i) + (ii) হতে পাই,

$2I = \int_0^{\pi/2} \frac{\sqrt{\cos x} + \sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx$

$= \int_0^{\pi/2} dx = [x]_0^{\pi/2} = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$ অর্থাৎ, $\int_0^{\pi/2} \frac{\sqrt{\cos x}}{\sqrt{\sin x} + \sqrt{\cos x}} dx = \frac{\pi}{4}$ **(Ans.)**

41. (i) $\int_{\frac{1}{4}}^{1} |2x-1|\, dx$

যখন $2x-1=0$ তখন $x=\frac{1}{2}$

$\frac{1}{4} \le x < \frac{1}{2}$ এর জন্য $2x-1<0 \therefore |2x-1| = 1-2x$

$\frac{1}{4} < x < 1$ এর জন্য $2x-1>0 \therefore |2x-1| = 2x-1$

$\therefore \int_{1/4}^{1} |2x-1|\, dx = \int_{1/4}^{1/2} |2x-1|\, dx + \int_{1/2}^{1} |2x-1|\, dx$

$= \int_{1/4}^{1/2} (1-2x)\, dx + \int_{1/2}^{1} (2x-1)\, dx$

$= [x-x^2]_{1/4}^{1/2} + [x^2-x]_{1/2}^{1}$

$= \left(\frac{1}{2}-\frac{1}{4}\right) - \left(\frac{1}{4}-\frac{1}{16}\right) + (1-1) - \left(\frac{1}{4}-\frac{1}{2}\right)$

$= \frac{1}{4} - \frac{3}{16} + \frac{1}{4}$

$= \frac{5}{16}$ **(Ans.)**

(ii) $\int_{-\pi/2}^{\pi/2} |\sin x|\, dx$

$\sin x = 0$ যখন $x=0$

$-\frac{\pi}{2} \le x < 0$ এর জন্য $\sin x < 0 \therefore |\sin x| = -\sin x$

$0 < x \le \frac{\pi}{2}$ এর জন্য $\sin x > 0 \therefore |\sin x| = \sin x$

$\therefore \int_{-\pi/2}^{\pi/2} |\sin x|\, dx = \int_{-\pi/2}^{0} |\sin x|\, dx + \int_{0}^{\pi/2} |\sin x|\, dx$

$= \int_{-\pi/2}^{0} -\sin x\, dx + \int_{0}^{\pi/2} \sin x\, dx$

$= \left[\cos x\right]_{-\pi/2}^{0} - \left[\cos x\right]_{0}^{\pi/2}$

$= \cos 0 - \cos\left(-\frac{\pi}{2}\right) - \left(\cos\frac{\pi}{2} - \cos 0\right)$

$= 1 - 0 + 1 = 2$ **(Ans.)**

(iii) $\int_{0}^{\pi/2} |\cos 2x|\, dx$

$\cos 2x = 0$ যখন $2x = \frac{\pi}{2}$ বা, $x = \frac{\pi}{4}$

$0 \le x < \frac{\pi}{4}$ বা, $0 \le 2x < \frac{\pi}{2}$ এর জন্য $\cos 2x > 0$

$\therefore |\cos 2x| = \cos 2x$

$\frac{\pi}{4} < x \le \frac{\pi}{2}$ বা, $\frac{\pi}{2} < 2x \le \pi$ এর জন্য $\cos 2x < 0$

$\therefore |\cos 2x| = -\cos 2x$

$\therefore \int_{0}^{\pi/2} |\cos 2x|\, dx = \int_{0}^{\pi/4} |\cos 2x|\, dx + \int_{\pi/4}^{\pi/2} |\cos 2x|\, dx$

$= \int_{0}^{\pi/4} \cos 2x\, dx + \int_{\pi/4}^{\pi/2} -\cos 2x\, dx$

$= \left[\frac{\sin 2x}{2}\right]_{0}^{\pi/4} - \left[\frac{\sin 2x}{2}\right]_{\pi/4}^{\pi/2}$

$= \frac{1}{2}\left(\sin\frac{\pi}{2} - \sin 0\right) - \frac{1}{2}\left(\sin\pi - \sin\frac{\pi}{2}\right)$

$= \frac{1}{2}(1-0) - \frac{1}{2}(0-1) = 1$ **(Ans.)**

(iv) $\int_{-2}^{2} |x|\, dx$

$= \int_{-2}^{0} (-x)\, dx + \int_{0}^{2} x\,dx$

$= \left[\frac{-x^2}{2}\right]_{-2}^{0} + \left[\frac{x^2}{2}\right]_{0}^{2}$

$= (0+2) + (2-0) = 4$ **(Ans.)**

(v) $\int_{-\pi}^{\pi} |\cos x| dx$

$= \int_{-\pi}^{-\pi/2} (-\cos x)dx + \int_{-\frac{\pi}{2}}^{\pi/2} \cos x dx + \int_{\pi/2}^{\pi} (-\cos x)dx$

$= -[\sin x]_{-\pi}^{-\pi/2} + [\sin x]_{-\pi/2}^{\pi/2} - [\sin x]_{\pi/2}^{\pi}$

$= -\left[\sin\left(-\frac{\pi}{2}\right) - \sin(-\pi)\right] + \left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right] - \left[\sin\pi - \sin\frac{\pi}{2}\right]$

$= -[-1+0] + [1+1] - [0-1]$

$= 1+2+1$

$= 4$ **(Ans.)**

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-10.8 | পৃষ্ঠা-৪৩২

$\frac{x^2}{16} + \frac{y^2}{9} = 1$ .... (i)

বা, $\frac{y^2}{9} = 1 - \frac{x^2}{16}$

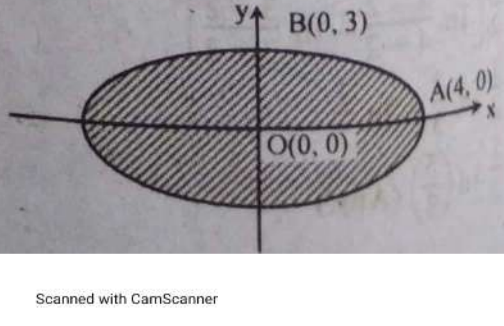
বা, $y^2 = 9\left(\frac{16-x^2}{16}\right)$

বা, $y^2 = \frac{9}{16}(16-x^2)$

বা, $y = \pm\frac{3}{4}\sqrt{16-x^2}$

$\therefore y = \frac{3}{4}\sqrt{4^2-x^2}$

[(–) বাদ দেওয়ার কারণ প্রথম চতুর্ভাগে y ধনাত্



(i) নং-এ $y = 0$ বসিয়ে পাই, $x^2 = 16 \therefore x = \pm 4$
অর্থাৎ, উপবৃত্তটি x-অক্ষকে $(-4, 0)$ ও $(4, 0)$ বিন্দুতে ছেদ করে।
আবার $x = 0$ বসিয়ে পাই, $y^2 = 9 \quad \therefore y = \pm 3$
অর্থাৎ, উপবৃত্তটি y-অক্ষকে $(0, -3)$ ও $(0, 3)$ বিন্দুতে ছেদ করে।

উপবৃত্তের ক্ষেত্রফল $= 4\int_0^4 y\,dx$

$= 4\int_0^4 \frac{3}{4}\sqrt{4^2 - x^2}\,dx$

$= 3\int_0^4 \sqrt{4^2 - x^2}\,dx$

$= 3\int_0^{\frac{\pi}{2}} \sqrt{4^2 - 4^2\sin^2\theta}\,.4\cos\theta\,d\theta$

$= 3.\int_0^{\frac{\pi}{2}} 4\sqrt{1 - \sin^2\theta}\,.4\cos\theta\,d\theta$

$= 24.\int_0^{\frac{\pi}{2}} 2\cos^2\theta\,d\theta$

$= 24.\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)\,d\theta$

$= 24\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

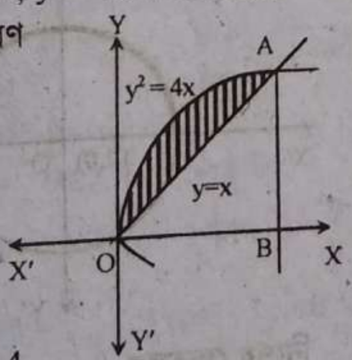
$= 24\left[\frac{\pi}{2} - 0\right]$

$= 12\pi$ বর্গ একক। **(Ans.)**

ধরি, $x = 4\sin\theta$
$\therefore dx = 4\cos\theta d\theta$
যখন, $x = 0$,
তখন $\theta = 0$
যখন, $x = 4$,
তখন $\theta = \frac{\pi}{2}$

অনুশীলনী-10(G) এর সমাধান

1.(i) প্রদত্ত পরাবৃত্তের সমীকরণ, $y^2 = 4x$ ... ... (i)
এবং সরলরেখার সমীকরণ
$y = x$ ... ... (ii)
(ii) নং ও (i) নং হতে,
$x^2 = 4x$
বা, $x^2 - 4x = 0$
বা, $x(x - 4) = 0$
$\therefore x = 0, 4$



$\therefore$ ছেদবিন্দুদ্বয়ের ভুজ 0 ও 4
$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_0^4 (y_1 - y_2)dx$

$= \int_0^4 (2\sqrt{x} - x)\,dx$

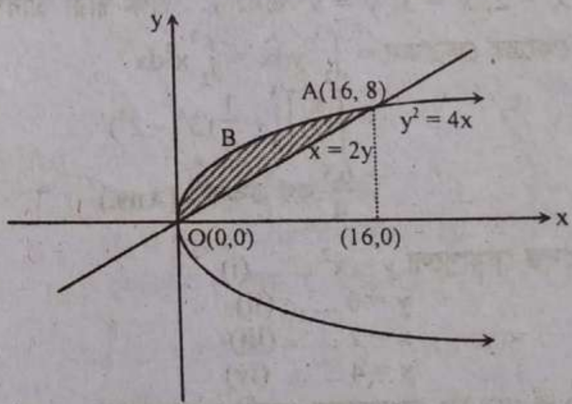
$= \left[2.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4$

$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^4$

$= \left[\frac{32}{3} - 8\right]$

$= \frac{8}{3}$ বর্গ একক। **(Ans.)**

(ii) $y^2 = 4x$ ... ... (i) পরাবৃত্ত ও $x = 2y$ ... ... (ii) সরলরেখার ছেদবিন্দু নির্ণয় করি।
(ii) নং হতে x-এর মান (i) নং এ বসিয়ে
$y^2 = 8y$ বা, $y(y - 8) = 0 \therefore y = 0, 8$
$\therefore$ (ii) বা, $x = 0, 16$
$\therefore$ ছেদবিন্দুদ্বয় $(0, 0)$ ও $(16, 8)$



নির্ণেয় ক্ষেত্রফল = OABO ক্ষেত্রের ক্ষেত্রফল

$= \int_0^{16} (y_1 - y_2)\,dx$

$= \int_0^{16} \left(2\sqrt{x} - \frac{x}{2}\right)dx$

$= \left[2.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{1}{2}.\frac{x^2}{2}\right]_0^{16}$

$= \frac{4}{3}[x^{\frac{3}{2}}]_0^{16} - \frac{1}{4}[x^2]_0^{16}$

$= \frac{4}{3}(64 - 0) - \frac{1}{4}(256 - 0)$

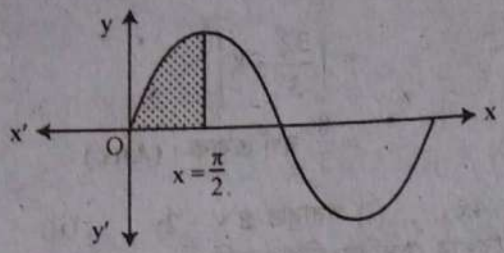
$= \frac{256}{3} - \frac{256}{4}$

$= 256\left(\frac{1}{3} - \frac{1}{4}\right)$

$= 256.\frac{1}{12}$

$= \frac{64}{3}$ বর্গ একক।

2. নির্ণেয় ক্ষেত্রফল, A = [$x = 0, x = \frac{\pi}{2}, y = \sin x$ এবং x অক্ষদ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল।]



$\therefore A = \int_0^{\pi/2} y\,dx = \int_0^{\pi/2} \sin x\,dx = -[\cos x]_0^{\frac{\pi}{2}}$

$= -\left[\cos\frac{\pi}{2} - \cos 0\right]$

= 1 বর্গ একক। **(Ans.)**

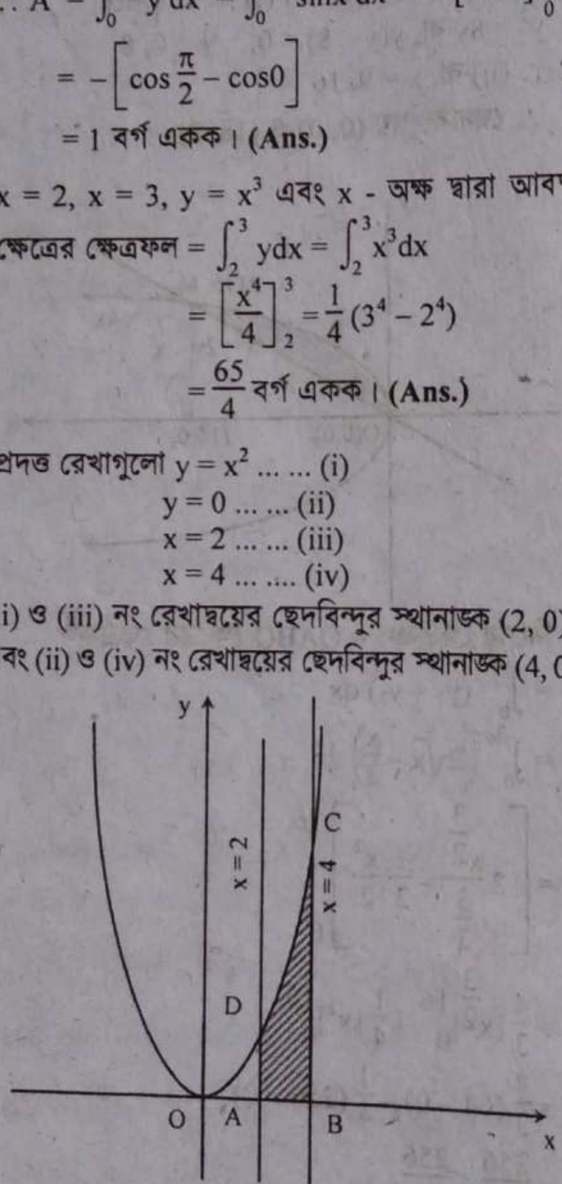
3.(i) $x = 2, x = 3, y = x^3$ এবং x - অক্ষ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= \int_2^3 y dx = \int_2^3 x^3 dx$

$= \left[\frac{x^4}{4}\right]_2^3 = \frac{1}{4}(3^4 - 2^4)$

$= \frac{65}{4}$ বর্গ একক। **(Ans.)**

(ii) প্রদত্ত রেখাগুলো $y = x^2$ ... ... (i)
$y = 0$ ... ... (ii)
$x = 2$ ... ... (iii)
$x = 4$ ... . (iv)

(ii) ও (iii) নং রেখাদ্বয়ের ছেদবিন্দুর স্থানাঙ্ক (2, 0) এবং (ii) ও (iv) নং রেখাদ্বয়ের ছেদবিন্দুর স্থানাঙ্ক (4, 0)

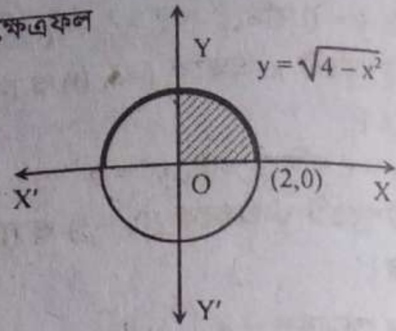


নির্ণেয় ক্ষেত্রফল = ABCD ক্ষেত্রের ক্ষেত্রফল

$= \int_2^4 y\,dx = \int_2^4 x^2\,dx = \left[\frac{x^3}{3}\right]_2^4$

$= \frac{64}{3} - \frac{8}{3} = \frac{56}{3}$ বর্গ একক।

4. নির্ণেয় ক্ষেত্রফল



$= 4 \times (y = \sqrt{4 - x^2}$, x অক্ষ এবং কোটি $x = 0$ ও $x = 2$ দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল)

$= 4\int_0^2 y dx$

$= 4\int_0^2 \sqrt{4 - x^2}\,dx$

ধরি, $x = 2\sin\theta$
$\therefore dx = 2\cos\theta d\theta$
সীমা: $x = 2$ হলে, $\theta = \frac{\pi}{2}$
$x = 0$ হলে, $\theta = 0$

$= 4\int_0^{\pi/2} \sqrt{4 - 4\sin^2\theta} \cdot 2\cos\theta\,d\theta$

$= 4 \cdot 4\int_0^{\pi/2} \cos^2\theta\,d\theta$

$= 8\int_0^{\pi/2} 2\cos^2\theta\,d\theta$

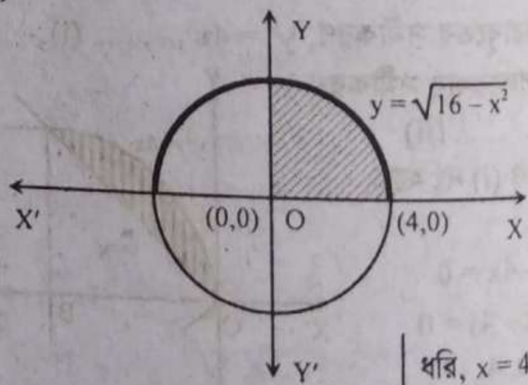
$= 8\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta$

$= 8\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= 8\cdot\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0\right)$ [$\because \sin\pi = 0$]

$= 4\pi$ বর্গ একক। **(Ans.)**

5. (i)



নির্ণেয় ক্ষেত্রফল = 4 × ($y = \sqrt{16 - x^2}$, x-অক্ষ এবং কোটি x = 0 ও x = 4 দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল)

ধরি, $x = 4\sin\theta$
$\therefore dx = 4\cos\theta\,d\theta$
এবং $\theta = \sin^{-1}\frac{x}{4}$

x	0	4
θ	0	$\frac{\pi}{2}$

$= 4\int_0^4 y\,dx = 4\int_0^4 \sqrt{16 - x^2}\,dx$

$= 4\int_0^{\frac{\pi}{2}} \sqrt{16 - 16\sin^2\theta}.4\cos\theta d\theta$

$= 64\int_0^{\pi/2} \sqrt{\cos^2\theta}\ \cos\theta\ d\theta$

$= 64\int_0^{\pi/2} \cos^2\theta\ d\theta = 32\int_0^{\pi/2} 2\cos^2\theta\ d\theta$

$= 32\int_0^{\pi/2} (1 + \cos 2\theta)\ d\theta$

$= 32\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}} = 32\left[\frac{\pi}{2} + 0 - 0\right]$

$= 16\pi$ বর্গ একক। **(Ans.)**

(ii) যেহেতু সম্পূর্ণ অংশের ক্ষেত্রফল 16π বর্গ একক, সুতরাং ১ম চতুর্ভাগে আবদ্ধ ক্ষেত্রফল হবে,

$\frac{1}{4} \times 16\pi$ বর্গ একক।

$= 4\pi$ বর্গ একক। **(Ans.)**

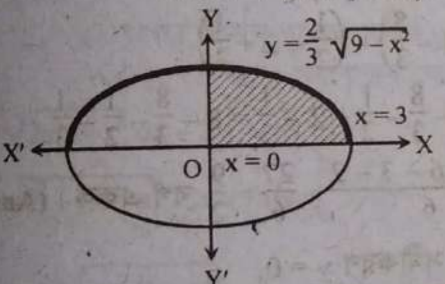
(iii) যেহেতু বৃত্তটির কেন্দ্র মূল বিন্দু, সুতরাং x-অক্ষের ওপরে ক্ষেত্রটি অর্ধেক অংশ অবস্থিত।

$\therefore$ x-অক্ষের ওপরে অবস্থিত অংশের

ক্ষেত্রফল $= \frac{1}{2} \times 16\pi$ বর্গ একক।

$= 8\pi$ বর্গ একক। **(Ans.)**

6.



উপবৃত্তটি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$= 4\int_0^3 y\ dx = 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\ dx = \frac{8}{3}\int_0^3 \sqrt{9 - x^2}\ dx$

$x = 3\sin\theta$ হলে, $dx = 3\cos\theta\ d\theta$

এবং $\sqrt{9 - x^2} = \sqrt{9 - 9\sin^2\theta} = 3\cos\theta$

সীমা: $x = 0$ হলে, $\theta = 0$ এবং $x = 3$ হলে, $\theta = \frac{\pi}{2}$

$\therefore$ উপবৃত্তটির ক্ষেত্রফল $= \frac{8}{3}\int_0^{\pi/2} 3\cos\theta \,.\, 3\cos\theta\ d\theta$

$= 12\int_0^{\pi/2} 2\cos^2\theta\ d\theta$

$= 12\int_0^{\pi/2} (1 + \cos 2\theta)\ d\theta$

$= 12\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$= 12\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right)$

$= 6\pi$ বর্গ একক। **(Ans.)**

7. দেওয়া আছে, $y^2 = x$ ...... (i)

এবং $x^2 = y$ ...... (ii)

(i) ও (ii) হতে পাই,

$(x^2)^2 = x$

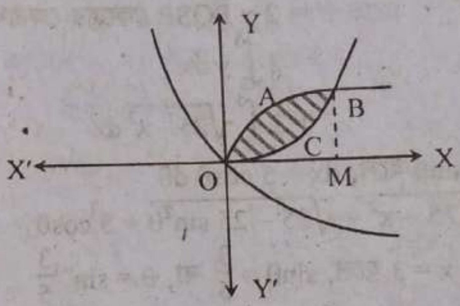
বা, $x^3 = 1\ (x \neq 0)$

বা, $(x^3 - 1) = 0$

বা, $(x - 1)(x^2 + x + 1) = 0$

হয়, $x = 1$ অথবা, $x^2 + x + 1 = 0$

[এখানে, $x^2 + x + 1 = 0$ গ্রহণযোগ্য নয় কারণ প্রাপ্ত মান কাল্পনিক]



(i) নং এ $x = 1$ বসিয়ে পাই, $y^2 = 1 \quad \therefore y = 1$

$\therefore$ (i) ও (ii) এর ছেদবিন্দু B(1, 1)

$\therefore$ ক্ষেত্রফল = ক্ষেত্র OABMO − ক্ষেত্র OCBMO

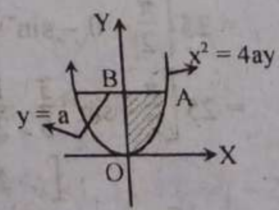
$= \int (y_1 - y_2)\ dx = \int_0^1 (x^{\frac{1}{2}} - x^2)\ dx$

$= \left[\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \frac{x^{2+1}}{2+1}\right]_0^1 = \left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^3}{3}\right]_0^1$

$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ বর্গ একক **(Ans.)**

8. $x^2 = 4ay$ পরাবৃত্তের উপকেন্দ্রিক লম্বের সমীকরণ $y = a$

ক্ষেত্রফল $= 2 \times$ OABO ক্ষেত্রের ক্ষেত্রফল



$\therefore$ ক্ষেত্রফল $= 2\int_{y=0}^{a} x dy$

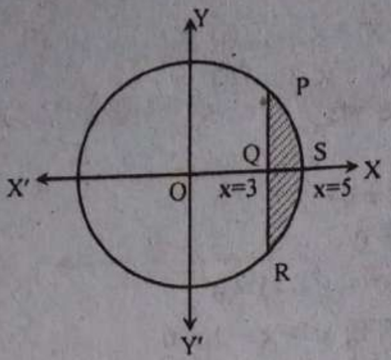
$= 2\int_0^a 2\sqrt{a}.\sqrt{y}dy$

$= 4\sqrt{a}\int_0^a \sqrt{y}dy$

$= 4\sqrt{a}\,\frac{2}{3}\left[y^{\frac{3}{2}}\right]_0^a = \frac{8\sqrt{a}}{3}\left(a^{\frac{3}{2}}\right)$

$= \frac{8a^2}{3}$ বর্গ একক **(Ans.)**

9.



$\therefore$ নির্ণেয় ক্ষেত্রফল = PQRS ক্ষেত্রের ক্ষেত্রফল

= 2 × PQSP ক্ষেত্রের ক্ষেত্রফল

$= 2\int_3^5 y\,dx$

$= 2\int_3^5 \sqrt{25-x^2}\,dx$

$x = 5\sin\theta$ হলে, $dx = 5\cos\theta\, d\theta$

এবং $\sqrt{25-x^2} = \sqrt{25-25\sin^2\theta} = 5\cos\theta$

আবার, $x = 3$ হলে, $\sin\theta = \frac{3}{5}$ বা, $\theta = \sin^{-1}\frac{3}{5}$

$x = 5$ হলে, $\sin\theta = 1$ বা, $\theta = \frac{\pi}{2}$

$\therefore$ নির্ণেয় ক্ষেত্রফল

$= 2\int_{\sin^{-1}\frac{3}{5}}^{\frac{\pi}{2}} 25\cos^2\theta\, d\theta$

$= 25\int_{\sin^{-1}\frac{3}{5}}^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$

$= 25\int_{\sin^{-1}\frac{3}{5}}^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta$

$= 25\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\sin^{-1}3/5}^{\pi/2}$

$= 25\left[\frac{\pi}{2} + 0 - \sin^{-1}\frac{3}{5} - \frac{1}{2}\sin\left(2\sin^{-1}\frac{3}{5}\right)\right]$

$= 25\left[\frac{\pi}{2} - \sin^{-1}\frac{3}{5} - \frac{1}{2}\sin.\sin^{-1}\left(2.\frac{3}{5}\sqrt{1-\frac{9}{25}}\right)\right]$

$\left[\because 2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right)\right]$

$= 25\left[\frac{\pi}{2} - \sin^{-1}\frac{3}{5} - \frac{1}{2}\sin.\sin^{-1}\left(\frac{6}{5}\cdot\frac{4}{5}\right)\right]$

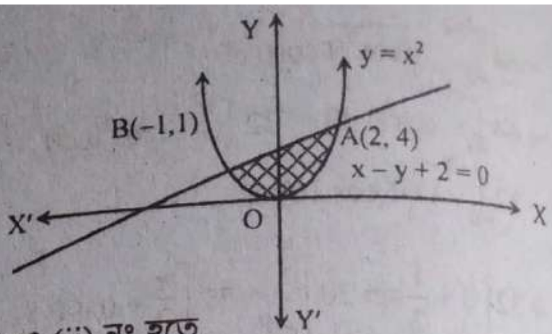
$= 25\left[\frac{\pi}{2} - \sin^{-1}\frac{3}{5} - \frac{1}{2}\cdot\frac{24}{25}\right]$

$= \left(\frac{25\pi}{2} - 25\sin^{-1}\frac{3}{5} - 12\right)$ বর্গ একক। **(Ans.)**

10. দেওয়া আছে,

$x - y + 2 = 0$ ....(i)

$y = x^2$ ....(ii)



(i) ও (ii) নং হতে,

$x - x^2 + 2 = 0$

বা, $x^2 - x - 2 = 0$

বা, $x^2 - 2x + x - 2 = 0$

বা, $x(x-2) + 1(x-2) = 0$

বা, $(x-2)(x+1) = 0$

$\therefore x = 2, -1$

$\therefore y = 4, 1$

$\therefore$ ছেদবিন্দুদ্বয় A(2, 4) ও B(−1, 1)

$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_{-1}^{2}(y_2 - y_1)\,dx$

$= \int_{-1}^{2}[x + 2 - x^2]\,dx$

$= \left[\frac{x^2}{2} + 2x - \frac{x^3}{3}\right]_{-1}^{2}$

$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} = 8 - \frac{8}{3} - \frac{1}{2} - \frac{1}{3}$

$= \frac{48 - 16 - 3 - 2}{6} = \frac{27}{6} = \frac{9}{2}$ বর্গ একক। (Ans.)

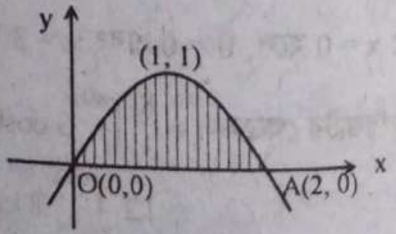
11. x-অক্ষের সমীকরণ $y = 0$.

আমরা পাই, $0 = 2x - x^2$

বা, $x(2-x) = 0$

$\therefore\ x = 0$ অথবা, 2

$\therefore$ পরাবৃত্তের সাথে x-অক্ষের ছেদবিন্দুদ্বয় O(0, 0) ও A(2, 0)



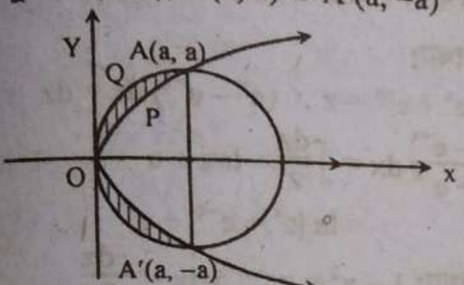
$\therefore$ ক্ষেত্রফল $= \int_0^2 y\,dx = \int_0^2 (2x - x^2)\,dx$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2 = 2^2 - \frac{8}{3} = 4 - \frac{8}{3}$

$= \frac{12-8}{3} = \frac{4}{3}$ বর্গ একক (Ans.)

12. $x^2 + y^2 = 2ax$ ... ... (i) বৃত্তের কেন্দ্রের স্থানাংক $(a, 0)$ এবং ব্যাসার্ধ $= a$.
$y^2 = ax$ ... ... (ii) পরাবৃত্তের শীর্ষবিন্দু $(0, 0)$ ও উপকেন্দ্র $\left(\frac{a}{4}, 0\right)$

(i) ও (ii) সমীকরণদ্বয় সমাধান করি
(ii)নং হতে y^2 এর মান (i)নং এ বসিয়ে
$x^2 + ax - 2ax = 0 \quad \Rightarrow x(x - a) = 0$
$\therefore x = 0, a$
$\therefore$ (ii) $\Rightarrow y = 0, \pm a$
$\therefore$ ছেদবিন্দুদ্বয় $O(0, 0)$, $A(a, a)$ ও $A'(a, -a)$



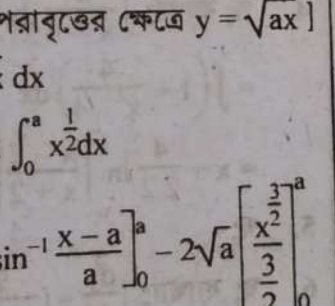
নির্ণেয় ক্ষেত্রফল $= 2 \times OPAQO$ ক্ষেত্রের ক্ষেত্রফল

$$= 2\int_{x=0}^{a}(y_1 - y_2)\,dx$$
$$= 2\int_{x=0}^{a}(\sqrt{2ax - x^2} - \sqrt{ax}\,)\,dx$$

[$\because$ x- অক্ষের উপরে বৃত্তের ক্ষেত্রে $y = \sqrt{2ax - x^2}$
এবং পরাবৃত্তের ক্ষেত্রে $y = \sqrt{ax}$]

$$= 2\int_0^a \sqrt{2ax - x^2}\,dx - 2\int_0^a \sqrt{ax}\,dx$$
$$= 2\int_0^a \sqrt{a^2 - (x - a)^2}\,dx - 2\sqrt{a}\int_0^a x^{\frac{1}{2}}dx$$
$$= 2\left[\frac{(x-a)\sqrt{a^2 - (x-a)^2}}{2} + \frac{a^2}{2}\sin^{-1}\frac{x - a}{a}\right]_0^a - 2\sqrt{a}\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^a$$
$$= 2\left[(0 + 0) - \left(0 + \frac{a^2}{2}\sin^{-1}(-1)\right)\right] - 2\sqrt{a}.\frac{2}{3}\left[a^{\frac{3}{2}} - 0\right]$$
$$= -a^2\sin^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right) - \frac{4a^2}{3}$$
$$= -a^2\left(\frac{-\pi}{2}\right) - \frac{4a^2}{3}$$
$$= a^2\left(\frac{\pi}{2} - \frac{4}{3}\right)$$ বর্গ একক **(Ans.)**

13. দেওয়া আছে, $xy = c^2$
$\therefore y = \frac{c^2}{x}$



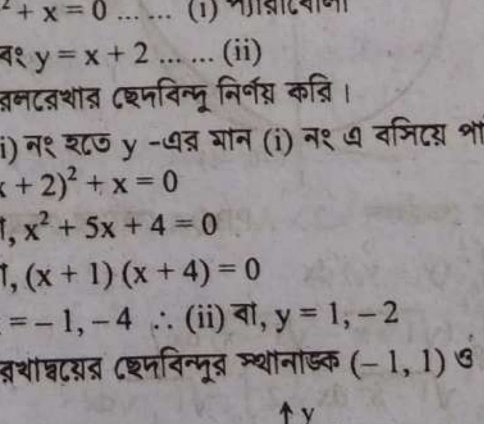
নির্ণেয় ক্ষেত্রফল = অধিবৃত্ত,
x-অক্ষ এবং $x = a$ ও $x = b$
দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= \int_a^b y\,dx$

$$= \int_a^b \frac{c^2}{x}\,dx$$
$$= c^2 \int_a^b \frac{1}{x}\,dx$$
$$= c^2\,[\ln x]_a^b$$
$$= c^2\,(\ln b - \ln a)$$
$$= c^2 \ln\left(\frac{b}{a}\right)$$ বর্গ একক **(Ans.)**

14. প্রদত্ত অধিবৃত্ত, $\sqrt{x} + \sqrt{y} = \sqrt{a}$
বা, $\sqrt{y} = \sqrt{a} - \sqrt{x}$
বা, $y = a + x - 2\sqrt{a}\sqrt{x}$ [বর্গ করে]
এখানে, $y = 0$ হলে $x = a$ হয়।
$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_0^a y\,dx$

$$= \int_0^a (a + x - 2\sqrt{a}\sqrt{x}\,)\,dx$$
$$= \left[ax + \frac{x^2}{2} - 2\sqrt{a}\,.\,\frac{2}{3}x^{\frac{3}{2}}\right]_0^a$$
$$= a.a + \frac{a^2}{2} - \frac{4\sqrt{a}}{3}a^{\frac{3}{2}} - 0$$
$$= a^2 + \frac{a^2}{2} - \frac{4\,a^2}{3}$$
$$= \frac{6a^2 + 3a^2 - 8a^2}{6}$$
$$= \frac{1}{6}a^2$$ বর্গ একক **(Ans.)**

15. $y^2 + x = 0$ ... ... (i) প্যারাবোলা
এবং $y = x + 2$ ... ... (ii)
সরলরেখার ছেদবিন্দু নির্ণয় করি।
(ii) নং হতে y -এর মান (i) নং এ বসিয়ে পাই,
$(x + 2)^2 + x = 0$
বা, $x^2 + 5x + 4 = 0$
বা, $(x + 1)(x + 4) = 0$
$\therefore \quad x = -1, -4 \;\; \therefore$ (ii) বা, $y = 1, -2$
$\therefore$ রেখাদ্বয়ের ছেদবিন্দুর স্থানাঙ্ক $(-1, 1)$ ও $(-4, -2)$



নির্ণেয় ক্ষেত্রফল = OABO ক্ষেত্রের ক্ষেত্রফল

$$= \int_{y=-2}^{1} (x_1 - x_2)\,dy$$
$$= \int_{-2}^{1} [-y^2-(y-2)]\,dy$$
$$= \left[-\frac{y^3}{3}-\frac{y^2}{2}+2y\right]_{-2}^{1}$$
$$= \left(-\frac{1}{3}-\frac{1}{2}+2\right)-\left(\frac{8}{3}-\frac{4}{2}-4\right)$$
$$= \frac{-2-3+12}{6}-\frac{16-12-24}{6}$$

$= \frac{7}{6}+\frac{20}{6}=\frac{9}{2}$ বর্গ একক।

16. $x^2+y^2=1$ ...... (i) ও

$y^2 = 1-x$ ...... (ii) সমীকরণদ্বয় সমাধান করি।

(ii) নং হতে y^2-এর মান (i) নং এ বসিয়ে পাই,

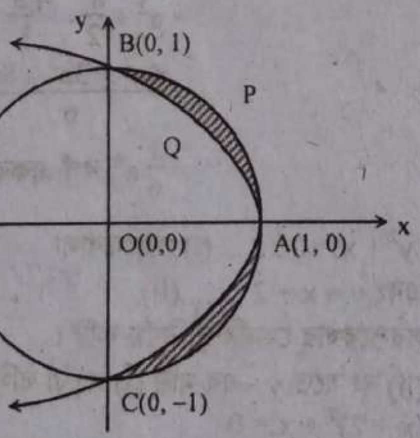
$x^2+1-x=1$ বা, $x(x-1)=0$

∴ $x=0, 1.$

$x=0$ হলে, (ii) নং হতে পাই, $y=\pm 1$

$x=1$ হলে, (ii) নং হতে পাই, $y=0$

∴ (i) নং বৃত্ত ও (ii) নং পরাবৃত্ত (1, 0), (0, 1) ও (0, − 1) বিন্দুতে ছেদ করে।



নির্ণেয় ক্ষেত্রফল = 2 × APBQA ক্ষেত্রের ক্ষেত্রফল

$$= 2\int_0^1 (y_1-y_2)\,dx$$
$$= 2\int_0^1 \left(\sqrt{1-x^2}-\sqrt{1-x}\right)dx$$
$$= 2\int_0^1 \sqrt{1-x^2}\,dx - 2\int_0^1 \sqrt{1-x}\,dx$$

ধরি, $x=\sin\theta \Rightarrow dx=\cos\theta\,d\theta$

যখন $x=0$, তখন $\theta=0$

এবং যখন $x=1$, তখন $\theta=\frac{\pi}{2}$

$$= 2I-2\left[\frac{(1-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right]_0^1 = 2I+\frac{4}{3}(0-1)=2I-\frac{4}{3}$$

$$I=\int_0^1\sqrt{1-x^2}\,dx$$

∴ $I=\int_0^{\pi/2}\sqrt{1-\sin^2\theta}\,.\cos\theta\,d\theta$

$$= \frac{1}{2}\int_0^{\pi/2}2\cos^2\theta\,d\theta = \frac{1}{2}\int_0^{\pi/2}(1+\cos2\theta)\,d\theta$$
$$= \frac{1}{2}\left[\theta+\frac{1}{2}\sin2\theta\right]_0^{\pi/2}=\frac{1}{2}\left[\left(\frac{\pi}{2}+0\right)-0\right]=\frac{\pi}{4}$$

∴ নির্ণেয় ক্ষেত্রফল $=2.\frac{\pi}{4}-\frac{4}{3}=\frac{\pi}{2}-\frac{4}{3}$ বর্গ একক।

বহুনির্বাচনি প্রশ্নের উত্তর ও ব্যাখ্যা

1. খ; ব্যাখ্যা:

ধরি, $e^x+e^{-x}=z$ ∴ $(e^x-e^{-x})\,dx=dz$

$$\int\frac{e^x-e^{-x}}{e^x+e^{-x}}\,dx=\int\frac{dz}{z}=\ln z+c$$
$$=\ln|e^x+e^{-x}|+c$$

2. খ; ব্যাখ্যা: $1-x^2=z$ ∴ $xdx=\frac{-dz}{2}$

∴ $\int\frac{-dz}{2\sqrt{z}}=-\sqrt{z}+c=-\sqrt{1-x^2}+c$

3. ক; ব্যাখ্যা: $\int e^x\{\tan x+\frac{d}{dx}(\tan x)\}\,dx=e^x\tan x+c$

4. গ; ব্যাখ্যা: $\int\frac{x^2}{x^2-4}\,dx=\int\left(\frac{x^2-4+4}{x^2-4}\right)dx$

$$=\int\left(1+\frac{4}{x^2-4}\right)dx$$
$$=x+\frac{4}{2.2}\ln\left|\frac{x-2}{x+2}\right|+c=x+\ln\left|\frac{x-2}{x+2}\right|+c$$

5. খ;

6. ঘ; ব্যাখ্যা: $\int\frac{dy}{dx}=\int\frac{2}{1+x^2}$

বা, $\int dy=\int\frac{2}{1+x^2}\,dx$

∴ $y=2\tan^{-1}x+c$

7. খ; ব্যাখ্যা: $\int\frac{dx}{2\cos^2\frac{x}{2}}=\frac{1}{2}\int\sec^2\frac{x}{2}\,dx$

$$=\frac{1}{2}\cdot\frac{\tan\frac{x}{2}}{\frac{d}{dx}\left(\frac{x}{2}\right)}+c=\tan\frac{x}{2}+c$$

8. ঘ; ব্যাখ্যা: ধরি $\sqrt{x}=z$ বা, $\frac{1}{2\sqrt{x}}\,dx=dz$ ∴ $\frac{dx}{\sqrt{x}}=2dz$

∴ $\int\cos z(2dz)=2\int\cos z\,dz$
$$=2\sin z+c$$
$$=2\sin\sqrt{x}+c$$

9. গ; ব্যাখ্যা: $\int \sin\frac{\pi x}{180}dx = \dfrac{-\cos\frac{\pi x}{180}}{\frac{\pi}{180}} + c$

$= -\frac{180}{\pi}\cos\frac{\pi x}{180} + c$

10. খ; ব্যাখ্যা: $\tan^{-1}x + 3 = z$ ধরে।

11. খ; ব্যাখ্যা: $\int \ell n(\sec^{-1}x)d(\sec^{-1}x)$

$= \sec^{-1}x \,.\, \ell n|\sec^{-1}x| - \sec^{-1}x + c$

12. গ; ব্যাখ্যা: $\int e^x\{\sin x + \frac{d}{dx}(\sin x)\}dx = e^x\sin x + c$

13. ঘ; ব্যাখ্যা: $\int x^n e^{ax}dx$

$= e^{ax}[\frac{x^n}{a} - \frac{nx^{n-1}}{a^2} + \frac{n(n-1)x^{n-2}}{a^3} \ldots + c]$

14. গ; ব্যাখ্যা: $2\int_0^1 \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx = 2[e^{\sqrt{x}}]_0^1$

$= 2(e-1)$

15. গ;

16. গ; ব্যাখ্যা: $A = \int_0^1 \frac{x\,dx}{1+(x^2)^2}$

ধরি, $x^2 = z$

বা, $2xdx = dz$

$\therefore xdx = \frac{dz}{2}$

যখন $x = 0$, তখন $z = 0$

যখন $x = 1$, তখন $z = 1$

$= \int_0^1 \frac{dz/2}{1+z^2}$

$= \frac{1}{2}[\tan^{-1}z]_0^1$

$= \frac{1}{2}(\tan^{-1}1 - \tan^{-1}0)$

$= \frac{1}{2}\times\frac{\pi}{4} = \frac{\pi}{8}$

17. ঘ; ব্যাখ্যা: $\int_{-1}^{1}|x+1|\,dx = \int_{-1}^{1}(x+1)dx$

$= \left[\frac{x^2}{2} + x\right]_{-1}^{1}$

$= \left(\frac{1}{2}+1\right) - \left(\frac{1}{2}-1\right)$

$= \frac{1}{2} + 1 - \frac{1}{2} + 1 = 2$

18. খ; ব্যাখ্যা: $\int_0^{\frac{\pi}{4}}\frac{dx}{2\cos^2x} = \frac{1}{2}[\tan x]_0^{\frac{\pi}{4}}$

$= \frac{1}{2}\left[\tan\frac{\pi}{4} - \tan 0\right] = \frac{1}{2}$

19. ঘ; ব্যাখ্যা: $-\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\cot^5x\, d(\cot x)$

$= -\frac{1}{6}[\cot^6x]_{\frac{\pi}{3}}^{\frac{\pi}{2}} = -\frac{1}{6}\left[0 - \frac{1}{(\sqrt{3})^6}\right] = \frac{1}{162}$

20. ঘ; ব্যাখ্যা: $x = a\tan\theta$ বসিয়ে $\int\frac{a\sec^2\theta d\theta}{\sqrt{a^2\tan^2\theta + a^2}}$

$= \int\frac{a\sec^2\theta\, d\theta}{a\sec\theta} = \int\sec\theta\, d\theta.$

21. ক; 22. ক; 23. ঘ;

24. খ; ব্যাখ্যা: $\int(11x^5 + 6x^4)\,dx = \frac{11x^6}{6} + \frac{6}{5}x^5 + c$

$\therefore A = \frac{11}{6}, B = \frac{6}{5}, D = 5$

25. খ;

26. গ; ব্যাখ্যা: (i) সঠিক নয়; কারণ $\int \ln x\,dx = x\ln x - x$

(ii) সঠিক; কারণ $\int_1^e \ln x\,dx = [x\ln x - x]_1^e = 1 = \ln e$

(iii) সঠিক; কারণ $\ln e = 1 = \frac{1}{\ln e}$

27. খ;

28. গ;

29. ঘ; ব্যাখ্যা: $\dfrac{\left(\frac{1}{2}-3\right)}{(1-2x)\left(1+\frac{1}{2}\right)} + \dfrac{-1-3}{\{1-2(-1)\}(1+x)}$

$= \dfrac{-\frac{5}{3}}{1-2x} + \dfrac{-\frac{4}{3}}{1+x}$

30. ঘ; ব্যাখ্যা: $-\frac{5}{3}\int\frac{dx}{1-2x} + \left(-\frac{4}{3}\right)\int\frac{dx}{1+x}$

$= \frac{-5}{3}\,\dfrac{\ell n|1-2x|}{\frac{d}{dx}(1-2x)} - \frac{4}{3}\ell n|1+x| + c$

$= \frac{5}{6}\ell n|1-2x| - \frac{4}{3}\ell n|1+x| + c$

31. খ; ব্যাখ্যা: $y = \int 2xdx = \frac{2x^2}{2} + c = x^2 + c$

32. ক; ব্যাখ্যা: $y = x^2 + c$ যা $(2, 5)$ বিন্দুগামী।

$5 = 4 + c \quad \therefore c = 1 \quad \therefore y = x^2 + 1$

33. ঘ; ব্যাখ্যা: $\int\dfrac{dx}{\sqrt{\frac{9}{4} - \left(x^2 - 2.\frac{3}{2}.x + \frac{9}{4}\right)}}$

$= \int\dfrac{dx}{\sqrt{\left(\frac{3}{2}\right)^2 - \left(x - \frac{3}{2}\right)^2}} = \sin^{-1}\dfrac{x - \frac{3}{2}}{\frac{3}{2}} + c$

$= \sin^{-1}\frac{2x-3}{3} + c$

34. খ; ব্যাখ্যা: $\left[\sin^{-1}\frac{2x-3}{3}\right]_0^3$

$= \sin^{-1}\left(\frac{2.3-3}{3}\right) - \sin^{-1}(-1)$

$= \sin^{-1}1 + \sin^{-1}1 = 2.\frac{\pi}{2} = \pi$

35. ঘ; ব্যাখ্যা: $(3)^2 = 9x$ বা, $9x = 9 \therefore x = 1$

36. গ; ব্যাখ্যা: $y^2 = 9x$ বা, $x = \frac{y^2}{9}$

y-অক্ষের সমীকরণ $x = 0$

$$\int_0^3 \left(\frac{y^2}{9} - 0\right) dy = \left[\frac{y^3}{27}\right]_0^3 = 1$$

37. ঘ; ব্যাখ্যা: $\int \frac{x}{\sqrt{1+x^2}} dx = \int \frac{d(1+x^2)}{2\sqrt{1+x^2}}$

$$= \sqrt{1+x^2} + c$$

$\therefore f(x) = \sqrt{1+x^2}$

38. গ; ব্যাখ্যা:

ধরি, $3x + 1 = z$

বা, $3dx = dz \therefore dx = \frac{dz}{3}$

x	0	1
z	1	4

$\int_0^1 f(3x+1)dx$,

$= \int_1^4 \frac{1}{3} f(z)dz$

$= \frac{1}{3}\int_1^4 f(x)dx$ [চলক পরিবর্তন করলে লিমিট পরিবর্তন হয় না]

$= \frac{1}{3} \times 5 = \frac{5}{3}$

39. ক; ব্যাখ্যা: $x^2 + y^2 = 16$

$\Rightarrow x^2 + y^2 = 4^2$

$\therefore$ প্রথম চতুর্ভাগে বৃত্তের ক্ষেত্রফল

$= \frac{1}{4}\pi.\ 4^2 = 4\pi$

y = x রেখা বৃত্তের চতুর্ভাগকে সমদ্বিখণ্ডিত করে, অতএব তার ক্ষেত্রফল $= \frac{1}{2} 4\pi = 2\pi$ sq. units.

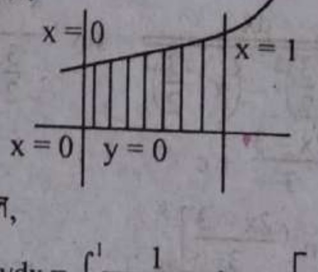
40. ক; ব্যাখ্যা: $\int_1^2 \frac{dx}{\sqrt{5-2x}} = \int_1^2 (5-2x)^{-\frac{1}{2}} dx$

$$= \left[\frac{(5-2x)^{\frac{1}{2}}}{\frac{1}{2}(-2)}\right]_1^2$$

$$= -(5-4)^{\frac{1}{2}} + (5-2)^{\frac{1}{2}}$$

$$= -1 + \sqrt{3}$$

41. ক; ব্যাখ্যা:



ক্ষেত্রফল,

$A = \int_0^1 y dx = \int_0^1 \frac{1}{(2x+1)^2} dx = -\left[\frac{1}{(2x+1).2}\right]_0^1$

$= \left[\frac{1}{2 \times 3} - \frac{1}{2}\right] = -\left[\frac{1}{6} - \frac{1}{2}\right] = -\frac{1-3}{6} = \frac{1}{3}$ বর্গ একক

42. খ; ব্যাখ্যা: $\int_0^{\frac{\pi}{4}} \tan^2 x dx = \int_0^{\frac{\pi}{4}} (\sec^2 x - 1)dx$

$$= [\tan x - x]_0^{\frac{\pi}{4}} = \left[\tan\frac{\pi}{4} - \frac{\pi}{4} - 0 + 0\right]$$

$$= 1 - \frac{\pi}{4}$$

43. গ;

44. গ;

45. ক;

46. খ; ব্যাখ্যা: ধরি, $xe^x = z$

$\therefore e^x (x+1) dx = dz$

$\therefore \int \frac{dz}{\cos^2 z} = \int \sec^2 z\, dz = \tan z + c = \tan(xe^x) + c$

47. ক;

48. খ;

49. খ; ব্যাখ্যা: $F(x) = \int_1^x \ln t\, dt$

$= [t \ln t - t]_1^x$

$= x \ln x - x - \ln 1 + 1$

$= x \ln x - x + 1$

$F'(x) = x.\frac{1}{x} + \ln x - 1 + 0 = \ln x$

50. খ;

51. ঘ;

52. ক; ব্যাখ্যা: $\int_1^{e^2} \frac{dx}{x(1+\ln x)}$

ধরি,

$z = 1 + \ln x$

$\therefore \frac{dz}{dx} = \frac{1}{x}$

x	e^2	1
z	3	1

$= \int_1^3 \frac{1}{z} dz$

$= [\ln z]_1^3 = \ln 3$

53. গ; 54. খ;

55. খ; 56. ঘ;

57. ঘ;

58. ঘ; ব্যাখ্যা: $= \int \frac{\sec^2(\cot^{-1} x)}{1+x^2} dx$

ধরি, $z = \cot^{-1} x$

$\therefore \frac{dz}{dx} = \frac{-1}{1+x^2}$

$= -\int \sec^2 z\,.\,dz$

$= -\tan z + c$

$= -\tan \cot^{-1} x + c$

$= -\tan\left(\tan^{-1}\frac{1}{x}\right) + c$

$= -\frac{1}{x} + c$

59. ক;

60. ঘ; ব্যাখ্যা: $\int \frac{1+\tan^2 x}{(1+\tan x)^2}dx = \int \frac{\sec^2 x}{(1+\tan x)^2}dx$

$= \int \frac{1}{z^2}dz$ | ধরি, $1 + \tan x = z$, $\therefore \frac{dz}{dx} = \sec^2 x$

$= \frac{z^{-1}}{-1} = -\frac{1}{1+\tan x} + c$

61. ক; 62. গ; 63. ঘ; 64. ক; 65. ক; 66. ক; 67. গ;

68. ঘ; ব্যাখ্যা: $\int_0^4 f(x+1)dx$

$= \int_0^4 f(z)dz$ | ধরি, $x + 1 = z$

$= \int_0^4 f(x)dx = 6$ | $dx = dz$

x	−1	3
z	0	4

69. ক; 70. ক;

71. খ; ব্যাখ্যা: $\int_1^\alpha \{2 + x\ln(x^2+5)\}dx + \int_1^\alpha \{3 - x\ln(x^2+5)\}dx = 30$

$\Rightarrow \int_1^\alpha 2dx + \int_1^\alpha x\ln(x^2+5)dx + \int_1^\alpha 3dx - \int_1^\alpha x\ln(x^2+5)dx = 30$

$\Rightarrow \int_1^\alpha 5dx = 30 \Rightarrow 5[x]_1^\alpha = 30$

$\Rightarrow \alpha - 1 = 6 \Rightarrow \alpha = 7$

72. ঘ; ব্যাখ্যা: ধরি, $y = x^x$ বা, $\ln y = x\ln x$

বা, $\frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + \ln x . 1$

$\therefore \frac{d}{dx}(x^x) = x^x(1 + \ln x)$

$\therefore \int x^x(1+\ln x)dx = \int d(x^x) = x^x + c$

সৃজনশীল প্রশ্নের সমাধান

1. ক $\int_0^{\pi/2} \cos 4x\,dx = \frac{1}{4}[\sin 4x]_0^{\frac{\pi}{2}}$

$= \frac{1}{4}[\sin 2\pi - \sin 0] = \frac{1}{4}[0 - 0] = 0$ **(Ans.)**

খ এখানে, $dI = dA.r^2 = r^2.dA$ ...... (i)

আবার, $A = \pi r^2$

$\frac{dA}{dr} = 2\pi r$

$dA = 2\pi r dr$

(i) নং হতে পাই, $dI = r^2 . 2\pi r dr$

বা, $dI = 2\pi r^3 dr$.

বা, $\int dI = 2\pi \int r^3 dr$

বা, $\int dI = \int 2\pi r^3 dr$

বা, $I = 2\pi \frac{r^4}{4}$

$\therefore I = \frac{\pi r^4}{2}$ **(Ans.)**

গ $x^2 + y^2 = 1$ ...... (i) ও

$y^2 = 1 - x$ ...... (ii) সমীকরণদ্বয় সমাধান করি।

(ii) নং হতে y^2-এর মান (i) নং এ বসিয়ে পাই,

$x^2 + 1 - x = 1$

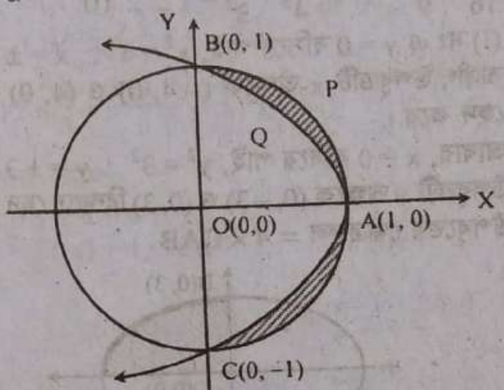
বা, $x(x - 1) = 0$

$\therefore x = 0, 1$.

$x = 0$ হলে, (ii) নং হতে পাই, $y = \pm 1$

$x = 1$ হলে, (ii) নং হতে পাই, $y = 0$

$\therefore$ (i) নং বৃত্ত ও (ii) নং পরাবৃত্ত (1, 0), (0, 1) ও (0, − 1) বিন্দুতে ছেদ করে।



নির্ণেয় ক্ষেত্রফল = 2 × APBQA ক্ষেত্রের ক্ষেত্রফল

$= 2\int_0^1 (y_1 - y_2)dx$

$= 2\int_0^1 (\sqrt{1-x^2} - \sqrt{1-x})dx$

$= 2\int_0^1 \sqrt{1-x^2}dx - 2\int_0^1 \sqrt{1-x}dx$

$= 2N - 2\left[\frac{(1-x)^{\frac{3}{2}}}{-\frac{3}{2}}\right]_0^1$ [এখানে, $N = \int_0^1 \sqrt{1-x^2}dx$]

$= 2N + \frac{4}{3}(0 - 1) = 2N - \frac{4}{3}$

$N = \int_0^1 \sqrt{1-x^2}dx$

ধরি, $x = \sin\theta \Rightarrow dx = \cos\theta\, d\theta$

যখন $x = 0$, তখন $\theta = 0$

এবং যখন $x = 1$, তখন $\theta = \frac{\pi}{2}$

$\therefore N = \int_0^{\pi/2} \sqrt{1-\sin^2\theta}.\cos\theta\, d\theta = \frac{1}{2}\int_0^{\pi/2} 2\cos^2\theta\, d\theta$

$= \frac{1}{2}\int_0^{\pi/2}(1 + \cos 2\theta)\, d\theta$

$= \frac{1}{2}\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = \frac{1}{2}\left[\left(\frac{\pi}{2} + 0\right) - 0\right] = \frac{\pi}{4}$

$\therefore$ নির্ণেয় ক্ষেত্রফল $= 2.\frac{\pi}{4} - \frac{4}{3} = \left(\frac{\pi}{2} - \frac{4}{3}\right)$ বর্গ একক। **(Ans.)**

2. ক $\int\frac{dx}{1-\sin x}=\int\frac{(1+\sin x)\,dx}{(1-\sin x)(1+\sin x)}$

$=\int\frac{1+\sin x}{1-\sin^2x}\,dx$

$=\int\frac{1+\sin x}{\cos^2x}\,dx$

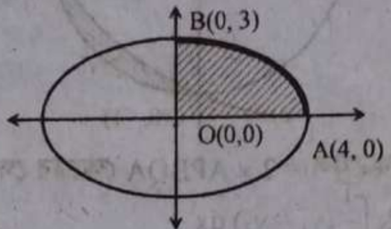
$=\int\left(\frac{1}{\cos^2x}+\frac{\sin x}{\cos^2x}\right)dx$

$=\int\sec^2x\,dx+\int\sec x\tan x\,dx$

$=\tan x+\sec x+c$ **(Ans.)**

খ $\frac{x^2}{16}+\frac{y^2}{9}=1$ বা, $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$ ... ... (i)

(i) নং এ $y=0$ বসিয়ে পাই, $x^2=4^2\ \therefore x=\pm4$
অর্থাৎ উপবৃত্তটি x-অক্ষকে $(-4,0)$ ও $(4,0)$ বিন্দুতে ছেদ করে।
আবার, $x=0$ বসিয়ে পাই, $y^2=3^2\ \therefore y=\pm3$
উপবৃত্তটি y অক্ষকে $(0,-3)$ ও $(0,3)$ বিন্দুতে ছেদ করে।
উপবৃত্তের ক্ষেত্রফল $=4\times OAB$



$=4\int_0^4 y\,dx$

$=4\int_0^4\frac{3}{4}\sqrt{4^2-x^2}\,dx$

$=3\int_0^4\sqrt{16-x^2}\,dx$

ধরি, $x=4\sin\theta$
$\therefore dx=4\cos\theta d\theta$
যখন, $x=0;\ \theta=0$
যখন, $x=4,\ \theta=\frac{\pi}{2}$

$=3\int_0^{\pi/2}\sqrt{16-4^2\sin^2\theta}\,.\,4\cos\theta d\theta$

$=3\int_0^{\pi/2}4.\cos\theta.\,4\cos\theta d\theta$

$=3.4^2\int_0^{\pi/2}\cos^2\theta d\theta=24\int_0^{\pi/2}2\cos^2\theta d\theta$

$=24\int_0^{\pi/2}(1+\cos2\theta)d\theta=24\left[\theta+\frac{1}{2}\sin2\theta\right]_0^{\pi/2}$

$=24\left(\frac{\pi}{2}+\frac{1}{2}\sin\pi-0-\frac{1}{2}\sin0\right)$

$=24\,.\,\frac{\pi}{2}=12\pi.$

ধরি, চৌবাচ্চাটির গভীরতা $=h$
$\therefore$ চৌবাচ্চাটির আয়তন $=12\pi h=10$
বা, $h=\frac{10}{12\pi}$
$\therefore h=0.27$ মিটার **(Ans.)**

গ এখানে, $\frac{ds}{dt}=9t-1$

$ds=(9t-1)dt$
যখন, $t=0$ তখন $s=0$
যখন, $t=t$ তখন $s=s$
বা, $\int_0^s ds=\int_0^t(9t-1)dt$

বা, $s=\left[\frac{9t^2}{2}-t\right]_0^t$

$\therefore s=\frac{9}{2}t^2-t$

$t=60$ সেকেন্ড হলে, $s=\frac{9}{2}\times60^2-60=16140$ মি.

$\therefore$ বিদ্যুৎ উৎপাদনে পানির শক্তি খরচ হবে $=w\times s$
$=(98\times16140)$ কিলোজুল
$=1581720$ কিলোজুল (Ans.)

3. ক

$\int\frac{1}{e^x+e^{-x}}dx=\int\frac{e^xdx}{e^x(e^x+e^{-x})}$

$=\int\frac{e^xdx}{(e^x)^2+1}$

ধরি, $e^x=z$
$\therefore e^xdx=dz$

$=\int\frac{dz}{z^2+1}$

$=\tan^{-1}z+c$

$=\tan^{-1}(e^x)+c$ **(Ans.)**

খ মনে করি, $\frac{x}{(x-1)(x^2+1)}\equiv\frac{A}{x-1}+\frac{Bx+C}{x^2+1}$

উভয়পক্ষকে $(x-1)(x^2+1)$ দ্বারা গুণ করে পাই,
$x\equiv A(x^2+1)+(Bx+C)(x-1)$ ... ... (i)

(i) নং এ $x=1$ বসিয়ে, $1=2A\Rightarrow A=\frac{1}{2}$

(i) নং এ $x=0$ বসিয়ে, $0=A-C\Rightarrow C=A=\frac{1}{2}$

(i) নং এর উভয়পক্ষ হতে x^2 এর সহগ সমীকৃত করে পাই,
$0=A+B\Rightarrow B=-A=-\frac{1}{2}$

$\therefore\frac{x}{(x-1)(x^2+1)}=\frac{\frac{1}{2}}{x-1}+\frac{-\frac{x}{2}+\frac{1}{2}}{x^2+1}$

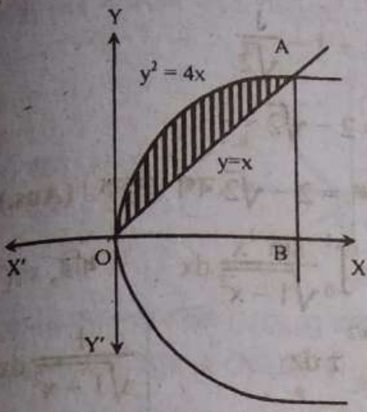
$=\frac{1}{2}.\frac{1}{x-1}-\frac{1}{2}.\frac{x}{x^2+1}+\frac{1}{2}.\frac{1}{x^2+1}$

$\therefore\int\frac{x}{(x-1)(x^2+1)}dx$

$=\frac{1}{2}\int\frac{dx}{x-1}-\frac{1}{4}\int\frac{2x}{x^2+1}dx+\frac{1}{2}\int\frac{dx}{x^2+1}$

$=\frac{1}{2}\ln|x-1|-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}\tan^{-1}x+c$

প্রদত্ত পরাবৃত্তের সমীকরণ, $y^2 = 4x$ ... ... (i)
এবং সরলরেখার সমীকরণ $y = x$ ... ... (ii)



[এখানে, $y_1{}^2 = 4x$ এবং $y_2 = x$
$\Rightarrow y_1 = \pm 2\sqrt{x}$ এবং x-অক্ষের উপরে y_1 ধনাত্মক]

(ii) নং ও (i) নং হতে, $x^2 = 4x$
বা, $x^2 - 4x = 0$
বা, $x(x-4) = 0$
$\therefore x = 0, 4$

$\therefore$ ছেদবিন্দুদ্বয়ের ভুজ 0 ও 4

$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_0^4 (y_1 - y_2)dx$

$$= \int_0^4 (2\sqrt{x} - x)\,dx$$

$$= \left[2.\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^2}{2}\right]_0^4$$

$$= \left[\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2\right]_0^4$$

$$= \left[\frac{32}{3} - 8\right] = \frac{8}{3}$$ বর্গ একক।

4. ক $\int_0^3 \frac{8}{3}\sqrt{9-x^2}\,dx$

ধরি, $x = 3\sin\theta$
$\therefore dx = 3\cos\theta d\theta$
$x = 0$ হলে, $\theta = 0$
$x = 3$ হলে, $\theta = \frac{\pi}{2}$

$$= \frac{8}{3}\int_0^{\frac{\pi}{2}} \sqrt{9 - 9\sin^2\theta}\,.3\cos\theta d\theta$$

$$= \frac{8}{3}.3.3\int_0^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 12\int_0^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$$

$$= 12\int_0^{\frac{\pi}{2}} (1 + \cos2\theta)\, d\theta$$

$$= 12\left[\theta + \frac{\sin2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

$$= 12\left(\frac{\pi}{2} + 0 - 0\right) = 6\pi$$ **(Ans.)**

খ $\int_{-3}^{3} |1 - f(x)|\,dx$

$$= \int_{-3}^{3} |1 - x^2|\,dx$$

$$= \int_{-3}^{-1} (x^2 - 1)dx + \int_{-1}^{1} (1 - x^2)dx + \int_{1}^{3} (x^2 - 1)dx$$

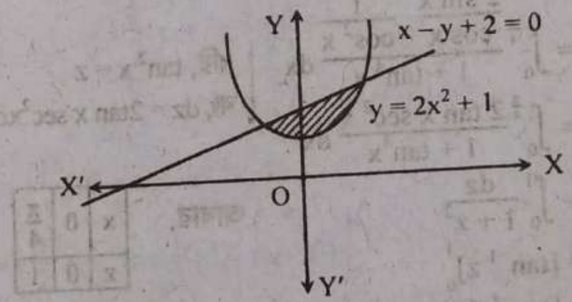
$$= \left[\frac{x^3}{3} - x\right]_{-3}^{-1} + \left[x - \frac{x^3}{3}\right]_{-1}^{1} + \left[\frac{x^3}{3} - x\right]_{1}^{3}$$

$$= \left[\left(\frac{-1}{3} + 1\right) - \left(\frac{-27}{3} + 3\right)\right] + \left[\left(1 - \frac{1}{3}\right) - \left(-1 + \frac{1}{3}\right)\right] + \left[\left(\frac{27}{3} - 3\right) - \left(\frac{1}{3} - 1\right)\right]$$

$$= \frac{2}{3} + 6 + \frac{2}{3} + \frac{2}{3} + 6 + \frac{2}{3}$$

$$= 12 + \frac{8}{3} = \frac{36 + 8}{3} = \frac{44}{3}$$ **(Ans.)**

গ দেওয়া আছে,
$x - y + 2 = 0$ ... ... (i)
$y = 2f(x) + 1 = 2x^2 + 1$ ... ... (ii)



(i) ও (ii) নং হতে,
$x - 2x^2 - 1 + 2 = 0$
বা, $2x^2 - x - 1 = 0$
বা, $2x^2 - 2x + x - 1 = 0$
বা, $2x(x - 1) + 1(x - 1) = 0$
বা, $(x - 1)(2x + 1) = 0$
$\therefore x = \frac{-1}{2}, 1$

$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_{\frac{-1}{2}}^{1} [x + 2 - 2x^2 - 1]\,dx$

$$= \left[\frac{x^2}{2} + x - \frac{2x^3}{3}\right]_{\frac{-1}{2}}^{1}$$

$$= \left(\frac{1}{2} + 1 - \frac{2}{3}\right) - \left(\frac{1}{8} - \frac{1}{2} + \frac{2}{24}\right)$$

$$= \frac{5}{6} - \frac{-7}{24}$$

$$= \frac{27}{24}$$

$$= \frac{9}{8}$$ বর্গ একক **(Ans.)**

5. ক $\int f(g+h)\,dx$

$= \int e^x (\sin x + \cos x)\,dx$

$= \int e^x \sin x\,dx + \int e^x \cos x\,dx$

$= \int e^x \sin x\,dx + e^x \int \cos x\,dx - \int\left(\frac{d}{dx}e^x \int \cos x\,dx\right)dx$

$= \int e^x \sin x\,dx + e^x \sin x - \int e^x \sin x\,dx + c$

$= e^x \sin x + c$

$= fg +$ ধ্রুবক (প্রমাণিত)

খ $\int_0^{\frac{\pi}{4}} \frac{2gh}{g^4+h^4}dx = \int_0^{\frac{\pi}{4}} \frac{2\sin x\cos x}{\sin^4 x + \cos^4 x}dx$

$= \int_0^{\frac{\pi}{4}} \frac{\frac{2\sin x\cos x}{\cos^4 x}}{\frac{\sin^4 x}{\cos^4 x}+\frac{\cos^4 x}{\cos^4 x}}dx$

$= \int_0^{\frac{\pi}{4}} \frac{\frac{2\sin x}{\cos x}\cdot\frac{1}{\cos^2 x}}{1+\tan^4 x}dx$ | ধরি, $\tan^2 x = z$ বা, $dz = 2\tan x \sec^2 x dx$

$= \int_0^{\frac{\pi}{4}} \frac{2\tan x\sec^2 x}{1+\tan^4 x}dx$

$= \int_0^1 \frac{dz}{1+z^2}$ আবার,

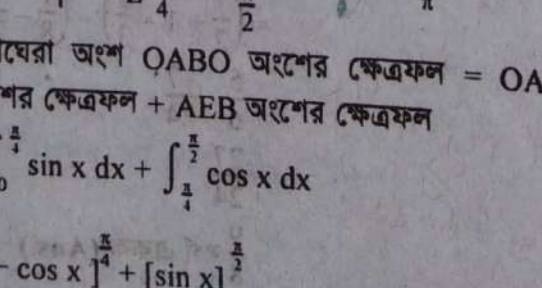
x	0	$\frac{\pi}{4}$
z	0	1

$= [\tan^{-1} z]_0^1$

$= [\tan^{-1} 1 - \tan^{-1} 0]$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$ **(Ans.)**

গ g, h, ও x অক্ষ অর্থাৎ sin x, cos x ও x অক্ষ দ্বারা আবদ্ধ অংশ নিচে স্কেচের মাধ্যমে দেখানো হলো:



ছায়াঘেরা অংশ OABO অংশের ক্ষেত্রফল = OAE অংশের ক্ষেত্রফল + AEB অংশের ক্ষেত্রফল

$= \int_0^{\frac{\pi}{4}} \sin x\,dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos x\,dx$

$= [-\cos x]_0^{\frac{\pi}{4}} + [\sin x]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$

$= \left(-\cos\frac{\pi}{4} + \cos 0\right) + \left(\sin\frac{\pi}{2} - \sin\frac{\pi}{4}\right)$

$= -\frac{1}{\sqrt{2}} + 1 + 1 - \frac{1}{\sqrt{2}}$

$= 2 - \frac{2}{\sqrt{2}} = 2 - \sqrt{2}$

$\therefore$ নির্ণেয় ক্ষেত্রফল $= 2-\sqrt{2}$ বর্গ একক। (Ans.)

6. ক এখানে, $I = \int_0^1 \frac{\sin^{-1}x}{\sqrt{1-x^2}}dx$ | ধরি, $\sin^{-1}x = z$

$= \int_0^{\pi/2} z\,dz$ | $\frac{1}{\sqrt{1-x^2}}dx = dz$

$= \left[\frac{z^2}{2}\right]_0^{\pi/2}$ | $x = 0$ হলে $z = 0$

$= \frac{1}{2}\left[\left(\frac{\pi}{2}\right)^2 - 0\right]$ | $x = 1$ হলে $z = \frac{\pi}{2}$

$= \frac{1}{2}\left[\frac{\pi^2}{4}\right]$

$= \frac{\pi^2}{8}$ **(Ans.)**

খ $I = \int \frac{dx}{y(x+1)}$

$= \int \frac{dx}{x^2(x+1)}$ [$\because y = x^2$ দেওয়া আছে]

ধরি, $\frac{1}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2}$

বা, $1 = Ax^2 + Bx(x+1) + C(x+1)$ .... (i)

$x = 0$ হলে, $1 = C$

$x = -1$ হলে $1 = A$

(i) নং এর উভয় পক্ষ হতে x^2 এর সহগ সমীকৃত করে পাই,

$A + B = 0$

বা, $1 + B = 0$

বা, $B = -1$

$\therefore \frac{1}{x^2(x+1)} = \frac{1}{x+1} - \frac{1}{x} + \frac{1}{x^2}$

$\int \frac{1}{x^2(x+1)}dx = \int \frac{1}{x+1}dx - \int \frac{1}{x}dx + \int x^{-2}dx$

$= ln(x+1) - ln(x) + \frac{x^{-1}}{-1} + c$

[যেখানে c যোগজীকরণ ধ্রুবক]

$= ln\frac{x+1}{x} - \frac{1}{x} + c$

$= ln\left(1+\frac{1}{x}\right) - \frac{1}{x} + c$ **(Ans.)**

প্রদত্ত সমীকরণদ্বয় $y = x^2$ ... ... (i)

এবং $y = x$ ... ... (ii)

(i) এবং (ii) সমাধান করে পাই, $x^2 = x$

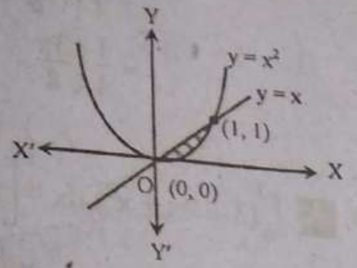
বা, $x^2 - x = 0$

বা, $x(x-1) = 0$

$\therefore x = 0, 1$

$x = 0$ হলে $y = 0$

$x = 1$ হলে $y = 1$



$\therefore$ ছেদবিন্দুর স্থানাঙ্ক O(0, 0) এবং A(1, 1)

(i) এবং (ii) নং দ্বারা আবদ্ধ ক্ষেত্রের

ক্ষেত্রফল $= \int_0^1 (y_1 - y_2)dx$ | এখানে, $y_1 = x$

$= \int_0^1 (x - x^2)dx$ | এবং $y_2 = x^2$

$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \left[\frac{1}{2} - \frac{1}{3}\right]$

$= \left[\frac{3-2}{6}\right] = \frac{1}{6}$ বর্গ একক **(Ans.)**

7. ক $\int \theta^7 dy = \int \cot^7 y\, dy$ [$\because \cot y = \theta$]

$= \int \cot^5 y(\text{cosec}^2 y - 1)\, dy$

$= \int \cot^5 y\, \text{cosec}^2 y dy - \int \cot^5 y dy$

$= -\frac{1}{6}\cot^6 y - \int \cot^3 y\, \text{cosec}^2 y dy + \int \cot^3 y dy$

$= -\frac{1}{6}\cot^6 y + \frac{1}{4}\cot^4 y + \int \cot y\, \text{cosec}^2 y\, dy - \int \cot y\, dy$

$= -\frac{1}{6}\cot^6 y + \frac{1}{4}\cot^4 y - \frac{1}{2}\cot^2 y - \ln \sin y + c.$ **(Ans.)**

খ $\int \sin^{-1}\sqrt{\frac{f(x-a)}{f(x)}}\, dx$

$= \int \sin^{-1}\sqrt{\frac{a+x-a}{x+a}}\, dx$

$= \int \sin^{-1}\sqrt{\frac{x}{x+a}}\, dx$

$= \int \sin^{-1}\sqrt{\frac{a\tan^2\theta}{a(1+\tan^2\theta)}} 2a\tan\theta\sec^2\theta\, d\theta$ | ধরি, $x = a\tan^2\theta$, $dx = 2a\tan\theta\sec^2\theta d\theta$

$= \int \sin^{-1}\left(\frac{\tan\theta}{\sec\theta}\right) 2a\tan\theta\sec^2\theta\, d\theta$

$= \int \sin^{-1}(\sin\theta) 2a\tan\theta\sec^2\theta\, d\theta$

$= 2a\int \theta\tan\theta\sec^2\theta\, d\theta$ | ধরি, $\tan\theta = z$

$= 2a\int z\tan^{-1} z\, dz$ | $\therefore \sec^2\theta\, d\theta = dz$

$= 2a\left[\tan^{-1} z\frac{z^2}{2} - \int \frac{1}{1+z^2}\frac{z^2}{2}dz\right]$ | $\theta = \tan^{-1} z$

$= 2a\left[\tan^{-1} z\frac{z^2}{2} - \frac{1}{2}\int \frac{z^2+1-1}{z^2+1}dz\right]$

$= az^2\tan^{-1} z - a\int\left(1 - \frac{1}{1+z^2}\right)dz$

$= az^2\tan^{-1} z - az + a\tan^{-1} z + c$

$= a\theta\tan^2\theta - a\tan\theta + a\theta + c$

$= x\tan^{-1}\sqrt{\frac{x}{a}} - a\sqrt{\frac{x}{a}} + a\tan^{-1}\sqrt{\frac{x}{a}} + c$

$= (x+a)\tan^{-1}\sqrt{\frac{x}{a}} - \sqrt{ax} + c$ **(Ans.)**

গ ধরি, $I = \int f(\theta - a)y\, d\theta$

$= \int (a + \theta - a)\cot^{-1}\theta\, d\theta + c$

$= \int \theta\cot^{-1}\theta\, d\theta$

$= \cot^{-1}\theta\int\theta\, d\theta - \int\left\{\frac{d}{d\theta}\cot^{-1}\theta\int\theta\, d\theta\right\}d\theta$

$= \cot^{-1}\theta.\frac{\theta^2}{2} - \int\left\{\frac{-1}{1+\theta^2}\cdot\frac{\theta^2}{2}\right\}d\theta$

$= \frac{\theta^2}{2}\cot^{-1}\theta + \frac{1}{2}\int\left[1 - \frac{1}{1+\theta^2}\right]d\theta$

$= \frac{\theta^2}{2}\cot^{-1}\theta + \frac{1}{2}(\theta - \tan^{-1}\theta) + c$

$\therefore \int_1^{\sqrt{3}} f(\theta - a)y\, d\theta$

$= \left[\frac{\theta^2}{2}\cot^{-1}\theta + \frac{1}{2}(\theta - \tan^{-1}\theta)\right]_1^{\sqrt{3}}$

$= \frac{3}{2}\cot^{-1}\sqrt{3} + \frac{1}{2}(\sqrt{3} - \tan^{-1}\sqrt{3}) - \frac{1}{2}\cot^{-1}1 - \frac{1}{2}(1 - \tan^{-1}1)$

$= \frac{3}{2}\times\frac{\pi}{6} + \frac{1}{2}\left(\sqrt{3} - \frac{\pi}{3}\right) - \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{2}\left(1 - \frac{\pi}{4}\right)$

$= \frac{3\pi}{12} + \frac{\sqrt{3}}{2} - \frac{\pi}{6} - \frac{\pi}{8} - \frac{1}{2} + \frac{\pi}{8}$

$= \frac{\pi}{12} + \frac{\sqrt{3}}{2} - \frac{1}{2}$ (দেখানো হলো)

8. ক $\int\frac{dx}{\sqrt{7 - 9x^2}} = \int\frac{dx}{\sqrt{9\left(\frac{7}{9} - x^2\right)}}$

$= \frac{1}{3}\int\frac{dx}{\sqrt{\left(\frac{\sqrt{7}}{3}\right)^2 - x^2}}$

$= \frac{1}{3}\sin^{-1}\frac{x}{\frac{\sqrt{7}}{3}} + c$

$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{\sqrt{7}}\right) + c$ **(Ans.)**

খ দেওয়া আছে, $f(x) = 1 - \sin^2 x = \cos^2 x$

$\therefore f(\theta) = \cos^2\theta$

$f\left(\frac{\pi}{2} - \theta\right) = \cos^2\left(\frac{\pi}{2} - \theta\right) = \sin^2\theta$

$$\therefore \int \frac{d\theta}{1 + \sqrt{\frac{f(\theta)}{f\left(\frac{\pi}{2} - \theta\right)}}} = \int \frac{d\theta}{1 + \sqrt{\frac{\cos^2\theta}{\sin^2\theta}}}$$

$$= \int \frac{d\theta}{1 + \frac{\cos\theta}{\sin\theta}}$$

$$= \int \frac{d\theta}{\frac{\sin\theta + \cos\theta}{\sin\theta}}$$

$$= \int \frac{\sin\theta\, d\theta}{\sin\theta + \cos\theta}$$

$$= \frac{1}{2}\int \frac{(\sin\theta + \cos\theta) - (\cos\theta - \sin\theta)}{\sin\theta + \cos\theta}\, d\theta$$

$$= \frac{1}{2}\int \left(\frac{\sin\theta + \cos\theta}{\sin\theta + \cos\theta} - \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\right) d\theta$$

$$= \frac{1}{2}\int d\theta - \frac{1}{2}\int \frac{\cos\theta - \sin\theta}{\sin\theta + \cos\theta}\, d\theta$$

$= \frac{1}{2}\theta - \frac{1}{2}\int \frac{dz}{z}$ [ধরি, $\sin\theta + \cos\theta = z$

$\therefore (\cos\theta - \sin\theta)\, d\theta = dz$]

$$= \frac{1}{2}\theta - \frac{1}{2}\ln|z| + c$$

$= \frac{1}{2}\theta - \frac{1}{2}\ln|\sin\theta + \cos\theta| + c$ **(Ans.)**

গ $\int_0^{\frac{\pi}{4}} \{f(x)\}^2\, dx = \int_0^{\frac{\pi}{4}} (1 - \sin^2 x)^2\, dx$

$$= \int_0^{\frac{\pi}{4}} (\cos^2 x)^2\, dx = \int_0^{\frac{\pi}{4}} \cos^4 x\, dx$$

এখন, $\int \cos^4 x\, dx = \frac{1}{4}\int (2\cos^2 x)^2\, dx$

$$= \frac{1}{4}\int (1 + \cos 2x)^2\, dx$$

$$= \frac{1}{4}\int (1 + 2\cos 2x + \cos^2 2x)\, dx$$

$$= \frac{1}{4}\left[x + 2 \cdot \frac{1}{2}\sin 2x + \frac{1}{2}\int 2\cos^2 2x\, dx\right]$$

$$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\int (1 + \cos 4x)\, dx\right]$$

$$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}x + \frac{1}{2}\cdot\frac{1}{4}\sin 4x\right] + c$$

$$= \frac{1}{4}\left[\frac{3x}{2} + \sin 2x + \frac{1}{8}\sin 4x\right] + c$$

$$\therefore \int_0^{\frac{\pi}{4}} \{f(x)\}^2\, dx = \frac{1}{4}\left[\frac{3x}{2} + \sin 2x + \frac{1}{8}\sin 4x\right]_0^{\pi/4}$$

$$= \frac{1}{4}\left\{\left(\frac{3\pi}{8} + \sin\frac{\pi}{2} + \frac{1}{8}\sin\pi\right) - 0\right\}$$

$$= \frac{1}{4}\left(\frac{3\pi}{8} + 1\right) = \frac{1}{32}(8 + 3\pi)$$

(দেখানো হলো)

9. **ক** $\int_0^3 (3 - 2x + x^2)\, dx = \left[3x - 2\frac{x^2}{2} + \frac{x^3}{3}\right]_0^3$

$$= \left(3.3 - 3^2 + \frac{3^3}{3}\right) - 0$$

$= 9 - 9 + 9 = 9$ (Ans.)

খ $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ... ... (i)

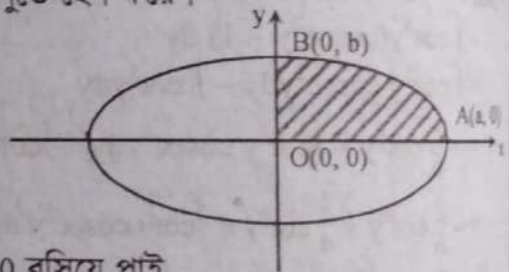
বা, $\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$ বা, $y^2 = \frac{b^2}{a^2}(a^2 - x^2)$

$\therefore y = \pm\frac{b}{a}\sqrt{a^2 - x^2}$

(i) নং এ $y = 0$ বসিয়ে পাই, $x^2 = a^2$

$\therefore x = \pm a$

অর্থাৎ উপবৃত্তটি x-অক্ষকে $(-a, 0)$ ও $(a, 0)$ বিন্দুতে ছেদ করে।



আবার, $x = 0$ বসিয়ে পাই,

$y^2 = b^2 \therefore y = \pm b$

$\therefore$ উপবৃত্তটি y-অক্ষকে $(0, -b)$ ও $(0, b)$ বিন্দুতে ছেদ করে।

উপবৃত্তের ক্ষেত্রফল

$= 4 \times$ OABO ক্ষেত্রের ক্ষেত্রফল

$= 4\int_0^a y\,dx$

$= 4\int_0^a \frac{b}{a}\sqrt{a^2 - x^2}\, dx$

$= \frac{4b}{a}\int_0^a \sqrt{a^2 - x^2}\, dx$

ধরি, $x = a\sin\theta \Rightarrow dx = a\cos\theta\, d\theta$

যখন $x = 0$, তখন $\theta = 0$

এবং যখন $x = a$, তখন $\theta = \frac{\pi}{2}$

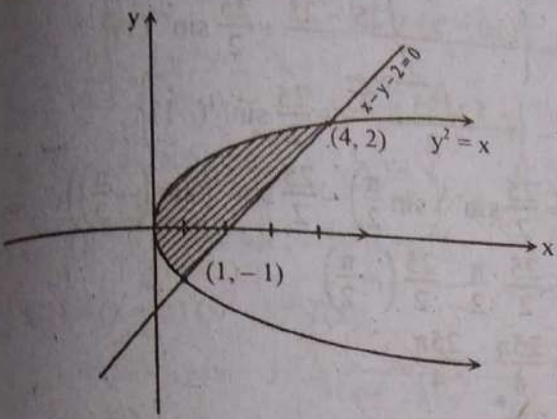
$$= \frac{4b}{a}\int_0^{\pi/2} \sqrt{a^2 - a^2\sin^2\theta}\, .a\cos\theta\, d\theta$$

$$= 2ab\int_0^{\pi/2} 2\cos^2\theta\, d\theta = 2ab\int_0^{\pi/2} (1 + \cos 2\theta)\, d\theta$$

$$= 2ab\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\pi/2} = 2ab\left[\frac{\pi}{2} - 0\right]$$

$= \pi ab$ বর্গ একক **(Ans.)**

প্রদত্ত পরাবৃত্ত, $y^2 = x$ ……… (i)
এবং সরলরেখা, $x - y - 2 = 0$ ……… (ii)
বা, $y^2 - y - 2 = 0$ [(i) নং দ্বারা]
বা, $y^2 - 2y + y - 2 = 0$
বা, $(y - 2)(y + 1) = 0$
$\therefore y = 2, -1$



[এখানে $y^2 = x_1$ এবং $x_2 - y - 2 = 0$]
y এর মান (i) নং এ বসিয়ে পাই, $x = 4, 1$
∴ ছেদ বিন্দুদ্বয় (4, 2) ও (1, −1)
∴ নির্ণেয় ক্ষেত্রফল $= \int_{-1}^{2} (x_1 - x_2)\, dy$

$= \int_{-1}^{2} (y + 2 - y^2)\, dy$

$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3}\right]_{-1}^{2}$

$= \left(\frac{1}{2} . 2^2 + 2.2 - \frac{1}{3} . 2^3\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$

$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$

$= \frac{48 - 16 - 3 - 2}{6}$

$= \frac{27}{6} = \frac{9}{2}$ বর্গ একক **(Ans.)**

10. ক দেওয়া আছে, $u = \ln y$

$\therefore \int u dy = \int \ln y\, dy$

$= \ln y \int dy - \int \left[\frac{d}{dy}(\ln y) \int dy\right] dy$

$= y \ln y - \int \frac{1}{y} . y\, dy$

$= y \ln y - \int dy$

$= y \ln y - y + c$ **(Ans.)**

খ দেওয়া আছে, $p = \tan^{-1} z$

$\therefore \int zp\, dz = \int z \tan^{-1} z\, dz$

$= \tan^{-1} z \int z\, dz - \int \left[\frac{d}{dz}(\tan^{-1} z) \int z\, dz\right] dz$

$= \frac{z^2}{2} \tan^{-1} z - \int \frac{1}{1 + z^2} . \frac{z^2}{2} dz$

$= \frac{z^2}{2} \tan^{-1} z - \frac{1}{2} \int \frac{1 + z^2 - 1}{1 + z^2} dz$

$= \frac{z^2}{2} \tan^{-1} z - \frac{1}{2} \int \left(1 - \frac{1}{1 + z^2}\right) dz$

$= \frac{z^2}{2} \tan^{-1} z - \frac{1}{2} (z - \tan^{-1} z) + c$

$= \frac{z^2}{2} \tan^{-1} z + \frac{1}{2} \tan^{-1} z - \frac{1}{2} z + c$

$= \frac{1}{2} (z^2 + 1) \tan^{-1} z - \frac{z}{2} + c$ **(Ans.)**

গ ধরি, $I = \int_4^9 \frac{u}{\sqrt{e^u}} dy$

$\therefore \int \frac{u}{\sqrt{e^u}} dy = \int \frac{\ln y}{\sqrt{e^{\ln y}}} dy = \int \frac{\ln y}{\sqrt{y}} dy$

$= \ln y \int y^{\frac{-1}{2}} dy - \int \left\{\frac{d}{dy}(\ln y) \int y^{\frac{-1}{2}} dy\right\} dy$

$= 2\sqrt{y}\, \ln y - \int \frac{1}{y} . 2\sqrt{y}\, dy$

$= 2\sqrt{y}\, \ln y - 2 \int \frac{1}{\sqrt{y}} dy$

$= 2\sqrt{y}\, \ln y - 2 . 2\sqrt{y} + c$

$= 2\sqrt{y}\, \ln y - 4\sqrt{y} + c$

$\therefore I = [2\sqrt{y}\, \ln y - 4\sqrt{y}]_4^9$

$= (2\sqrt{9}\, \ln 9 - 4\sqrt{9}) - (2\sqrt{4}\, \ln 4 - 4\sqrt{4})$

$= (2.3\, \ln 9 - 4.3) - (2.2\, \ln 4 - 4.2)$

$= 6 \ln 9 - 12 - 4 \ln 4 + 8$

$= 12 \ln 3 - 8 \ln 2 - 4$ **(Ans.)**

11. ক $\int \sin x^\circ\, dx = \int \sin \frac{x\pi}{180} dx\ [\because 1^\circ = \frac{\pi^c}{180}]$

$= \frac{-\cos \frac{x\pi}{180}}{\frac{\pi}{180}} + c$ [c যোগজীকরণ ধ্রুবক]

$= \frac{-180}{\pi} \cos \frac{x\pi}{180} + c$ **(Ans.)**

খ দেওয়া আছে, $g(x) = \sin x - \cos x$

$\therefore \int \frac{1 - g(x)}{1 + g(x)} dx = \int \frac{1 - \sin x + \cos x}{1 + \sin x - \cos x} dx$

$= \int \frac{-(1 + \sin x - \cos x) + 2}{(1 + \sin x - \cos x)} dx$

$= -\int dx + 2 \int \frac{1}{1 + \sin x - \cos x} dx$

$$= -x + 2\int \frac{dx}{1 + \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}} - \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}}$$

$$= -x + 2\int \frac{\sec^2\frac{x}{2}\,dx}{2\tan^2\frac{x}{2} + 2\tan\frac{x}{2}}$$

$$= -x + 2\int \frac{dz}{z^2+z}$$ [ধরি, $z = \tan\frac{x}{2}$

$\therefore dz = \frac{1}{2}\sec^2\frac{x}{2}\,dx$]

$$= -x + 2\int\left(\frac{1}{z} - \frac{1}{z+1}\right)dz$$

$$= -x + 2\ln z - 2\ln(z+1) + c$$

$$= -x + 2\ln\frac{z}{z+1} + c$$

$$= -x + 2\ln\frac{\tan\frac{x}{2}}{1+\tan\frac{x}{2}} + c \text{ (Ans.)}$$

গ দেওয়া আছে, $f(x) = (x-5)^2$

$$\therefore \int_0^{10}\sqrt{25 - f(x)}\,dx = \int_0^{10}\sqrt{25-(x-5)^2}\,dx$$

$$= \int_0^{10}\sqrt{25 - x^2 + 10x - 25}\,dx = \int_0^{10}\sqrt{10x - x^2}\,dx$$

[ধরি, $x = 10\sin^2\theta$, $dx = 10 \times 2\sin\theta\cos\theta\,d\theta$

$= 10\sin 2\theta\,d\theta$]

x	0	10
θ	0	$\frac{\pi}{2}$

$$= \int_0^{\frac{\pi}{2}}\sqrt{100\sin^2\theta - 100\sin^4\theta}.\ 10\sin2\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}}\sqrt{100\sin^2\theta(1-\sin^2\theta)}\ 10\sin2\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}}\sqrt{100\sin^2\theta\cos^2\theta}\ 10\sin2\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} 10\sin\theta\cos\theta.\ 10\sin2\theta\,d\theta$$

$$= \int_0^{\frac{\pi}{2}} 5\sin2\theta.\ 10\sin2\theta\,d\theta$$

$$= 25\int_0^{\frac{\pi}{2}} 2\sin^2 2\theta\,d\theta = 25\int_0^{\frac{\pi}{2}}(1-\cos4\theta)\,d\theta$$

$$= 25\left[\theta - \frac{\sin4\theta}{4}\right]_0^{\frac{\pi}{2}} = 25\left[\left(\frac{\pi}{2} - \frac{\sin4.\frac{\pi}{2}}{4}\right) - 0\right]$$

$$= 25\left(\frac{\pi}{2} - 0\right) = \frac{25\pi}{2} \text{ (প্রমাণিত)}$$

বিকল্প পদ্ধতি: $\int_0^{10}\sqrt{25 - f(x)}\,dx$

$$= \int_0^{10}\sqrt{5^2 - (x-5)^2}\,dx$$

$$= \left[\frac{(x-5)\sqrt{5^2-(x-5)^2}}{2} + \frac{5^2}{2}\sin^{-1}\left(\frac{x-5}{5}\right)\right]_0^{10}$$

$$= \left\{\frac{(10-5)\sqrt{25-25}}{2} + \frac{25}{2}\sin^{-1}(1)\right\}$$

$$- \left\{\frac{-5\sqrt{25-25}}{2} + \frac{25}{2}\sin^{-1}(-1)\right\}$$

$$= \frac{25}{2}\sin^{-1}\left(\sin\frac{\pi}{2}\right) - \frac{25}{2}\sin^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right)$$

$$= \frac{25}{2}.\frac{\pi}{2} - \frac{25}{2}\left(-\frac{\pi}{2}\right)$$

$$= \frac{25\pi}{4} + \frac{25\pi}{4}$$

$$= \frac{25\pi}{2} \text{ (প্রমাণিত)}$$

12. ক নির্ণেয় আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$$= \int_0^3 y\,dx = \int_0^3 x^2\,dx = \left[\frac{x^{2+1}}{2+1}\right]_0^3 = \frac{1}{3}[x^3]_0^3$$

$$= \frac{1}{3}(3^3 - 0) = 9 \text{ বর্গ একক (Ans.)}$$

খ দেওয়া আছে, $f(x) = \frac{2x+1}{(x+2)(x-3)^2}$

ধরি, $\frac{2x+1}{(x+2)(x-3)^2} \equiv \frac{A}{x+2} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$

বা, $2x + 1 \equiv A(x-3)^2 + B(x+2)(x-3) + C(x+2)$ ...... (i)

(i) এ $x \equiv 3$ বসিয়ে পাই, $2\times3 + 1 = C\times5 \Rightarrow C = \frac{7}{5}$.

(i) এ $x = -2$ বসিয়ে পাই, $-4 + 1 = A(-5)^2 \Rightarrow A = \frac{-3}{25}$

(i) হতে x^2 এর সহগ সমীকৃত করে পাই, $0 = A + B$

বা, $B = -A = \frac{3}{25}$

$$\therefore \frac{2x+1}{(x+2)(x-3)^2} = \frac{-\frac{3}{25}}{x+2} + \frac{\frac{3}{25}}{x-3} + \frac{\frac{7}{5}}{(x-3)^2}$$

$$\int_0^2 f(x)\,dx = \int_0^2 \frac{2x+1}{(x+2)(x-3)^2}\,dx$$

$$= \int_0^2\left\{\frac{\frac{-3}{25}}{x+2} + \frac{\frac{3}{25}}{x-3} + \frac{\frac{7}{5}}{(x-3)^2}\right\}dx$$

$$= \left[-\frac{3}{25}\ln|x+2| + \frac{3}{25}\ln|x-3| + \frac{7}{5}\times\frac{(x-3)^{-1}}{(-1)}\right]_0^2$$

$= \left[\frac{3}{25}\ln\left|\frac{x-3}{x+2}\right| - \frac{7}{5(x-3)}\right]_0^2$

$= \frac{3}{25}\ln\left|\frac{1}{4}\right| + \frac{7}{5} - \frac{3}{25}\ln\left|\frac{3}{2}\right| - \frac{7}{15}$

$= \frac{3}{25}\ln\left|\frac{\frac{1}{4}}{\frac{3}{2}}\right| + \frac{14}{15}$

$= \frac{3}{25}\ln\left|\frac{1}{6}\right| + \frac{14}{15}$

$= -\frac{3}{25}(\ln 1 - \ln 6) + \frac{14}{15}$

$= -\frac{3}{25}\ln 6 + \frac{14}{15}$ **(Ans.)** $[\because \ln 1 = 0]$

গ $h(x) = (x+2)(x-3)^2 f(x)$

$= (x+2)(x-3)^2 \times \frac{2x+1}{(x+2)(x-3)^2}$

$= 2x + 1$

ধরি, $y = h(x) = 2x + 1$

এখন, $y^2 + x = 0$ ……… (i)

ও $y = 2x + 1$ …… (ii) এর ছেদবিন্দু নির্ণয় করি।

(ii) নং হতে y এর মান (i) এ বসিয়ে পাই,

$(2x+1)^2 + x = 0$

বা, $4x^2 + 4x + 1 + x = 0$

বা, $4x^2 + 4x + x + 1 = 0$

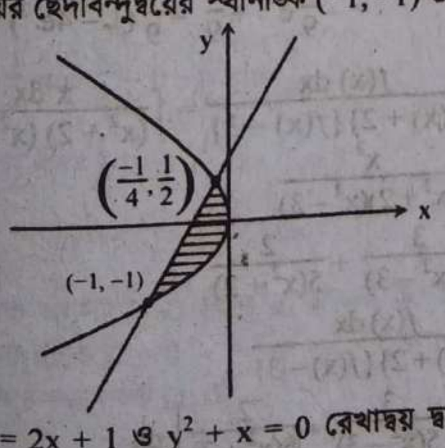
বা, $4x(x+1) + 1(x+1) = 0$

বা, $(x+1)(4x+1) = 0$

হয়, $x = -1$ অথবা, $x = -\frac{1}{4}$

$\therefore y = -1$ বা, $\frac{1}{2}$

∴ রেখাদ্বয়ের ছেদবিন্দুদ্বয়ের স্থানাঙ্ক $(-1, -1)$ ও $\left(\frac{-1}{4}, \frac{1}{2}\right)$



∴ $h(x) = 2x + 1$ ও $y^2 + x = 0$ রেখাদ্বয় দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= \int_{-1}^{\frac{1}{2}} (x_1 - x_2)\, dy$

এখানে, $y = 2x_2 + 1 \therefore x_2 = \frac{y-1}{2}$

এবং $y^2 + x_1 = 0 \therefore x_1 = -y^2$

$= \int_{-1}^{\frac{1}{2}} \left[-y^2 - \frac{y-1}{2}\right] dy$

$= \int_{-1}^{\frac{1}{2}} -y^2 dy - \int_{-1}^{\frac{1}{2}} \frac{y}{2} dy + \frac{1}{2}\int_{-1}^{\frac{1}{2}} dy$

$= -\left[\frac{y^3}{3}\right]_{-1}^{\frac{1}{2}} - \frac{1}{4}[y^2]_{-1}^{\frac{1}{2}} + \frac{1}{2}[y]_{-1}^{\frac{1}{2}}$

$= -\frac{1}{3}\left[\frac{1}{8} + 1\right] - \frac{1}{4}\left[\frac{1}{4} - 1\right] + \frac{1}{2}\left[\frac{1}{2} + 1\right]$

$= -\frac{1}{3}\cdot\frac{9}{8} + \frac{1}{4}\cdot\frac{3}{4} + \frac{3}{4}$

$= -\frac{3}{8} + \frac{3}{16} + \frac{3}{4} = \frac{-6+3+12}{16}$

$= \frac{9}{16}$ বর্গ একক **(Ans.)**

13.

ক এখানে, $I = \int_0^{\pi/2} \frac{\cos x}{9 - \sin^2 x} dx$

ধরি, $\sin x = z$
$\cos x\, dx = dz$
$x = 0$ হলে $z = 0$
$x = \frac{\pi}{2}$ হলে $z = 1$

$= \int_0^1 \frac{dz}{9 - z^2}$

$= \int_0^1 \frac{dz}{(3)^2 - z^2} = \frac{1}{2.3}\left[\ln\left|\frac{3+z}{3-z}\right|\right]_0^1$

$= \frac{1}{6}\left[\ln\frac{4}{2} - \ln 1\right]$

$= \frac{1}{6}[\ln 2 - 0]$

$= \frac{1}{6}\ln 2$ **(Ans.)**

খ এখানে $I = \int_0^{\pi/4} f(\cos\theta)\, d\theta$

$= \int_0^{\pi/4} \cos^4\theta\, d\theta$

$= \frac{1}{4}\int_0^{\pi/4} (2\cos^2\theta)^2 d\theta$

$= \frac{1}{4}\int_0^{\pi/4} (1 + \cos 2\theta)^2 d\theta$

$= \frac{1}{4}\int_0^{\pi/4} (1 + 2\cos 2\theta + \cos^2 2\theta)\, d\theta$

$= \frac{1}{4}\int_0^{\pi/4} \{1 + 2\cos 2\theta + \frac{1}{2}(1 + \cos 4\theta)\}\, d\theta$

$= \frac{1}{4}\int_0^{\pi/4} \left(\frac{3}{2} + 2\cos 2\theta + \frac{1}{2}\cos 4\theta\right) d\theta$

$$=\frac{1}{4}\left[\frac{3}{2}\theta+\frac{2\sin 2\theta}{2}+\frac{1}{2}\frac{\sin 4\theta}{4}\right]_0^{\pi/4}$$

$$=\frac{1}{4}\left[\frac{3}{2}\theta+\sin 2\theta+\frac{1}{8}\sin 4\theta\right]_0^{\pi/4}$$

$$=\frac{1}{4}\left[\frac{3}{2}\cdot\frac{\pi}{4}+\sin\frac{\pi}{2}+\frac{1}{8}\sin\pi-0\right]$$

$$=\frac{1}{4}\left[\frac{3\pi}{8}+1+\frac{1}{8}\cdot 0\right]$$

$$=\frac{1}{4}\left[\frac{3\pi}{8}+1\right]$$

এখন, $I-\frac{1}{4}=\frac{3\pi}{32}+\frac{1}{4}-\frac{1}{4}=\frac{3\pi}{32}$ (দেখানো হলো)

গ চিত্রে প্রদর্শিত উপবৃত্তের $a=4$ এবং $b=3$

এ উপবৃত্তের সমীকরণ, $\frac{x^2}{4^2}+\frac{y^2}{3^2}=1$

বা, $\frac{y^2}{3^2}=1-\frac{x^2}{4^2}$

বা, $y^2=3^2\left(1-\frac{x^2}{4^2}\right)$

$\therefore y=\pm 3\sqrt{1-\frac{x^2}{4^2}}$

$\therefore$ উপবৃত্তের ক্ষেত্রফল $= 4\times$ ১ম চতুর্ভাগে উপবৃত্ত করা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$=4\int_0^4 y\,dx$ [x-অক্ষের উপরে y ধনাত্মক]

$$=4\int_0^4 3\sqrt{1-\frac{x^2}{4^2}}\,dx$$

$$=12\int_0^{\pi/2}\sqrt{1-\frac{4^2\sin^2\theta}{4^2}}\cdot 4\cos\theta\,d\theta$$

$$=12\int_0^{\pi/2}\sqrt{1-\sin^2\theta}\cdot 4\cos\theta\,d\theta$$

$x=4\sin\theta$

$dx=4\cos\theta\,d\theta$

$x=0$ হলে $\theta=0$

$x=4$ হলে $\theta=\frac{\pi}{2}$

$$=12\int_0^{\pi/2}\cos\theta\cdot 4\cos\theta\,d\theta$$

$$=24\int_0^{\pi/2}2\cos^2\theta\,d\theta$$

$$=24\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta$$

$$=24\left[\theta+\frac{\sin 2\theta}{2}\right]_0^{\pi/2}$$

$$=24\left[\frac{\pi}{2}+\frac{1}{2}\cdot\sin 2\cdot\frac{\pi}{2}-0\right]=24\left[\frac{\pi}{2}+\frac{1}{2}\sin\pi\right]$$

$$=24\left[\frac{\pi}{2}+\frac{1}{2}\cdot 0\right]=12\pi \textbf{ (Ans.)}$$

14. ক $\int\frac{dx}{1+\cos^2 x}$

$=\int\frac{\sec^2 x}{\sec^2 x+1}dx$ [লব ও হরকে $\cos^2 x$ দ্বারা ভাগ করে]

$$=\int\frac{\sec^2 x}{1+\tan^2 x+1}dx$$

$$=\int\frac{\sec^2 x}{2+\tan^2 x}dx$$

$$=\int\frac{dz}{(\sqrt{2})^2+z^2}$$

ধরি, $\tan x=z$

$\therefore \sec^2 x\,dx=dz$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\frac{z}{\sqrt{2}}+c$$

$$=\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right)+c \textbf{ (Ans.)}$$

খ এখানে, $\int xe^{-3x}dx$

$$=x\int e^{-3x}dx-\int\left\{\frac{d}{dx}(x)\int e^{-3x}dx\right\}dx$$

$$=x\left(\frac{e^{-3x}}{-3}\right)-\int\frac{e^{-3x}}{-3}dx$$

$$=-\frac{1}{3}xe^{-3x}+\frac{1}{3}\int e^{-3x}dx$$

$$=-\frac{1}{3}xe^{-3x}+\frac{1}{3}\left(\frac{e^{-3x}}{-3}\right)+c$$

$$=-\frac{1}{3}xe^{-3x}-\frac{1}{9}e^{-3x}+c$$

$$=-\frac{1}{3}e^{-3x}\left(x+\frac{1}{3}\right)+c$$

$$\therefore \int_0^1 xe^{-3x}dx=\left[-\frac{1}{3}e^{-3x}\left(x+\frac{1}{3}\right)\right]_0^1$$

$$=-\frac{1}{3}e^{-3}\left(1+\frac{1}{3}\right)+\frac{1}{3}e^{-3\times 0}\left(0+\frac{1}{3}\right)$$

$$=-\frac{4}{9}e^{-3}+\frac{1}{9}=\frac{1}{9}(1-4e^{-3}) \textbf{ (Ans.)}$$

গ $\int\frac{f(x)\,dx}{\{f(x)+2\}\{f(x)-3\}}=\int\frac{x^2dx}{(x^2+2)(x^2-3)}$

এখানে, $\frac{x^2}{(x^2+2)(x^2-3)}$

$$=\frac{3}{5(x^2-3)}+\frac{2}{5(x^2+2)}$$

$$\therefore \int\frac{f(x)\,dx}{\{f(x)+2\}\{f(x)-3\}}$$

$$=\int\left\{\frac{3}{5(x^2-3)}+\frac{2}{5(x^2+2)}\right\}dx$$

$$=\frac{3}{5}\int\frac{dx}{x^2-(\sqrt{3})^2}+\frac{2}{5}\int\frac{dx}{x^2+(\sqrt{2})^2}$$

$= \frac{3}{5} \cdot \frac{1}{2\sqrt{3}} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + \frac{2}{5} \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$

$= \frac{\sqrt{3}}{10} \ln\left|\frac{x-\sqrt{3}}{x+\sqrt{3}}\right| + \frac{\sqrt{2}}{5} \tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + c$ **(Ans.)**

15. **ক** $\int lnx\, dx = lnx \int dx - \int\left\{\frac{d}{dx}(lnx) \int dx\right\} dx$

$= xlnx - \int \frac{1}{x} \cdot x\, dx$

$= xlnx - \int dx$

$= xlnx - x + c$

খ বক্ররেখাটি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$= 4 \times$ OABO ক্ষেত্রের ক্ষেত্রফল

$= 4\int_0^3 y\, dx. = 4\int_0^3 \frac{2}{3}\sqrt{9-x^2}dx$

[$y = \pm\frac{2}{3}\sqrt{9-x^2}$ এর অংশ x অক্ষের উপর ধনাত্মক]

$\because \frac{x^2}{9} + \frac{y^2}{4} = 1$ বা, $\frac{y^2}{4} = 1 - \frac{x^2}{9}$ বা, $y^2 = \frac{4}{9}(9-x^2)$

$\therefore y = \pm\frac{2}{3}\sqrt{9-x^2}$

ধরি, $x = 3\sin\theta \therefore dx = 3\cos\theta d\theta$

x	0	3
θ	0	$\frac{\pi}{2}$

$\therefore \sqrt{9-x^2} = \sqrt{9-9\sin^2\theta}$

$= 3\sqrt{1-\sin^2\theta}$

$= 3\cos\theta$

$\therefore$ নির্ণেয় আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$= 4 \times \frac{2}{3} \times \int_0^{\frac{\pi}{2}} 3\cos\theta.3\cos\theta\, d\theta$

$= 12\int_0^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$

$= 12\int_0^{\frac{\pi}{2}} (1+\cos 2\theta)\, d\theta$

$= 12\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= 12\left(\frac{\pi}{2} + \frac{1}{2}\sin 2.\frac{\pi}{2} - 0 - \frac{1}{2}\sin 0\right)$

$= 12\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right)$

$= 12 \times \frac{\pi}{2} [\because \sin\pi = 0] = 6\pi$ **(Ans.)**

গ উদ্দীপক অনুসারে, D বিন্দু x অক্ষের উপর অবস্থিত অর্থাৎ D বিন্দুর কোটি = 0

ধরি, D বিন্দুর স্থানাঙ্ক (a, 0)

$\because$ BD সরলরেখার ঢাল $= \frac{0-2}{a-0}$ বা, $-1 = \frac{-2}{a} \therefore a = 2$

$\therefore$ D বিন্দুর স্থানাঙ্ক (2, 0)

B(0, 2) ও D(2, 0) বিন্দুগামী সরলরেখার সমীকরণ,

$\frac{x-0}{0-2} = \frac{y-2}{2-0}$

বা, $x = -y + 2$ বা, $x + y = 2 \therefore y = 2 - x$

$\therefore$ ABD অংশের ক্ষেত্রফল = OAB অংশের ক্ষেত্রফল

$-$ ODB অংশের ক্ষেত্রফল $= \int_0^3 y_1\, dx - \int_0^2 y_2\, dx$

$= \int_0^3 \frac{2}{3}\sqrt{9-x^2}\, dx - \int_0^2 (2-x)\, dx$

$= \frac{1}{4} \cdot 4\int_0^3 \frac{2}{3}\sqrt{9-x^2}\, dx - \left[2x - \frac{x^2}{2}\right]_0^2$

$= \frac{1}{4} \times 6\pi - 2 \times 2 + \frac{2^2}{2} - 0$ ['খ' হতে পাই]

$= \frac{3}{2}\pi - 2$

$= \left(\frac{3\pi}{2} - 2\right)$ বর্গ একক। **(Ans.)**

16. **ক** ধরি, $I = \int \frac{dx}{e^x + e^{-x}} = \int \frac{e^x}{(e^x)^2 + 1} dx$

[লব ও হরকে e^x দ্বারা গুণ করে]

মনে করি, $e^x = z$ বা, $e^x dx = dz$.

$\therefore I = \int \frac{dz}{z^2+1} = \tan^{-1}z + c$

$= \tan^{-1}(e^x) + c.$ **(Ans.)**

খ বক্ররেখার সমীকরণ, $y = \sqrt{x}.\ln x$

$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sqrt{x}.\ln x) = \sqrt{x}\frac{d}{dx}\ln x + \ln x\frac{d}{dx}\sqrt{x}$

$= \sqrt{x}.\frac{1}{x} + \ln x.\frac{1}{2\sqrt{x}} = \frac{1}{\sqrt{x}}\left(1 + \frac{1}{2}\ln x\right)$

$\therefore$ বক্ররেখার সর্বনিম্ন বিন্দুতে স্পর্শক x-অক্ষের সমান্তরাল বলে $\frac{dy}{dx} = 0$

$\therefore \frac{1}{\sqrt{x}}\left(1 + \frac{1}{2}\ln x\right) = 0$

বা, $\frac{1}{2}\ln x + 1 = 0 \quad \left[\because \frac{1}{\sqrt{x}} \neq 0\right]$

বা, $\ln x = -2$

বা, $x = e^{-2}$

$\therefore x = e^{-2}$ হলে $y = \sqrt{e^{-2}}.\ln e^{-2} = e^{-\frac{2}{2}}.(-2) = -\frac{2}{e}$.

$\therefore$ বক্ররেখার সর্বনিম্ন বিন্দু $= \left(e^{-2}, -\frac{2}{e}\right)$. **(Ans.)**

গ আমরা জানি, x অক্ষে y এর মান শূন্য

$\therefore 0 = \sqrt{x} \times \ln x$

$\therefore \sqrt{x} = 0$ এবং $\ln x = 0 = \ln 1$

$\therefore x = 0 \quad \therefore x = 1$

$\therefore$ x = 1 থেকে x = 4 সরলরেখা দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$$= \int_1^4 y\,dx = \int_1^4 \sqrt{x}\ \ln x\ dx$$

এখন, $\int \sqrt{x}\ \ln x\ dx = \ln x \int \sqrt{x}\ dx - \int \left\{\frac{d}{dx} \ln x \int \sqrt{x}\ dx\right\} dx$

$$= \ln x . \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} - \int \frac{1}{x} \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} dx$$

$$= \frac{2}{3} \ln x . x^{\frac{3}{2}} - \int \frac{1}{x} \times \frac{2}{3} x^{\frac{3}{2}} dx$$

$$= \frac{2}{3} \ln x.\ x^{\frac{3}{2}} - \frac{2}{3} \int x^{\frac{1}{2}} dx$$

$$= \frac{2}{3} \ln x.\ x^{\frac{3}{2}} - \frac{2}{3} . \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \frac{2}{3} \ln x.\ x^{\frac{3}{2}} - \frac{4}{9} x^{\frac{3}{2}}$$

$\therefore$ আবদ্ধ ক্ষেত্রের ক্ষেত্রফল $= \int_1^4 \sqrt{x}\ \ln x\ dx$

$$= \left[\frac{2}{3} \ln x.\ x^{\frac{3}{2}} - \frac{4}{9} x^{\frac{3}{2}}\right]_1^4$$

$$= \frac{2}{3} \ln 4. \left(4^{\frac{3}{2}}\right) - \frac{4}{9} \times 4^{\frac{3}{2}} - \frac{2}{3} \ln 1 . 1^{\frac{3}{2}} + \frac{4}{9} \times 1^{\frac{3}{2}}$$

$$= \frac{2}{3} \ln 2^2 . 8 - \frac{32}{9} - \frac{2}{3} . 0 + \frac{4}{9}$$

$= \frac{32}{3} \ln 2 - \frac{28}{9}$ বর্গ একক **(Ans.)**

17. ক

$$\int_2^5 \frac{x^3}{1 + x^8} dx$$

ধরি, $x^4 = z$

$\therefore 4x^3 dx = dz$

$\therefore x^3 dx = \frac{dz}{4}$

$x = 2$ হলে, $z = 16$

$x = 5$ হলে, $z = 625$

$$= \int_2^5 \frac{x^3}{1 + (x^4)^2} dx$$

$$= \frac{1}{4} \int_{16}^{625} \frac{dz}{1 + z^2}$$

$$= \frac{1}{4} [\tan^{-1} z]_{16}^{625}$$

$= \frac{1}{4} (\tan^{-1} 625 - \tan^{-1} 16)$ **(Ans.)**

খ দেওয়া আছে, $f(x) = 4 - x^2$

$$\therefore \int_{-1}^1 x^2 \sqrt{f(x)}\ dx = \int_{-1}^1 x^2 \sqrt{4 - x^2}\ dx$$

ধরি, $x = 2\sin\theta$

$\therefore dx = 2\cos\theta\ d\theta$

x	-1	1
θ	$-\frac{\pi}{6}$	$\frac{\pi}{6}$

$$\therefore \int_{-1}^1 x^2 \sqrt{f(x)}\ dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 4\sin^2\theta \sqrt{4 - 4\sin^2\theta}\ .2\cos\theta\ d\theta$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 8\sin^2\theta\ .2\sqrt{1 - \sin^2\theta}\ .\cos\theta\ d\theta$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} 16\sin^2\theta \cos^2\theta\ d\theta$$

$$= 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (2\sin\theta \cos\theta)^2 d\theta$$

$$= 4\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^2 2\theta\ d\theta$$

$$= 2\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} (1 - \cos 4\theta)\ d\theta$$

$$= 2\left[\theta - \frac{\sin 4\theta}{4}\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}}$$

$$= 2\left[\frac{\pi}{6} - \frac{1}{4}\sin\frac{2\pi}{3} + \frac{\pi}{6} + \frac{1}{4}\sin\left(\frac{-2\pi}{3}\right)\right]$$

$$= 2\left[\frac{\pi}{3} - \frac{1}{4} . \frac{\sqrt{3}}{2} - \frac{1}{4} . \frac{\sqrt{3}}{2}\right] = 2\left[\frac{\pi}{3} - \frac{2\sqrt{3}}{8}\right]$$

$= \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$ **(Ans.)**

গ $f(x) = 4 - x^2$ হলে পাই,

$y^2 = 4 - x^2$ [$\because y^2 = f(x)$]

$\therefore y = \pm\sqrt{4 - x^2}$

$y = 0$ হলে পাই, $x = \pm 2$

নির্ণেয় ক্ষেত্রফল

$= 4 \times$ ($y = \sqrt{4 - x^2}$, x অক্ষ এবং ভুজ x = 0 ও x

= 2 দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল)

$$= 4\int_0^2 y\,dx$$

$$= 4\int_0^2 \sqrt{4 - x^2}\ dx$$

ধরি, $x = 2\sin\theta$

$\therefore dx = 2\cos\theta d\theta$

সীমা : $x = 2$ হলে, $\theta = \frac{\pi}{2}$

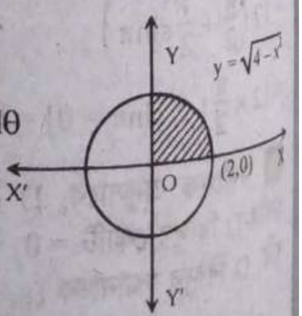
$x = 0$ হলে, $\theta = 0$

$$= 4\int_0^{\pi/2} \sqrt{4 - 4\sin^2\theta} \cdot 2\cos\theta\ d\theta$$

$$= 4 \cdot 4 \int_0^{\pi/2} \cos^2\theta\ d\theta$$

$$= 8\int_0^{\pi/2} 2\cos^2\theta\ d\theta$$

$$= 8\int_0^{\pi/2} (1 + \cos 2\theta)\ d\theta$$



$$= 8\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$$

$= 8\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi - 0\right)$ [$\because \sin\pi = 0$]

$= 4\pi$ বর্গ একক **(Ans.)**

18. ক $\int \sin 7x \sin 5x\, dx$

$=\frac{1}{2}\int 2\sin 7x \sin 5x\, dx$

$=\frac{1}{2}\int \{\cos(7x-5x)-\cos(7x+5x)\}\, dx$

$=\frac{1}{2}\int (\cos 2x-\cos 12x)\, dx$

$=\frac{1}{2}\left(\frac{\sin 2x}{2}-\frac{\sin 12x}{12}\right)+c$

$=\frac{1}{24}(6\sin 2x-\sin 12x)+c$ **(Ans.)**

খ দেওয়া আছে, $f(x)=\ln x$

$\therefore (fof)(x)=f(\ln x)$
$=\ln(\ln x)$

$\therefore \int \frac{(fof)(x)}{x}dx=\int \frac{\ln(\ln x)}{x}dx$

$=\ln(\ln x)\int \frac{1}{x}dx-\int [\frac{d}{dx}\{\ln(\ln x)\}\int \frac{1}{x}dx]\, dx$

$=\ln x\ln(\ln x)-\int (\frac{1}{\ln x}\times\frac{1}{x}\times \ln x)\, dx$

$=\ln x\ln(\ln x)-\int \frac{dx}{x}$

$=\ln x\ln(\ln x)-\ln x+c$

$=\ln x\{\ln(\ln x)-1\}+c$

$=f(x)\{(fof)(x)-1\}+c$ (দেখানো হলো)

গ ধরি, $I=\int_0^1 f(y^2+1)\,dy=\int_0^1 \ln(y^2+1)\,dy$

এখন, $\int \ln(y^2+1)\,dy$

$=\ln(y^2+1)\int dy-\int\left[\frac{d}{dy}\{\ln(y^2+1)\}\int dy\right]dy$

$=\ln(y^2+1).\, y-\int \frac{2y^2}{y^2+1}dy$

$=y\ln(y^2+1)-2\int\left(1-\frac{1}{1+y^2}\right)dy$

$=y\ln(y^2+1)-2(y-\tan^{-1}y)+c$

$=y\ln(y^2+1)-2y+2\tan^{-1}y+c$

[যেখানে, c যোগজীকরণ ধ্রুবক]

$\therefore I=\int_0^1 \ln(y^2+1)\,dy$

$=[y\ln(y^2+1)-2y+2\tan^{-1}y]_0^1$

$=\ln 2-2+2\tan^{-1}1$

$=\ln 2-2+2.\frac{\pi}{4}$

$=\ln 2-2+\frac{\pi}{2}$ **(Ans.)**

19. ক দেওয়া আছে, $f(x)=\sin x+\cos x$

$\therefore f(A)=\sin A+\cos A$

এবং $f(B)=\sin B+\cos B$

$f(A)=f(B)$ হলে পাই,

$\sin A+\cos A=\sin B+\cos B$

বা, $\sin A-\sin B=\cos B-\cos A$

বা, $2\cos\frac{A+B}{2}\sin\frac{A-B}{2}=2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

বা, $\cos\frac{A+B}{2}=\sin\frac{A+B}{2}$

বা, $\sin\left(\frac{\pi}{2}-\frac{A+B}{2}\right)=\sin\frac{A+B}{2}$

বা, $\frac{\pi}{2}-\frac{A+B}{2}=\frac{A+B}{2}$ বা, $\frac{\pi}{2}=\frac{A+B}{2}+\frac{A+B}{2}$

বা, $\frac{\pi}{2}=A+B$ $\therefore A+B=\frac{\pi}{2}$ (দেখানো হলো)

খ দেওয়া আছে, $f(x)=\sin x+\cos x$

$\therefore f(-\theta)=\sin(-\theta)+\cos(-\theta)$
$=-\sin\theta+\cos\theta$

$\therefore 1-f(-\theta)=1+\sin\theta-\cos\theta$

$$=1+\frac{2\tan\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}-\frac{1-\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$=\frac{1+\tan^2\frac{\theta}{2}+2\tan\frac{\theta}{2}-1+\tan^2\frac{\theta}{2}}{1+\tan^2\frac{\theta}{2}}$$

$$=\frac{2\tan^2\frac{\theta}{2}+2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$$\therefore \int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{d\theta}{1-f(-\theta)}=\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\frac{\sec^2\frac{\theta}{2}\,d\theta}{2\tan^2\frac{\theta}{2}+2\tan\frac{\theta}{2}}$$

$=\int_{\frac{1}{\sqrt{3}}}^{1}\frac{2\,dz}{2z^2+2z}$

$=\int_{\frac{1}{\sqrt{3}}}^{1}\frac{dz}{z(z+1)}$

$=\int_{\frac{1}{\sqrt{3}}}^{1}\left(\frac{1}{z}-\frac{1}{z+1}\right)dz$

$=[\ln z-\ln(z+1)]_{\frac{1}{\sqrt{3}}}^{1}$

ধরি, $\tan\frac{\theta}{2}=z$

$\therefore \sec^2\frac{\theta}{2}d\theta=2dz$

$\theta=\frac{\pi}{3}$ হলে, $z=\frac{1}{\sqrt{3}}$

$\theta=\frac{\pi}{2}$ হলে, $z=1$

$= [\ln 1 - \ln(1+1)] - \left[\ln\frac{1}{\sqrt{3}} - \ln\left(1 + \frac{1}{\sqrt{3}}\right)\right]$

$= -\ln 2 - \left[\ln 1 - \ln\sqrt{3} - \ln\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)\right]$

$= -\ln 2 - 0 + \ln\sqrt{3} + \ln(1+\sqrt{3}) - \ln\sqrt{3}$

$= \ln\left(\frac{1+\sqrt{3}}{2}\right)$ **(Ans.)**

গ ধরি, $\frac{1}{2}\sin^{-1}\sqrt{49-x^2} = \theta$

বা, $\sin^{-1}\sqrt{49-x^2} = 2\theta$

$\therefore \sin 2\theta = \sqrt{49-x^2}$ ……… (i)

দেওয়া আছে, $f(x) = \sin x + \cos x$

$\therefore \{f(\theta)\}^2 = (\sin\theta + \cos\theta)^2$

বা, $\left\{f\left(\frac{1}{2}\sin^{-1}\sqrt{49-x^2}\right)\right\}^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$

$= 1 + \sin 2\theta$

$= 1 + \sqrt{49-x^2}$ [(i) নং দ্বারা]

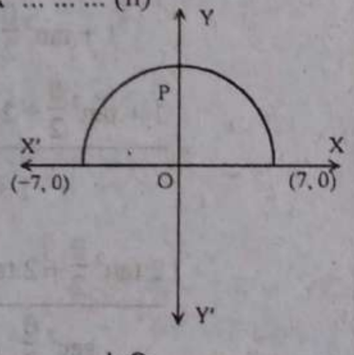
প্রদত্ত সমীকরণ, $y = \left\{f\left(\frac{1}{2}\sin^{-1}\sqrt{49-x^2}\right)\right\}^2 - 1$

বা, $y = |1 + \sqrt{49-x^2} - 1|$

$\therefore y = \sqrt{49-x^2}$ ……… (ii)

$y = 0$ হলে পাই, $x = \pm 7$

$\therefore$ (ii) নং রেখা x-অক্ষকে $(-7, 0)$ ও $(7, 0)$ বিন্দুতে ছেদ করে।



$\therefore$ নির্ণেয় ক্ষেত্রফল $= 2\int_0^7 y\,dx$

ধরি, $x = 7\sin\theta$

$\therefore dx = 7\cos\theta\, d\theta$

$x = 0$ হলে, $\theta = 0$

$x = 7$ হলে, $\theta = \frac{\pi}{2}$

$= 2\int_0^7 \sqrt{49-x^2}\,dx$

$= 2\int_0^{\frac{\pi}{2}} 7\cos\theta . 7\cos\theta\, d\theta$

$= 49\int_0^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta = 49\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)\, d\theta$

$= 49\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}} = 49\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$

$= \frac{49\pi}{2}$ বর্গ একক। **(Ans.)**

20. **ক** $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x]_0^1$

$= \tan^{-1}(1) - \tan^{-1}(0)$

$= \frac{\pi}{4} - 0 = \frac{\pi}{4}$ **(Ans.)**

খ চিত্রানুযায়ী, $AB = c$, $BC = a$ এবং $AC = b$

$BC = 2AC$ হলে পাই, $a = 2b$

ত্রিভুজের সাইন সূত্রানুসারে পাই, $\frac{a}{\sin\alpha} = \frac{b}{\sin\beta}$

বা, $\frac{2b}{\sin\alpha} = \frac{b}{\sin\beta}$ [$\because a = 2b$]

বা, $\frac{2}{\sin\alpha} = \frac{1}{\sin\beta}$

বা, $\frac{2}{\sin 3\beta} = \frac{1}{\sin\beta}$ [$\because \alpha = 3\beta$]

বা, $\sin 3\beta = 2\sin\beta$

বা, $3\sin\beta - 4\sin^3\beta = 2\sin\beta$

বা, $-4\sin^3\beta = 2\sin\beta - 3\sin\beta$

বা, $-4\sin^3\beta = -\sin\beta$

বা, $4\sin^2\beta = 1$ [$\because \sin\beta \neq 0$]

বা, $\sin^2\beta = \frac{1}{4}$

বা, $\sin\beta = \frac{1}{2}$

বা, $\sin\beta = \sin 30°$

$\therefore \beta = 30°$

$\therefore \alpha = 3\beta = 3 \times 30° = 90°$ [$\because \beta = 30°$]

আমরা জানি, $\alpha + \beta + \gamma = 180°$

বা, $\gamma = 180° - (\alpha + \beta)$

বা, $\gamma = 180° - (90° + 30°)$

বা, $\gamma = 180° - 120°$

$\therefore \gamma = 60°$

$\therefore$ $\alpha = 90°, \beta = 30°, \gamma = 60°$ **(Ans.)**

গ চিত্রানুযায়ী পাই, $C = \gamma$

$\therefore C = 60°$ [$\because \gamma = 60°$]

বা, $\cos C = \cos 60°$

বা, $\frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}$ $\left[\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}\right]$

বা, $\frac{a^2 + b^2 - c^2}{ab} = 1$

বা, $a^2 + b^2 - c^2 = ab$

বা, $a^2 + b^2 + c^2 - ab - 2c^2 = 0$

বা, $(a + b + c)^2 - 2ab - 2bc - 2ca - ab - 2c^2 = 0$

বা, $(a + b + c)^2 + ac + bc + c^2 = 3ab + 3bc + 3ac + 3c^2$

বা, $(a+b+c)^2+c(a+b+c)=3b(a+c)+3c(a+c)$

বা, $(a+b+c)(a+b+c+c)=(a+c)(3b+3c)$

বা, $(a+b+c)\{(a+c)+(b+c)\}=3(a+c)(b+c)$

বা, $\frac{(a+c)+(b+c)}{(a+c)(b+c)}=\frac{3}{a+b+c}$

$\therefore \frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ (প্রমাণিত)

21.

ক এখানে, $I=\int_0^1 \frac{x}{\sqrt{4-x^2}}dx$

ধরি, $4-x^2=z$

বা, $0-2x\,dx=dz$

$\therefore x\,dx=-\frac{dz}{2}$

$x=0$ হলে $z=4$

$x=1$ হলে $z=3$

$=\int_4^3 \frac{-\frac{dz}{2}}{\sqrt{z}}$

$=\frac{1}{2}\int_3^4 z^{\frac{-1}{2}}dz$

$=\frac{1}{2}\left[\frac{z^{\frac{1}{2}}}{\frac{1}{2}}\right]_3^4$

$=[\sqrt{z}]_3^4$

$=\sqrt{4}-\sqrt{3}$

$=2-\sqrt{3}$ **(Ans.)**

খ ধরি, $g(x)=[f(x)-1]^{\frac{n}{2}}$

$=[x^2+1-1]^{\frac{n}{2}}$ [$\because$ দেওয়া আছে, $f(x)=x^2+1$]

$=x^n$

$\therefore g(x+h)=(x+h)^n$

অন্তরীকরণের সংজ্ঞানুসারে,

$\therefore \frac{d}{dx}\{g(x)\}=\lim_{h\to 0}\frac{g(x+h)-g(x)}{h}$

বা, $\frac{d}{dx}(x^n)=\lim_{h\to 0}\frac{(x+h)^n-x^n}{h}$

$=\lim_{h\to 0}\left[\frac{x^n+\frac{n}{1!}x^{n-1}h+\frac{n(n-1)}{2!}x^{n-2}h^2+\ldots+h^n-x^n}{h}\right]$

$=\lim_{h\to 0}h\left[\frac{nx^{n-1}+\frac{n(n-1)}{2!}x^{n-2}h+\ldots+h^{n-1}}{h}\right]$

$=nx^{n-1}+0+0+\ldots+0=nx^{n-1}$ **(Ans.)**

গ এখানে, $I=\int\frac{x}{(x-5)f(x)}dx$

$=\int\frac{x}{(x-5)(x^2+1)}dx$

$\therefore \frac{x}{(x-5)(x^2+1)}\equiv\frac{A}{x-5}+\frac{Bx+C}{x^2+1}$

বা, $x\equiv A(x^2+1)+(Bx+C)(x-5)$ .... (i)

$x=5$ বসিয়ে, $5=26A+0 \quad \therefore A=\frac{5}{26}$

(i) হতে পাই, $x\equiv Ax^2+A+Bx^2+Cx-5Bx-5C$

উভয় পক্ষ হতে x^2 এবং x এর সহগ সমীকৃত করে পাই,

$0=A+B$

বা, $\frac{5}{26}+B=0$

$\therefore B=-\frac{5}{26}$

এবং $C-5B=1$ বা, $C+\frac{25}{26}=1 \ \therefore C=\frac{1}{26}$

$\therefore \frac{x}{(x-5)(x^2+1)}=\frac{\frac{5}{26}}{x-5}+\frac{-\frac{5}{26}x+\frac{1}{26}}{x^2+1}$

$=\frac{5}{26(x-5)}-\frac{5}{26}\cdot\frac{x}{x^2+1}+\frac{1}{26}\cdot\frac{1}{x^2+1}$

$\therefore \int\frac{x}{(x-5)(x^2+1)}dx=\int\frac{5}{26(x-1)}dx-\frac{5}{26}\int\frac{x}{x^2+1}dx$

$+\frac{1}{26}\int\frac{1}{x^2+1}dx$

$=\frac{5}{26}\ln|x-5|-\frac{5}{26}\cdot\frac{1}{2}\int\frac{2x}{x^2+1}dx+\frac{1}{26}\int\frac{1}{x^2+1}dx$

$=\frac{5}{26}\ln|x-5|-\frac{5}{52}\ln|x^2+1|+\frac{1}{26}\tan^{-1}x+c$

[যেখানে, c যোগজীকরণ ধ্রুবক]

$\therefore \int\frac{x}{(x-5)f(x)}dx$

$=\frac{5}{26}\ln|x-5|-\frac{5}{52}\ln|x^2+1|+\frac{1}{26}\tan^{-1}x+c$ **(Ans.)**

22. ক $\frac{d}{dq}\left(\frac{p}{q^2}+\frac{1}{pq}+\frac{q}{p^2}\right)$

$=\frac{d}{dq}\left(\frac{p}{q^2}\right)+\frac{d}{dq}\left(\frac{1}{pq}\right)+\frac{d}{dq}\left(\frac{q}{p^2}\right)$

$=p\frac{d}{dq}\left(\frac{1}{q^2}\right)+\frac{1}{p}\cdot\frac{d}{dq}\left(\frac{1}{q}\right)+\frac{1}{p^2}\cdot\frac{d}{dq}(q)$

$=p.\frac{-2}{q^3}+\frac{1}{p}\cdot\frac{-1}{q^2}+\frac{1}{p^2}$

$=\frac{1}{p^2}-\frac{1}{pq^2}-\frac{2p}{q^3}$ **(Ans.)**

খ $\int\frac{f(5x)+f(3x)}{f(x)+g(x)}dx$

$=\int\frac{e^{5x}+e^{3x}}{e^x+e^{-x}}dx$ [$\because f(x)=e^x$ ও $g(x)=e^{-x}$]

$$= \int \frac{e^{5x}+e^{3x}}{e^{-x}(e^{2x}+1)}dx = \int \frac{e^{6x}+e^{4x}}{e^{2x}+1}dx$$
$$= \int \frac{e^{4x}(1+e^{2x})}{(1+e^{2x})}dx = \int e^{4x}dx$$
$$= \frac{e^{4x}}{4} + c \textbf{ (Ans.)}$$

গ ধরি, $y = g(x) = e^{-x}$ এবং $y = f(x) = e^{x}$

বা, $\ln y = \ln e^{-x}$ বা, $\ln y = \ln e^{x}$

$\therefore x = -\ln y$ ……… (i) $\therefore x = \ln y$ ……… (ii)

(i) ও (ii) নং কে সমাধান করি।

$\ln y = -\ln y$

বা, $2\ln y = 0$

বা, $\ln y = 0 = \ln 1$

$\therefore y = 1$

y এর মান (ii) নং এ বসিয়ে পাই, $x = 0$

$\therefore$ বক্ররেখাদ্বয় পরস্পরকে (0, 1) বিন্দুতে ছেদ করে।

বক্ররেখাদ্বয় y-অক্ষের সাপেক্ষে প্রতিসম।

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল

$$= 2\int_1^2 x dy$$
$$= 2\int_1^2 \ln y \, dy$$
$$= 2\left[\ln y \int_1^2 dy - \int_1^2 \left\{\frac{d}{dy}(\ln y)\int dy\right\}dy\right]$$
$$= 2\left\{[y \ln y]_1^2 - \int_1^2 \frac{1}{y}.\, y dy\right\}$$
$$= 2\left\{(2\ln 2 - 1.\ln 1) - \int_1^2 dy\right\}$$
$$= 2\left\{(\ln 2^2 - 0) - [y]_1^2\right\}$$
$$= 2\{\ln 4 - (2-1)\}$$
$$= 2\ln 4 - 2$$
$$= \ln 16 - 2 \textbf{ (Ans.)}$$

23. **ক** $\frac{d}{dx}\{f(x)\} = \frac{d}{dx}$ (sinpx cosqx)

$$= \sin px \frac{d}{dx}(\cos qx) + \cos qx \frac{d}{dx}(\sin px)$$
$$= \sin px(-\sin qx.q) + \cos qx(\cos px.p)$$
$$= p\cos px \cos qx - q \sin px \sin qx$$

(Ans.)

খ $\int f(x)dx = \int \sin px \cos qx \, dx$

$$= \frac{1}{2}\int 2\sin px \cos qx \, dx$$
$$= \frac{1}{2}\int [\sin\{(p+q)x\} + \sin\{(p-q)x\}]dx$$
$$= \frac{1}{2}\left[\frac{-\cos(p+q)x}{p+q} + \frac{-\cos(p-q)x}{p-q}\right] + c$$
$$= \frac{1}{2}\left[\frac{\cos(p-q)x}{q-p} - \frac{\cos(p+q)x}{p+q}\right] + c$$
$$= g(x) + \text{ধ্রুবক}$$

যেখানে, $g(x) = \frac{1}{2}\left[\frac{\cos(p-q)x}{q-p} - \frac{\cos(p+q)x}{p+q}\right]$

(Ans.)

গ $\int_0^4 h(x)\sqrt{4-h(x)}\,dx$

$$= \int_0^4 x\sqrt{4-x}\,dx \; [\because h(x) = x]$$

ধরি, $4 - x = z^2$

বা, $x = 4 - z^2$

$\therefore dx = -2zdz$

$x = 0$ হলে, $z = 2$

$x = 4$ হলে, $z = 0$

$$\therefore \int_0^4 h(x)\sqrt{4-h(x)}\,dx = \int_2^0 (4-z^2).\,z.\,(-2zdz)$$
$$= -2\int_2^0 (4-z^2)z^2dz$$
$$= 2\int_0^2 (4z^2 - z^4)dz$$
$$= 2\left[\frac{4}{3}z^3 - \frac{1}{5}z^5\right]_0^2$$
$$= 2\left[\frac{4}{3}.8 - \frac{1}{5}.32 - 0 + 0\right]$$
$$= 2\left[\frac{32}{3} - \frac{32}{5}\right]$$
$$= 2\left[\frac{160-96}{15}\right]$$
$$= 2 \times \frac{64}{15} = \frac{128}{15} \text{ (দেখানো হলো)}$$

24. **ক** দেওয়া আছে, $f(x) = \tan^{-1}x$.

ধরি, $y = x^{f(x)}$

বা, $y = x^{\tan^{-1}x}$

$\therefore \ln y = \ln\left(x^{\tan^{-1}x}\right)$

বা, $\ln y = \tan^{-1}x \ln x$

x-এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{y}\frac{dy}{dx} = (\tan^{-1}x)\frac{1}{x} + (\ln x)\frac{1}{1+x^2}$$

বা, $\frac{dy}{dx} = y\left\{\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right\}$

$$= x^{\tan^{-1}x}\left(\frac{\tan^{-1}x}{x} + \frac{\ln x}{1+x^2}\right) \textbf{ (Ans.)}$$

খ দেওয়া আছে, $ln y = f(x)$

বা, $ln y = \tan^{-1}x$

বা, $y = e^{\tan^{-1}x}$ ... ... (i)

$\therefore \frac{dy}{dx} = e^{\tan^{-1}x} \cdot \frac{1}{1+x^2}$ [x-এর সাপেক্ষে অন্তরীকরণ করে]

বা, $(1+x^2)\frac{dy}{dx} = e^{\tan^{-1}x}$

বা, $(1+x^2)\frac{dy}{dx} = y$ [(i) নং এর সাহায্যে]

x-এর সাপেক্ষে পুনরায় অন্তরীকরণ করে পাই,

$(1+x^2)\frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 2x = \frac{dy}{dx}$

$\therefore (1+x^2)\frac{d^2y}{dx^2} + (2x-1)\frac{dy}{dx} = 0$ (দেখানো হলো)

গ $\int x f(x)\, dx = \int x\tan^{-1}x\, dx$

$= \tan^{-1}x \int x\, dx - \int\left\{\frac{d}{dx}(\tan^{-1}x)\int x\, dx\right\} dx$

$= \frac{1}{2}x^2 \tan^{-1}x - \frac{1}{2}\int\frac{x^2}{1+x^2}dx$

$= \frac{1}{2}x^2 \tan^{-1}x - \frac{1}{2}\int\frac{(1+x^2)-1}{1+x^2}dx$

$= \frac{1}{2}x^2 \tan^{-1}x - \frac{1}{2}\int dx + \frac{1}{2}\int\frac{dx}{1+x^2}$

$= \frac{1}{2}x^2 \tan^{-1}x - \frac{x}{2} + \frac{1}{2}\tan^{-1}x + c$

$= \frac{1}{2}(x^2+1)\tan^{-1}x - \frac{x}{2} + c$ **(Ans.)**

25. **ক** $\int \log_a x\, dx = \log_a x \int dx + \int\left\{\frac{d}{dx}(\log_a x)\int dx\right\}dx$

$= x\log_a x + \int\left(\frac{1}{x}\log_a e\right).x dx + c$

$= x\log_a x + x\log_a e + c$ **(Ans.)**

খ দেওয়া আছে, $y = \ln\{x + f(x)\}$.

$\therefore y = ln\left(x + \sqrt{a^2+x^2}\right)$

x এর সাপেক্ষে পর্যায়ক্রমিক অন্তরীকরণ করে পাই,

বা, $y_1 = \frac{1}{x+\sqrt{a^2+x^2}}\left(1 + \frac{1}{2\sqrt{a^2+x^2}}2x\right)$

বা, $y_1 = \frac{1}{x+\sqrt{a^2+x^2}} \times \frac{x+\sqrt{a^2+x^2}}{\sqrt{a^2+x^2}}$

বা, $\sqrt{a^2+x^2}\,.y_1 = 1$

বা, $(a^2+x^2)y_1^2 = 1$ [উভয়পক্ষকে বর্গ করে]

$\therefore (a^2+x^2)2y_1y_2 + 2xy_1^2 = 0$

$\therefore (a^2+x^2)\,y_2 + xy_1 = 0$ (প্রমাণিত)

গ $\int\frac{1}{f(x)}dx = \int\frac{dx}{\sqrt{a^2+x^2}}$ [$\because f(x) = \sqrt{a^2+x^2}$]

$= \int\frac{a\sec^2\theta\, d\theta}{a\sec\theta}$

$= \int\sec\theta\, d\theta$

$= \ln(\sec\theta + \tan\theta) + c$

$= \ln\left(\frac{1}{a}\sqrt{a^2+x^2} + \frac{x}{a}\right) + c$

$= \ln\left(\frac{x+\sqrt{a^2+x^2}}{a}\right) + c$

$= \ln\left(x+\sqrt{a^2+x^2}\right) - \ln a + c$

$= \ln\{x + f(x)\}$ + ধ্রুবক

$= g(x)$ + ধ্রুবক (প্রমাণিত)

ধরি, $x = a\tan\theta$

$\therefore dx = a\sec^2\theta\, d\theta$

$\therefore \tan\theta = \frac{x}{a}$

বা, $\tan^2\theta = \frac{x^2}{a^2}$

বা, $\sec^2\theta - 1 = \frac{x^2}{a^2}$

বা, $\sec\theta = \sqrt{\frac{x^2}{a^2}+1}$

$\therefore \sec\theta = \frac{1}{a}\sqrt{a^2+x^2}$

26. **ক** $\lim_{x\to 0}\frac{x}{\sqrt{x+9}-3}$

$= \lim_{x\to 0}\frac{x(\sqrt{x+9}+3)}{(\sqrt{x+9}-3)(\sqrt{x+9}+3)}$

$= \lim_{x\to 0}\frac{x(\sqrt{x+9}+3)}{x+9-9}$

$= \mathrm{Lim}_{x\to 0}\frac{x(\sqrt{x+9}+3)}{x}$

$= \lim_{x\to 0}\left(\sqrt{x+9}+3\right)$

$= \sqrt{0+9}+3$

$= 3+3 = 6$ **(Ans.)**

খ দেওয়া আছে, বৃত্তের সমীকরণ, $x^2+y^2-16=0$

এবং সরলরেখার সমীকরণ, $x+y-4=0$

$\therefore$ এদের সমষ্টি, $x^2+y^2-16+x+y-4=0$

$\therefore x^2+y^2+x+y-20=0$ ... ... (i)

যা বক্ররেখার সমীকরণ।

x এর সাপেক্ষে অন্তরজ করে পাই,

$2x + 2y\frac{dy}{dx} + 1 + \frac{dy}{dx} = 0$

বা, $\frac{dy}{dx}(2y+1) = -(1+2x)$

বা, $\frac{dy}{dx} = \frac{-(1+2x)}{1+2y}$ $\therefore \frac{dx}{dy} = -\frac{1+2y}{1+2x}$

যেহেতু স্পর্শক x-অক্ষের উপর লম্ব $\therefore \frac{dx}{dy} = 0$

বা, $-\frac{1+2y}{1+2x} = 0$

বা, $1+2y=0$ বা, $2y=-1$

$\therefore y = -\frac{1}{2}$

y এর মান (i)নং সমীকরণে বসিয়ে পাই,

$$x^2+\left(-\frac{1}{2}\right)^2+x+\left(-\frac{1}{2}\right)-20=0$$

বা, $x^2+x+\frac{1}{4}-\frac{1}{2}-20=0$

বা, $x^2+x+\frac{1-2-80}{4}=0$

বা, $x^2+x-\frac{81}{4}=0$

বা, $4x^2+4x-81=0$

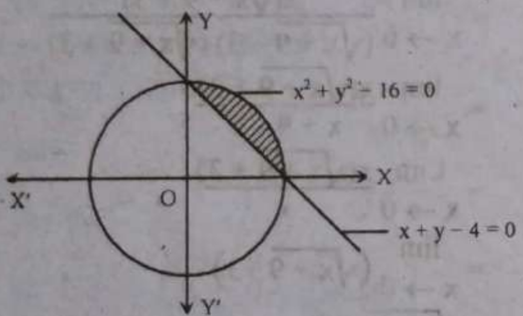
$$\therefore\ x=\frac{-4\pm\sqrt{16-4.4.(-81)}}{2.4}=\frac{-4\pm\sqrt{16+16.81}}{8}$$

$$=\frac{-4\pm4\sqrt{1+81}}{8}=\frac{-1\pm\sqrt{82}}{2}$$

∴ বিন্দুসমূহ $\left(\frac{-1+\sqrt{82}}{2},-\frac{1}{2}\right),\left(\frac{-1-\sqrt{82}}{2},-\frac{1}{2}\right)$

(Ans.)

গ



দেওয়া আছে, বৃত্তের সমীকরণ, $x^2+y^2-16=0$

$$\therefore\ x^2+y^2=16=4^2$$

∴ বৃত্তের কেন্দ্র (0, 0) ও বৃত্তের ব্যাসার্ধ, $r=4$

এবং সরলরেখার সমীকরণ $x+y-4=0$

বা, $x+y=4\ \therefore\ \frac{x}{4}+\frac{y}{4}=1$

∴ সরলরেখা দ্বারা উভয় অক্ষের ছেদাংশ = 4

এখন, $x^2+y^2=16$

বা, $y=\sqrt{16-x^2}$ [যেহেতু x অক্ষের উপরে y ধনাত্মক]

এবং $x+y=4\ \therefore\ y=4-x$

∴ ছায়াঘেরা অংশের ক্ষেত্রফল $=\int_0^4 (y_1-y_2)\,dx$

$$=\int_0^4\{\sqrt{16-x^2}-(4-x)\}\,dx$$

$$=\int_0^4\sqrt{16-x^2}\,dx-\int_0^4(4-x)\,dx$$

$$=I_1-I_2 \text{ (ধরি)}$$

এখন, $I_1=\int_0^4\sqrt{16-x^2}\,dx$

ধরি, $x=4\sin\theta$

$\therefore\ dx=4\cos\theta\,d\theta$

x	0	4
θ	0	$\frac{\pi}{2}$

$$=\int_0^{\frac{\pi}{2}}\sqrt{16-16\sin^2\theta}.4\cos\theta\,d\theta$$

$$=16\int_0^{\frac{\pi}{2}}\sqrt{1-\sin^2\theta}\cos\theta\,d\theta$$

$$=16\int_0^{\frac{\pi}{2}}\cos\theta.\cos\theta\,d\theta=8\int_0^{\frac{\pi}{2}}2\cos^2\theta\,d\theta$$

$$=8\int_0^{\frac{\pi}{2}}(1+\cos2\theta)\,d\theta=8\left[\theta+\frac{1}{2}\sin2\theta\right]_0^{\frac{\pi}{2}}$$

$$=8\left\{\left(\frac{\pi}{2}+\frac{1}{2}.0\right)-0\right\}=4\pi$$

এবং $I_2=\int_0^4(4-x)\,dx=\left[4x-\frac{x^2}{2}\right]_0^4$

$$=(16-8)-0=8$$

∴ ছায়াঘেরা অংশের ক্ষেত্রফল $=(4\pi-8)$ বর্গ একক।

(Ans.)

27. ক এখানে, $I=\int\frac{dx}{\sqrt{5-4x^2}}$

$$=\int\frac{dx}{\sqrt{4\left(\frac{5}{4}-x^2\right)}}$$

$$=\frac{1}{2}\int\frac{dx}{\sqrt{\left(\frac{\sqrt{5}}{2}\right)^2-x^2}}$$

$$=\frac{1}{2}.\sin^{-1}\frac{x}{\frac{\sqrt{5}}{2}}+c$$

[যেখানে, c একটি যোগজীকরণ ধ্রুবক]

$$=\frac{1}{2}\sin^{-1}\frac{2x}{\sqrt{5}}+c\ \textbf{(Ans.)}$$

খ দেওয়া আছে, $f(x)=\frac{2}{3}x^3+2x^2-2x$

$$\therefore\ f(\sin\alpha)=\frac{2}{3}(\sin\alpha)^3+2.\sin^2\alpha-2\sin\alpha$$

বা, $f(\sin\alpha)=\frac{2}{3}\sin^3\alpha+2\sin^2\alpha-2\sin\alpha$

$$\therefore\ \int_0^{\pi/2}f(\sin\alpha)\,d\alpha$$

$$=\int_0^{\pi/2}\left(\frac{2}{3}\sin^3\alpha+2\sin^2\alpha-2\sin\alpha\right)d\alpha$$

$$=\int_0^{\pi/2}\left[\frac{2}{3}\cdot\frac{1}{4}[3\sin\alpha-\sin3\alpha]+(1-\cos2\alpha)-2\sin\alpha\right]d\alpha$$

$$=\int_0^{\pi/2}\left[\frac{1}{6}[3\sin\alpha-\sin3\alpha]+1-\cos2\alpha-2\sin\alpha\right]d\alpha$$

$$=\int_0^{\pi/2}\left[\frac{1}{2}\sin\alpha-\frac{1}{6}\sin3\alpha+1-\cos2\alpha-2\sin\alpha\right]d\alpha$$

$$=\left[\frac{1}{2}(-\cos\alpha)-\frac{1}{6}\left(-\frac{\cos3\alpha}{3}\right)+\alpha-\frac{\sin2\alpha}{2}+2\cos\alpha\right]_0^{\pi/2}$$

$$=\left[-\frac{1}{2}\cos\alpha+\frac{1}{18}\cos3\alpha+\alpha-\frac{\sin2\alpha}{2}+2\cos\alpha\right]_0^{\pi/2}$$

$$=\left[\left(-\frac{1}{2}\cos\frac{\pi}{2}+\frac{1}{18}\cos\frac{3\pi}{2}+\frac{\pi}{2}-\frac{1}{2}\sin 2.\frac{\pi}{2}+2\cos\frac{\pi}{2}\right)-\left(-\frac{1}{2}\cos 0+\frac{1}{18}\cos 3.0+0-0+2\cos 0\right)\right]$$

$$=0+\frac{1}{18}.0+\frac{\pi}{2}-\frac{1}{2}.0+2.0-\left(-\frac{1}{2}.1+\frac{1}{18}.1+2.1\right)$$

$$=\frac{\pi}{2}+\frac{1}{2}-\frac{1}{18}-2=\frac{\pi}{2}-\frac{14}{9}$$ **(Ans.)**

গ প্রদত্ত ফাংশন, $f(x)=\frac{2}{3}x^3+2x^2-2x$

$$f'(x)=2x^2+4x-2$$

সর্বোচ্চ ও সর্বনিম্ন মানের জন্য, $f'(x)=0$

$\therefore\ 2x^2+4x-2=0$

বা, $x^2+2x-1=0$

বা, $x^2+2x+(-1)=0$

$$\therefore\ x=\frac{-2\pm\sqrt{(2)^2-4.1.(-1)}}{2.1}=\frac{-2\pm\sqrt{4+4}}{2}=\frac{-2\pm 2\sqrt{2}}{2}=-1\pm\sqrt{2}$$

$\therefore\ x=-1-\sqrt{2}$ এবং $-1+\sqrt{2}$

এখানে, স্পষ্টতই: $-1-\sqrt{2}\notin[0,5]$

কিন্তু, $-1+\sqrt{2}\in[0,5]$

$f''(x)=4x+4$

$\therefore\ x=-1+\sqrt{2}$ এর জন্য

$$f''(-1+\sqrt{2})=4(-1+\sqrt{2})+4=4\sqrt{2}>0$$

যেহেতু $x=-1+\sqrt{2}$ এর জন্য $f''(x)>0$, সেহেতু উক্ত বিন্দুতে প্রদত্ত ফাংশনের সর্বনিম্ন মান বিদ্যমান।

$\therefore$ সর্বনিম্ন মান $=\frac{2}{3}(-1+\sqrt{2})^3+2(-1+\sqrt{2})^2-2(-1+\sqrt{2})$ **(Ans.)**

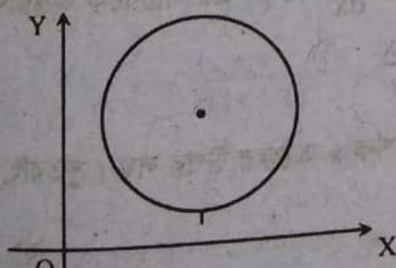
28. ক মনে করি, $y=\ln(e^p+e^{-p})$

p এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{dy}{dp}=\frac{d}{dp}\{\ln(e^p+e^{-p})\}=\frac{1}{e^p+e^{-p}}\frac{d}{dp}(e^p+e^{-p})$$

$$=\frac{1}{e^p+e^{-p}}(e^p-e^{-p})=\frac{e^p-e^{-p}}{e^p+e^{-p}}$$ **(Ans.)**

খ



চিত্রের বৃত্তের কেন্দ্র (a, b) এবং ব্যাসার্ধ = c

$\therefore$ বৃত্তের সমীকরণ $(x-a)^2+(y-b)^2=c^2$

$\therefore\ x^2-2ax+a^2+y^2-2by+b^2=c^2$ ……… (i)

x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$2x-2a+0+2y\frac{dy}{dx}-2b\frac{dy}{dx}+0=0$$

বা, $\frac{dy}{dx}(2y-2b)=2a-2x\ \therefore\ \frac{dy}{dx}=\frac{2a-2x}{2y-2b}$

x-অক্ষের সমান্তরাল হলে, $\frac{dy}{dx}=0\ \therefore\ \frac{2a-2x}{2y-2b}=0$

বা, $2a-2x=0\ \therefore\ x=a$

x এর মান (i)নং এ বসিয়ে পাই,

$a^2-2a^2+a^2+y^2-2by+b^2=c^2$

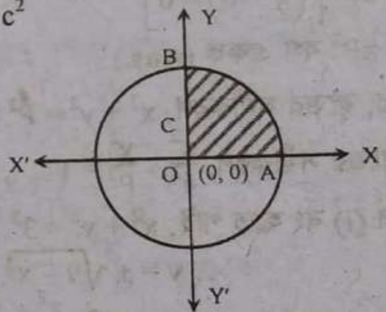
বা, $y^2-2by+b^2-c^2=0$

$$\therefore\ y=\frac{-(-2b)\pm\sqrt{(-2b)^2-4.1.(b^2-c^2)}}{2.1}=\frac{2b\pm\sqrt{4b^2-4b^2+4c^2}}{2}=\frac{2b\pm\sqrt{4c^2}}{2}=\frac{2b\pm 2c}{2}=b\pm c$$

$\therefore$ বিন্দুসমূহ (a, b + c) এবং (a, b − c) **(Ans.)**

গ a = b = 0 হলে, বৃত্তের সমীকরণ হবে

$x^2+y^2=c^2$



$x^2+y^2=c^2$ বৃত্তের কেন্দ্রের স্থানাঙ্ক (0, 0) এবং ব্যাসার্ধ = c.

এখানে, $x^2+y^2=c^2$

$\therefore\ y=\pm\sqrt{c^2-x^2}$, x-অক্ষের উপরের অংশে y ধনাত্মক অর্থাৎ $y=\sqrt{c^2-x^2}$

$\therefore$ বৃত্তের ক্ষেত্রফল = 4 × OABO ক্ষেত্রের ক্ষেত্রফল

$=4\int_0^c y\,dx$

$=4\int_0^c\sqrt{c^2-x^2}\,dx$

ধরি, $x=c\sin\theta$

$\therefore\ dx=c\cos\theta\,d\theta$,

যখন $x=0$, তখন $\theta=0$

এবং যখন $x=c$, তখন $\theta=\frac{\pi}{2}$

$=4\int_0^{\pi/2}\sqrt{c^2-c^2\sin^2\theta}\,.c\cos\theta\,d\theta$

$=2c^2\int_0^{\pi/2}2\cos^2\theta\,d\theta$

$=2c^2\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta$

$=2c^2\left[\theta+\frac{1}{2}\sin 2\theta\right]_0^{\pi/2}$

$=2c^2\left(\frac{\pi}{2}-0\right)=\pi c^2$ বর্গ একক। **(Ans.)**

29. ক $\frac{d}{da}(7^a) = 7^a \ln 7$

$\therefore$ a = e বিন্দুতে, $\frac{d}{da}(7^a) = 7^e \ln 7$ **(Ans.)**

খ চিত্রের বৃত্তটির কেন্দ্র (0, 0) এবং ব্যাসার্ধ P.

$\therefore$ বৃত্তটির সমীকরণ, $x^2 + y^2 = P^2$

বা, $y^2 = P^2 - x^2$

$\therefore y = \pm\sqrt{P^2 - x^2}$

$\therefore$ বৃত্তটির ক্ষেত্রফল $= 4\int_0^P ydx$

$= 4\int_0^P \sqrt{P^2 - x^2}\,dx$

$= 4\int_0^{\frac{\pi}{2}} P\cos\theta.P\cos\theta d\theta$

$= 2P^2\int_0^{\frac{\pi}{2}} 2\cos^2\theta d\theta$

ধরি, $x = P\sin\theta$

$\therefore dx = P\cos\theta d\theta$

$x = 0$ হলে, $\theta = 0$

$x = P$ হলে, $\theta = \frac{\pi}{2}$

$= 2P^2.\int_0^{\frac{\pi}{2}}(1 + \cos2\theta)d\theta$

$= 2P^2\left[\theta + \frac{\sin2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 2P^2\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$

$= \pi P^2$ বর্গ একক **(Ans.)**

গ চিত্রের, বৃত্তের সমীকরণ, $x^2 + y^2 = P^2$ ... ... (i)

এবং উপবৃত্তের সমীকরণ, $\frac{x^2}{4^2} + \frac{y^2}{P^2} = 1$ ... ... (ii)

P = 3 হলে (i) নং হতে পাই, $x^2 + y^2 = 3^2$

$\therefore y = \pm\sqrt{9 - x^2}$ ... ... (iii)

আবার, P = 3 হলে (ii) নং হতে পাই, $\frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$

বা, $\frac{y^2}{3^2} = 1 - \frac{x^2}{4^2}$

বা, $y^2 = \frac{3^2}{4^2}(4^2 - x^2)$

$\therefore y = \frac{3}{4}\sqrt{4^2 - x^2}$ ... ... (iv)

y = 0 হলে (iii) নং হতে পাই, $x = \pm 3$

এবং y = 0 হলে (iv) নং হতে পাই, $x = \pm 4$

ছায়াঘেরা অংশের ক্ষেত্রফল = উপবৃত্তের ক্ষেত্রফল – বৃত্তের ক্ষেত্রফল

$\therefore$ বৃত্তের ক্ষেত্রফল $= 4\int_0^3 \sqrt{9 - x^2}\,dx$

$= 4\int_0^{\frac{\pi}{2}} 3\cos\theta.3\cos\theta d\theta$

$= 18\int_0^{\frac{\pi}{2}} 2\cos^2\theta d\theta$

ধরি, $x = 3\sin\theta$

$\therefore dx = 3\cos\theta d\theta$

$x = 0$ হলে, $\theta = 0$

$x = 3$ হলে, $\theta = \frac{\pi}{2}$

$= 18\int_0^{\frac{\pi}{2}}(1 + \cos2\theta)d\theta$

$= 18\left[\theta + \frac{\sin2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 18\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$

$= 9\pi$ বর্গ একক

আবার, উপবৃত্তের ক্ষেত্রফল $= 4\int_0^4 \frac{3}{4}\sqrt{4^2 - x^2}dx$

$= 3\int_0^{\frac{\pi}{2}} 4\cos\theta.4\cos\theta d\theta$

$= 24\int_0^{\frac{\pi}{2}}(1 + \cos2\theta)d\theta$

ধরি, $x = 4\sin\theta$

$\therefore dx = 4\cos\theta d\theta$

$x = 0$ হলে, $\theta = 0$

$x = 4$ হলে, $\theta = \frac{\pi}{2}$

$= 24\left[\theta + \frac{\sin2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 24\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$

$= 12\pi$ বর্গ একক

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল $= (12\pi - 9\pi)$ বর্গ একক

$= 3\pi$ বর্গ একক **(Ans.)**

30. ক দেওয়া আছে, $y = \frac{1}{2}(e^x + e^{-x})$

বা, $\frac{d}{dx}(y) = \frac{d}{dx}\left\{\frac{1}{2}(e^x + e^{-x})\right\}$

বা, $y_1 = \frac{1}{2}(e^x - e^{-x})$

বা, $y_1^2 = \frac{1}{4}(e^x - e^{-x})^2$ [বর্গ করে]

বা, $y_1^2 = \frac{1}{4}\{(e^x + e^{-x})^2 - 4.e^x.e^{-x}\}$

বা, $y_1^2 = \frac{1}{4}(e^x + e^{-x})^2 - 1$

$\therefore$ $y_1^2 = y^2 - 1$ **(দেখানো হলো)**

খ দেওয়া আছে, $x^2 + y^2 - 16 = 0$ ... ... (i)

বা, $2x + 2y\frac{dy}{dx} = 0$ [x এর সাপেক্ষে অন্তরীকরণ করে]

$\therefore \frac{dy}{dx} = \frac{-x}{y}$

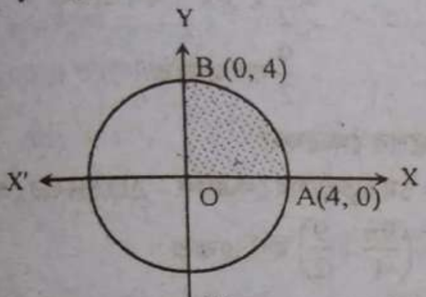
যেহেতু স্পর্শক x-অক্ষের উপর লম্ব। সুতরাং, $\frac{dx}{dy} = 0$

বা, $\frac{y}{-x} = 0$

$\therefore y = 0$

y এর মান (i) নং এ বসিয়ে,
$x^2 + 0^2 - 16 = 0$
বা, $x^2 = 16$
$\therefore x = \pm 4$
$\therefore$ স্পর্শবিন্দু $(\pm 4, 0)$ **(Ans.)**

গ দেওয়া আছে, $x^2 + y^2 - 16 = 0$
বা, $y^2 = 16 - x^2$
$\therefore y = \pm\sqrt{16 - x^2}$
বৃত্তের কেন্দ্র (0, 0) এবং ব্যাসার্ধ = 4
x অক্ষের উপরের অংশে y ধনাত্মক।
$\therefore y = \sqrt{16 - x^2}$



নির্ণেয় ক্ষেত্রফল = 4 × OABO ক্ষেত্রের ক্ষেত্রফল

$= 4\int_0^4 y\,dx$

$= 4\int_0^4 \sqrt{16 - x^2}\,dx$

$= 4\int_0^{\frac{\pi}{2}} \sqrt{16 - 16\sin^2\theta}\,.4\cos\theta\,d\theta$

ধরি, $x = 4\sin\theta$
$\therefore dx = 4\cos\theta\,d\theta$

লিমিট:

x	0	4
θ	0	$\frac{\pi}{2}$

$= 64\int_0^{\frac{\pi}{2}} \sqrt{\cos^2\theta}\,.\cos\theta\,d\theta = 32\int_0^{\frac{\pi}{2}} 2\cos^2\theta\,d\theta$

$= 32\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)\,d\theta = 32\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= 32\left[\frac{\pi}{2} + 0 - 0\right] = 16\pi$ বর্গ একক **(Ans.)**

31. **ক** $\frac{d}{dx}\left(\frac{4x}{3x+1}\right) = \frac{(3x+1)\frac{d}{dx}(4x) - 4x\frac{d}{dx}(3x+1)}{(3x+1)^2}$

$= \frac{(3x+1).4 - 4x.3}{(3x+1)^2}$

$= \frac{12x + 4 - 12x}{(3x+1)^2}$

$= \frac{4}{(3x+1)^2}$ **(Ans.)**

খ $f(x)$ এর অন্তরক সহগ $= \frac{d}{dx}[f(x)]$

$= \frac{d}{dx}\left\{\frac{4x}{(3x+1)(x+1)^2}\right\}$

$= \frac{(3x+1)(x+1)^2\frac{d}{dx}(4x) - 4x\frac{d}{dx}\{(3x+1)(x+1)^2\}}{\{(3x+1)(x+1)^2\}^2}$

$= \frac{4(3x+1)(x+1)^2 - 4x\left\{(3x+1)\frac{d}{dx}(x+1)^2 + (x+1)^2\frac{d}{dx}(3x+1)\right\}}{\{(3x+1)(x+1)^2\}^2}$

$= \frac{4(3x+1)(x+1)^2 - 4x\{2(3x+1)(x+1) + 3(x+1)^2\}}{\{(3x+1)(x+1)^2\}^2}$ **(Ans.)**

গ ধরি, $\frac{4x}{(3x+1)(x+1)^2} \equiv \frac{A}{3x+1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ ... (i)

বা, $4x \equiv A(x+1)^2 + B(3x+1)(x+1) + C(3x+1)$ ... (ii)

$\therefore x = -1$ হলে (ii) নং হতে, $4(-1) = 0 + 0 + C(-3 + 1)$
বা, $-4 = -2C \therefore C = 2$

$x = -\frac{1}{3}$ হলে, (ii) নং হতে,

$4\left(-\frac{1}{3}\right) = A\left(-\frac{1}{3} + 1\right)^2 + 0 + 0$

বা, $-\frac{4}{3} = A \times \left(\frac{2}{3}\right)^2$

$\therefore A = -3$

(ii) নং হতে x^2 এর সহগ সমীকৃত করে পাই,

$0 = A + 3B$ বা, $B = -\frac{A}{3} = -\frac{(-3)}{3} = 1$

(i) নং হতে, $\frac{4x}{(3x+1)(x+1)^2} = \frac{-3}{3x+1} + \frac{1}{x+1} + \frac{2}{(x+1)^2}$

$\int_0^1 f(x)\,dx = \int_0^1 \frac{4x}{(3x+1)(x+1)^2}\,dx$

$= \int_0^1 \left\{\frac{-3}{3x+1} + \frac{1}{x+1} + \frac{2}{(x+1)^2}\right\}dx$

$= \left[-3.\ln|3x+1|.\frac{1}{3} + \ln|x+1| + 2.\frac{(x+1)^{-1}}{(-1)}\right]_0^1$

$= \left[\ln\left|\frac{x+1}{3x+1}\right| - \frac{2}{x+1}\right]_0^1$

$= \ln\left|\frac{1+1}{3.1+1}\right| - \frac{2}{1+1} - \ln\left|\frac{0+1}{0+1}\right| + \frac{2}{0+1}$

$= \ln\frac{1}{2} - 1 - \ln 1 + 2$

$= \ln 2^{-1} + 1$

$= 1 - \ln 2$ [$\because \ln 1 = 0$]

$\therefore \int_0^1 f(x)\,dx = 1 - \ln 2$ **(দেখানো হলো)**

32. **ক** ব্যাসার্ধ, r = 4 এবং কেন্দ্রে উৎপন্ন কোণ,

$\theta = 50° = 50 \times \frac{\pi}{180}$ রেডিয়ান $= \frac{5\pi}{18}$ রেডিয়ান

$\therefore$ বৃত্তকলার ক্ষেত্রফল $= \frac{1}{2} r^2\theta$ বর্গ একক

$= \frac{1}{2} \times 4^2 \times \frac{5\pi}{18}$ বর্গ একক

$= \frac{20\pi}{9}$ বর্গ একক **(Ans.)**

ঘ প্রদত্ত বক্ররেখা, $x^2 + y^2 = 9$ .... (i)

বা, $2x + 2y\frac{dy}{dx} = 0$

বা, $\frac{dy}{dx} = -\frac{x}{y}$ ...... (ii)

$\therefore \frac{dx}{dy} = -\frac{y}{x}$

স্পর্শক x অক্ষের উপর লম্ব হলে, $\frac{dx}{dy} = 0$

বা, $-\frac{y}{x} = 0$

$\therefore y = 0$

y এর মান (i) নং এ বসিয়ে পাই, $x^2 = 9 \therefore x = \pm 3$

$\therefore$ বিন্দুগুলো হলো, $(\pm 3, 0)$

$\therefore (\pm 3, 0)$ বিন্দুতে, $\frac{dx}{dy} = 0$

$\therefore (\pm 3, 0)$ বিন্দুতে স্পর্শকের সমীকরণ,

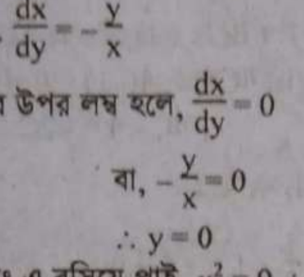
$x \pm 3 = 0\,(y - 0)$

$\therefore x \pm 3 = 0$ **(Ans.)**

গ প্রদত্ত বৃত্তের সমীকরণ, $x^2 + y^2 = 9$

$\therefore y = \pm\sqrt{9 - x^2}$

$y = 0$ হলে পাই, $x = \pm 3$



$\therefore$ ১ম চতুর্ভাগে বক্ররেখা দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$= \int_0^3 y\,dx$

$= \int_0^3 \sqrt{9 - x^2}\,dx$

$= \int_0^{\frac{\pi}{2}} 3\cos\theta \,.\, 3\cos\theta\, d\theta$

$= \frac{9}{2}\int_0^{\frac{\pi}{2}} 2\cos^2\theta\, d\theta$

$= \frac{9}{2}\int_0^{\frac{\pi}{2}} (1 + \cos 2\theta)\, d\theta$

ধরি, $x = 3\sin\theta$
$\therefore dx = 3\cos\theta d\theta$
$x = 0$ হলে, $\theta = 0$
$x = 3$ হলে, $\theta = \frac{\pi}{2}$

$= \frac{9}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= \frac{9}{2}\left[\left(\frac{\pi}{2} + 0\right) - 0\right]$

$= \frac{9\pi}{4}$ বর্গ একক

আবার, প্রদত্ত বক্ররেখা, $x + y = 3 \therefore \frac{x}{3} + \frac{y}{3} = 0$

$\therefore OA = 3$ ও $OB = 3$

$\therefore \Delta OAB$ এর ক্ষেত্রফল $= \frac{1}{2} \times OA \times OB$ বর্গ একক

$= \frac{1}{2} \times 3 \times 3$ বর্গ একক

$= \frac{9}{2}$ বর্গ একক

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল

= ১ম চতুর্ভাগের ক্ষেত্রফল – ΔOAB এর ক্ষেত্রফল

$= \left(\frac{9\pi}{4} - \frac{9}{2}\right)$ বর্গ একক

$= \frac{9}{4}(\pi - 2)$ বর্গ একক **(Ans.)**

33. **ক** পৃথিবীর কেন্দ্রে উৎপন্ন কোণ, $\theta = 1.5°$

$= 1.5 \times \frac{\pi}{180}$ রেডিয়ান

$= \frac{\pi}{120}$ রেডিয়ান

পৃথিবীর ব্যাসার্ধ, $R = 6440$ কি. মি.

$\therefore$ মধ্যবর্তী দূরত্ব, $s = r\theta = \left(6440 \times \frac{\pi}{120}\right)$ কি. মি.

$= 168.6$ কি. মি. (প্রায়) **(Ans.)**

খ দেওয়া আছে, $f(x, y) = x^2 + y^2$ এবং $g(x, y) = \ln(x + y)$

$f(x, y) = g(x, y)$ হলে পাই,

$x^2 + y^2 = \ln(x + y)$

বা, $2x + 2y\frac{dy}{dx} = \frac{1}{x + y}\left(1 + \frac{dy}{dx}\right)$

[x এর সাপেক্ষে অন্তরীকরণ করে]

বা, $2x + 2y\frac{dy}{dx} = \frac{1}{x + y} + \frac{1}{x + y}\cdot\frac{dy}{dx}$

বা, $\left(2y - \frac{1}{x + y}\right)\frac{dy}{dx} = \frac{1}{x + y} - 2x$

বা, $\left(\frac{2xy + 2y^2 - 1}{x + y}\right)\frac{dy}{dx} = \frac{1 - 2x^2 - 2xy}{x + y}$

$\therefore \frac{dy}{dx} = \frac{1 - 2x^2 - 2xy}{2xy + 2y^2 - 1}$

$\therefore (\sqrt{2}, \sqrt{2})$ বিন্দুতে, $\frac{dy}{dx} = \frac{1 - 2.(\sqrt{2})^2 - 2.\sqrt{2}.\sqrt{2}}{2.\sqrt{2}.\sqrt{2} + 2(\sqrt{2})^2 - 1}$

$= \frac{1 - 4 - 4}{4 + 4 - 1} = \frac{-7}{7} = -1$

$\therefore$ অভিলম্বের সমীকরণ, $(x - \sqrt{2}) - 1(y - \sqrt{2}) = 0$

$\therefore x - y = 0$ **(Ans.)**

গ দেওয়া আছে,

$f(x, y) = 9$

বা, $x^2 + y^2 = 9$

বা, $x^2 = 9 - y^2$

$\therefore x = \pm\sqrt{9 - y^2}$

$x = 0$ হলে পাই, $y = \pm 3$

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল

$= 4\int_2^3 x dy$

$= 4\int_2^3 \sqrt{9 - y^2}\, dy$

$= 4\int_{\sin^{-1}\frac{2}{3}}^{\frac{\pi}{2}} 3\cos\theta \,.\, 3\cos\theta d\theta$

ধরি, $y = 3\sin\theta$

$\therefore dy = 3\cos\theta d\theta$

$y = 2$ হলে, $\theta = \sin^{-1}\frac{2}{3}$

$y = 3$ হলে, $\theta = \frac{\pi}{2}$

$= 18\int_{\sin^{-1}\frac{2}{3}}^{\frac{\pi}{2}} 2\cos^2\theta d\theta$

$= 18\int_{\sin^{-1}\frac{2}{3}}^{\frac{\pi}{2}} (1 + \cos 2\theta) d\theta$

$= 18\left[\theta + \frac{\sin 2\theta}{2}\right]_{\sin^{-1}\frac{2}{3}}^{\frac{\pi}{2}}$

$= 18\left[\frac{\pi}{2} + 0 - \sin^{-1}\frac{2}{3} - \frac{1}{2}\sin\left(2\sin^{-1}\frac{2}{3}\right)\right]$

$= 9\pi - 18\sin^{-1}\frac{2}{3} - 9\sin\left(2\sin^{-1}\frac{2}{3}\right)$ বর্গএকক। **(Ans.)**

34. **ক** দেওয়া আছে, $P(x) = \sin 2x$ ও $R(x) = \cos 2x$

$\therefore \lim_{x \to \frac{\pi}{4}} \frac{1 - P(x)}{R(x)} = \lim_{x \to \frac{\pi}{4}} \frac{1 - \sin 2x}{\cos 2x}$

$= \lim_{x \to \frac{\pi}{4}} \frac{(1 - \sin 2x)(1 + \sin 2x)}{\cos 2x (1 + \sin 2x)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \sin^2 2x}{\cos 2x (1 + \sin 2x)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{\cos^2 2x}{\cos 2x (1 + \sin 2x)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{\cos 2x}{1 + \sin 2x}$

$= \frac{\cos\frac{\pi}{2}}{1 + \sin\frac{\pi}{2}} = \frac{0}{1 + 1} = 0$ **(Ans.)**

খ ধরি, $I = \int e^x P(x) dx = \int e^x \sin 2x\, dx$

$= e^x \int \sin 2x\, dx - \int \{\frac{d}{dx}(e^x) \int \sin 2x\, dx\} dx$

$= \frac{e^x(-\cos 2x)}{2} - \int \frac{\{e^x(-\cos 2x)\}}{2} dx$

$= -\frac{1}{2} e^x \cos 2x + \frac{1}{2} \int e^x \cos 2x\, dx$

$= -\frac{1}{2} e^x \cos 2x + \frac{1}{2} [e^x . \frac{\sin 2x}{2} - \frac{1}{2} \int e^x \sin 2x\, dx]$

$\therefore I = -\frac{1}{2} e^x \cos 2x + \frac{1}{4} e^x \sin 2x - \frac{1}{4} I$

বা, $I + \frac{1}{4} I = \frac{1}{4} e^x (\sin 2x - 2\cos 2x)$

বা, $I = \frac{4}{5} . \frac{1}{4} . e^x (\sin 2x - 2\cos 2x)$

$= \frac{1}{5} e^x (\sin 2x - 2\cos 2x) + c$

$= \frac{1}{5} . e^x . \cos 2x \left(\frac{\sin 2x}{\cos 2x} - 2\right) + c$

$= \frac{1}{5} e^x R(x) (\tan 2x - 2) + c$

$\therefore \int e^x P(x)\, dx = \frac{1}{5} e^x R(x) (\tan 2x - 2) + c$

(দেখানো হলো)

গ দেওয়া আছে, $P(x) - R(x) = 0; 0 \le x \le \frac{9\pi}{8}$

বা, $\sin 2x - \cos 2x = 0$

$\therefore \sin 2x = \cos 2x$

সমাধানের জন্য ধরি, $y = \sin 2x$ .... (i)

এবং $y = \cos 2x$ .... (ii)

x এর বিভিন্ন মানের জন্য y বা sin 2x এর মান পাওয়া যায়। $x = 0$ থেকে $x = \frac{9\pi}{8}$ সীমার মধ্যে x এর কতিপয় মানের জন্য $y = \sin 2x$ এর আনুষঙ্গিক মান তালিকাবদ্ধ করি।

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{9\pi}{8}$
$y = \sin 2x$	0	1	0	-1	0	0.707

উপরোক্ত মানসমূহ ছক কাগজে স্থাপন করে (i) নং এর লেখচিত্র অংকন করি।

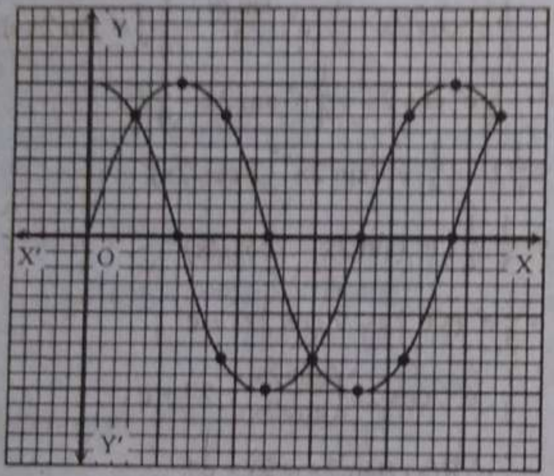
Scanned with CamScanner

আবার, x এর বিভিন্ন মানের জন্য y বা cos 2x এর মান পাওয়া যায়। $x = 0$ থেকে $x = \frac{9\pi}{8}$ সীমার মধ্যে x এর কতিপয় মানের জন্য $y = \cos 2x$ এর আনুষঙ্গিক মান তালিকাবদ্ধ করি।

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{9\pi}{8}$
$y = \cos 2x$	1	0	−1	0	1	0.707

উপরোক্ত মানসমূহ একই ছক কাগজে স্থাপন করে (ii) নং এর লেখচিত্র অংকন করি।

স্কেল: x অক্ষ বরাবর ছোট বর্গের 3 বাহু $= \frac{\pi}{8}$ একক এবং y অক্ষ বরাবর ছোট বর্গের 10 বাহু = 1 একক ধরে লেখচিত্র অংকন করা হলো।



লেখচিত্রে দেখা যায় $0 \le x \le \frac{9\pi}{8}$ ব্যবধিতে উভয় লেখ $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}$ ব্যবধিতে পরস্পরকে ছেদ করে।

$\therefore$ নির্ণেয় সমাধান, $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}$

35. **ক** দেওয়া আছে, $\frac{1}{M(A)} + \frac{1}{M(B)} + \frac{1}{M(C)} = 0$

বা, $\frac{1}{\cot A} + \frac{1}{\cot B} + \frac{1}{\cot C} = 0 \quad [\because M(x) = \cot x]$

বা, $\frac{\cot B \cot C + \cot A \cot C + \cot A \cot B}{\cot A \cot B \cot C} = 0$

$\therefore \cot A \cot B + \cot B \cot C + \cot A \cot C = 0 \ldots\ldots\ldots (i)$

এখন, $\{\Sigma M(A)\}^2 = (\Sigma \cot A)^2 \quad [\because M(x) = \cot x]$

$= (\cot A + \cot B + \cot C)^2$

$= \cot^2 A + \cot^2 B + \cot^2 C + 2(\cot A \cot B + \cot B \cot C + \cot A \cot C)$

$= \Sigma \cot^2 A + 2.0 \quad$ [(i) নং দ্বারা]

$= \Sigma\{M(A)\}^2$ (দেখানো হলো)

খ দেওয়া আছে, $y = M\left(a \tan^{-1}\frac{1}{x}\right)$

বা, $y = \cot\left(a \tan^{-1}\frac{1}{x}\right) \quad [\because M(x) = \cot x]$

$\therefore y = \cot(a \cot^{-1} x) \ldots\ldots\ldots (i)$

x এর সাপেক্ষে পর্যায়ক্রমিক অন্তরীকরণ করে পাই,

$y_1 = -\operatorname{cosec}^2(a \cot^{-1} x) \cdot \frac{-a}{1+x^2}$

বা, $y_1(1+x^2) = a \operatorname{cosec}^2(a \cot^{-1} x)$

বা, $(1+x^2) y_1 = a\{1 + \cot^2(a \cot^{-1} x)\}$

বা, $(1+x^2) y_1 = a(1+y^2) \quad$ [(i) নং দ্বারা]

বা, $(1+x^2) y_2 + 2xy_1 = a.2y\, y_1$

বা, $(1+x^2) y_2 + 2xy_1 - 2ay\, y_1 = 0$

$\therefore (1+x^2) y_2 + 2(x - ay) y_1 = 0$ (প্রমাণিত)

গ ধরি, $I = \int_{\frac{1}{\sqrt{3}}}^{1} x M\{\cot^{-1}(\cot^{-1} x)\}\, dx$

$= \int_{\frac{1}{\sqrt{3}}}^{1} x \cot\{\cot^{-1}(\cot^{-1} x)\}\, dx$

$= \int_{\frac{1}{\sqrt{3}}}^{1} x \cot^{-1} x\, dx$

$\therefore \int x\cot^{-1} x dx = \cot^{-1} x \int x\, dx - \int\left\{\frac{d}{dx}(\cot^{-1} x)\int x\, dx\right\} dx$

$= \frac{x^2}{2}\cot^{-1} x - \int \frac{-1}{1+x^2} \cdot \frac{x^2}{2}\, dx$

$= \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}\int \frac{1+x^2-1}{1+x^2}\, dx$

$= \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}\int\left(1 - \frac{1}{1+x^2}\right) dx$

$= \frac{x^2}{2}\cot^{-1} x + \frac{1}{2}x - \frac{1}{2}\tan^{-1} x + c$

$\therefore I = \int_{\frac{1}{\sqrt{3}}}^{1} x \cot^{-1} x\, dx$

$= \left[\frac{x^2}{2}\cot^{-1} x + \frac{1}{2}x - \frac{1}{2}\tan^{-1} x\right]_{\frac{1}{\sqrt{3}}}^{1}$

$= \left(\frac{1}{2}\cot^{-1} 1 + \frac{1}{2} - \frac{1}{2}\tan^{-1} 1\right) - \left(\frac{1}{6}\cot^{-1}\frac{1}{\sqrt{3}} + \frac{1}{2\sqrt{3}} - \frac{1}{2}\tan^{-1}\frac{1}{\sqrt{3}}\right)$

$= \frac{1}{2}\cdot\frac{\pi}{4} + \frac{1}{2} - \frac{1}{2}\cdot\frac{\pi}{4} - \frac{1}{6}\cdot\frac{\pi}{3} - \frac{1}{2\sqrt{3}} + \frac{1}{2}\cdot\frac{\pi}{6}$

$= \frac{1}{2} - \frac{\sqrt{3}}{6} + \frac{\pi}{36}$

$= \frac{1}{36}(18 - 6\sqrt{3} + \pi)$ **(Ans.)**

36. ক $\int \ln\{g(x)\}dx = \int \ln x^x\, dx$ $[\because g(a) = a^x \therefore g(x) = x^x]$

$$= \int x \ln x\, dx$$
$$= \ln x \int x dx - \int\left\{\frac{d}{dx}(\ln x)\int x dx\right\}dx$$
$$= \ln x.\frac{x^2}{2} - \int \frac{1}{x}.\frac{x^2}{2}dx$$
$$= \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\, dx$$
$$= \frac{1}{2}x^2 \ln x - \frac{x^2}{4} + c$$
$$= \frac{x^2}{4}(2\ln x - 1) + c \text{ (Ans.)}$$

খ দেওয়া আছে, $y = x^{g(x)}$

$\therefore\ y = x^{x^x}$ $[\because g(a) = a^x \therefore g(x) = x^x]$

বা, $\ln y = \ln x^{x^x}$ [উভয়পক্ষে ln নিয়ে]

বা, $\ln y = x^x \ln x$

বা, $\ln(\ln y) = \ln(x^x \ln x)$ [পুনরায় ln নিয়ে]

বা, $\ln(\ln y) = \ln x^x + \ln(\ln x)$

বা, $\ln(\ln y) = x\ln x + \ln(\ln x)$

x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$$\frac{1}{\ln y}.\frac{d}{dx}(\ln y) = \ln x.1 + x.\frac{1}{x} + \frac{1}{\ln x}.\frac{d}{dx}(\ln x)$$

বা, $\frac{1}{\ln y}.\frac{1}{y}.\frac{dy}{dx} = \ln x + 1 + \frac{1}{\ln x}.\frac{1}{x}$

বা, $\frac{dy}{dx} = y\ln y\left(1 + \ln x + \frac{1}{x\ln x}\right)$

বা, $\frac{dy}{dx} = x^{x^x} \ln x^{x^x}\left(1 + \ln x + \frac{1}{x\ln x}\right)$

বা, $\frac{dy}{dx} = x^{x^x}.x^x.\ln x.\left\{\frac{x\ln x + x(\ln x)^2 + 1}{x\ln x}\right\}$

$\therefore\ \frac{dy}{dx} = x^{x^x}.x^{x-1}\ \{1 + x\ln x + x(\ln x)^2\}$ **(Ans.)**

গ দেওয়া আছে, $g(a) = a^x$

$\therefore\ g(e) = e^x$

$\therefore\ \tan x = e^x$ $[\because \tan x = g(e)]$

সমাধানের জন্য ধরি, $y = \tan x$........(i)

এবং $y = e^x$ .... (ii)

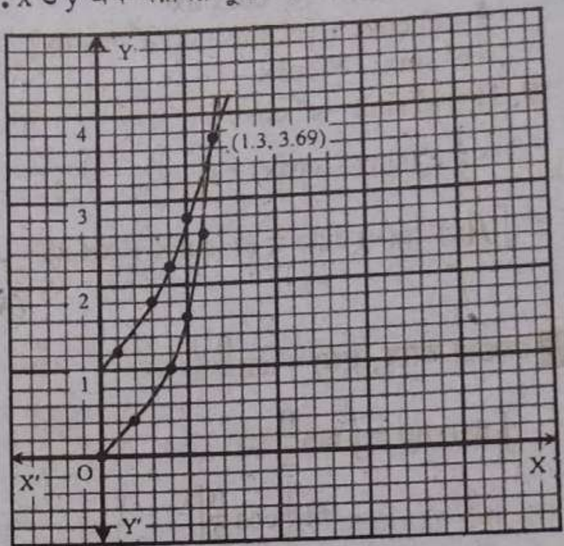
এখন (i) ও (ii) নং সমীকরণের জন্য (0, 2) ব্যবধিতে x এর কয়েকটি মান নিয়ে y এর প্রতিসংগী মান নির্ণয় করে দুটি তালিকা তৈরী করি।

x	0.4	0.8	1	1.2	1.3	1.4
y = tanx	0.42	1.03	1.56	2.58	3.6	5.8

x	0.2	0.6	0.8	1	1.3	1.6
$y = e^x$	1.2	1.8	2.23	2.72	3.67	4.95

১ম তালিকা হতে প্রাপ্ত বিন্দুগুলো ছক কাগজে বসিয়ে y = tanx এর লেখচিত্র অঙ্কন করা হলো। ২য় তালিকা হতে প্রাপ্ত বিন্দুগুলো একই ছক কাগজে বসিয়ে $y = e^x$ এর লেখচিত্র অঙ্কন করা হলো।

স্কেল: x ও y অক্ষ বরাবর ক্ষুদ্রতম বর্গের 1 বাহু = 0.2 একক



লেখচিত্র হতে দেখা যায় যে, উভয় লেখ x = 1.3 বিন্দুতে ছেদ করে।

$\therefore$ নির্ণেয় সমাধান, x = 1.3 **(Ans.)**

37. ক আমরা জানি,

sin (A + B) = sinA cosB + cosA sinB

A = 540° ও B = 210° বসিয়ে পাই,

sin (540° + 210°) = sin540° cos210° + cos540° sin210°

$\therefore$ sin750° = sin540° cos210° + cos540° sin210°

(প্রমাণিত)

খ দেওয়া আছে, $f(x) = e^x$

$$\therefore f(x+h) = e^{x+h} = e^x.e^h$$

$$\therefore \lim_{h\to 0}\frac{f(x+h) - f(x)}{h} = \lim_{h\to 0}\frac{e^x.e^h - e^x}{h}$$
$$= \lim_{h\to 0}\frac{e^x}{h}(e^h - 1)$$
$$= \lim_{h\to 0}\frac{e^x}{h}\left(1 + h + \frac{h^2}{2!} + \frac{h^3}{3!} + \ldots\ldots - 1\right)$$
$$= \lim_{h\to 0}\frac{e^x}{h}\left(h + \frac{h^2}{2!} + \frac{h^3}{3!} + \ldots\ldots\right)$$
$$= \lim_{h\to 0} e^x\left(1 + \frac{h}{2!} + \frac{h^2}{3!} + \ldots\ldots\right)$$
$$= e^x\left(1 + \frac{0}{2!} + \frac{0}{3!} + \ldots\ldots\right)$$
$$= e^x \text{ (Ans.)}$$

গ দেওয়া আছে, $f(x) = e^x$ ও $g(x) = \sin x$

ধরি, $I = \int f(x) \,.\, g(x)\, dx$.

$$\therefore I = \int e^x \sin x\, dx$$
$$= e^x \int \sin x\, dx - \int \left\{\frac{d}{dx}(e^x) \int \sin x\, dx\right\} dx$$
$$= -e^x \cos x + \int e^x \cos x\, dx$$
$$= -e^x \cos x + [e^x \sin x - \int e^x \sin x\, dx]$$
$$= e^x \sin x - e^x \cos x - I$$

বা, $2I = e^x \sin x - e^x \cos x$

$$\therefore I = \frac{1}{2} e^x(\sin x - \cos x) + c$$
$$\therefore \int f(x) \,.\, g(x)\, dx = \frac{1}{2} e^x (\sin x - \cos x) + c$$

(Ans.)

38. **ক** দেওয়া আছে, $f(\theta) = \tan\frac{\theta}{2}$

$$\therefore \int f(\theta)\, d\theta = \int \tan\frac{\theta}{2}\, d\theta$$
$$= \frac{-\ln\left|\cos\frac{\theta}{2}\right|}{\frac{1}{2}} + c$$
$$= -2\ln\left|\cos\frac{\theta}{2}\right| + c \text{ (Ans.)}$$

খ দেওয়া আছে, $f(\theta) = \sqrt{\frac{1-e}{1+e}} \tan\frac{\varphi}{2}$

বা, $\tan\frac{\theta}{2} = \sqrt{\frac{1-e}{1+e}} \tan\frac{\varphi}{2}$

বা, $\tan^2\frac{\theta}{2} = \frac{1-e}{1+e} \tan^2\frac{\varphi}{2}$ [বর্গ করে]

বা, $\tan^2\frac{\varphi}{2} = \frac{1+e}{1-e} \tan^2\frac{\theta}{2}$

বা, $\frac{\sin^2\frac{\varphi}{2}}{\cos^2\frac{\varphi}{2}} = \frac{1+e}{1-e} \cdot \frac{\sin^2\frac{\theta}{2}}{\cos^2\frac{\theta}{2}}$

বা, $\frac{\cos^2\frac{\varphi}{2}}{\sin^2\frac{\varphi}{2}} = \frac{(1-e)}{(1+e)} \cdot \frac{\cos^2\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}$

বা, $\frac{\cos^2\frac{\varphi}{2} - \sin^2\frac{\varphi}{2}}{\cos^2\frac{\varphi}{2} + \sin^2\frac{\varphi}{2}} = \frac{(1-e)\cos^2\frac{\theta}{2} - (1+e)\sin^2\frac{\theta}{2}}{(1-e)\cos^2\frac{\theta}{2} + (1+e)\sin^2\frac{\theta}{2}}$

[বিয়োজন-যোজন করে]

বা, $\frac{\cos 2.\frac{\varphi}{2}}{1} = \frac{\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right) - e\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)}{\left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right) - e\left(\cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}\right)}$

$[\because \cos 2A = \cos^2 A - \sin^2 A]$

$\therefore \cos\varphi = \frac{\cos\theta - e}{1 - e\cos\theta}$ (প্রমাণিত)

গ $f(\theta) = \tan\frac{\theta}{2}$

$$\therefore \quad f(\theta + h) = \tan\frac{\theta + h}{2}$$

আমরা জানি, $\frac{d}{d\theta}\{f(\theta)\} = \lim\limits_{h \to 0} \frac{f(\theta + h) - f(\theta)}{h}$

$$\therefore \frac{d}{d\theta}\left(\tan\frac{\theta}{2}\right) = \lim_{h \to 0} \frac{\tan\frac{\theta + h}{2} - \tan\frac{\theta}{2}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{\sin\frac{\theta+h}{2}}{\cos\frac{\theta+h}{2}} - \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}}{h}$$
$$= \lim_{h \to 0} \frac{\frac{\sin\frac{\theta+h}{2}\cos\frac{\theta}{2} - \cos\frac{\theta+h}{2}\sin\frac{\theta}{2}}{\cos\frac{\theta+h}{2}\cos\frac{\theta}{2}}}{h}$$
$$= \lim_{h \to 0} \frac{\sin\left(\frac{\theta+h}{2} - \frac{\theta}{2}\right)}{h\cos\frac{\theta+h}{2}\cos\frac{\theta}{2}}$$
$$= \frac{1}{2}\lim_{h \to 0} \frac{\sin\frac{h}{2}}{\frac{h}{2}} \lim_{h \to 0} \frac{1}{\cos\frac{\theta+h}{2}\cos\frac{\theta}{2}}$$
$$= \frac{1}{2} .1. \frac{1}{\cos\frac{\theta}{2}\cos\frac{\theta}{2}}$$
$$= \frac{1}{2} . \frac{1}{\cos^2\frac{\theta}{2}}$$
$$= \frac{1}{2}\sec^2\frac{\theta}{2}$$
$$\therefore \frac{d}{d\theta}\left(\tan\frac{\theta}{2}\right) = \frac{1}{2}\sec^2\frac{\theta}{2} \text{ (Ans.)}$$

39. ক $\frac{f(\theta)}{g(\theta)} = \frac{\sin\theta}{\cos\theta} = \tan\theta$

কাজেই $\tan\theta$ অসংজ্ঞায়িত হয়, যখন $\cos\theta = 0$.

অর্থাৎ $\theta = \pm(2n-1)\frac{\pi}{2}, n \in \mathbb{N}$.

সুতরাং $\tan\theta$ এর ডোমেন $= \mathbb{R} - \{\pm(2n-1)\frac{\pi}{2}, n \in \mathbb{N}\}$

খ ধরি, $y = f(\theta)\{1 + g(\theta)\}$

$= \sin\theta(1+\cos\theta)$

$= \sin\theta + \sin\theta\cos\theta$

$\frac{dy}{d\theta} = \cos\theta + \cos^2\theta - \sin^2\theta$

ক্ষুদ্রতম ও বৃহত্তম মানের জন্য, $\frac{dy}{d\theta} = 0$

বা, $\cos\theta + \cos^2\theta - \sin^2\theta = 0$

বা, $\cos\theta + 2\cos^2\theta - 1 = 0$

বা, $2\cos^2\theta + \cos\theta - 1 = 0$

$\therefore \cos\theta = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4}$

'+' চিহ্ন নিয়ে, $\therefore \cos\theta = \frac{1}{2}$ বা, $\theta = \frac{\pi}{3}$

$\frac{d^2y}{d\theta^2} = -\sin\theta - 2\cos\theta\sin\theta - 2\sin\theta\cos\theta$

$= -\sin\theta - 2\sin 2\theta$

যখন, $\theta = \frac{\pi}{3}$ তখন $\frac{d^2y}{d\theta^2} = -\sin\left(\frac{\pi}{3}\right) - 2\sin\left(\frac{2\pi}{3}\right)$

$= -\frac{\sqrt{3}}{2} - \frac{2\sqrt{3}}{2}$

$= -\frac{3\sqrt{3}}{2} < 0$

$\therefore \theta = \frac{\pi}{3}$ বিন্দুতে ফাংশনটির আপেক্ষিক বৃহত্তম মান রয়েছে। (দেখানো হলো)

গ $\int f^2(2g^2-1)\,d\theta$

$= \int \sin^2\theta(2\cos^2\theta - 1)\,d\theta$

$= \int \sin^2\theta \cos 2\theta\,d\theta$

$= \frac{1}{2}\int(2\sin^2\theta)\cos 2\theta\,d\theta$

$= \frac{1}{2}\int(1-\cos 2\theta)\cos 2\theta\,d\theta$

$= \frac{1}{2}\int(\cos 2\theta - \cos^2 2\theta)\,d\theta$

$= \frac{1}{2}\int\cos 2\theta\,d\theta - \frac{1}{4}\int 2\cos^2 2\theta\,d\theta$

$= \frac{1}{2}\int\cos 2\theta\,d\theta - \frac{1}{4}\int(1+\cos 4\theta)\,d\theta$

$= \frac{1}{2}\int\cos 2\theta\,d\theta - \frac{1}{4}\int d\theta - \frac{1}{4}\int\cos 4\theta\,d\theta$

$= \frac{\sin 2\theta}{4} - \frac{1}{4}\theta - \frac{\sin 4\theta}{16} + c$ **(Ans.)**

40. ক দেওয়া আছে, $f(x) = 1 + x^2$ ও $g(x) = 3 - x^2$

$\therefore \frac{f(x)}{g(x)} = \frac{1+x^2}{3-x^2}$

$\therefore \frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{d}{dx}\left(\frac{1+x^2}{3-x^2}\right)$

$= \frac{(3-x^2)\frac{d}{dx}(1+x^2) - (1+x^2)\frac{d}{dx}(3-x^2)}{(3-x^2)^2}$

$= \frac{(3-x^2).2x - (1+x^2)(-2x)}{(3-x^2)^2}$

$= \frac{2x(3-x^2+1+x^2)}{(3-x^2)^2} = \frac{8x}{(3-x^2)^2}$ **(Ans.)**

খ $\frac{x}{f(x)} = \frac{x}{1+x^2} = h(x)$ (ধরি) $[\because f(x) = 1 + x^2]$

x এর সকল বাস্তব মানের জন্য h(x) সংজ্ঞায়িত।

$\therefore$ h(x) এর ডোমেন $= \mathbb{R}$ **(Ans.)**

ধরি, $y = h(x)$

$\therefore y = \frac{x}{1+x^2}$

বা, $y + x^2y = x$

বা, $yx^2 - x + y = 0$

বা, $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4.y.y}}{2y}$

$\therefore x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$ ... ... (i)

যেহেতু h(x) এর ডোমেন $\mathbb{R}$ সুতরাং (i) নং এর বাস্তব মান বিদ্যমান থাকবে যদি,

$1 - 4y^2 \geq 0$ হয় এবং $y \neq 0$ হয়।

বা, $4y^2 \leq 1$

বা, $y^2 \leq \frac{1}{4}$

বা, $|y|^2 \leq \frac{1}{4}$

বা, $|y| \leq \frac{1}{2}$

$\therefore -\frac{1}{2} \leq y \leq \frac{1}{2}$ এবং $y \neq 0$

$\therefore \frac{x}{f(x)}$ এর রেঞ্জ $= \left[\frac{-1}{2}, 0\right) \cup \left(0, \frac{1}{2}\right]$ **(Ans.)**

গ দেওয়া আছে, $f(x) = 1 + x^2$ এবং $g(x) = 3 - x^2$
ধরি, $y = f(x) = 1 + x^2$.....(i) এবং $y = g(x) = 3 - x^2$ ... (ii)
(i) ও (ii) সমাধান করি। সুতরাং
$1 + x^2 = 3 - x^2$
বা, $2x^2 = 2$
বা, $x^2 = 1$
$\therefore x = \pm 1$
$\therefore$ নির্ণেয় ক্ষেত্রফল $= \int_{-1}^{1}(1 + x^2 - 3 + x^2)dx$
$= \int_{-1}^{1}(2x^2 - 2)dx$
$= 2\int_{-1}^{1}(x^2 - 1)dx$
$= 2\left[\frac{x^3}{3} - x\right]_{-1}^{1}$
$= 2\left[\left(\frac{1}{3} - 1\right) - \left(-\frac{1}{3} + 1\right)\right]$
$= 2\left[\frac{-2}{3} - \frac{2}{3}\right]$
$= 2.\frac{-4}{3}$
$= \frac{-8}{3}$
$\therefore$ নির্ণেয় ক্ষেত্রফল $= \left|\frac{-8}{3}\right| = \frac{8}{3}$ বর্গ একক **(Ans.)**

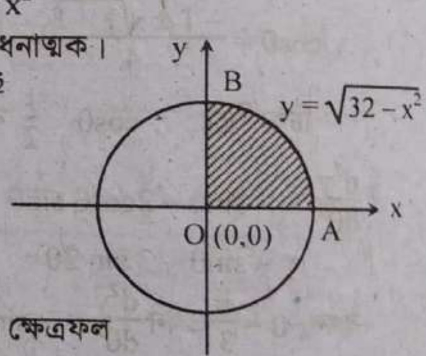
41. **ক** $\int \ln x\, dx = \ln x \int dx - \int \left[\frac{d}{dx}(\ln x) \int dx\right] dx$
$= x \ln x - \int \frac{1}{x} . x dx$
$= x \ln x - \int dx$
$= x \ln x - x + c$ [c যোগজীকরণ ধ্রুবক]

খ $\int f(x)\, dx = \int \frac{x}{(x - 1)(x^2 + 1)} dx$

মনে করি, $\frac{x}{(x - 1)(x^2 + 1)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + 1}$
উভয়পক্ষকে $(x - 1)(x^2 + 1)$ দ্বারা গুণ করে পাই,
$x = A(x^2 + 1) + (Bx + C)(x - 1)$ .... (i)
(i) নং এ $x = 1$ বসিয়ে,
$1 = 2A \Rightarrow A = \frac{1}{2}$
(i) নং এ $x = 0$ বসিয়ে,
$0 = A - C \Rightarrow C = A = \frac{1}{2}$
(i) নং এর উভয় পক্ষ হতে x^2 এর সহগ সমীকৃত করে পাই,
$0 = A + B \Rightarrow B = -A = -\frac{1}{2}$

$\therefore \frac{x}{(x - 1)(x^2 + 1)} = \frac{\frac{1}{2}}{x - 1} + \frac{-\frac{x}{2} + \frac{1}{2}}{x^2 + 1}$
$= \frac{1}{2} . \frac{1}{x - 1} - \frac{1}{2} . \frac{x}{x^2 + 1} + \frac{1}{2} . \frac{1}{x^2 + 1}$
$\therefore \int \frac{x}{(x - 1)(x^2 + 1)} dx$
$= \frac{1}{2}\int \frac{dx}{x - 1} - \frac{1}{4}\int \frac{2x}{x^2 + 1} dx + \frac{1}{2}\int \frac{dx}{x^2 + 1}$
$= \frac{1}{2} \ln|x - 1| - \frac{1}{4} \ln(x^2 + 1) + \frac{1}{2}\tan^{-1}x + c$. **(Ans.)**

গ $2(x^2 + y^2) = 64$
বা, $x^2 + y^2 = 32$ .... (i)
$\therefore x^2 + y^2 = (\sqrt{32})^2$
বৃত্তের কেন্দ্র (0, 0) এবং ব্যাসার্ধ $= \sqrt{32}$
(i) নং হতে পাই,
$y^2 = 32 - x^2$
$\therefore y = \pm\sqrt{32 - x^2}$
প্রথম চতুর্ভাগে y ধনাত্মক।
$\therefore y = \sqrt{32 - x^2}$



$\therefore$ OAB ক্ষেত্রের ক্ষেত্রফল
$= \int_0^{\sqrt{32}} \sqrt{32 - x^2}\, dx$
$= \int_0^{\sqrt{32}} \sqrt{(\sqrt{32})^2 - x^2}\, dx$
$= \left[\frac{x\sqrt{(\sqrt{32})^2 - x^2}}{2} + \frac{(\sqrt{32})^2}{2}\sin^{-1}\frac{x}{\sqrt{32}}\right]_0^{\sqrt{32}}$
$= \left(0 + \frac{32}{2}\sin^{-1} 1\right) - (0 + 0)$
$= 16.\frac{\pi}{2}$
$= 8\pi$ বর্গ একক **(Ans.)**

42. **ক** $\int \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2 dx$
$= \int \left(\sin^2\frac{x}{2} + \cos^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}\right) dx$
$= \int \left(1 + \sin 2.\frac{x}{2}\right) dx$
$= \int (1 + \sin x)dx$
$= x - \cos x + c$ [c একটি সমাকলন ধ্রুবক] **(Ans.)**

(i) ও (ii) নং দ্বারা আবন্ধ ক্ষুদ্রতর অংশের ক্ষেত্রফল

$$= 2\int_3^6 \frac{5}{6}\sqrt{6^2 - x^2}\,dx$$

$$= \frac{5}{3}\left[\frac{x}{2}\sqrt{6^2 - x^2} + \frac{6^2}{2}\sin^{-1}\frac{x}{6}\right]_3^6$$

$$= \frac{5}{3}\left[\left(0 - \frac{3}{2}\sqrt{27}\right) + 18\left(\sin^{-1}1 - \sin^{-1}\frac{1}{2}\right)\right]$$

$$= \frac{-5}{2} \times 3\sqrt{3} + 30\left(\frac{\pi}{2} - \frac{\pi}{6}\right)$$

$$= 30 \times \frac{\pi}{3} - \frac{15\sqrt{3}}{2}$$

$$= 10\pi - \frac{15\sqrt{3}}{2}$$ বর্গ একক **(Ans.)**

44. ক $\lim\limits_{x\to 0} \frac{e^{2x} - 1}{ax}$

$$= \lim_{x\to 0} \frac{1 + 2x + \frac{(2x)^2}{2!} + \ldots\ldots\ldots - 1}{ax}$$

$$= \lim_{x\to 0} \frac{x(2 + 2x + \ldots\ldots\ldots)}{ax}$$

$$= \lim_{x\to 0} \frac{2 + 2x + \ldots\ldots\ldots}{a}$$

$$= \lim_{x\to 0} \frac{2}{a}(1 + x + \ldots\ldots\ldots\ldots)$$

$$= \frac{2}{a}$$ **(Ans.)**

খ দেওয়া আছে, $f(x) = \ln x$

$\therefore f(2x) = \ln 2x$

ধরি, $y = \frac{f(2x)}{x} = \frac{\ln 2x}{x}$

$$\therefore \frac{dy}{dx} = \frac{x\frac{d}{dx}\ln 2x - \ln 2x\frac{d}{dx}x}{x^2}$$

$$= \frac{x.\frac{1}{2x}.2 - \ln 2x . 1}{x^2}$$

$$= \frac{1 - \ln 2x}{x^2}$$

আবার, $$\frac{d^2y}{dx^2} = \frac{x^2\frac{d}{dx}(1 - \ln 2x) - (1 - \ln 2x)\frac{d}{dx}x^2}{(x^2)^2}$$

$$= \frac{x^2\left(\frac{-1}{2x}.2\right) - (1 - \ln 2x).2x}{x^4}$$

$$= \frac{-x - 2x(1 - \ln 2x)}{x^4}$$

$$= \frac{-1 - 2 + 2\ln 2x}{x^3}$$

$$= \frac{-3 + 2\ln 2x}{x^3}$$

লঘুমান ও গুরুমানের জন্য, $\frac{dy}{dx} = 0$

বা, $\frac{1 - \ln 2x}{x^2} = 0$

বা, $1 = \ln 2x$

বা, $\ln 2x = \ln e$

বা, $2x = e$

$\therefore x = \frac{e}{2}$

$x = \frac{e}{2}$ হলে, $$\frac{d^2y}{dx^2} = \frac{-3 + 2\ln 2.\frac{e}{2}}{\left(\frac{e}{2}\right)^3}$$

$$= \frac{-1}{\frac{e^3}{8}} = \frac{-8}{e^3} < 0$$

$x = \frac{e}{2}$ এর জন্য প্রদত্ত ফাংশনটির গুরুমান বিদ্যমান।

$\therefore$ গুরুমান $$= \frac{\ln 2.\frac{e}{2}}{\frac{e}{2}} = \frac{1}{\frac{e}{2}} = \frac{2}{e}$$ **(Ans.)**

গ $\int_1^{e^2} \frac{f(x)}{x}\,dx + \int_1^2 g(x)\,dx$

$$= \int_1^{e^2} \frac{\ln x}{x}\,dx + \int_1^2 e^x\,dx$$

$$= \int_0^2 z\,dz + [e^x]_1^2$$

$$= \left[\frac{z^2}{2}\right]_0^2 + (e^2 - e^1)$$

$$= \frac{4}{2} + e^2 - e$$

$$= 2 + e^2 - e$$ **(Ans.)**

ধরি, $\ln x = z$

$\therefore \frac{1}{x}\,dx = dz$

x	e^2	1
z	2	0

45. ক $\int_1^2 \frac{1}{z}\cos(\ln z)\,dz$

$$= \int_0^{\ln 2} \cos x\,dx$$

$$= [\sin x]_0^{\ln 2}$$

$$= \sin(\ln 2) - \sin 0$$

$$= \sin(\ln 2)$$ **(Ans.)**

ধরি, $\ln z = x$

$\therefore \frac{1}{z}\,dz = dx$

z	2	1
x	ln2	0

খ $\int g(x)\,dx = \int mx \sin^{-1}x\,dx$ [$\because g(z) = mz\sin^{-1}z$]

$\therefore \int x\sin^{-1}x\,dx$

$= \sin^{-1}x \int x\,dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int x\,dx\right\}dx$

$= \sin^{-1}x.\frac{x^2}{2} - \int\frac{1}{\sqrt{1-x^2}}.\frac{x^2}{2}\,dx$

$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int\frac{x^2}{\sqrt{1-x^2}}\,dx$

$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}I$ [$I = \int\frac{x^2}{\sqrt{1-x^2}}dx$ ধরে]

এখন, $I = \int\frac{x^2}{\sqrt{1-x^2}}\,dx$

$= \int\frac{\sin^2\theta}{\sqrt{1-\sin^2\theta}}.\cos\theta\,d\theta$

$= \int\sin^2\theta\,d\theta$

$= \int\frac{1}{2}(1-\cos2\theta)\,d\theta$

| ধরি, $x = \sin\theta$
| বা, $\theta = \sin^{-1}x$
| $\therefore dx = \cos\theta\,d\theta$

$= \frac{1}{2}\left(\theta - \frac{1}{2}\sin2\theta\right)$

$= \frac{1}{2}(\sin^{-1}x - \frac{1}{2}.2\sin\theta\cos\theta)$

$\therefore I = \frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}$

$\therefore \int g(x)\,dx = m\int x\sin^{-1}x\,dx$

$= m\left[\frac{1}{2}x^2\sin^{-1}x - \frac{1}{2}\left(\frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1-x^2}\right)\right] + c$

$= \frac{1}{2}mx^2\sin^{-1}x - \frac{1}{4}m\sin^{-1}x + \frac{1}{4}mx\sqrt{1-x^2} + c$ **(Ans.)**

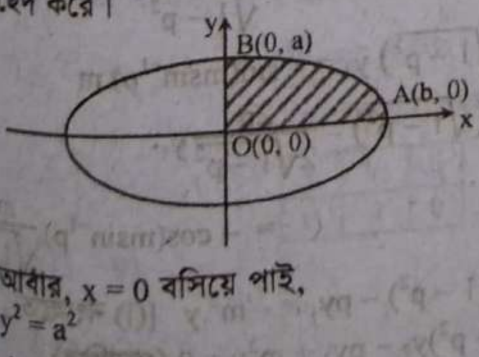
গ $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ ... ... (i)

(i) নং এ $y = 0$ বসিয়ে পাই,

$x^2 = b^2$

$\therefore x = \pm b$

অর্থাৎ উপবৃত্তটি x-অক্ষকে $(-b, 0)$ ও $(b, 0)$ বিন্দুতে ছেদ করে।



আবার, $x = 0$ বসিয়ে পাই,

$y^2 = a^2$

$\therefore y = \pm a$

$\therefore$ উপবৃত্তটি y-অক্ষকে $(0, -a)$ ও $(0, a)$ বিন্দুতে ছেদ করে।

উপবৃত্ত দ্বারা আবদ্ধ ক্ষেত্রের অর্ধাংশের ক্ষেত্রফল

$= 2 \times$ OABO ক্ষেত্রের ক্ষেত্রফল

$= 2\int_0^b y\,dx$

$= 2\int_0^b \frac{a}{b}\sqrt{b^2-x^2}\,dx$

$= \frac{2a}{b}\int_0^b\sqrt{b^2-x^2}\,dx$

| ধরি, $x = b\sin\theta \Rightarrow dx = b\cos\theta\,d\theta$
| যখন $x = 0$, তখন $\theta = 0$
| এবং যখন $x = b$, তখন $\theta = \frac{\pi}{2}$

$= \frac{2a}{b}\int_0^{\pi/2}\sqrt{b^2 - b^2\sin^2\theta}\,.b\cos\theta\,d\theta$

$= 2a\int_0^{\pi/2}\sqrt{b^2(1-\sin^2\theta)}\,.\cos\theta\,d\theta$

$= 2ab\int_0^{\pi/2}\sqrt{\cos^2\theta}\,.\cos\theta\,d\theta$

$= ab\int_0^{\pi/2}2\cos^2\theta\,d\theta$

$= ab\int_0^{\pi/2}(1+\cos2\theta)\,d\theta$

$= ab\left[\theta + \frac{1}{2}\sin2\theta\right]_0^{\pi/2}$

$= ab\left[\frac{\pi}{2} - 0\right]$

$= \frac{\pi ab}{2}$ বর্গ একক **(Ans.)**

46. ক $\int \ln x\,dx = \ln x\int dx - \int\left\{\frac{d}{dx}(\ln x)\int dx\right\}dx$

$= x\ln x - \int\frac{1}{x}.x\,dx$

$= x\ln x - \int dx$

$= x\ln x - x + c$ **(দেখানো হলো)**

খ দেওয়া আছে, $u = e^x$

$\therefore \int_0^{\ln2}\frac{u}{1+u}\,dx$

$= \int_0^{\ln2}\frac{e^x}{1+e^x}\,dx$

$= \int_2^3\frac{1}{z}\,dz$

$= [\ln z]_2^3$

$= \ln 3 - \ln 2$

$= \ln\frac{3}{2}$ **(Ans.)**

ধরি, $z = 1 + e^x$

$\therefore dz = e^x dx$

x	ln2	0
z	3	2

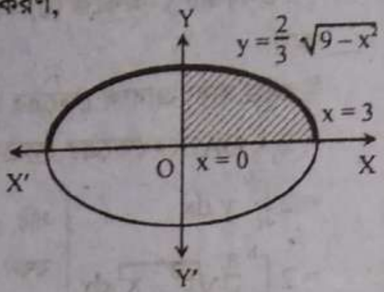
গ প্রদত্ত উপবৃত্তের সমীকরণ,

$4x^2 + 9y^2 = 36$

বা, $9y^2 = 36 - 4x^2$

বা, $y^2 = \frac{4}{9}(9 - x^2)$

$\therefore y = \frac{2}{3}\sqrt{9 - x^2}$



উপবৃত্তটি দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল

$= 4\int_0^3 y\,dx = 4\int_0^3 \frac{2}{3}\sqrt{9 - x^2}\,dx$

$= \frac{8}{3}\int_0^3 \sqrt{9 - x^2}\,dx$

$x = 3\sin\theta$ হলে, $dx = 3\cos\theta\,d\theta$

সীমা: $x = 0$ হলে, $\theta = 0$

এবং $x = 3$ হলে, $\theta = \frac{\pi}{2}$

$\therefore$ উপবৃত্তটির ক্ষেত্রফল $= \frac{8}{3}\int_0^{\pi/2}\sqrt{9 - 9\sin^2\theta}\,.\,3\cos\theta\,d\theta$

$= \frac{8}{3}\int_0^{\pi/2}\sqrt{9(1 - \sin^2\theta)}\,.\,3\cos\theta\,d\theta$

$= 8\int_0^{\pi/2}\sqrt{9\cos^2\theta}\,.\,\cos\theta\,d\theta$

$= 8\int_0^{\pi/2} 3\cos\theta \times \cos\theta\,d\theta$

$= 12\int_0^{\pi/2} 2\cos^2\theta\,d\theta$

$= 12\int_0^{\pi/2} (1 + \cos 2\theta)\,d\theta$

$= 12\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= 12\left(\frac{\pi}{2} + \frac{1}{2}\sin\pi\right)$

$= 6\pi$ বর্গ একক। **(Ans.)**

47. **ক** $\frac{d}{dx}\{\ln(x + \sqrt{x^2 + a^2})\}$

$= \frac{1}{x + \sqrt{x^2 + a^2}}\frac{d}{dx}(x + \sqrt{x^2 + a^2})$

$= \frac{1}{x + \sqrt{x^2 + a^2}}\left\{1 + \frac{1}{2\sqrt{x^2 + a^2}}\frac{d}{dx}(x^2 + a^2)\right\}$

$= \frac{1}{x + \sqrt{x^2 + a^2}}\left(1 + \frac{2x}{2\sqrt{x^2 + a^2}}\right)$

$= \frac{1}{x + \sqrt{x^2 + a^2}}\cdot\frac{\sqrt{x^2 + a^2} + x}{\sqrt{x^2 + a^2}}$

$= \frac{1}{\sqrt{x^2 + a^2}}$ **(Ans.)**

খ দেওয়া আছে,

$g(x) = x\,\text{cosec}^{-1}\frac{1}{x} = x\sin^{-1}x$

$\therefore \int g(x)\,dx = \int x\sin^{-1}x\,dx$

$= \sin^{-1}x\int x\,dx - \int\left\{\frac{d}{dx}(\sin^{-1}x)\int x\,dx\right\}dx$

$= \sin^{-1}x.\frac{x^2}{2} - \int\frac{1}{\sqrt{1 - x^2}}.\frac{x^2}{2}\,dx$

$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}\int\frac{x^2}{\sqrt{1 - x^2}}\,dx$

$= \frac{x^2}{2}\sin^{-1}x - \frac{1}{2}I$ $\quad [I = \int\frac{x^2}{\sqrt{1 - x^2}}\,dx$ ধরে]

এখন, $I = \int\frac{x^2}{\sqrt{1 - x^2}}\,dx$

$= \int\frac{\sin^2\theta}{\sqrt{1 - \sin^2\theta}}.\cos\theta\,d\theta$

$= \int\sin^2\theta\,d\theta$

$= \int\frac{1}{2}(1 - \cos 2\theta)\,d\theta$

$= \frac{1}{2}\left(\theta - \frac{1}{2}\sin 2\theta\right)$

ধরি, $x = \sin\theta$
বা, $\theta = \sin^{-1}x$
$\therefore dx = \cos\theta\,d\theta$

$= \frac{1}{2}(\sin^{-1}x - \frac{1}{2}.\,2\sin\theta\cos\theta)$

$\therefore I = \frac{1}{2}\sin^{-1}x - \frac{1}{2}\,x\sqrt{1 - x^2}$

$\therefore \int x\sin^{-1}x\,dx = \frac{1}{2}x^2\sin^{-1}x - \frac{1}{2}\left(\frac{1}{2}\sin^{-1}x - \frac{1}{2}x\sqrt{1 - x^2}\right) + c$

$= \frac{1}{2}x^2\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1 - x^2} + c$ **(Ans.)**

গ দেওয়া আছে, $y = \cos(m\sin^{-1}p)$ ... ... (i)

p এর সাপেক্ষে পর্যায়ক্রমিক অন্তরীকরণ করে পাই,

$y_1 = -\sin(m\sin^{-1}p).m\frac{1}{\sqrt{1 - p^2}}$

বা, $(\sqrt{1 - p^2})\,y_1 = -\sin(m\sin^{-1}p).m$

বা, $y_2(\sqrt{1 - p^2}) - \frac{2p}{2\sqrt{1 - p^2}}y_1 = -\cos(m\sin^{-1}p).\frac{m.m}{\sqrt{1 - p^2}}$

বা, $y_2(1 - p^2) - py_1 = -m^2.y$ [(i) নং হতে]

$\therefore (1 - p^2)y_2 - py_1 + m^2y = 0$ (প্রমাণিত)

48. **ক** $\int \frac{xdx}{x-1} = \int \frac{x-1+1}{x-1} dx$

$= \int \left(1 + \frac{1}{x-1}\right) dx$

$= x + \ln|x-1| + c$ **(Ans.)**

খ (i) $\int_0^2 f(x)\tan^{-1}(x-2)dx = \int_0^2 (x-2)\tan^{-1}(x-2)dx$

$[\because f(x) = x-2]$

$\therefore \int (x-2)\tan^{-1}(x-2)dx$

$= \tan^{-1}(x-2)\int(x-2)dx - \int\left[\frac{d}{dx}\tan^{-1}(x-2)\int(x-2)\,dx\right]dx$

$= \left(\frac{x^2}{2} - 2x\right)\tan^{-1}(x-2) - \int \frac{1}{1+(x-2)^2}.1.\left(\frac{x^2}{2} - 2x\right)dx$

$= \frac{1}{2}(x^2-4x)\tan^{-1}(x-2) - \frac{1}{2}\int\frac{x^2-4x}{x^2-4x+5}dx$

$= \frac{1}{2}(x^2-4x)\tan^{-1}(x-2) - \frac{1}{2}\int\frac{x^2-4x+5-5}{x^2-4x+5}dx$

$= \frac{1}{2}(x^2-4x)\tan^{-1}(x-2) - \frac{1}{2}\int\left(1 - \frac{5}{(x-2)^2+1^2}\right)dx$

$= \frac{1}{2}(x^2-4x)\tan^{-1}(x-2) - \frac{1}{2}x + \frac{1}{2}.5\tan^{-1}(x-2) + c$

$= \frac{1}{2}\{(x^2-4x+5)\tan^{-1}(x-2) - x\} + c$

$\therefore \int_0^2 f(x)\tan^{-1}(x-2)dx$

$= \frac{1}{2}\left[(x^2-4x+5)\tan^{-1}(x-2) - x\right]_0^2$

$= \frac{1}{2}[\{(2^2-4\times2+5)\tan^{-1}(2-2) - 2\}$

$- \{(0^2-4.0+5)\tan^{-1}(0-2) - 0\}]$

$= \frac{1}{2}\{0 - 2 - 5\tan^{-1}(-2)\}$

$= -1 - \frac{5}{2}\tan^{-1}(-2)$ **(Ans.)**

(ii) $\int_0^{\frac{\pi}{2}} g(x)\cos x\,dx$

$= \int_0^{\frac{\pi}{2}} \sin^6 x\cos x\,dx\ [\because g(x) = \sin^6 x]$

$= \int_0^1 z^6\,dz$

ধরি, $\sin x = z$

$\therefore \cos x\,dx = dz$

x	$\pi/2$	0
z	1	0

$= \left[\frac{z^7}{7}\right]_0^1$

$= \frac{1}{7}(1-0)$

$= \frac{1}{7}$ **(Ans.)**

গ দেওয়া আছে, $\phi(x, y) = 0$

বা, $9x^2 + 16y^2 - 144 = 0$

বা, $\frac{9x^2}{144} + \frac{16y^2}{144} = 1$

বা, $\frac{x^2}{16} + \frac{y^2}{9} = 1$ ...... (i)

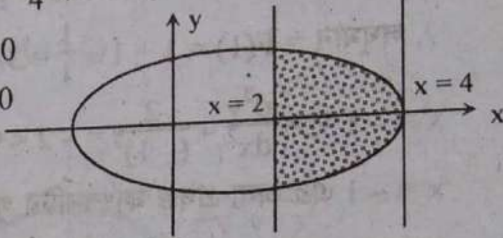
বা, $\frac{y^2}{9} = 1 - \frac{x^2}{16}$

$\therefore y = \frac{3}{4}\sqrt{16-x^2}$

এবং $f(x) = 0$

বা, $x - 2 = 0$

$\therefore x = 2$



y = 0 হলে (i) নং হতে পাই,

$\frac{x^2}{16} = 1$

$\therefore x = \pm 4$

আবদ্ধ ক্ষুদ্রতর অংশের ক্ষেত্রফল

$= 2\int_2^4 \frac{3}{4}\sqrt{4^2-x^2}\,dx$

$= \frac{3}{2}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_2^4$

$= \frac{3}{2}\left[\left(0 + 8\times\frac{\pi}{2}\right) - \left(\sqrt{12} + 8\times\frac{\pi}{6}\right)\right]$

$= \frac{3}{2}\left(4\pi - \frac{4\pi}{3} - \sqrt{12}\right)$

$= \frac{3}{2}\left(\frac{8\pi}{3} - \sqrt{12}\right)$ বর্গ একক **(Ans.)**

49. **ক** $y = (x-2)(x+1)$

$= x^2 + x - 2x - 2$

$= x^2 - x - 2$

$\therefore \frac{dy}{dx} = 2x - 1$

x = 2 বিন্দুতে স্পর্শকের ঢাল $= 2\times2 - 1$

$= 4 - 1$

$= 3$ **(Ans.)**

খ ধরি, $y = F(x) = \frac{x^2+x+1}{x} = x + 1 + \frac{1}{x}$

$\therefore \frac{dy}{dx} = 1 + 0 - \frac{1}{x^2} = 1 - \frac{1}{x^2}$

$\therefore \frac{d^2y}{dx^2} = \frac{2}{x^3}$

লঘুমান ও গুরুমানের জন্য, $\frac{dy}{dx} = 0$

বা, $1 - \frac{1}{x^2} = 0$

বা, $1 = \frac{1}{x^2}$

$\therefore x = \pm 1$

$x = 1$ হলে $\frac{d^2y}{dx^2} = \frac{2}{1^3} = 2 > 0$

$x = 1$ এর জন্য প্রদত্ত ফাংশনটির লঘুমান বিদ্যমান।

$\therefore$ লঘুমান $= F(1) = 1 + 1 + \frac{1}{1} = 3$

$x = -1$ হলে $\frac{d^2y}{dx^2} = \frac{2}{(-1)^3} = -2 < 0$

$x = -1$ এর জন্য প্রদত্ত ফাংশনটির গুরুমান বিদ্যমান।

গুরুমান $= F(-1) = -1 + 1 + \frac{1}{-1} = -1$

$\therefore F(x)$ ফাংশনটির লঘুমান, গুরুমান অপেক্ষা, বৃহত্তর।

(দেখানো হলো)

গ দেওয়া আছে, $H(x) = \frac{xe^x}{(x+1)^2}$

$\therefore \int \frac{xe^x}{(x+1)^2} dx$

$= \int \frac{\{(x+1) - 1\}e^x}{(x+1)^2} dx$

$= \int e^x . \frac{1}{x+1} dx - \int \frac{e^x}{(x+1)^2} dx.$

$= \frac{1}{x+1} \int e^x dx - \int \left\{\frac{d}{dx}\left(\frac{1}{x+1}\right) \int e^x dx\right\} dx - \int \frac{e^x}{(x+1)^2} dx$

$= \frac{e^x}{x+1} + \int \frac{e^x}{(x+1)^2} dx - \int \frac{e^x}{(x+1)^2} dx$

$= \frac{e^x}{x+1} + c.$

$\therefore \int_0^1 H(x) dx = \int_0^1 \frac{xe^x}{(x+1)^2} dx$

$= \left[\frac{e^x}{x+1}\right]_0^1$

$= \frac{e^1}{1+1} - \frac{e^0}{0+1}$

$= \frac{e}{2} - 1$ **(Ans.)**

50. **ক** ধরি, $y = x^x$

উভয়পক্ষে ln নিয়ে পাই,

$\ln y = x \ln x$

উভয়পক্ষকে x এর সাপেক্ষে অন্তরীকরণ করে পাই,

$\frac{1}{y}\frac{dy}{dx} = x.\frac{1}{x} + \ln x$

বা, $\frac{dy}{dx} = y(1 + \ln x)$

$\therefore \frac{d}{dx}(x^x) = x^x(1 + \ln x)$ **(Ans.)**

খ দেওয়া আছে, $f(x) = x^3 - 9x^2 + 24x - 12$

$\therefore f'(x) = 3x^2 - 18x + 24$

$\therefore f''(x) = 6x - 18$

এখন, লঘিষ্ঠ ও গরিষ্ঠ মানের জন্য $f'(x) = 0$

$3x^2 - 18x + 24 = 0$

বা, $x^2 - 6x + 8 = 0$

বা, $x^2 - 4x - 2x + 8 = 0$

বা, $x(x-4) - 2(x-4) = 0$

বা, $(x-2)(x-4) = 0$

$\therefore x = 2, 4$

$x = 2$ হলে $f''(x) = 6.2 - 18 = 12 - 18 = -6 < 0$

$\therefore x = 2$ এর জন্য ফাংশনটির গরিষ্ঠমান বিদ্যমান

এবং গরিষ্ঠমান $f(2) = 2^3 - 9.2^2 + 24.2 - 12$

$= 8 - 36 + 48 - 12$

$= 8$

আবার, $x = 4$ হলে $f''(x) = 6.4 - 18 = 24 - 18 = 6 > 0$

$\therefore x = 4$ এর জন্য ফাংশনটির লঘিষ্ঠমান বিদ্যমান

এবং লঘিষ্ঠমান $f(4) = 4^3 - 9.4^2 + 24.4 - 12$

$= 64 - 144 + 96 - 12 = 4$

$\therefore$ নির্ণেয় লঘিষ্ঠমান 4 এবং গরিষ্ঠমান 8. **(Ans.)**

গ (i) $\int \phi(x) dx = \int \frac{1}{\sqrt{12 - 16x^2}} dx$

$= \int \frac{1}{\sqrt{16\left(\frac{12}{16} - x^2\right)}} dx$

$= \int \frac{dx}{4 \times \sqrt{\frac{3}{4} - x^2}}$

$= \int \frac{dx}{4\sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 - x^2}}$

$= \frac{1}{4} \sin^{-1}\left(\frac{x}{\frac{\sqrt{3}}{2}}\right) + c$

$= \frac{1}{4} \sin^{-1}\left(\frac{2x}{\sqrt{3}}\right) + c$ **(Ans.)**

(ii) $\int \Psi(x)\,dx = \int \tan^{-1}\left(\frac{x}{5}\right) dx$

$= \tan^{-1}\left(\frac{x}{5}\right)\int dx - \int \left[\frac{d}{dx}\left\{\tan^{-1}\left(\frac{x}{5}\right)\right\}\int dx\right] dx$

$= x\tan^{-1}\frac{x}{5} - \int \frac{1}{1+\frac{x^2}{25}}\cdot\frac{1}{5}\cdot x\,dx$

$= x\tan^{-1}\frac{x}{5} - \int \frac{5x}{25+x^2}\,dx$

$= x\tan^{-1}\frac{x}{5} - \frac{5}{2}\int -\frac{2x}{5^2+x^2}\,dx$

$= x\tan^{-1}\frac{x}{5} - \frac{5}{2}\ln|5^2+x^2| + c$ **(Ans.)**

51. ক $\int \frac{dx}{1+e^x} = \int \frac{e^{-x}\,dx}{e^{-x}+1}$

ধরি, $e^{-x}+1 = z$

$\therefore -e^{-x}\,dx = dz$

$= \int \frac{-dz}{z}$

$= -\ln z + c$

$= -\ln(e^{-x}+1) + c$ **(Ans.)**

খ (i) $\int_0^{\pi/2} \sqrt{1+g(\theta)}\,d\theta$

$= \int_0^{\pi/2} \sqrt{1+\sin\theta}\,d\theta$

$= \int_0^{\pi/2} \sqrt{\sin^2\frac{\theta}{2}+\cos^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\,d\theta$

$= \int_0^{\pi/2} \sqrt{\left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right)^2}\,d\theta$

$= \int_0^{\pi/2} \left(\sin\frac{\theta}{2}+\cos\frac{\theta}{2}\right) d\theta$

$= \left[-\frac{\cos\frac{\theta}{2}}{\frac{1}{2}} + \frac{\sin\frac{\theta}{2}}{\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$

$= -2\cos\frac{\pi}{4} + 2\sin\frac{\pi}{4} + 2\cos 0 - 2\sin 0$

$= -2\cdot\frac{1}{\sqrt{2}} + 2\cdot\frac{1}{\sqrt{2}} + 2 - 0$

$= 2$ **(Ans.)**

(ii) $\int_0^{\pi/2} f(\theta)\sqrt[3]{g(\theta)}\,d\theta$

$= \int_0^{\pi/2} \cos^3\theta\sqrt[3]{\sin\theta}\,d\theta$

$= \int_0^{\pi/2} (1-\sin^2\theta)\sqrt[3]{\sin\theta}\cdot\cos\theta\,d\theta$

$= \int_0^1 (1-z^2)\,z^{\frac{1}{3}}\,dz$

ধরি, $z = \sin\theta$

$\therefore dz = \cos\theta\,d\theta$

θ	0	$\frac{\pi}{2}$
z	0	1

$= \int_0^1 (z^{\frac{1}{3}} - z^{\frac{7}{3}})\,dz$

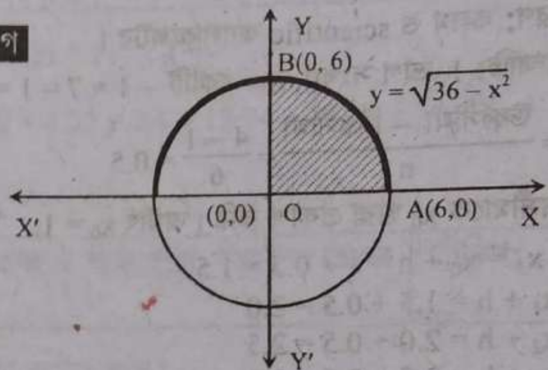
$= \left[\frac{z^{\frac{1}{3}+1}}{\frac{1}{3}+1} - \frac{z^{\frac{7}{3}+1}}{\frac{7}{3}+1}\right]_0^1$

$= \left[\frac{3}{4}\cdot z^{\frac{4}{3}} - \frac{3}{10}z^{\frac{10}{3}}\right]_0^1$

$= \frac{3}{4} - \frac{3}{10} - 0 + 0$

$= \frac{15-6}{20} = \frac{9}{20}$ **(Ans.)**

গ



নির্ণেয় ক্ষেত্রফল

$= 4 \times (y = \sqrt{36-x^2}$, x অক্ষ

এবং কোটি $x = 0$ ও $x = 6$

দ্বারা আবদ্ধ ক্ষেত্রের ক্ষেত্রফল)

$= 4\int_0^4 y\,dx$

$= 4\int_0^4 \sqrt{36-x^2}\,dx$

$= 4\int_0^{\pi/2} \sqrt{36-36\sin^2\theta}\,.6\cos\theta\,d\theta$

ধরি, $x = 6\sin\theta$

$\therefore dx = 6\cos\theta\,d\theta$

এবং $\theta = \sin^{-1}\frac{x}{6}$

x	0	6
θ	0	$\frac{\pi}{2}$

$= 144\int_0^{\pi/2} \sqrt{\cos^2\theta}\,\cos\theta\,d\theta$

$= 144\int_0^{\pi/2} \cos^2\theta\,d\theta$

$= 72\int_0^{\pi/2} 2\cos^2\theta\,d\theta$

$= 72\int_0^{\pi/2} (1+\cos 2\theta)\,d\theta$

$= 72\left[\theta + \frac{1}{2}\sin 2\theta\right]_0^{\frac{\pi}{2}}$

$= 72\left[\frac{\pi}{2} + 0 - 0\right]$

$= 36\pi$ বর্গ একক। **(Ans.)**

খ ধরি, $y = f(x) = \frac{\ln x}{x^2+1}$

$$f'(x) = \frac{(x^2+1)\frac{d}{dx}(\ln x) - \ln x \frac{d}{dx}(x^2+1)}{(x^2+1)^2}$$

$$= \frac{(x^2+1)\frac{1}{x} - \ln x \, . \, 2x}{(x^2+1)^2}$$

$$= \frac{\frac{x^2+1-2x^2\ln x}{x}}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2\ln x}{x(x^2+1)^2}$$

$x = 2$ বিন্দুতে $f'(x) = \frac{2^2+1-2\times 2^2 \times \ln 2}{2(2^2+1)^2}$

$$= \frac{5-8\ln 2}{50}$$

$x = 2$ হলে $y = \frac{\ln 2}{2^2+1} = \frac{\ln 2}{5}$

$\left(2, \frac{\ln 2}{5}\right)$ বিন্দুতে স্পর্শকের সমীকরণ,

$y - \frac{\ln 2}{5} = \frac{5 - 8\ln 2}{50}(x-2)$

বা, $50y - 10\ln 2 = (5 - 8\ln 2)x - 10 + 16\ln 2$

$\therefore (5 - 8\ln 2)x - 50y - 10 + 26\ln 2 = 0$ **(Ans.)**

গ $\int_0^1 (x).g(x)\,dx = \int_0^1 \frac{\ln x}{x^2+1}(x^2+1)\,dx$

$$= \int_0^1 \ln x \, dx$$

$\therefore \int \ln x\,dx = \ln x \int dx - \int \left[\frac{d}{dx}(\ln x)\int dx\right]dx$

$$= x\ln x - \int \frac{1}{x}.\,x\,dx$$

$$= x\ln x - x + c$$

$\therefore \int_0^1 f(x)g(x)\,dx = [x\ln x - x]_0^1$

$$= (0-1) - 0$$

$$= -1 \textbf{ (Ans.)}$$

43. **ক** দেওয়া আছে, $\cot x = \frac{1}{9}$

বা, $\cot^2 x = \frac{1}{81}$

বা, $\frac{\cos^2 x}{\sin^2 x} = \frac{1}{81}$

বা, $81\cos^2 x = 1 - \cos^2 x$

বা, $\cos^2 x = \frac{1}{82}$

বা, $2\cos^2 x = \frac{2}{82}$

বা, $2\cos^2 x - 1 = \frac{1}{41} - 1$

বা, $\cos 2x = \frac{-40}{41}$

বা, $\frac{1}{\sec 2x} = \frac{-40}{41}$

$\therefore \sec 2x = \frac{-41}{40}$ **(Ans.)**

খ $\int_0^3 \frac{f(x)}{\frac{d}{dx}\{g(x)\}}dx = \int_0^3 \frac{x e^x}{\frac{d}{dx}(x+1)^3}dx$

$$= \int_0^3 \frac{x e^x}{3(x+1)^2}$$

$\therefore \int \frac{xe^x}{(x+1)^2}dx = \int \frac{\{(x+1)-1\}e^x}{(x+1)^2}dx$

$= \int e^x . \frac{1}{x+1}dx - \int \frac{e^x}{(x+1)^2}dx.$

$= \frac{1}{x+1}\int e^x dx - \int\left\{\frac{d}{dx}\left(\frac{1}{x+1}\right)\int e^x dx\right\}dx - \int \frac{e^x}{(x+1)^2}dx$

$= \frac{e^x}{x+1} + \int \frac{e^x}{(x+1)^2}dx - \int \frac{e^x}{(x+1)^2}dx$

$= \frac{e^x}{x+1} + c$

$\therefore \int_0^3 \frac{x e^x}{3(x+1)^2} = \frac{1}{3}\left[\frac{e^x}{x+1}\right]_0^3 = \frac{1}{3}\left(\frac{e^3}{3+1} - \frac{e^0}{0+1}\right)$

$$= \frac{1}{3}\left(\frac{e^3}{4} - 1\right)$$

$$= \frac{1}{12}(e^3 - 4) \textbf{ (Ans.)}$$

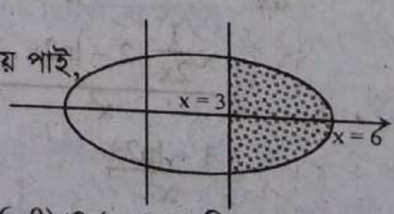
গ $\frac{x^2}{36} + \frac{y^2}{25} = 1$ ……… (i)

$x = 3$ ……… (ii)

(i) নং এ $y = 0$ বসিয়ে পাই,

$\frac{x^2}{36} = 1$

$\therefore x = \pm 6$



উপবৃত্তটি x-অক্ষকে (6, 0) ও (– 6, 0) বিন্দুতে ছেদ করে।

(i) নং হতে, $\frac{y^2}{25} = 1 - \frac{x^2}{36}$

বা, $\frac{y^2}{25} = \frac{1}{36}(36 - x^2)$

$\therefore y = \frac{5}{6}\sqrt{36 - x^2}$