

Inverse Circular Function

(বিপরীত ত্রিকোণমিতিক ফাংশন)

১। বিপরীত ত্রিকোণমিতিক ফাংশন :

$$(i) \sin^{-1}x = \arcsin x \quad (ii) \cos^{-1}x = \arccos x \quad (iii) \tan^{-1}x = \arctan x$$

$$(iv) \cot^{-1}x = \operatorname{arccot} x \quad (v) \sec^{-1}x = \operatorname{arcsec} x \quad (vi) \operatorname{cosec}^{-1}x = \operatorname{arccosec} x$$

২। মুখ্যমানঃ নির্দিষ্ট ব্যবধিতে বিপরীত ত্রিকোণমিতিক ফাংশনের ধনাত্মক বা ঋণাত্মক ক্ষুদ্রতম মান।

$$\text{যেমনঃ (i) } \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) \text{ এর মুখ্যমান } 60^\circ \quad (ii) \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) \text{ এর মুখ্যমান } -60^\circ$$

$$(iii) \cos^{-1}\left(\frac{-1}{2}\right) \text{ এর মুখ্যমান } 120^\circ \because \cos^{-1}\left(-\frac{1}{2}\right) = \pi - \cos^{-1}\frac{1}{2} = 180^\circ - 60^\circ = 120^\circ [\because \cos^{-1}(-x) = \pi - \cos^{-1}x]$$

$$৩। \sin^{-1}(-x) = -\sin^{-1}(x)$$

$$৪। \cos^{-1}(-x) = \pi - \cos^{-1}x$$

$$৫। \tan^{-1}(-x) = -\tan^{-1}x$$

$$৬। \sin^{-1}x = \operatorname{cosec}^{-1}\frac{1}{x}$$

$$৭। \cos^{-1}x = \sec^{-1}\frac{1}{x}$$

$$৮। \tan^{-1}x = \cot^{-1}\frac{1}{x}$$

$$৯। \sin^{-1}x + \cos^{-1}x = \pi/2$$

$$১০। \tan^{-1}x + \cot^{-1}x = \pi/2$$

$$১১। \sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$$

$$১২। \sin^{-1}\sin x = \sin \sin^{-1}x = x$$

$$১৩। \cos^{-1}\cos x = \cos \cos^{-1}x = x$$

$$১৪। \tan^{-1}\tan x = \tan \tan^{-1}x = x$$

১৫। (i) $\tan^{-1}x + \tan^{-1}y = \tan^{-1} \frac{x+y}{1-xy}$ (ii) $\tan^{-1}x - \tan^{-1}y = \tan^{-1} \frac{x-y}{1+xy}$

Function	Domain	Range
$\sin^{-1}x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1}x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1}x$	$(-\infty, +\infty)$	$(-\pi/2, \pi/2)$
$\cot^{-1}x$	$(-\infty, +\infty)$	$(0, \pi)$
$\sec^{-1}x$	$(-\infty, -1] \cup [1, +\infty)$	$[0, \pi/2) \cup [\pi, 3\pi/2)$
$\operatorname{cosec}^{-1}x$	$(-\infty, -1] \cup [1, +\infty)$	$(0, \pi/2] \cup (\pi, 3\pi/2]$

১৬। $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \tan^{-1} \frac{x+y+z-xyz}{1-xy-yz-zx}$

১৭। (i) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

(ii) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} - y\sqrt{1-x^2})$

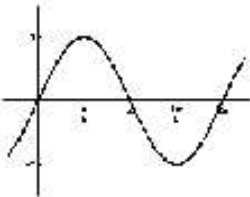
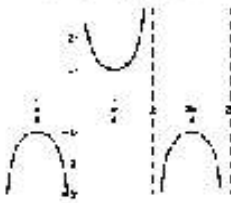
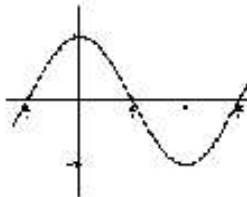
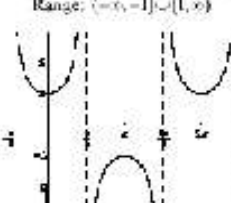
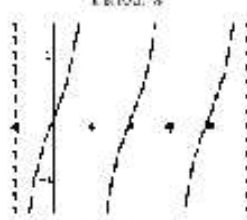

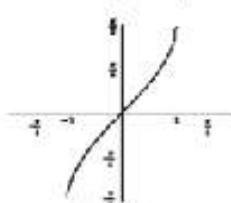
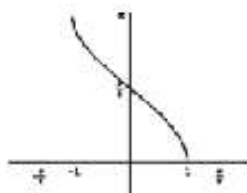
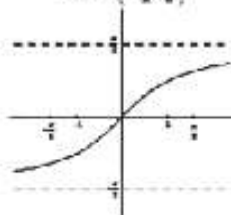
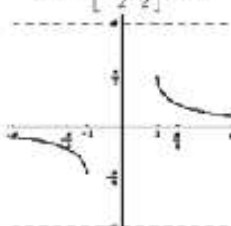
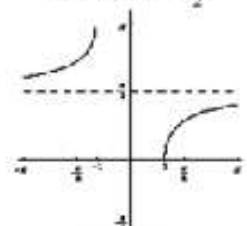
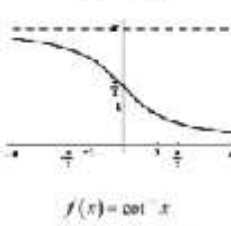
১৮। (i) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\{xy - \sqrt{(1-x^2)(1-y^2)}\}$

(ii) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\{xy + \sqrt{(1-x^2)(1-y^2)}\}$

১৯। $2\tan^{-1}x = \tan^{-1} \frac{2x}{1-x^2} = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2}$ (V.V. Important law)

২০। বিপরীত ত্রিকোণমিতিক ফাংশনের ডোমেন, রেঞ্জ :

৯। বৃত্তীয় ও বিপরীত বৃত্তীয় ফাংশনের লেখচিত্র :

<p>Domain: $(-\infty, \infty)$ Range: $[-1, 1]$ Period: 2π</p>  <p>$f(x) = \sin x$</p> <p>Domain: $\{(k+1)\pi, k\pi\}$ Range: $(-\infty, -1] \cup [1, \infty)$</p>  <p>$f(x) = \csc x = \frac{1}{\sin x}$</p>	<p>Domain: $(-\infty, \infty)$ Range: $[-1, 1]$ Period: 2π</p>  <p>$f(x) = \cos x$</p> <p>Domain: $\{(k-\frac{1}{2})\pi, (k+\frac{1}{2})\pi\}$ Range: $(-\infty, -1] \cup [1, \infty)$</p>  <p>$f(x) = \sec x = \frac{1}{\cos x}$</p>	<p>Domain: $\{(k-\frac{1}{2})\pi, (k+\frac{1}{2})\pi\}$ Range: $(-\infty, \infty)$ Period: π</p>  <p>$f(x) = \tan x$</p> <p>Domain: $\{(k+1)\pi, k\pi\}$ Range: $(-\infty, \infty)$</p>  <p>$f(x) = \cot x = \frac{1}{\tan x}$</p>
<p>Domain: $[-1, 1]$ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}]$</p>  <p>$f(x) = \sin^{-1} x$ $f(x) = \arcsin x$</p>	<p>Domain: $[-1, 1]$ Range: $[0, \pi]$</p>  <p>$f(x) = \cos^{-1} x$ $f(x) = \arccos x$</p>	<p>Domain: $(-\infty, \infty)$ Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$</p>  <p>$f(x) = \tan^{-1} x$ $f(x) = \operatorname{arctan} x$</p>
<p>Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $[-\frac{\pi}{2}, \frac{\pi}{2}], y \neq 0$</p>  <p>$f(x) = \csc^{-1} x$ $f(x) = \operatorname{arccsc} x$</p>	<p>Domain: $(-\infty, -1] \cup [1, \infty)$ Range: $[0, \pi], y \neq \frac{\pi}{2}$</p>  <p>$f(x) = \sec^{-1} x$ $f(x) = \operatorname{arcsec} x$</p>	<p>Domain: $(-\infty, \infty)$ Range: $(0, \pi)$</p>  <p>$f(x) = \cot^{-1} x$ $f(x) = \operatorname{arccot} x$</p>

Trigonometric Equation

(ত্রিকোণমিতিক সমীকরণ)

১। (i) $\sin\theta = 0$ হলে $\theta = n\pi$

(ii) $\cos\theta = 0$ হলে $\theta = (2n + 1)\pi/2$

(iii) $\tan\theta = 0$ হলে $\theta = n\pi$

(iv) $\sin\theta = 1$ হলে $\theta = (4n + 1)\pi/2$

(v) $\sin\theta = -1$ হলে $\theta = (4n - 1)\pi/2$

(vi) $\cos\theta = 1$ হলে $\theta = 2n\pi$

(vii) $\cos\theta = -1$ হলে $\theta = (2n + 1)\pi$

(viii) $\sin\theta = k = \sin\alpha$ হলে $\theta = n\pi + (-1)^n\alpha$

যেমন : $\sin 2\theta = \frac{\sqrt{3}}{2} \Rightarrow \sin 2\theta = \frac{\sqrt{3}}{2} = \sin \frac{\pi}{3} \Rightarrow \therefore 2\theta = n\pi + (-1)^n \frac{\pi}{3} \therefore \theta = \frac{n\pi}{2} + (-1)^n \frac{\pi}{6}$

(ix) $\cos\theta = K = \cos\alpha$ হলে $\theta = 2n\pi \pm \alpha$

(x) $\tan\theta = K = \tan\alpha$ হলে $\theta = n\pi + \alpha$

২। $a \cos\theta + b \sin\theta = c$ এই ধরনের সমীকরণের সমাধানে উভয়পক্ষকে $\sqrt{a^2 + b^2}$ দ্বারা ভাগ করতে হবে।