সপ্তম অধ্যায়

সংযুক্ত কোণের ত্রিকোণমিতিক অনুপাত
Trigonometric Ratios of Associated Angles

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-7.1.6 | পৃষ্ঠা-২৩৮

$(270° - \theta)$ কোণের ত্রিকোণমিতিক অনুপাত:

$\sin(270° - \theta) = \sin(3 \times 90° - \theta) = -\cos\theta$

$\cos(270° - \theta) = \cos\{180° + (90° - \theta)\}$
$= -\cos(90° - \theta) = -\sin\theta$

$\tan(270° - \theta) = \tan\{180° + (90° - \theta)\}$
$= \tan(90° - \theta) = \cot\theta$

$\text{cosec}\,(270° - \theta) = \text{cosec}\{180° + (90° - \theta)\}$
$= -\text{cosec}(90° - \theta) = -\sec\theta$

$\sec(270° - \theta) = \sec\{180° + (90° - \theta)\}$
$= -\sec(90° - \theta) = -\text{cosec}\,\theta$

$\cot(270° - \theta) = \cot\{180° + (90° - \theta)\}$
$= \cot(90° - \theta) = \tan\theta$

$(360° - \theta)$ কোণের ত্রিকোণমিতিক অনুপাত :

$\sin(360° - \theta) = \sin\{270° + (90° - \theta)\}$
$= -\cos(90° - \theta) = -\sin\theta$

$\cos(360° - \theta) = \cos\{270° + (90° - \theta)\}$
$= \sin(90° - \theta) = \cos\theta$

$\tan(360° - \theta) = \tan\{270° + (90° - \theta)\}$
$= -\cot(90° - \theta) = -\tan\theta$

$\text{cosec}\,(360° - \theta) = \text{cosec}\,\{270° + (90° - \theta)\}$
$= -\sec(90° - \theta) = -\text{cosec}\,\theta$

$\sec(360° - \theta) = \sec\{270° + (90° - \theta)\}$
$= \text{cosec}(90° - \theta) = \sec\theta$

$\cot(360° - \theta) = \cot\{270° + (90° - \theta)\}$
$= -\tan(90° - \theta) = -\cot\theta$

অনুচ্ছেদ-7.1.8 | পৃষ্ঠা-২৩৯

(i) $\cos 1050° = \cos(12 \times 90° - 30°)$

$= \cos 30° = \frac{\sqrt{3}}{2}$ **(Ans.)**

(ii) $\sin 420° \cos 390° + \sin(-300°)\cos(-330°)$

$= \sin 420° \cos 390° - \sin 300° \cos 330°$

$= \sin(4 \times 90° + 60°)\cos(4 \times 90° + 30°)$
$\quad - \sin(4 \times 90° - 60°)\cos(4 \times 90° - 30°)$

$= \sin 60°.\cos 30° - (-\sin 60°).\cos 30°$

$= \sin 60°.\cos 30° + \sin 60°.\cos 30°$

$= 2\sin 60° \cos 30° = 2.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = \frac{3}{2}$ **(Ans.)**

(iii) $\sec\left(\frac{\pi}{2} - \theta\right)\sec\left(\frac{\pi}{2} + \theta\right) + \tan\left(\frac{\pi}{2} + \theta\right)\tan\left(\frac{3\pi}{2} + \theta\right)$

$= \text{cosec}\theta\,.\,(-\text{cosec}\theta) + (-\cot\theta)(-\cot\theta)$

$= -\text{cosec}^2\theta + \cot^2\theta$

$= -(\text{cosec}^2\theta - \cot^2\theta) = -1$ **(Ans.)**

অনুশীলনী-7(A) এর সমাধান

1. (i) $\sec 3630° = \sec(40° \times 90 + 30°)$

$= \sec 30° = \frac{2}{\sqrt{3}}$ **(Ans.)**

(ii) $\cot(-1575°) = -\cot 1575°$
$= -\cot(17 \times 90° + 45°)$
$= -(-\tan 45°)$
$= \tan 45° = 1$ **(Ans.)**

(iii) $\cos\left(\frac{5\pi}{2} - \frac{19\pi}{3}\right) = \sin\frac{19\pi}{3} = \sin\left(12 \times \frac{\pi}{2} + \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ **(Ans.)**

(iv) $\sin\left\{n\pi + (-1)^n\frac{\pi}{6}\right\}$

$n = -1$ হলে, $\sin\left\{n\pi + (-1)^n\frac{\pi}{6}\right\}$

$= \sin\left(-\pi - \frac{\pi}{6}\right) = -\sin\left(\pi + \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$n = 0$ হলে, $\sin\left\{n\pi + (-1)^n\frac{\pi}{6}\right\} = \sin\frac{\pi}{6} = \frac{1}{2}$

যখন $n = 2m, m \in \mathbb{N}$

অর্থাৎ, n এর সাংখ্যিক মান জোড়।

তখন $\sin\left\{n\pi + (-1)^n\frac{\pi}{6}\right\} = -\sin\left(2m\pi + \frac{1}{6}\pi\right)$

$= \sin\frac{\pi}{6} = \frac{1}{2}$

যখন $n = 2m + 1, m \in \mathbb{N}$

অর্থাৎ n এর সাংখ্যিক মান বিজোড়।

তখন $\sin\left\{n\pi + (-1)^n\frac{\pi}{6}\right\}$

$= \sin\left\{(2m + 1)\pi + (-1)^{2m+1}\frac{\pi}{6}\right\}$

$= \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$

$\therefore$ n একটি পূর্ণসংখ্যা হলে নির্ণেয় মান $= \frac{1}{2}$ **(Ans.)**

(v) $\tan\left\{\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}\right\}$

যখন $n=0$, তখন $\tan\left\{\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}\right\}=\tan\frac{\pi}{4}=1$

যদি n জোড় সংখ্যা এবং m একটি ধনাত্মক পূর্ণ সংখ্যা হয়, তবে $n=2m$ ধরে

$$\tan\left\{\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}\right\}=\tan\left\{\frac{2m\pi}{2}+(-1)^{2m}\frac{\pi}{4}\right\}$$
$$=\tan\left\{m\pi+\frac{\pi}{4}\right\}=\tan\frac{\pi}{4}=1$$

আবার, যখন n একটি বিজোড় সংখ্যা এবং m ধনাত্মক পূর্ণসংখ্যা হয়, তখন $n=2m+1$ ধরে

$$\tan\left\{\frac{n\pi}{2}+(-1)^n\frac{\pi}{4}\right\}=\tan\left\{\frac{(2m+1)\pi}{2}+(-1)^{2m+1}\frac{\pi}{4}\right\}$$
$$=\tan\left\{m\pi+\frac{\pi}{2}-\frac{\pi}{4}\right\}$$
$$=\tan\left(m\pi+\frac{\pi}{4}\right)=\tan\frac{\pi}{4}=1$$

সুতরাং n এর মান শূন্য কিংবা ধনাত্মক পূর্ণ সংখ্যা হলে, নির্ণেয় মান = 1 **(Ans.)**

(vi) $\cos\left(2n\pi\pm\frac{\pi}{4}\right)$

$n=1$ হলে, $\cos\left(2\pi\pm\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

$n=0$ হলে, $\cos\left(0\pm\frac{\pi}{4}\right)=\cos\left(\pm\frac{\pi}{4}\right)=\frac{1}{\sqrt{2}}$

$n=-1$ হলে, $\cos\left(-2\pi\pm\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$

যখন $n=2m,\ m\in\mathbb{Z}$

অর্থাৎ n এর সাংখ্যিক মান জোড়।

$$\therefore\cos\left(2n\pi\pm\frac{\pi}{4}\right)=\cos\left(4m\pi\pm\frac{\pi}{4}\right)=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

যখন $n=2m+1,\ m\in\mathbb{Z}$

অর্থাৎ n এর সাংখ্যিক মান বিজোড়।

$$\therefore\cos\left(2n\pi\pm\frac{\pi}{4}\right)=\cos\left\{2(2m+1)\pi\pm\frac{\pi}{4}\right\}$$
$$=\cos\left\{(4m+2)\pi\pm\frac{\pi}{4}\right\}$$
$$=\cos\frac{\pi}{4}=\frac{1}{\sqrt{2}}$$

$\therefore$ n এর যেকোনো পূর্ণ মানের জন্য $\cos\left(2n\pi\pm\frac{\pi}{4}\right)$ এর মান $\frac{1}{\sqrt{2}}$ **(Ans.)**

2. (i) $\sin 780^\circ\cos 390^\circ+\sin(-330^\circ)\cos(-300^\circ)$

$=\sin780^\circ\cos390^\circ-\sin330^\circ\cos300^\circ$

$=\sin(8\times90^\circ+60^\circ)\cos(4\times90^\circ+30^\circ)$
$\quad-\sin(4\times90^\circ-30^\circ)\cos(4\times90^\circ-60^\circ)$

$=\sin60^\circ\cos30^\circ-(-\sin30^\circ)\cos60^\circ$

$=\sin60^\circ\cos30^\circ+\sin30^\circ\cos60^\circ$

$=\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2}+\frac{1}{2}.\frac{1}{2}$

$=\frac{3}{4}+\frac{1}{4}=1$ **(Ans.)**

(ii) $\tan18^\circ+\cos102^\circ+\tan162^\circ+\cos438^\circ$

$=\tan18^\circ+\cos(90^\circ+12^\circ)+\tan(2\times90^\circ-18^\circ)$
$\quad+\cos(5\times90^\circ-12^\circ)$

$=\tan18^\circ-\sin12^\circ-\tan18^\circ+\sin12^\circ$

$=0$ **(Ans.)**

(iii) $\tan\frac{17\pi}{4}\cos\left(-\frac{11\pi}{4}\right)+\sec\left(-\frac{34\pi}{3}\right)\operatorname{cosec}\left(\frac{25\pi}{6}\right)$

$=\tan\frac{17\pi}{4}\cos\frac{11\pi}{4}+\sec\frac{34\pi}{3}\operatorname{cosec}\frac{25\pi}{6}$

$=\tan\left(8\times\frac{\pi}{2}+\frac{\pi}{4}\right)\cos\left(6\times\frac{\pi}{2}-\frac{\pi}{4}\right)$
$\quad+\sec\left(22\times\frac{\pi}{2}+\frac{\pi}{3}\right)\operatorname{cosec}\left(8\times\frac{\pi}{2}+\frac{\pi}{6}\right)$

$=\tan\frac{\pi}{4}\left(-\cos\frac{\pi}{4}\right)-\sec\frac{\pi}{3}\operatorname{cosec}\frac{\pi}{6}$

$=1.\left(-\frac{1}{\sqrt{2}}\right)-2.2$

$=-\left(4+\frac{1}{\sqrt{2}}\right)$ **(Ans.)**

(iv) $\cot\frac{\pi}{20}\cot\frac{3\pi}{20}\cot\frac{5\pi}{20}\cot\frac{7\pi}{20}\cot\frac{9\pi}{20}$

$=\cot\frac{\pi}{20}.\cot\frac{9\pi}{20}.\cot\frac{3\pi}{20}.\cot\frac{7\pi}{20}.\cot\frac{5\pi}{20}$

$=\cot\frac{\pi}{20}.\cot\left(\frac{\pi}{2}-\frac{\pi}{20}\right).\cot\frac{3\pi}{20}.\cot\left(\frac{\pi}{2}-\frac{3\pi}{20}\right).\cot\frac{5\pi}{20}$

$=\cot\frac{\pi}{20}.\tan\frac{\pi}{20}.\cot\frac{3\pi}{20}.\tan\frac{3\pi}{20}.\cot\frac{\pi}{4}$

$=\frac{\tan\frac{\pi}{20}}{\tan\frac{\pi}{20}}.\frac{\tan\frac{3\pi}{20}}{\tan\frac{3\pi}{20}}.1\quad[\because\cot\frac{\pi}{4}=1]$

$=1.1.1=1$ **(Ans.)**

(v) $\tan\frac{\pi}{12}\tan\frac{5\pi}{12}\tan\frac{7\pi}{12}\tan\frac{11\pi}{12}$

$=\tan\frac{\pi}{12}\tan\left(\frac{\pi}{2}-\frac{\pi}{12}\right)\tan\left(\frac{\pi}{2}+\frac{\pi}{12}\right)\tan\left(\pi-\frac{\pi}{12}\right)$

$=\tan\frac{\pi}{12}\cot\frac{\pi}{12}\left(-\cot\frac{\pi}{12}\right)\left(-\tan\frac{\pi}{12}\right)$

$=1$ **(Ans.)**

3. (i) $\sin^2\frac{\pi}{7}+\sin^2\frac{5\pi}{14}+\sin^2\frac{8\pi}{7}+\sin^2\frac{9\pi}{14}$

$=\sin^2\frac{\pi}{7}+\left\{\sin\left(\frac{\pi}{2}-\frac{\pi}{7}\right)\right\}^2+\left\{\sin\left(\pi+\frac{\pi}{7}\right)\right\}^2+\left\{\sin\left(\frac{\pi}{2}+\frac{\pi}{7}\right)\right\}^2$

$=\sin^2\frac{\pi}{7}+\cos^2\frac{\pi}{7}+\sin^2\frac{\pi}{7}+\cos^2\frac{\pi}{7}$

$=1+1$

$=2$ **(Ans.)**

(ii) $\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{5\pi}{8}+\cos^2\frac{7\pi}{8}$

$=\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\left\{\cos\left(\pi-\frac{3\pi}{8}\right)\right\}^2+\left\{\cos\left(\pi-\frac{\pi}{8}\right)\right\}^2$

$=\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{3\pi}{8}+\cos^2\frac{\pi}{8}$

$=2\left[\cos^2\frac{\pi}{8}+\cos^2\frac{3\pi}{8}\right]$

$=2\left[\cos^2\frac{\pi}{8}+\left\{\cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right)\right\}^2\right]$

$=2\left(\cos^2\frac{\pi}{8}+\sin^2\frac{\pi}{8}\right)$

$=2\times1$

$=2$ **(Ans.)**

(iii) $\sin^2\frac{\pi}{4}+\sin^2\frac{3\pi}{4}+\sin^2\frac{5\pi}{4}+\sin^2\frac{7\pi}{4}$

$=\sin^2\frac{\pi}{4}+\sin^2\left(\frac{\pi}{2}+\frac{\pi}{4}\right)+\sin^2\frac{5\pi}{4}+\sin^2\left(\frac{\pi}{2}+\frac{5\pi}{4}\right)$

$=\left(\sin^2\frac{\pi}{4}+\cos^2\frac{\pi}{4}\right)+\left(\sin^2\frac{5\pi}{4}+\cos^2\frac{5\pi}{4}\right)$

$=1+1=2$ **(Ans.)**

(iv) $\cos^2\frac{\pi}{24}+\cos^2\frac{19\pi}{24}+\cos^2\frac{31\pi}{24}+\cos^2\frac{37\pi}{24}$

$=\cos^2\frac{\pi}{24}+\left\{\cos\left(\frac{\pi}{2}+\frac{7\pi}{24}\right)\right\}^2+\left\{\cos\left(\pi+\frac{7\pi}{24}\right)\right\}^2+\left\{\cos\left(\pi+\frac{13\pi}{24}\right)\right\}^2$

$=\cos^2\frac{\pi}{24}+\sin^2\frac{7\pi}{24}+\cos^2\frac{7\pi}{24}+\cos^2\frac{13\pi}{24}$

$=\cos^2\frac{\pi}{24}+\sin^2\frac{7\pi}{24}+\cos^2\frac{7\pi}{24}+\left\{\cos\left(\frac{\pi}{2}+\frac{\pi}{24}\right)\right\}^2$

$=\cos^2\frac{\pi}{24}+1+\sin^2\frac{\pi}{24}=1+\left(\sin^2\frac{\pi}{24}+\cos^2\frac{\pi}{24}\right)$

$=1+1=2$ **(Ans.)**

(v) $\sec^2\frac{14\pi}{17}-\sec^2\frac{39\pi}{17}+\cot^2\frac{41\pi}{34}-\cot^2\frac{23\pi}{34}$

$=\left\{\sec\left(\pi-\frac{3\pi}{17}\right)\right\}^2-\left\{\sec\left(2\pi+\frac{5\pi}{17}\right)\right\}^2+\left\{\cot\left(\frac{3\pi}{2}-\frac{5\pi}{17}\right)\right\}^2-\left\{\cot\left(\frac{\pi}{2}+\frac{3\pi}{17}\right)\right\}^2$

$=\sec^2\frac{3\pi}{17}-\sec^2\frac{5\pi}{17}+\tan^2\frac{5\pi}{17}-\tan^2\frac{3\pi}{17}$

$=\left(\sec^2\frac{3\pi}{17}-\tan^2\frac{3\pi}{17}\right)-\left(\sec^2\frac{5\pi}{17}-\tan^2\frac{5\pi}{17}\right)$

$=1-1=0$ **(Ans.)**

4.(i) বামপক্ষ $=\sin^2 15^\circ+\sin^2 20^\circ+\sin^2 25^\circ+\dots+\sin^2 75^\circ$

$=\sin^2 15^\circ+\sin^2 75^\circ+\sin^2 20^\circ+\sin^2 70^\circ+\sin^2 25^\circ+\sin^2 65^\circ+\sin^2 30^\circ+\sin^2 60^\circ+\sin^2 35^\circ+\sin^2 55^\circ+\sin^2 40^\circ+\sin^2 50^\circ+\sin^2 45^\circ$

$=\sin^2 15^\circ+\sin^2(90^\circ-15^\circ)+\sin^2 20^\circ+\sin^2(90^\circ-20^\circ)+\sin^2 25^\circ+\sin^2(90^\circ-25^\circ)+\sin^2 30^\circ+\sin^2(90^\circ-30^\circ)+\sin^2 35^\circ+\sin^2(90^\circ-35^\circ)+\sin^2 40^\circ+\sin^2(90^\circ-40^\circ)+\left(\frac{1}{\sqrt{2}}\right)^2$

$=(\sin^2 15^\circ+\cos^2 15^\circ)+(\sin^2 20^\circ+\cos^2 20^\circ)+(\sin^2 25^\circ+\cos^2 25^\circ)+(\sin^2 30^\circ+\cos^2 30^\circ)+(\sin^2 35^\circ+\cos^2 35^\circ)+(\sin^2 40^\circ+\cos^2 40^\circ)+\frac{1}{2}$

$=1+1+1+1+1+1+\frac{1}{2}$

$=6+\frac{1}{2}=\frac{12+1}{2}=\frac{13}{2}$

= ডানপক্ষ (দেখানো হলো)

(ii) বামপক্ষ $=\cos^2 3^\circ+\cos^2 9^\circ+\cos^2 15^\circ+\dots\dots+\cos^2 177^\circ$

$=\cos^2 3^\circ+\cos^2 9^\circ+\cos^2 15^\circ+\dots\dots+\cos^2 93^\circ+\cos^2 99^\circ+\dots\dots+\cos^2 177^\circ$

$=(\cos^2 3^\circ+\cos^2 93^\circ)+(\cos^2 9^\circ+\cos^2 99^\circ)+(\cos^2 15^\circ+\cos^2 105^\circ)+\dots\dots+(\cos^2 87^\circ+\cos^2 177^\circ)$

$=\cos^2 3^\circ+\cos^2(90^\circ+3^\circ)+\cos^2 9^\circ+\cos^2(90^\circ+9^\circ)+\cos^2 15^\circ+\cos^2(90^\circ+15^\circ)+\dots\dots+\cos^2 87^\circ+\cos^2(90^\circ+87^\circ)$

$=(\cos^2 3^\circ+\sin^2 3^\circ)+(\cos^2 9^\circ+\sin^2 9^\circ)+(\cos^2 15^\circ+\sin^2 15^\circ)+\dots\dots$ 15 তম পদ পর্যন্ত

$=1+1+1+\dots\dots$ 15 তম পদ পর্যন্ত

$=15$ = ডানপক্ষ (দেখানো হলো)

(iii) $\cos^2 10^\circ+\cos^2 20^\circ+\cos^2 30^\circ+\dots+\cos^2 80^\circ$

$=\cos^2(90^\circ-80^\circ)+\cos^2(90^\circ-70^\circ)+\cos^2 30^\circ+\cos^2(90^\circ-50^\circ)+\cos^2 50^\circ+\cos^2 60^\circ+\cos^2 70^\circ+\cos^2 80^\circ$

$= \sin^2 80° + \sin^2 70° + \left(\frac{\sqrt{3}}{2}\right)^2 + \sin^2 50°$

$+ \cos^2 50° + \left(\frac{1}{2}\right)^2 + \cos^2 70° + \cos^2 80°$

$= 1 + 1 + 1 + \frac{3}{4} + \frac{1}{4} = \frac{12 + 3 + 1}{4} = 4$ (প্রমাণিত)

5. (i) দেওয়া আছে, $\tan\theta = \frac{5}{12}$

যেহেতু $\tan\theta$ ও $\cos\theta$ ধনাত্মক। সুতরাং θ এর মান প্রথম চতুর্থ ভাগে পড়বে।

১ম চতুর্ভাগে ত্রিকোণমিতিক সকল অনুপাতের মান ধনাত্মক।

আমরা জানি, $\sec^2\theta = 1 + \tan^2\theta = 1 + \left(\frac{5}{12}\right)^2$

$= 1 + \frac{25}{144} = \frac{169}{144}$

$\therefore \sec\theta = \frac{13}{12}$ [যেহেতু $\cos\theta$ ধণাত্মক]

সুতরাং, $\cos\theta = \frac{12}{13}$

এখন, $\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{144}{169}}$

$= \sqrt{\frac{25}{169}} = \frac{5}{13}$

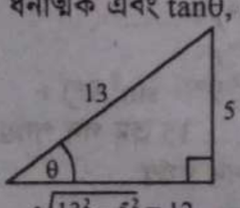
$\therefore \sin\theta = \frac{5}{13}$

এখন, $\frac{\sin\theta + \cos(-\theta)}{\sec(\theta) + \tan\theta} = \frac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}$

$= \frac{\frac{5}{13} + \frac{12}{13}}{\frac{13}{12} + \frac{5}{12}} = \frac{\frac{17}{13}}{\frac{18}{12}} = \frac{17}{13} \times \frac{12}{18} = \frac{34}{39}$ **(Ans.)**

(ii) দেওয়া আছে, $\sin\theta = \frac{5}{13}$ এবং $\frac{\pi}{2} < \theta < \pi$

যেহেতু θ দ্বিতীয় চতুর্ভাগে অবস্থিত। কাজেই $\operatorname{cosec}\theta$ ধনাত্মক এবং $\tan\theta$, $\sec\theta$ ও $\cot\theta$ ঋণাত্মক।



$\operatorname{cosec}\theta = \frac{13}{5}$, $\cot\theta = \frac{-12}{5}$

$\tan\theta = \frac{-5}{12}$, $\sec\theta = \frac{-13}{12}$

প্রদত্ত রাশি $= \frac{\tan\theta + \sec(-\theta)}{\cot\theta + \operatorname{cosec}(-\theta)}$

$= \frac{\tan\theta + \sec\theta}{\cot\theta - \operatorname{cosec}\theta}$

$= \frac{\frac{-5}{12} - \frac{13}{12}}{\frac{-12}{5} - \frac{13}{5}} = \frac{\frac{5}{12} + \frac{13}{12}}{\frac{12}{5} + \frac{13}{5}}$

$= \frac{\cancel{18}^{\,3}}{\cancel{12}_{\,2}} \times \frac{\cancel{5}}{\cancel{25}_{\,5}} = \frac{3}{10}$ **(Ans.)**

(iii) দেওয়া আছে,

$\alpha = \frac{11\pi}{4}$

প্রদত্ত রাশি $= \sin^2\alpha - \cos^2\alpha - 2\tan\alpha - \sec^2\alpha$

$= \sin^2\frac{11\pi}{4} - \cos^2\frac{11\pi}{4} - 2\tan\frac{11\pi}{4} - \sec^2\frac{11\pi}{4}$

$= \sin^2\left(3\pi - \frac{\pi}{4}\right) - \cos^2\left(3\pi - \frac{\pi}{4}\right) - 2\tan\left(3\pi - \frac{\pi}{4}\right)$

$- \sec^2\left(3\pi - \frac{\pi}{4}\right)$

$= \sin^2\frac{\pi}{4} - \cos^2\frac{\pi}{4} + 2\tan\frac{\pi}{4} - \sec^2\frac{\pi}{4}$

$= \left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2 + 2.1 - (\sqrt{2})^2$

$= 2 - 2$

$= 0$ **(Ans.)**

6. $\cot\theta \cot3\theta \cot5\theta \cot7\theta \ldots \ldots \ldots \cot19\theta$

$= \cot\frac{\pi}{20} \cot\frac{3\pi}{20} \cot\frac{5\pi}{20} \cot\frac{7\pi}{20} \cot\frac{9\pi}{20} \cot\frac{11\pi}{20}$

$\cot\frac{13\pi}{20} \cot\frac{15\pi}{20} \cot\frac{17\pi}{20} \cot\frac{19\pi}{20}$ $[\because \theta = \frac{\pi}{20}]$

$= \cot\frac{\pi}{20} \cot\frac{3\pi}{20} \cot\frac{5\pi}{20} \cot\frac{7\pi}{20} \cot\frac{9\pi}{20} \cot\left(\frac{\pi}{2} + \frac{\pi}{20}\right)$

$\cot\left(\frac{\pi}{2} + \frac{3\pi}{20}\right) \cot\left(\frac{\pi}{2} + \frac{5\pi}{20}\right) \cot\left(\frac{\pi}{2} + \frac{7\pi}{20}\right) \cot\left(\frac{\pi}{2} + \frac{9\pi}{20}\right)$

$= \cot\frac{\pi}{20}\left(-\tan\frac{\pi}{20}\right) . \cot\frac{3\pi}{20}\left(-\tan\frac{3\pi}{20}\right) \cot\frac{5\pi}{20}$

$\left(-\tan\frac{5\pi}{20}\right) \cot\frac{7\pi}{20}\left(-\tan\frac{7\pi}{20}\right) \cot\frac{9\pi}{20}\left(-\tan\frac{9\pi}{20}\right)$

$= -1$ (দেখানো হলো)

7. দেওয়া আছে,

$x = r\sin(\theta + 45°)$

$y = r\sin(\theta - 45°)$

$\therefore x^2 + y^2 = r^2\sin^2(\theta + 45°) + r^2\sin^2(\theta - 45°)$

$= r^2\{\sin^2(90° + \theta - 45°) + \sin^2(\theta - 45°)\}$

$= r^2\{\cos^2(\theta - 45°) + \sin^2(\theta - 45°)\}$

$= r^2 \times 1$

$\therefore x^2 + y^2 = r^2$ (প্রমাণিত)

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-7.2.3 | পৃষ্ঠা-২৮২

প্রমাণ: (i) $\cot(A+B) = \dfrac{\cos(A+B)}{\sin(A+B)}$

$= \dfrac{\cos A\cos B - \sin A \sin B}{\sin A\cos B + \cos A\sin B}$

$= \dfrac{\dfrac{\cos A\cos B}{\sin A\sin B} - \dfrac{\sin A\sin B}{\sin A\sin B}}{\dfrac{\sin A\cos B}{\sin A\sin B} + \dfrac{\cos A\sin B}{\sin A\sin B}}$

$= \dfrac{\cot A\cot B - 1}{\cot B + \cot A}$

[লব ও হরকে sinAsinB দ্বারা ভাগ করে]

(ii) $\cot(A-B) = \dfrac{\cos(A-B)}{\sin(A-B)}$

$= \dfrac{\cos A\cos B + \sin A \sin B}{\sin A\cos B - \cos A\sin B}$

$= \dfrac{\dfrac{\cos A\cos B}{\sin A\sin B} + \dfrac{\sin A\sin B}{\sin A\sin B}}{\dfrac{\sin A\cos B}{\sin A\sin B} - \dfrac{\cos A\sin B}{\sin A\sin B}}$

[লব ও হরকে sinAsinB দ্বারা ভাগ করে]

$= \dfrac{\cot A\cot B + 1}{\cot B - \cot A}$ (i) ও (ii) নং সূত্র, দুইটি কোণের যোগফল ও বিয়োগফলের কোট্যানজেন্ট সূত্র নামে পরিচিত।

অনুচ্ছেদ-7.2.3 | পৃষ্ঠা-২৮৩

(i) $\cos 81°26' \cos 21°26' + \cos 8°34' \cos 68°34'$

$= \cos(90° - 8°34') \cos 21°26' + \cos 8°34' \cos(90° - 21°26')$

$= \sin 8°34' \cos 21°26' + \cos 8°34'.\sin 21°26'$

$= \sin(8°34' + 21°26') = \sin 30° = \frac{1}{2}$ **(Ans.)**

(ii) $\cos 68°20' \cos 8°20' + \cos 81°40' . \cos 21°40'$

$= \cos(90° - 21°40') . \cos 8°20' + \cos(90° - 8°20') \cos 21°40'$

$= \sin 21°40' \cos 8°20' + \sin 8°20' \cos 21°40'$

$= \sin 21°40' \cos 8°20' + \cos 21°40' \sin 8°20'$

$= \sin(21°40' + 8°20') = \sin 30° = \frac{1}{2}$ **(Ans.)**

(iii) বামপক্ষ $= \tan 23° = \tan(45° - 22°)$

$= \dfrac{\tan 45° - \tan 22°}{1 + \tan 45° \tan 22°} = \dfrac{1 - \tan 22°}{1 + \tan 22°} = \dfrac{1 - \dfrac{\sin 22°}{\cos 22°}}{1 + \dfrac{\sin 22°}{\cos 22°}}$

$= \dfrac{\cos 22° - \sin 22°}{\cos 22° + \sin 22°} = \dfrac{\cos(90° - 68°) - \sin 22°}{\cos(90° - 68°) + \sin 22°}$

$= \dfrac{\sin 68° - \sin 22°}{\sin 68° + \sin 22°}$

= ডানপক্ষ

(iv) আমরা জানি,

$\tan 20° = \tan(55° - 35°)$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1 + \tan 55°.\tan 35°}$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1 + \tan(90° - 35°) \tan 35°}$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1 + \cot 35°. \dfrac{1}{\cot 35°}}$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1+1}$

বা, $\tan 55° - \tan 35° = 2 \tan 20°$

$\therefore \tan 55° = \tan 35° + 2 \tan 20°$. (প্রমাণিত)

অনুশীলনী-7(B) এর সমাধান

1. (i) $\cos 15° = \cos(60° - 45°)$

$= \cos 60° \cos 45° + \sin 60° \cos 45°$

$= \frac{1}{2}.\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}}$

$= \dfrac{1+\sqrt{3}}{2\sqrt{2}} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$ **(Ans.)**

(ii) $\sin 105° = \sin(60° + 45°)$

$= \sin 60° \cos 45° + \cos 60° \sin 45°$

$= \frac{\sqrt{3}}{2}.\frac{1}{\sqrt{2}} + \frac{1}{2}.\frac{1}{\sqrt{2}} = \dfrac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{1}{4}(\sqrt{6} + \sqrt{2})$ **(Ans.)**

(iii) $\operatorname{cosec} 375° = \operatorname{cosec}(360° + 15°) = \operatorname{cosec} 15°$

$= \dfrac{1}{\sin 15°} = \dfrac{1}{\sin(45° - 30°)}$

$= \dfrac{1}{\sin 45° \cos 30° - \cos 45° \sin 30°}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2}} = \dfrac{2\sqrt{2}}{\sqrt{3}-1}$

$= \dfrac{2\sqrt{2}(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \dfrac{2(\sqrt{6}+\sqrt{2})}{2}$

$= \sqrt{6} + \sqrt{2}$ **(Ans.)**

(iv) $\cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

(v) $\tan 15° = \tan(45° - 30°) = \dfrac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \dfrac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = \dfrac{(\sqrt{3}-1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \dfrac{3 + 1 - 2\sqrt{3}}{3 - 1} = \dfrac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$ **(Ans.)**

2. দেওয়া আছে, $\cos A = \frac{4}{5}$

বা, $\cos^2 A = \frac{16}{25}$ বা, $1 - \sin^2 A = \frac{16}{25}$

বা, $\sin^2 A = 1 - \frac{16}{25} = \frac{25-16}{25} = \frac{9}{25}$

$\therefore \sin A = \frac{3}{5}$ [$\because$ A ধনাত্মক সূক্ষ্মকোণ]

আবার, $\sin B = \frac{5}{13}$

বা, $\sin^2 B = \frac{25}{169}$

বা, $1 - \cos^2 B = \frac{25}{169}$

বা, $\cos^2 B = 1 - \frac{25}{169} = \frac{169 - 25}{169} = \frac{144}{169}$

$\because \cos B = \frac{12}{13}$ [$\because$ B ধনাত্মক সূক্ষ্মকোণ]

এখন, $\sin(A - B) = \sin A \cos B - \cos A \sin B$

$= \frac{3}{5} \cdot \frac{12}{13} - \frac{4}{5} \cdot \frac{5}{13} = \frac{36}{65} - \frac{4}{13}$

$= \frac{36 - 20}{65} = \frac{16}{65}$ **(Ans.)**

এবং $\cos(A + B) = \cos A \cos B - \sin A \sin B$

$= \frac{4}{5} \cdot \frac{12}{13} - \frac{3}{5} \cdot \frac{5}{13} = \frac{48}{65} - \frac{3}{13}$

$= \frac{48 - 15}{65} = \frac{33}{65}$ **(Ans.)**

3. দেওয়া আছে, $\tan A = \frac{2}{11}$ এবং $\tan B = \frac{7}{24}$

$\therefore \cot A = \frac{11}{2}$ এবং $\cot B = \frac{24}{7}$

$\therefore \cot(A - B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A} = \dfrac{\frac{11}{2} \cdot \frac{24}{7} + 1}{\frac{24}{7} - \frac{11}{2}}$

$= \frac{264 + 14}{48 - 77} = -\frac{278}{29}$ **(Ans.)**

এবং $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = \dfrac{\frac{2}{11} + \frac{7}{24}}{1 - \frac{2}{11} \cdot \frac{7}{24}}$

$= \frac{48 + 77}{264 - 14} = \frac{125}{250} = \frac{1}{2}$ **(Ans.)**

4. (i) $\cos 17°40' \sin 77°40' + \cos 107°40' \sin 12°20'$
$= \cos 17°40' \sin 77°40' + \cos(90° + 17°40') \sin(90° - 77°40')$
$= \sin 77°40' \cos 17°40' - \cos 77°40' \sin 17°40'$
$= \sin(77°40' - 17°40')$
$= \sin 60° = \frac{\sqrt{3}}{2}$ **(Ans.)**

(ii) $\dfrac{\tan 65°35' - \cot 69°25'}{1 + \tan 65°35' \cot 69°25'}$

$= \dfrac{\tan 65°35' - \cot(90° - 20°35')}{1 + \tan 65°35' \cot(90° - 20°35')}$

$= \dfrac{\tan 65°35' - \tan 20°35'}{1 + \tan 65°35' \tan 20°35'}$

$= \tan(65°35' - 20°35')$
$= \tan 45° = 1$ **(Ans.)**

5.(i) বামপক্ষ $= \cos(x - 60°) \cos(x - 30°) - \sin(x - 60°) \sin(x + 330°)$
$= \cos(x - 60°) \cos(x - 30°) - \sin(x - 60°) \sin\{4 \times 90° + (x - 30°)\}$
$= \cos(x - 60°) \cos(x - 30°) - \sin(x - 60°) \sin(x - 30°)$
$= \cos(x - 60° + x - 30°)$
$= \cos(2x - 90°) = \cos\{-(90° - 2x)\}$
$= \cos(90° - 2x) = \sin 2x$
= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin x \sin(x + 30°) + \cos x \sin(x + 120°)$
$= \sin x \sin(x + 30°) + \cos x \sin(90° + x + 30°)$
$= \sin x \sin(x + 30°) + \cos x \cos(x + 30°)$
$= \cos(x + 30° - x)$
$= \cos 30°$
$= \frac{\sqrt{3}}{2}$
= ডানপক্ষ

$\therefore \sin x \sin(x + 30°) + \cos x \sin(x + 120°) = \frac{\sqrt{3}}{2}$ (প্রমাণিত)

6. বামপক্ষ $= \cos A + \cos(120° - A) + \cos(120° + A)$
$= \cos A + 2\cos 120° \cos A$
$= \cos A + 2 \cdot \frac{-1}{2} \cdot \cos A$
$= \cos A - \cos A = 0$
= ডানপক্ষ (প্রমাণিত)

7. বামপক্ষ $= \frac{\sin(B-C)}{\cos B \cos C} + \frac{\sin(C-A)}{\cos C \cos A} + \frac{\sin(A-B)}{\cos A \cos B}$

$= \frac{\sin B\cos C - \cos B\sin C}{\cos B \cos C} + \frac{\sin C\cos A - \cos C\sin A}{\cos C\cos A} + \frac{\sin A\cos B - \cos A\sin B}{\cos A\cos B}$

$= \frac{\sin B\cos C}{\cos B\cos C} - \frac{\cos B\sin C}{\cos B\cos C} + \frac{\sin C\cos A}{\cos C\cos A} - \frac{\cos C\sin A}{\cos C\cos A} + \frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}$

$= \tan B - \tan C + \tan C - \tan A + \tan A - \tan B$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

8. (i) বামপক্ষ $= \frac{\cos(45° + A) + \cos(45° - A)}{\cos(45° - A) - \cos(45° + A)}$

$= \frac{2\cos 45° \cos A}{2\sin 45° \sin A} = \frac{\frac{1}{\sqrt{2}}.\cos A}{\frac{1}{\sqrt{2}}.\sin A}$

$= \cot A =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \frac{\tan\left(\frac{\pi}{4}+\theta\right) - \tan\left(\frac{\pi}{4}-\theta\right)}{\tan\left(\frac{\pi}{4}+\theta\right) + \tan\left(\frac{\pi}{4}-\theta\right)}$

$= \frac{\frac{\sin\left(\frac{\pi}{4}+\theta\right)}{\cos\left(\frac{\pi}{4}+\theta\right)} - \frac{\sin\left(\frac{\pi}{4}-\theta\right)}{\cos\left(\frac{\pi}{4}-\theta\right)}}{\frac{\sin\left(\frac{\pi}{4}+\theta\right)}{\cos\left(\frac{\pi}{4}+\theta\right)} + \frac{\sin\left(\frac{\pi}{4}-\theta\right)}{\cos\left(\frac{\pi}{4}-\theta\right)}}$

$= \frac{\sin\left(\frac{\pi}{4}+\theta\right)\cos\left(\frac{\pi}{4}-\theta\right) - \cos\left(\frac{\pi}{4}+\theta\right)\sin\left(\frac{\pi}{4}-\theta\right)}{\sin\left(\frac{\pi}{4}+\theta\right)\cos\left(\frac{\pi}{4}-\theta\right) + \cos\left(\frac{\pi}{4}+\theta\right)\sin\left(\frac{\pi}{4}-\theta\right)}$

$= \frac{\sin\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right)}{\sin\left(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right)} = \frac{\sin 2\theta}{\sin\frac{\pi}{2}} = \sin 2\theta$

$=$ ডানপক্ষ (প্রমাণিত)

9. বামপক্ষ $= 2\sin\left(\frac{\pi}{4}+A\right)\cos\left(\frac{\pi}{4}+B\right)$

$= \sin\left(\frac{\pi}{4}+A+\frac{\pi}{4}+B\right) + \sin\left(\frac{\pi}{4}+A-\frac{\pi}{4}-B\right)$

$= \sin\left(\frac{\pi}{2}+A+B\right) + \sin(A-B)$

$= \cos(A+B) + \sin(A-B) =$ ডানপক্ষ (প্রমাণিত)

10. (i) আমরা পাই,

$\tan 70° = \tan(50° + 20°)$

বা, $\tan 70° = \frac{\tan 50° + \tan 20°}{1 - \tan 50° \tan 20°}$

বা, $\tan 70° - \tan 70° \tan 50° \tan 20° = \tan 50° + \tan 20°$

বা, $\tan 70° - \tan(90° - 20°)\tan 50° \tan 20° = \tan 50° + \tan 20°$

বা, $\tan 70° - \cot 20° \tan 50°.\frac{1}{\cot 20°} = \tan 50° + \tan 20°$

বা, $\tan 70° - \tan 50° = \tan 50° + \tan 20°$

$\therefore \tan 70° = \tan 20° + 2\tan 50°$ (প্রমাণিত)

(ii) আমরা পাই,

$\tan 18° = \tan(54° - 36°)$

$= \frac{\tan 54° - \tan 36°}{1 + \tan 54° \tan 36°}$

$= \frac{\tan 54° - \tan 36°}{1 + \tan(90° - 36°)\tan 36°}$

$= \frac{\tan 54° - \tan 36°}{1 + \cot 36°.\frac{1}{\cot 36°}}$

$= \frac{\tan 54° - \tan 36°}{1 + 1}$

বা, $\tan 54° - \tan 36° = 2\tan 18°$

$\therefore \tan 54° = \tan 36° + 2\tan 18°$ (প্রমাণিত)

(iii) $\tan\frac{\pi}{20} + \tan\frac{\pi}{5} + \tan\frac{\pi}{20}\tan\frac{\pi}{5} = 1$

আমরা জানি, $\tan\frac{\pi}{4} = 1$

বা, $\tan\frac{5\pi}{20} = 1$

বা, $\tan\left(\frac{\pi}{20} + \frac{4\pi}{20}\right) = 1$

বা, $\frac{\tan\frac{\pi}{20} + \tan\frac{\pi}{5}}{1 - \frac{\pi}{20}\tan\frac{\pi}{20}\tan\frac{\pi}{5}}$

$\therefore \tan\frac{\pi}{20} + \tan\frac{\pi}{5} + \tan\frac{\pi}{20}\tan\frac{\pi}{5} = 1$ (প্রমাণিত)

11. বামপক্ষ $= \tan\left(\alpha + \frac{\pi}{3}\right) + \tan\left(\alpha - \frac{\pi}{3}\right)$

$= \frac{\sin\left(\alpha+\frac{\pi}{3}\right)}{\cos\left(\alpha+\frac{\pi}{3}\right)} + \frac{\sin\left(\alpha-\frac{\pi}{3}\right)}{\cos\left(\alpha-\frac{\pi}{3}\right)}$

$$= \frac{\sin\left(\alpha+\frac{\pi}{3}\right)\cos\left(\alpha-\frac{\pi}{3}\right)+\cos\left(\alpha+\frac{\pi}{3}\right)\sin\left(\alpha-\frac{\pi}{3}\right)}{\cos\left(\alpha+\frac{\pi}{3}\right)\cos\left(\alpha-\frac{\pi}{3}\right)}$$

$$= \frac{\sin\left(\alpha+\frac{\pi}{3}+\alpha-\frac{\pi}{3}\right)}{\cos\left(\alpha+\frac{\pi}{3}\right)\cos\left(\alpha-\frac{\pi}{3}\right)}$$

$$= \frac{\sin 2\alpha}{\cos^2\frac{\pi}{3}-\sin^2\alpha}$$

$[\cos(A+B)\cos(A-B) = \cos^2 B - \sin^2 A]$

$$= \frac{\sin 2\alpha}{\frac{1}{4}-\sin^2\alpha} = \frac{4\sin 2\alpha}{1-4\sin^2\alpha}$$ = ডানপক্ষ (প্রমাণিত)

12. বামপক্ষ $= \cot(A+B)+\cot(A-B)$

$$= \frac{\cos(A+B)}{\sin(A+B)}+\frac{\cos(A-B)}{\sin(A-B)}$$

$$= \frac{\cos(A+B)\sin(A-B)+\cos(A-B)\sin(A+B)}{\sin(A+B)\sin(A-B)}$$

$$= \frac{\sin(A+B+A-B)}{\sin(A+B)\sin(A-B)}$$

$$= \frac{\sin 2A}{\sin^2 A-\sin^2 B}$$ = ডানপক্ষ (প্রমাণিত)

13. (i) বামপক্ষ $= \frac{\cos 8^\circ+\sin 8^\circ}{\cos 8^\circ-\sin 8^\circ}$

$$= \frac{1+\tan 8^\circ}{1-\tan 8^\circ}$$

[লব ও হরকে $\cos 8^\circ$ দ্বারা ভাগ করে]

$$= \frac{\tan 45^\circ+\tan 8^\circ}{1-\tan 45^\circ \tan 8^\circ} \quad [\because \tan 45^\circ = 1]$$

$= \tan(45^\circ+8^\circ)$

$= \tan 53^\circ$ = ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \frac{\cos 25^\circ+\sin 25^\circ}{\cos 25^\circ-\sin 25^\circ}$

$$= \frac{1+\tan 25^\circ}{1-\tan 25^\circ}$$

[হর ও লবকে $\cos 25^\circ$ দ্বারা ভাগ করে]

$$= \frac{\tan 45^\circ+\tan 25^\circ}{1-\tan 45^\circ.\tan 25^\circ}$$

$$= \frac{1+\tan 45^\circ.\tan 25^\circ}{\tan 45^\circ-\tan 25^\circ}$$

$$= \frac{1}{\tan(45^\circ-25^\circ)} = \frac{1}{\tan 20^\circ}$$

$= \cot 20^\circ$ = ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \frac{\sin 75^\circ+\sin 15^\circ}{\sin 75^\circ-\sin 15^\circ}$

$$= \frac{2\sin\frac{75^\circ+15^\circ}{2}\cos\frac{75^\circ-15^\circ}{2}}{2\cos\frac{75^\circ+15^\circ}{2}\sin\frac{75^\circ-15^\circ}{2}}$$

$$= \frac{2\sin 45^\circ\cos 30^\circ}{2\cos 45^\circ\sin 30^\circ}$$

$= \tan 45^\circ \cot 30^\circ$

$= 1.\sqrt{3} = \sqrt{3}$ = ডানপক্ষ (প্রমাণিত)

(iv) বামপক্ষ $= \frac{\cos 27^\circ-\cos(90^\circ-27^\circ)}{\cos 27^\circ+\cos(90^\circ-27^\circ)}$

$$= \frac{\cos 27^\circ-\sin 27^\circ}{\cos 27^\circ+\sin 27^\circ}$$

$$= \frac{1-\tan 27^\circ}{1+\tan 27^\circ} = \frac{\tan 45^\circ-\tan 27^\circ}{1+\tan 45^\circ\tan 27^\circ}$$

$= \tan(45^\circ-27^\circ) = \tan 18^\circ$ = ডানপক্ষ

14. দেওয়া আছে, $\cot\alpha+\cot\beta = a$ $[\because \tan\alpha+\tan\beta = b]$

বা, $\frac{1}{\tan\alpha}+\frac{1}{\tan\beta} = a$

বা, $\frac{\tan\beta+\tan\alpha}{\tan\alpha\tan\beta} = a$

বা, $\frac{b}{\tan\alpha\tan\beta} = a$

বা, $a\tan\alpha\tan\beta = b$

বা, $\tan\alpha\ \text{tab}\ \beta = \frac{b}{a}$

আবার, দেওয়া আছে, $\alpha+\beta = \theta$

বা, $\tan(\alpha+\beta) = \tan\theta$

বা, $\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta} = \tan\theta$

বা, $\frac{b}{1-\frac{b}{a}} = \tan\theta$

বা, $\frac{ab}{a-b} = \tan\theta$

$\therefore \tan\theta = \frac{ab}{a-b}$ (প্রমাণিত)

15. দেওয়া আছে, $\cot\beta-\cot\alpha = b$

বা, $\frac{1}{\tan\beta}-\frac{1}{\tan\alpha} = b$

বা, $\frac{\tan\alpha-\tan\beta}{\tan\alpha\tan\beta} = b$ $[\because \tan\alpha-\tan\beta = a]$

বা, $\frac{a}{\tan\alpha\tan\beta} = b$

বা, $\tan\alpha\tan\beta = \frac{a}{b}$

$\cot(\alpha-\beta) = \dfrac{1}{\tan(\alpha-\beta)}$

$= \dfrac{1}{\dfrac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}} = \dfrac{1+\tan\alpha\tan\beta}{\tan\alpha - \tan\beta}$

$= \dfrac{1+\dfrac{a}{b}}{a} = \dfrac{\dfrac{a+b}{b}}{a} = \dfrac{a+b}{ab} = \dfrac{a}{ab}+\dfrac{b}{ab} = \dfrac{1}{b}+\dfrac{1}{a}$

$= \dfrac{1}{a}+\dfrac{1}{b}$ (প্রমাণিত)

16. বামপক্ষ = $(1+\tan A)(1+\tan B)$

$= \left\{1+\tan\left(\dfrac{\pi}{4}-B\right)\right\}(1+\tan B)$

$[\because A+B=\dfrac{\pi}{4} \Rightarrow A=\dfrac{\pi}{4}-B]$

$= \left\{1+\dfrac{\tan\dfrac{\pi}{4}-\tan B}{1+\tan\dfrac{\pi}{4}.\tan B}\right\}(1+\tan B)$

$= \left(1+\dfrac{1-\tan B}{1+\tan B}\right)(1+\tan B)$

$= \dfrac{1+\tan B+1-\tan B}{(1+\tan B)}(1+\tan B)$

$= 2$

= ডানপক্ষ (দেখানো হলো)

17. দেওয়া আছে,

$\cos\alpha\cos\beta - \sin\alpha\sin\beta = 1$

বা, $\cos(\alpha+\beta) = 1$

বা, $\cos(\alpha+\beta) = \cos 0°$

$\therefore \alpha+\beta = 0$

বামপক্ষ = $1+\cot\alpha\tan\beta$

$= 1+\dfrac{\cos\alpha}{\sin\alpha}\dfrac{\sin\beta}{\cos\beta}$

$= \dfrac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$= \dfrac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta}$

$= \dfrac{0}{\sin\alpha\cos\beta}$ $[\because \alpha+\beta=0]$

$= 0$

= ডানপক্ষ (দেখানো হলো)

18. দেওয়া আছে, $a\sin(\theta+\alpha) = b\sin(\theta+\beta)$

বা, $a(\sin\theta\cos\alpha+\cos\theta\sin\alpha) = b(\sin\theta\cos\beta + \cos\theta\sin\beta)$

বা, $\sin\theta(a\cos\alpha - b\cos\beta) = \cos\theta(b\sin\beta - a\sin\alpha)$

বা, $\dfrac{\cos\theta}{\sin\theta} = \dfrac{a\cos\alpha - b\cos\beta}{b\sin\beta - a\sin\alpha}$

$\therefore \cot\theta = \dfrac{a\cos\alpha - b\cos\beta}{b\sin\beta - a\sin\alpha}$ (দেখানো হলো)

19. বামপক্ষ = $\dfrac{\tan(\alpha-\beta)+\tan\beta}{1-\tan(\alpha-\beta)\tan\beta}$

$= \tan(\alpha-\beta+\beta)$

$= \tan\alpha$

দেওয়া আছে, $\sin\alpha = \dfrac{m^2-n^2}{m^2+n^2}$

বা, $\sin^2\alpha = \left(\dfrac{m^2-n^2}{m^2+n^2}\right)^2$

বা, $1-\cos^2\alpha = \dfrac{(m^2-n^2)^2}{(m^2+n^2)^2}$

বা, $\cos^2\alpha = 1-\dfrac{(m^2-n^2)^2}{(m^2+n^2)^2}$

বা, $\cos^2\alpha = \dfrac{(m^2+n^2)^2-(m^2-n^2)^2}{(m^2+n^2)^2}$

বা, $\cos^2\alpha = \dfrac{4m^2n^2}{(m^2+n^2)^2}$

$\therefore \cos\alpha = \dfrac{2mn}{m^2+n^2}$

$\therefore \tan\alpha = \dfrac{\sin\alpha}{\cos\alpha} = \dfrac{m^2-n^2}{2mn}$ (দেখানো হলো)

20. দেওয়া আছে, $\tan\theta = k\tan\phi$

বা, $\dfrac{\tan\theta}{\tan\phi} = k$ বা, $\dfrac{\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\sin\phi}{\cos\phi}} = k$

বা, $\dfrac{\sin\theta\cos\phi}{\cos\theta\sin\phi} = k$

বা, $\dfrac{\sin\theta\cos\phi - \cos\theta\sin\phi}{\sin\theta\cos\phi+\cos\theta\sin\phi} = \dfrac{k-1}{k+1}$

[বিয়োজন-যোজন করে]

বা, $\dfrac{\sin(\theta-\phi)}{\sin(\theta+\phi)} = \dfrac{k-1}{k+1}$

বা, $\sin(\theta-\phi) = \dfrac{k-1}{k+1}\sin(\theta+\phi)$

$\therefore \sin(\theta-\phi) = \dfrac{k-1}{k+1}\sin\alpha$ $[\because \alpha = \theta+\phi]$ (প্রমাণিত)

21. দেওয়া আছে, $\dfrac{\sin(\alpha+\gamma)}{\sin\alpha} = \dfrac{2\sin(\beta+\gamma)}{\sin\beta}$

বা, $\dfrac{\sin\alpha\cos\gamma+\cos\alpha\sin\gamma}{\sin\alpha} = \dfrac{2(\sin\beta\cos\gamma+\cos\beta\sin\gamma)}{\sin\beta}$

বা, $\cos\gamma + \cot\alpha\sin\gamma = 2\cos\gamma + 2\cot\beta\sin\gamma$

বা, $\cot\alpha\sin\gamma - 2\cot\beta\sin\gamma = \cos\gamma$

বা, $\cot\alpha - 2\cot\beta = \cot\gamma$ [$\sin\gamma$ দ্বারা ভাগ করে]

$\therefore \cot\alpha - \cot\gamma = 2\cot\beta$ (প্রমাণিত)

22. দেওয়া আছে, $\tan\beta = \dfrac{2\sin\alpha\sin\gamma}{\sin(\alpha+\gamma)}$

বা, $\dfrac{\sin(\alpha+\gamma)}{\sin\alpha\sin\gamma} = \dfrac{2}{\tan\beta}$

বা, $\dfrac{\sin\alpha\cos\gamma + \cos\alpha\sin\gamma}{\sin\alpha\sin\gamma} = \dfrac{2}{\tan\beta}$

$\therefore \cot\gamma + \cot\alpha = 2\cot\beta$ (প্রমাণিত)

23. বামপক্ষ $= a\cos\theta + b\sin\theta$

$= a\left(\cos\theta + \dfrac{b}{a}\sin\theta\right)$

$= a(\cos\theta + \tan\alpha\sin\theta)$ [$\therefore \tan\alpha = \dfrac{b}{a}$]

$= \dfrac{a}{\cos\alpha}(\cos\theta\cos\alpha + \sin\alpha\sin\theta)$

$= \dfrac{a}{\cos\alpha}\cos(\theta-\alpha)$

$= a\sec\alpha\cos(\theta-\alpha)$

$= a\sqrt{\sec^2\alpha}\cos(\theta-\alpha)$

$= a\sqrt{1+\tan^2\alpha}\cos(\theta-\alpha)$

$= a\sqrt{1+\dfrac{b^2}{a^2}}\cos(\theta-\alpha)$

$= \sqrt{a^2+b^2}\cos(\theta-\alpha)$

= ডানপক্ষ (প্রমাণিত)

24. এখানে, $\sqrt{2}\cos A = \cos B + \cos^3 B$ ... ... (i)

$\sqrt{2}\sin A = \sin B - \sin^3 B$ ... ... (ii)

(i) ও (ii) কে বর্গ করে যোগ করে পাই,

$2(\cos^2 A + \sin^2 A) = \cos^2 B + \sin^2 B + 2(\cos^4 B - \sin^4 B) + \cos^6 B + \sin^6 B$

বা, $2 = 1 + 2(\cos2B + \sin2B)(\cos2B - \sin2B) + (\cos2B + \sin2B)3 - 3\cos2B.\sin2B.(\cos2B + \sin2B)$

বা, $1 = 2.1(\cos^2B - \sin^2B) + 1^3 - 3\cos^2B.\sin^2B$

বা, $3\cos^2B(1-\cos^2B) - 2(\cos^2B - 1 + \cos^2B) = 0$

বা, $3\cos^2B - 3\cos^4B - 2\cos^2B + 2 - 2\cos^2B = 0$

বা, $3\cos^4B + \cos^2B - 2 = 0$

বা, $(3\cos^2B - 2)(\cos^2B + 1) = 0$

কিন্তু $\cos^2B + 1 \neq 0$ $\therefore 3\cos^2B - 2 = 0$

$\therefore \cos B = \pm\sqrt{\dfrac{2}{3}}$

এখন, $\sin(A-B)$

$= \sin A\cos B - \cos A\sin B$

$= \cos B.\dfrac{1}{\sqrt{2}}(\sin B - \sin^3 B) - \sin B\dfrac{1}{\sqrt{2}}(\cos B + \cos^3 B)$ [(i) ও (ii) থেকে]

$= -\dfrac{1}{\sqrt{2}}\sin B\cos B(\sin^2B + \cos^2B)$

$= -\dfrac{1}{\sqrt{2}}\sin B\cos B$

$= -\dfrac{1}{\sqrt{2}}\cos B\sqrt{1-\cos^2B}$

$= -\dfrac{1}{\sqrt{2}}\left(\pm\sqrt{\dfrac{2}{3}}\right).\sqrt{1-\left(\pm\sqrt{\dfrac{2}{3}}\right)2}$

[$\cos B$ এর মান বসিয়ে]

$= \pm\dfrac{1}{\sqrt{2}}.\dfrac{\sqrt{2}}{\sqrt{3}}.\dfrac{1}{\sqrt{3}} = \pm\dfrac{1}{3}$

$\therefore \sin(A-B) = \pm\dfrac{1}{3}$ (প্রমাণিত)

25.(i) দেওয়া আছে, $a\cos(x+\alpha) = b\cos(x-\alpha)$

বা, $\dfrac{a}{b} = \dfrac{\cos(x-\alpha)}{\cos(x+\alpha)}$

বা, $\dfrac{a+b}{a-b} = \dfrac{\cos(x-\alpha)+\cos(x+\alpha)}{\cos(x-\alpha)-\cos(x+\alpha)}$

[যোজন-বিয়োজন করে]

বা, $\dfrac{a+b}{a-b} = \dfrac{\cos x\cos\alpha + \sin x\sin\alpha + \cos x\cos\alpha - \sin x\sin\alpha}{\cos x\cos\alpha + \sin x\sin\alpha - \cos x\cos\alpha + \sin x\sin\alpha}$

বা, $\dfrac{a+b}{a-b} = \dfrac{2\cos x\cos\alpha}{2\sin x\sin\alpha}$

বা, $(a+b)\sin x\sin\alpha = (a-b)\cos x\cos\alpha$

বা, $(a+b)\dfrac{\sin x}{\cos x} = (a-b)\dfrac{\cos\alpha}{\sin\alpha}$

$\therefore (a+b)\tan x = (a-b)\cot\alpha$. (প্রমাণিত)

(ii) দেওয়া আছে, $a\sin(x+\theta) = b\sin(x-\theta)$

বা, $a(\sin x\cos\theta + \cos x\sin\theta) = b(\sin x\cos\theta - \cos x\sin\theta)$

বা, $(a-b)\sin x\cos\theta + (a+b)\cos x\sin\theta = 0$

বা, $(a-b)\tan x + (a+b)\tan\theta = 0$.

[উভয় পক্ষকে $\cos\theta\cos x$ দ্বারা ভাগ করে] (প্রমাণিত)

26. $\tan\beta = \dfrac{n\sin\alpha\cos\alpha}{1-n\sin^2\alpha}$

$= \dfrac{\dfrac{n\sin\alpha\cos\alpha}{\cos^2\alpha}}{\dfrac{1}{\cos^2\alpha} - \dfrac{n\sin^2\alpha}{\cos^2\alpha}}$

[লব ও হরকে $\cos^2\alpha$ দ্বারা ভাগ করে]

$= \dfrac{n\tan\alpha}{\sec^2\alpha - n\tan^2\alpha}$

$= \dfrac{n\tan\alpha}{1+\tan^2\alpha - n\tan^2\alpha}$

বামপক্ষ $= \tan(\alpha - \beta)$

$= \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \dfrac{\tan\alpha - \dfrac{n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}{1 + \tan\alpha.\dfrac{n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}$

$= \dfrac{\dfrac{\tan\alpha + \tan^3\alpha - n\tan^3\alpha - n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}{\dfrac{1 + \tan^2\alpha - n\tan^2\alpha - n\tan^2\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}$

$= \dfrac{\tan\alpha\,(1 + \tan^2\alpha) - n\tan\alpha(1 + \tan^2\alpha)}{1 + \tan^2\alpha}$

$= \dfrac{(1 + \tan^2\alpha)\,(\tan\alpha - n\tan\alpha)}{1 + \tan^2\alpha}$

$= (1 - n)\tan\alpha$

= ডানপক্ষ

$\therefore$ বামপক্ষ = ডানপক্ষ (দেখানো হলো)

27. যেহেতু θ কোণকে α ও β অংশে বিভক্ত করা হয়েছে,

$\therefore \theta = \alpha + \beta.$

আবার, $\tan\alpha$ ঃ $\tan\beta = x$ ঃ y

বা, $\dfrac{\tan\alpha}{\tan\beta} = \dfrac{x}{y}$

বা, $\dfrac{\sin\alpha\cos\beta}{\cos\alpha\sin\beta} = \dfrac{x}{y}$

$\therefore \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\cos\beta + \cos\alpha\sin\beta} = \dfrac{x - y}{x + y}$

[যোজন - বিয়োজন প্রক্রিয়ায়]

বা, $\dfrac{\sin(\alpha - \beta)}{\sin(\alpha + \beta)} = \dfrac{x - y}{x + y}$

$\therefore \sin(\alpha - \beta) = \dfrac{x - y}{x + y}\sin(\alpha + \beta)$

$= \dfrac{x - y}{x + y}\sin\theta.$ (প্রমাণিত)

28. দেওয়া আছে,

$\cot\theta = \dfrac{a\cos x - b\cos y}{a\sin x + b\sin y}$

বা, $\dfrac{\cos\theta}{\sin\theta} = \dfrac{a\cos x - b\cos y}{a\sin x + b\sin y}$

বা, $a\sin\theta\cos x - b\sin\theta\cos y = a\cos\theta\sin x + b\cos\theta\sin y$

বা, $a\sin\theta\cos x - a\cos\theta\sin x = b\sin\theta\cos y + b\cos\theta\sin y$

বা, $a\sin(\theta - x) = b\sin(\theta + y)$

$\therefore \dfrac{\sin(\theta - x)}{\sin(\theta + y)} = \dfrac{b}{a}$ (দেখানো হলো)

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-7.2.6 | পৃষ্ঠা-২৮৬

$\cos A = \cos B\cos C$

$\Rightarrow \cos\{\pi - (B + C)\} = \cos B\cos C\ [\because A+B+C=\pi]$

$\Rightarrow -\cos(B + C) = \cos B\cos C$

$\Rightarrow \cos B\cos C + \cos(B + C) = 0$

$\Rightarrow \cos B\cos C + \cos B\cos C - \sin B\sin C = 0$

$\Rightarrow 2\cos B\cos C = \sin B\sin C$

$\Rightarrow \dfrac{2\cos B\cos C}{\sin B\sin C} = 1$

$\Rightarrow 2\,.\cot B\cot C = 1$

$\therefore \cot B\cot C = \dfrac{1}{2}$ (প্রমাণিত)

অনুশীলনী-7(C) এর সমাধান

1. (i) বামপক্ষ $= \cos40° + \cos80° + \cos160°$

$= \cos40° + (\cos160° + \cos80°)$

$= \cos40° + 2\cos\left(\dfrac{160° + 80°}{2}\right)\cos\left(\dfrac{160° - 80°}{2}\right)$

$= \cos40° + 2\cos120°\cos40°$

$= \cos40° + 2\left(-\dfrac{1}{2}\right)\cos40°\ [\because \cos120° = -\dfrac{1}{2}]$

$= \cos40° - \cos40°$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin50° - \sin70° + \sin10°$

$= (\sin50° - \sin70°) + \sin10°$

$= -(\sin70° - \sin50°) + \sin10°$

$= -2\cos\left(\dfrac{70° + 50°}{2}\right).\sin\left(\dfrac{70° - 50°}{2}\right) + \sin10°$

$= -2\cos60°.\sin10° + \sin10°$

$= -2\left(\dfrac{1}{2}\right)\sin10° + \sin10°\ [\because \cos60° = \dfrac{1}{2}]$

$= -\sin10° + \sin10°$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

2. (i) বামপক্ষ $= \cos(60° - \theta) + \cos(60° + \theta) - \cos\theta$

$= 2\cos\left(\dfrac{60° - \theta + 60° + \theta}{2}\right)\cos\left(\dfrac{60° - \theta - 60° - \theta}{2}\right) - \cos\theta$

$= 2\cos60°\cos(-\theta) - \cos\theta$

$= 2\left(\dfrac{1}{2}\right).\cos\theta - \cos\theta$

$= \cos\theta - \cos\theta = 0 =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin\theta + \sin(120° + \theta) + \sin(240° + \theta)$

$= \sin(240° + \theta) + \sin\theta + \sin(120° + \theta)$

$= 2\sin(120° + \theta)\cos120° + \sin(120° + \theta)$

$= 2\sin(120° + \theta).\left(-\dfrac{1}{2}\right) + \sin(120° + \theta)$

$= -\sin(120° + \theta) + \sin(120° + \theta)$
$= 0$
$=$ ডানপক্ষ
$\therefore \sin\theta + \sin(120° + \theta) + \sin(240° + \theta) = 0$
(প্রমাণিত)

(iii) বামপক্ষ $= \sin\theta \sin(60° - \theta) \sin(60° + \theta)$

$= \frac{1}{2}\sin\theta \{2\sin(60° - \theta)\sin(60° + \theta)\}$

$= \frac{1}{2}\sin\theta \{\cos(60° - \theta - 60° - \theta)$
$- \cos(60° - \theta + 60° + \theta)\}$

$= \frac{1}{2}\sin\theta [\{\cos(-2\theta)\} - \cos 120°]$

$= \frac{1}{2}\sin\theta\left[\cos 2\theta - \left(-\frac{1}{2}\right)\right]$

$= \frac{1}{2}\sin\theta\cos 2\theta + \frac{1}{4}\sin\theta$

$= \frac{1}{2}\cdot\frac{1}{2}\cdot 2\cos 2\theta\sin\theta + \frac{1}{4}\sin\theta$

$= \frac{1}{4}\{\sin(2\theta + \theta) - \sin(2\theta - \theta)\} + \frac{1}{4}\sin\theta$

$= \frac{1}{4}\sin 3\theta - \frac{1}{4}\sin\theta + \frac{1}{4}\sin\theta$

$= \frac{1}{4}\sin 3\theta =$ ডানপক্ষ (প্রমাণিত)

3. (i) বামপক্ষ $= \cos 10° \cos 30° \cos 50° \cos 70°$

$= \cos 10°. \frac{\sqrt{3}}{2}\cos 50° \cos 70°$

$= \frac{\sqrt{3}}{2}.\cos 10°.\frac{1}{2}(2\cos 70° \cos 50°)$

$= \frac{\sqrt{3}}{4}\cos 10° \{\cos(70° + 50°) + \cos(70° - 50°)\}$

$= \frac{\sqrt{3}}{4}\cos 10° (\cos 120° + \cos 20°)$

$= \frac{\sqrt{3}}{4}\cos 10° \cos 120° + \frac{\sqrt{3}}{4}\cos 10° \cos 20°$

$= \frac{\sqrt{3}}{4}\cos 10°\left(-\frac{1}{2}\right) + \frac{\sqrt{3}}{4}.\frac{1}{2}(2\cos 20° \cos 10°)$

$= -\frac{\sqrt{3}}{8}\cos 10° + \frac{\sqrt{3}}{8}\{\cos(20° + 10°)$
$+ \cos(20° - 10°)\}$

$= -\frac{\sqrt{3}}{8}\cos 10° + \frac{\sqrt{3}}{8}\cos 30° + \frac{\sqrt{3}}{8}\cos 10°$

$= \frac{\sqrt{3}}{8}\cos 30° = \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}$

$= \frac{3}{16} =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin 20° \sin 40° \sin 60° \sin 80°$

$= \sin 20° \sin 40°.\frac{\sqrt{3}}{2}\sin 80°$

$= \frac{\sqrt{3}}{2}\sin 20°.\frac{1}{2}(2\sin 80° \sin 40°)$

$= \frac{\sqrt{3}}{4}\sin 20° \{\cos(80° - 40°) - \cos(80° + 40°)\}$

$= \frac{\sqrt{3}}{4}\sin 20° (\cos 40° - \cos 120°)$

$= \frac{\sqrt{3}}{4}\sin 20° \cos 40° - \frac{\sqrt{3}}{4}\sin 20° \cos 120°$

$= \frac{\sqrt{3}}{4}.\frac{1}{2}(2\cos 40° \sin 20°) - \frac{\sqrt{3}}{4}\sin 20°\left(-\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{8}\{\sin(40° + 20°) - \sin(40° - 20°)\} + \frac{\sqrt{3}}{8}\sin 20°$

$= \frac{\sqrt{3}}{8}\sin 60° - \frac{\sqrt{3}}{8}\sin 20° + \frac{\sqrt{3}}{8}\sin 20°$

$= \frac{\sqrt{3}}{8}\sin 60° = \frac{\sqrt{3}}{8}.\frac{\sqrt{3}}{2}$

$= \frac{3}{16} =$ ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \cos 20° \cos 40° \cos 60° \cos 80°$

$= \frac{1}{2}\cos 20°\left(\frac{1}{2}\right)(2\cos 80° \cos 40°)$

$= \frac{1}{4}\cos 20° (\cos 120° + \cos 40°)$

$= \frac{1}{4}\cos 20°\left(\frac{-1}{2} + \cos 40°\right)$

$= \frac{-1}{8}\cos 20° + \frac{1}{4}\cos 40° \cos 20°$

$= \frac{-1}{8}\cos 20° + \frac{1}{8}.2\cos 40° \cos 20°$

$= \frac{-1}{8}\cos 20° + \frac{1}{8}(\cos 60° + \cos 20°)$

$= \frac{-1}{8}\cos 20° + \frac{1}{8}\left(\frac{1}{2} + \cos 20°\right)$

$= \frac{-1}{8}\cos 20° + \frac{1}{16} + \frac{1}{8}\cos 20°$

$= \frac{1}{16} =$ ডানপক্ষ

$\therefore \cos 20° \cos 40° \cos 60° \cos 80° = \frac{1}{16}$ (প্রমাণিত)

4. (i) বামপক্ষ $= \sin 18° + \cos 18°$
$= \sin 18° + \cos(90° - 72°)$
$= \sin 18° + \sin 72°$
$= \sin 72° + \sin 18°$
$= 2\sin\frac{72° + 18°}{2}\cos\frac{72° - 18°}{2}$

$= 2\sin 45^\circ \cos 27^\circ = 2.\frac{1}{\sqrt{2}}.\cos 27^\circ$

$= \sqrt{2}\cos 27^\circ =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin27^\circ + \cos27^\circ$

$= \sin(90^\circ - 63^\circ) + \cos27^\circ$

$= \cos63^\circ + \cos27^\circ)$

$= 2\cos\frac{63^\circ + 27^\circ}{2}\cos\frac{63^\circ - 27^\circ}{2}$

$= 2\cos45^\circ \cos18^\circ$

$= 2.\frac{1}{\sqrt{2}}\cos18^\circ$

$= \sqrt{2}\cos18^\circ$

= ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \sin105^\circ + \cos105^\circ$

$= \sin105^\circ + \cos(90^\circ + 15^\circ)$

$= \sin105^\circ - \sin15^\circ$

$= 2.\cos\left(\frac{105^\circ + 15^\circ}{2}\right)\sin\left(\frac{105^\circ - 15^\circ}{2}\right)$

$= 2.\cos60^\circ.\sin45^\circ = 2.\frac{1}{2}.\frac{1}{\sqrt{2}}$

$= \frac{1}{\sqrt{2}} =$ ডানপক্ষ (প্রমাণিত)

(iv) বামপক্ষ $= \cos 85^\circ + \sin 85^\circ$

$= \cos 85^\circ + \sin(90^\circ - 5^\circ)$

$= \cos 85^\circ + \cos 5^\circ$

$= 2\cos\frac{85^\circ + 5^\circ}{2}\cos\frac{85^\circ - 5^\circ}{2}$

$= 2\cos 45^\circ \cos 40^\circ$

$= 2.\frac{1}{\sqrt{2}}\cos 40^\circ$

$= \sqrt{2}\cos 40^\circ$

= ডানপক্ষ

$\therefore \cos 85^\circ + \sin 85^\circ = \sqrt{2}\cos 40^\circ$ (প্রমাণিত)

5. বামপক্ষ $= \frac{1}{2}\operatorname{cosec}10^\circ - 2\sin70^\circ$

$= \frac{1}{2\sin 10^\circ} - 2\sin 70^\circ$

$= \frac{1 - 4\sin 70^\circ \sin 10^\circ}{2\sin 10^\circ}$

$= \frac{1 - 2(\cos 60^\circ - \cos 80^\circ)}{2\sin 10^\circ}$

$= \frac{1 - 2\left(\frac{1}{2} - \cos 80^\circ\right)}{2\sin 10^\circ} = \frac{2\cos 80^\circ}{2\sin 10^\circ}$

$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} = \frac{\sin 10^\circ}{\sin 10^\circ} = 1$

= ডানপক্ষ (প্রমাণিত)

6. বামপক্ষ $= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \left(\cos\frac{5\pi}{13} + \cos\frac{3\pi}{13}\right)$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\left(\frac{\frac{5\pi}{13} + \frac{3\pi}{13}}{2}\right)\cos\left(\frac{\frac{5\pi}{13} - \frac{3\pi}{13}}{2}\right)$

$= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + 2\cos\frac{4\pi}{13}.\cos\frac{\pi}{13}$

$= 2\cos\frac{\pi}{13}\left(\cos\frac{9\pi}{13} + \cos\frac{4\pi}{13}\right)$

$= 2\cos\frac{\pi}{13}.2\cos\left(\frac{\frac{9\pi}{13} + \frac{4\pi}{13}}{2}\right).\cos\left(\frac{\frac{9\pi}{13} - \frac{4\pi}{13}}{2}\right)$

$= 4\cos\frac{\pi}{13}\cos\frac{\pi}{2}.\cos\frac{5\pi}{26}$

$= 4\cos\frac{\pi}{13}.0.\cos\frac{5\pi}{26}\ [\because \cos\frac{\pi}{2} = 0]$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

7. বামপক্ষ $= \frac{\sin\theta + \sin5\theta + \sin9\theta + \sin13\theta}{\cos\theta + \cos5\theta + \cos9\theta + \cos13\theta}$

$= \frac{(\sin9\theta + \sin5\theta) + (\sin13\theta + \sin\theta)}{(\cos9\theta + \cos5\theta) + (\cos13\theta + \cos\theta)}$

$= \frac{2\sin7\theta\cos2\theta + 2\sin7\theta\cos6\theta}{2\cos7\theta\cos2\theta + 2\cos7\theta\cos6\theta}$

$= \frac{2\sin7\theta(\cos2\theta + \cos6\theta)}{2\cos7\theta(\cos2\theta + \cos6\theta)}$

$= \frac{\sin7\theta}{\cos7\theta} = \tan7\theta =$ ডানপক্ষ (প্রমাণিত)

8. বামপক্ষ $= \cos A + \cos B + \cos C + \cos(A + B + C)$

$= \{\cos(A + B + C) + \cos A\} + (\cos B + \cos C)$

$= 2\cos\left(\frac{A + B + C + A}{2}\right)\cos\left(\frac{A + B + C - A}{2}\right) + 2\cos\left(\frac{B + C}{2}\right)\cos\left(\frac{B - C}{2}\right)$

$= 2\cos\left(\frac{2A + B + C}{2}\right)\cos\left(\frac{B + C}{2}\right) + 2\cos\left(\frac{B + C}{2}\right)\cos\left(\frac{B - C}{2}\right)$

$= 2\cos\left(\frac{B + C}{2}\right)\left\{\cos\left(\frac{2A + B + C}{2}\right) + \cos\left(\frac{B - C}{2}\right)\right\}$

$= 2\cos\left(\frac{B + C}{2}\right).2\cos\left(\frac{\frac{2A + B + C}{2} + \frac{B - C}{2}}{2}\right).\cos\left(\frac{\frac{2A + B + C}{2} - \frac{B - C}{2}}{2}\right)$

$= 4\cos\left(\frac{B+C}{2}\right)\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A+C}{2}\right)$

$= 4\cos\frac{B+C}{2}\cos\frac{C+A}{2}\cos\frac{A+B}{2}$

= ডানপক্ষ (প্রমাণিত)

9. (i) বামপক্ষ $= \tan\frac{45^\circ+\theta}{2}\tan\frac{45^\circ-\theta}{2}$

$= \frac{\sin\frac{1}{2}(45^\circ+\theta)\sin\frac{1}{2}(45^\circ-\theta)}{\cos\frac{1}{2}(45^\circ+\theta)\cos\frac{1}{2}(45^\circ-\theta)}$

$= \frac{2\sin\left(\frac{45^\circ}{2}+\frac{\theta}{2}\right)\sin\left(\frac{45^\circ}{2}-\frac{\theta}{2}\right)}{2\cos\left(\frac{45^\circ}{2}+\frac{\theta}{2}\right)\cos\left(\frac{45^\circ}{2}-\frac{\theta}{2}\right)}$

$= \frac{\cos\left(\frac{45^\circ}{2}+\frac{\theta}{2}-\frac{45^\circ}{2}+\frac{\theta}{2}\right)-\cos\left(\frac{45^\circ}{2}+\frac{\theta}{2}+\frac{45^\circ}{2}-\frac{\theta}{2}\right)}{\cos\left(\frac{45^\circ}{2}+\frac{\theta}{2}+\frac{45^\circ}{2}-\frac{\theta}{2}\right)+\cos\left(\frac{45^\circ}{2}+\frac{\theta}{2}-\frac{45^\circ}{2}+\frac{\theta}{2}\right)}$

$= \frac{\cos\theta-\cos45^\circ}{\cos45^\circ+\cos\theta} = \frac{\cos\theta-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}+\cos\theta} = \frac{\sqrt{2}\cos\theta-1}{\sqrt{2}\cos\theta+1}$

= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \cot\frac{60^\circ+\theta}{2}\cot\frac{60^\circ-\theta}{2}$

$= \frac{\cos\frac{60^\circ+\theta}{2}}{\sin\frac{60^\circ+\theta}{2}}\cdot\frac{\cos\frac{60^\circ-\theta}{2}}{\sin\frac{60^\circ-\theta}{2}}$

$= \frac{2\cos\left(\frac{60^\circ}{2}+\frac{\theta}{2}\right)\cos\left(\frac{60^\circ}{2}-\frac{\theta}{2}\right)}{2\sin\left(\frac{60^\circ}{2}+\frac{\theta}{2}\right)\sin\left(\frac{60^\circ}{2}-\frac{\theta}{2}\right)}$

$= \frac{\cos\left(\frac{60^\circ}{2}+\frac{\theta}{2}+\frac{60^\circ}{2}-\frac{\theta}{2}\right)+\cos\left(\frac{60^\circ}{2}+\frac{\theta}{2}-\frac{60^\circ}{2}+\frac{\theta}{2}\right)}{\cos\left(\frac{60^\circ}{2}+\frac{\theta}{2}-\frac{60^\circ}{2}+\frac{\theta}{2}\right)-\cos\left(\frac{60^\circ}{2}+\frac{\theta}{2}+\frac{60^\circ}{2}-\frac{\theta}{2}\right)}$

$= \frac{\cos60^\circ+\cos\theta}{\cos\theta-\cos60^\circ} = \frac{\frac{1}{2}+\cos\theta}{\cos\theta-\frac{1}{2}}$

$= \frac{1+2\cos\theta}{2\cos\theta-1} = \frac{2\cos\theta+1}{2\cos\theta-1}$

= ডানপক্ষ (প্রমাণিত)

10. (i) বামপক্ষ $= \tan(45^\circ+\theta)+\tan(45^\circ-\theta)$

$= \frac{\sin(45^\circ+\theta)}{\cos(45^\circ+\theta)}+\frac{\sin(45^\circ-\theta)}{\cos(45^\circ-\theta)}$

$= \frac{\sin(45^\circ+\theta)\cos(45^\circ-\theta)+\cos(45^\circ+\theta)\sin(45^\circ-\theta)}{\cos(45^\circ+\theta)\cos(45^\circ-\theta)}$

$= \frac{\sin(45^\circ+\theta+45^\circ-\theta)}{\cos(45^\circ+\theta)\cos(45^\circ-\theta)}$

$= \frac{\sin90^\circ}{\cos(45^\circ+\theta)\cos(45^\circ-\theta)}$

$= \frac{2.1}{2\cos(45^\circ+\theta)\cos(45^\circ-\theta)}$

$= \frac{2}{\cos(45^\circ+\theta+45^\circ-\theta)+\cos(45^\circ+\theta-45^\circ+\theta)}$

$= \frac{2}{\cos90^\circ+\cos2\theta} = \frac{2}{\cos2\theta}$

$= 2\sec2\theta$ = ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \cot(\alpha+15^\circ)-\tan(\alpha-15^\circ)$

$= \frac{\cos(\alpha+15^\circ)}{\sin(\alpha+15^\circ)}-\frac{\sin(\alpha-15^\circ)}{\cos(\alpha-15^\circ)}$

$= \frac{\cos(\alpha+15^\circ)\cos(\alpha-15^\circ)-\sin(\alpha+15^\circ)\sin(\alpha-15^\circ)}{\sin(\alpha+15^\circ)\cos(\alpha-15^\circ)}$

$= \frac{\cos(\alpha+15^\circ+\alpha-15^\circ)}{\sin(\alpha+15^\circ)\cos(\alpha-15^\circ)}$

$= \frac{2\cos2\alpha}{2\sin(\alpha+15^\circ)\cos(\alpha-15^\circ)}$

$= \frac{2\cos2\alpha}{\sin(\alpha+15^\circ+\alpha-15^\circ)+\sin(\alpha+15^\circ-\alpha+15^\circ)}$

$= \frac{2\cos2\alpha}{\sin2\alpha+\sin30^\circ} = \frac{2\cos2\alpha}{\sin2\alpha+\frac{1}{2}} = \frac{4\cos2\alpha}{2\sin2\alpha+1}$

= ডানপক্ষ (প্রমাণিত)

11. বামপক্ষ $= \sec\left(\frac{\pi}{4}+\theta\right)\sec\left(\frac{\pi}{4}-\theta\right)$

$= \frac{1}{\cos\left(\frac{\pi}{4}+\theta\right)}\cdot\frac{1}{\cos\left(\frac{\pi}{4}-\theta\right)}$

$= \frac{2}{2\cos\left(\frac{\pi}{4}+\theta\right)\cos\left(\frac{\pi}{4}-\theta\right)}$

$= \frac{2}{\cos\left(\frac{\pi}{4}+\theta+\frac{\pi}{4}-\theta\right)+\cos\left(\frac{\pi}{4}+\theta-\frac{\pi}{4}+\theta\right)}$

$= \frac{2}{\cos\frac{\pi}{2}+\cos2\theta} = \frac{2}{\cos2\theta}$

$= 2\sec2\theta$ = ডানপক্ষ (প্রমাণিত)

12. এখানে, $\sin A + \cos A = \sin B + \cos B$

বা, $\sin A - \sin B = \cos B - \cos A$

বা, $2\cos\frac{A+B}{2}\sin\frac{A-B}{2} = 2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$

বা, $\cos\frac{A+B}{2} = \sin\frac{A+B}{2}$ $[\because \sin\frac{A-B}{2} \neq 0]$

বা, $\tan\frac{A+B}{2} = 1$

বা, $\tan\frac{A+B}{2} = \tan\frac{\pi}{4}$

বা, $\frac{A+B}{2} = \frac{\pi}{4}$

$\therefore A + B = \frac{\pi}{2}$ (দেখানো হলো)

13. (i) দেওয়া আছে, $\text{cosec}A + \sec A = \text{cosec}B + \sec B$

বা, $\sec A - \sec B = \text{cosec } B - \text{cosec } A$

বা, $\frac{1}{\cos A} - \frac{1}{\cos B} = \frac{1}{\sin B} - \frac{1}{\sin A}$

বা, $\frac{\cos B - \cos A}{\cos A \cos B} = \frac{\sin A - \sin B}{\sin A \sin B}$

বা, $\frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{\sin A \sin B} = \frac{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}}{\cos A \cos B}$

বা, $\frac{\sin A \sin B}{\cos A \cos B} = \frac{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}{2\sin\frac{A+B}{2}\sin\frac{A-B}{2}}$ [পক্ষান্তর করে]

$\therefore \tan A \tan B = \cot\left(\frac{A+B}{2}\right)$ (দেখানো হলো)

(ii) দেওয়া আছে, $\sin x = m \sin y$

বা, $\frac{\sin x}{\sin y} = \frac{m}{1}$

বা, $\frac{\sin x - \sin y}{\sin x + \sin y} = \frac{m-1}{m+1}$ [বিয়োজন-যোজন করে]

বা, $\frac{2\cos\frac{x+y}{2}\sin\frac{x-y}{2}}{2\sin\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{m-1}{m+1}$

বা, $\cot\frac{x+y}{2}\tan\frac{x-y}{2} = \frac{m-1}{m+1}$

$\therefore \tan\frac{1}{2}(x-y) = \frac{m-1}{m+1}\tan\frac{1}{2}(x+y)$ (প্রমাণিত)

(iii) দেওয়া আছে, $\sin 2\alpha = k \sin 2\theta$

বা, $\frac{\sin 2\alpha}{\sin 2\theta} = k$

বা, $\frac{\sin 2\alpha - \sin 2\theta}{\sin 2\alpha + \sin 2\theta} = \frac{k-1}{k+1}$ [বিয়োজন-যোজন করে]

বা, $\frac{2\cos\left(\frac{2\alpha+2\theta}{2}\right)\sin\left(\frac{2\alpha-2\theta}{2}\right)}{2\sin\left(\frac{2\alpha+2\theta}{2}\right)\cos\left(\frac{2\alpha-2\theta}{2}\right)} = \frac{k-1}{k+1}$

বা, $\frac{\cos(\alpha+\theta)\sin(\alpha-\theta)}{\sin(\alpha+\theta)\cos(\alpha-\theta)} = \frac{k-1}{k+1}$

বা, $\cot(\alpha+\theta)\tan(\alpha-\theta) = \frac{k-1}{k+1}$

বা, $\frac{1}{\tan(\alpha+\theta)}.\tan(\alpha-\theta) = \frac{k-1}{k+1}$

$\therefore \tan(\alpha-\theta) = \frac{k-1}{k+1}\tan(\alpha+\theta)$ (দেখানো হলো)

(iv) দেওয়া আছে, $x + y = \theta$ এবং $\cos x = m \cos y$

বা, $\frac{\cos x}{\cos y} = m$

বা, $\frac{\cos x - \cos y}{\cos x + \cos y} = \frac{m-1}{m+1}$ [বিয়োজন-যোজন করে]

বা, $\frac{2\sin\frac{x+y}{2}\sin\frac{y-x}{2}}{2\cos\frac{x+y}{2}\cos\frac{x-y}{2}} = \frac{m-1}{m+1}$

বা, $-\tan\frac{x+y}{2}.\tan\frac{x-y}{2} = -\left(\frac{1-m}{1+m}\right)$

বা, $\tan\frac{\theta}{2}.\tan\frac{x-y}{2} = \frac{1-m}{1+m}$ $[\because x + y = \theta]$

$\therefore \tan\frac{1}{2}(x-y) = \frac{1-m}{1+m}\cot\frac{\theta}{2}$ (প্রমাণিত)

14. এখানে, $\sin\alpha + \sin\beta = a$

$\therefore 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = a$ ... ... (i)

এবং $\cos\alpha + \cos\beta = b$

$\therefore 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = b$ ... ... (ii)

এখন, (i) নং ও (ii) নং কে বর্গ করে যোগ করে পাই,

$$4\cos^2\left(\frac{\alpha-\beta}{2}\right)\sin^2\left(\frac{\alpha+\beta}{2}\right) + 4\cos^2\left(\frac{\alpha-\beta}{2}\right)\cos^2\left(\frac{\alpha+\beta}{2}\right) = a^2 + b^2$$

বা, $4\cos^2\left(\frac{\alpha-\beta}{2}\right)\left\{\sin^2\left(\frac{\alpha+\beta}{2}\right) + \cos^2\left(\frac{\alpha+\beta}{2}\right)\right\} = a^2 + b^2$

বা, $4\cos^2\left(\frac{\alpha-\beta}{2}\right) = a^2 + b^2$

বা, $4\left\{1 - \sin^2\left(\frac{\alpha-\beta}{2}\right)\right\} = a^2 + b^2$

বা, $4\sin^2\left(\frac{\alpha-\beta}{2}\right) = 4 - a^2 - b^2$

বা, $\sin^2\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{4}(4-a^2-b^2)$

$\therefore\ \sin\frac{1}{2}(\alpha-\beta)=\pm\frac{1}{2}\sqrt{4-a^2-b^2}$ (প্রমাণিত)

15. এখানে, $\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n$

$$=\left\{\frac{2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right\}^n+\left\{\frac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)}\right\}^n$$

$$=\cot^n\left(\frac{A-B}{2}\right)+(-1)^n\cot^n\left(\frac{A-B}{2}\right)$$

এখন, n যদি জোড় হয়, তবে

$$=\cot^n\left(\frac{A-B}{2}\right)+\cot^n\left(\frac{A-B}{2}\right)$$

$$=2\cot^n\left(\frac{A-B}{2}\right)$$

অথবা, n যদি বিজোড় হয়, তবে

$$=\cot^n\left(\frac{A-B}{2}\right)-\cot^n\left(\frac{A-B}{2}\right)=0$$

$$\therefore\left(\frac{\cos A+\cos B}{\sin A-\sin B}\right)^n+\left(\frac{\sin A+\sin B}{\cos A-\cos B}\right)^n=\begin{cases}2\cot^n\left(\frac{A-B}{2}\right); & \text{যখন n জোড় সংখ্যা}\\ 0; & \text{যখন n বিজোড় সংখ্যা}\end{cases}$$ (প্রমাণিত)

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-7.2.7 | পৃষ্ঠা-২৪৯

(i) $\tan\theta=\sec 2\alpha=\frac{1}{\cos 2\alpha}=\frac{1}{\frac{1-\tan^2\alpha}{1+\tan^2\alpha}}=\frac{1+\tan^2\alpha}{1-\tan^2\alpha}$

বামপক্ষ $=\sin 2\theta=\frac{2\tan\theta}{1+\tan^2\theta}$

$$=\frac{2\left(\frac{1+\tan^2\alpha}{1-\tan^2\alpha}\right)}{1+\left(\frac{1+\tan^2\alpha}{1-\tan^2\alpha}\right)^2}$$

$$=\frac{2\left(\frac{1+\tan^2\alpha}{1-\tan^2\alpha}\right)}{\frac{(1-\tan^2\alpha)^2+(1+\tan^2\alpha)^2}{(1-\tan^2\alpha)^2}}$$

$$=\frac{2(1+\tan^2\alpha)}{1-\tan^2\alpha}\times\frac{(1-\tan^2\alpha)^2}{1-2\tan^2\alpha+\tan^4\alpha+1+2\tan^2\alpha+\tan^4\alpha}$$

$$=\frac{2(1+\tan^2\alpha)(1-\tan^2\alpha)}{2+2\tan^4\alpha}$$

$$=\frac{1-\tan^4\alpha}{1+\tan^4\alpha}$$

= ডানপক্ষ (প্রমাণিত)

(ii) দেওয়া আছে, $\tan x=\frac{b}{a}$

বামপক্ষ $=a\cos 2x+b\sin 2x$

$$=a.\frac{1-\tan^2x}{1+\tan^2x}+b.\frac{2\tan x}{1+\tan^2x}$$

$$=a.\frac{1-\left(\frac{b}{a}\right)^2}{1+\left(\frac{b}{a}\right)^2}+b.\frac{2.\frac{b}{a}}{1+\left(\frac{b}{a}\right)^2}$$

$$=a.\frac{1-\frac{b^2}{a^2}}{1+\frac{b^2}{a^2}}+\frac{\frac{2b^2}{a}}{1+\frac{b^2}{a^2}}$$

$$=a.\frac{a^2-b^2}{a^2+b^2}+\frac{2b^2}{a}\times\frac{a^2}{a^2+b^2}$$

$$=a.\frac{a^2-b^2}{a^2+b^2}+\frac{2ab^2}{a^2+b^2}$$

$$=\frac{a^3-ab^2+2ab^2}{a^2+b^2}$$

$$=\frac{a^3+ab^2}{a^2+b^2}=\frac{a(a^2+b^2)}{a^2+b^2}$$

$=a$

= ডানপক্ষ (দেখানো হলো)

(iii) দেওয়া আছে, $\tan\theta=\frac{1}{2}$

বামপক্ষ $=10\sin 2\theta-6\tan 2\theta+5\cos 2\theta$

$$=10.\frac{2\tan\theta}{1+\tan^2\theta}-6.\frac{2\tan\theta}{1-\tan^2\theta}+5.\frac{1-\tan^2\theta}{1+\tan^2\theta}$$

$$=10.\frac{2.\frac{1}{2}}{1+\left(\frac{1}{2}\right)^2}-6.\frac{2.\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}+5\frac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}$$

$$=10.\frac{1}{1+\frac{1}{4}}-6.\frac{1}{1-\frac{1}{4}}+5.\frac{1-\frac{1}{4}}{1+\frac{1}{4}}$$

$$=10.\frac{4}{5}-6.\frac{4}{3}+5.\left(\frac{3}{4}\times\frac{4}{5}\right)$$

$= 8 - 8 + 3 = 3$
= ডানপক্ষ

$\therefore 10\sin2\theta - 6\tan2\theta + 5\cos2\theta = 3$ (প্রমাণিত)

(iv) বামপক্ষ $= \dfrac{\sqrt{3}}{\sin20°} - \dfrac{1}{\cos20°}$

$= \dfrac{\sqrt{3}.\cos20° - \sin20°}{\sin20°.\cos20°}$

$= \dfrac{\frac{\sqrt{3}}{2}\cos20° - \frac{1}{2}\sin20°}{\frac{1}{4}.2\sin20°.\cos20°}$

$= \dfrac{\sin60°\cos20° - \cos60°\sin20°}{\frac{1}{4}.\sin40°}$

$= \dfrac{\sin(60° - 20°)}{\frac{1}{4}\sin40°} = \dfrac{\sin40°}{\frac{1}{4}.\sin40°}$

$= 4$
= ডানপক্ষ

$\therefore \dfrac{\sqrt{3}}{\sin20°} - \dfrac{1}{\cos20°} = 4$ (প্রমাণিত)

অনুশীলনী-7(D) এর সমাধান

1. বামপক্ষ $= \sqrt{\dfrac{1-\cos2\theta}{1+\cos2\theta}} = \sqrt{\dfrac{2\sin^2\theta}{2\cos^2\theta}}$

$= \sqrt{\dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \sqrt{\tan^2\theta}$

$= \tan\theta$
= ডানপক্ষ (প্রমাণিত)

2. বামপক্ষ $= \dfrac{1+\cos2A+\sin2A}{1-\cos2A+\sin2A}$

$= \dfrac{2\cos^2A + 2\sin A\cos A}{2\sin^2A + 2\sin A\cos A}$

$= \dfrac{2\cos A(\cos A + \sin A)}{2\sin A(\cos A + \sin A)}$

$= \dfrac{\cos A}{\sin A}$

$= \cot A$ = ডানপক্ষ (প্রমাণিত)

3. বামপক্ষ $= 2\cos2A + 1$

$= 2(2\cos^2A - 1) + 1 [\because 1 + \cos2A = 2\cos^2A]$

$= 4\cos^2A - 2 + 1$

$= 4\cos^2A - 1$

$= (2\cos A)^2 - (1)^2$

$= (2\cos A + 1)(2\cos A - 1)$
= ডানপক্ষ (প্রমাণিত)

4. বামপক্ষ $= \sec2\theta - \tan2\theta$

$= \dfrac{1}{\cos2\theta} - \dfrac{\sin2\theta}{\cos2\theta}$

$= \dfrac{1-\sin2\theta}{\cos2\theta}$

$= \dfrac{\cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$

$= \dfrac{(\cos\theta - \sin\theta)^2}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$

$= \dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$ = মধ্যপক্ষ

$= \dfrac{\frac{1}{\sqrt{2}}(\cos\theta - \sin\theta)}{\frac{1}{\sqrt{2}}(\cos\theta + \sin\theta)}$

$= \dfrac{\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta}{\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta}$

$= \dfrac{\cos\frac{\pi}{4}\cos\theta - \sin\frac{\pi}{4}\sin\theta}{\cos\frac{\pi}{4}\cos\theta + \sin\frac{\pi}{4}\sin\theta}$

$= \dfrac{\cos\left(\frac{\pi}{4}+\theta\right)}{\cos\left(\frac{\pi}{4}-\theta\right)}$
= ডানপক্ষ (প্রমাণিত)

5. বামপক্ষ $= \cos^6\theta + \sin^6\theta$

$= (\cos^2\theta)^3 + (\sin^2\theta)^3$

$= (\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)$

$= 1 - \frac{3}{4}(2\sin\theta\cos\theta)^2$

$= 1 - \frac{3}{4}\sin^22\theta$ = মধ্যপক্ষ

$= 1 - \frac{3}{4}(1 - \cos^22\theta)$

$= 1 - \frac{3}{4} + \frac{3}{4}\cos^22\theta$

$= \frac{1}{4} + \frac{3}{4}\cos^22\theta$

$= \frac{1}{4}(1 + 3\cos^22\theta)$
= ডানপক্ষ (প্রমাণিত)

6. (i) বামপক্ষ $= \sin5\theta$

$= \sin(4\theta + \theta)$

$= \sin4\theta\cos\theta + \cos4\theta\sin\theta$
$= 2\sin2\theta\cos2\theta\cos\theta + \cos4\theta\sin\theta$
$= 2.2\sin\theta\cos\theta\,(1 - 2\sin^2\theta).\cos\theta + \cos4\theta\sin\theta$
$= 4\sin\theta\,(1 - 2\sin^2\theta)\cos^2\theta + \sin\theta\,(1 - 2\sin^2 2\theta)$
$= (4\sin\theta - 8\sin^3\theta)(1 - \sin^2\theta) + \sin\theta\,(1 - 2.4\sin^2\theta\cos^2\theta)$
$= 4\sin\theta - 8\sin^3\theta + 8\sin^5\theta - 4\sin^3\theta + \sin\theta - 8\sin^3\theta\,(1 - \sin^2\theta)$
$= 5\sin\theta - 12\sin^3\theta + 8\sin^5\theta - 8\sin^3\theta + 8\sin^5\theta$
$= 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta.$
= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \cos5\theta = \cos(4\theta + \theta)$
$= \cos4\theta\cos\theta - \sin4\theta\sin\theta$
$= (2\cos^2 2\theta - 1)\cos\theta - 2\sin2\theta\cos2\theta\sin\theta$
$= \{2(2\cos^2\theta - 1)^2 - 1\}\cos\theta - 4\sin^2\theta\cos\theta\,(2\cos^2\theta - 1)$
$= \{2(4\cos^4\theta - 4\cos^2\theta + 1) - 1\}\cos\theta - 4(1 - \cos^2\theta)\cos\theta\,(2\cos^2\theta - 1)$
$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta - 4(\cos\theta - \cos^3\theta)(2\cos^2\theta - 1)$
$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta - 4(2\cos^3\theta - 2\cos^5\theta - \cos\theta + \cos^3\theta)$
$= 8\cos^5\theta - 8\cos^3\theta + \cos\theta - 12\cos^3\theta + 8\cos^5\theta + 4\cos\theta$
$= 16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$
= ডানপক্ষ (প্রমাণিত)

7. (i) বামপক্ষ $= \dfrac{\sin\theta + \cos\theta}{\sqrt{1 + \sin2\theta}}$
$= \dfrac{\sin\theta + \cos\theta}{\sqrt{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}}$
$= \dfrac{\sin\theta + \cos\theta}{\sqrt{(\cos\theta + \sin\theta)^2}}$
$= \dfrac{\sin\theta + \cos\theta}{\cos\theta + \sin\theta}$
$= 1$ = ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \dfrac{\cos x - \cos2x}{1 - \cos x}$
$= \dfrac{\cos x - 2\cos^2 x + 1}{1 - \cos x}$
$= \dfrac{2\cos x - 2\cos^2 x + 1 - \cos x}{1 - \cos x}$
$= \dfrac{2\cos x(1 - \cos x) + 1(1 - \cos x)}{1 - \cos x}$
$= \dfrac{(1 - \cos x)(2\cos x + 1)}{(1 - \cos x)}$
$= 2\cos x + 1$
= ডানপক্ষ (প্রমাণিত)

8. ডানপক্ষ $= \dfrac{2}{\sqrt{2 + \sqrt{2 + 2\cos 4x}}}$
$= \dfrac{2}{\sqrt{2 + \sqrt{2(1 + \cos 2.2x)}}}$
$= \dfrac{2}{\sqrt{2 + \sqrt{2.2\cos^2 2x}}} = \dfrac{2}{\sqrt{2 + 2\cos 2x}}$
$= \dfrac{2}{\sqrt{2(1 + \cos 2x)}} = \dfrac{2}{\sqrt{2.2\cos^2 x}}$
$= \dfrac{2}{2\cos x} = \dfrac{1}{\cos x}$
$= \sec x$
= বামপক্ষ (প্রমাণিত)

9. বামপক্ষ $= \sin^2\left(\dfrac{\pi}{8} + \dfrac{\theta}{2}\right) - \sin^2\left(\dfrac{\pi}{8} - \dfrac{\theta}{2}\right)$
$= \dfrac{1}{2}\left\{2\sin^2\left(\dfrac{\pi}{8} + \dfrac{\theta}{2}\right) - 2\sin^2\left(\dfrac{\pi}{8} - \dfrac{\theta}{2}\right)\right\}$
$= \dfrac{1}{2}\left\{1 - \cos 2\left(\dfrac{\pi}{8} + \dfrac{\theta}{2}\right) - 1 + \cos 2\left(\dfrac{\pi}{8} - \dfrac{\theta}{2}\right)\right\}$
$= \dfrac{1}{2}\left\{\cos\left(\dfrac{\pi}{4} - \theta\right) - \cos\left(\dfrac{\pi}{4} + \theta\right)\right\}$
$= \dfrac{1}{2}.2\sin\dfrac{\pi}{4}\sin\theta$
$= \dfrac{1}{\sqrt{2}}\sin\theta$
= ডানপক্ষ (প্রমাণিত)

10. বামপক্ষ $= \cos^2(A - 120^\circ) + \cos^2 A + \cos^2(A + 120^\circ)$
$= \dfrac{1}{2}\{2\cos^2(A - 120^\circ) + 2\cos^2 A + 2\cos^2(A + 120^\circ)\}$
$= \dfrac{1}{2}\{1 + \cos 2(A - 120^\circ) + 1 + \cos 2A + 1 + \cos 2(A + 120^\circ)\}$
$= \dfrac{1}{2}\{3 + \cos(2A - 240^\circ) + \cos(2A + 240^\circ) + \cos 2A\}$
$= \dfrac{1}{2}(3 + 2\cos 2A\cos 240^\circ + \cos 2A)$
$= \dfrac{1}{2}\{3 + 2\cos 2A\left(-\dfrac{1}{2}\right) + \cos 2A\}\ \left[\because \cos 240^\circ = -\dfrac{1}{2}\right]$
$= \dfrac{1}{2}\{3 - \cos 2A + \cos 2A\}$
$= \dfrac{3}{2}$
= ডানপক্ষ (প্রমাণিত)

11. (i) বামপক্ষ $= \cos^3 x + \cos^3(60^\circ - x) + \cos^3(60^\circ + x)$

$= \frac{\cos 3x + 3\cos x}{4} + \frac{\cos(180^\circ - 3x) + 3\cos(60^\circ - x)}{4} + \frac{\cos(180^\circ + 3x) + 3\cos(60^\circ + x)}{4}$

$= \frac{\cos 3x + 3\cos x}{4} + \frac{3\cos(60^\circ - x) - \cos 3x}{4} + \frac{3\cos(60^\circ + x) - \cos 3x}{4}$

$= \frac{\cos 3x + 3\cos x + 3\cos 60^\circ \cos x + 3\sin 60^\circ \sin x - \cos 3x + 3\cos 60^\circ \cos x - 3\sin 60^\circ \sin x - \cos 3x}{4}$

$= \frac{3\cos x + \frac{3}{2}\cos x + \frac{3\sqrt{3}}{2}\sin x + \frac{3}{2}\cos x - \frac{3\sqrt{3}}{2}\sin x - \cos 3x}{4}$

$= \frac{1}{4}(6\cos x - \cos 3x) =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin^3 x + \sin^3(120^\circ + x) + \sin^3(240^\circ + x)$

$= \frac{1}{4}\{4\sin^3 x + 4\sin^3(120^\circ + x) + 4\sin^3(240^\circ + x)\}$

$= \frac{1}{4}\{3\sin x - \sin 3x + 3\sin(120^\circ + x) - \sin 3(120^\circ + x) + 3\sin(240^\circ + x) - \sin 3(240^\circ + x)\}$

$= \frac{1}{4}[3\sin x - \sin 3x + 3\{\sin(120^\circ + x) + \sin(240^\circ + x)\} - \sin(360^\circ + 3x) - \sin(720^\circ + 3x)]$

$= \frac{1}{4}[3\sin x - \sin 3x \;\; 3.2\sin(180^\circ + x)\cos 60^\circ - \sin 3x - \sin 3x]$

$= \frac{1}{4}\left[3\sin x - 3\sin 3x - 3.2\sin x \,.\, \frac{1}{2}\right]$

$= \frac{1}{4}[3\sin x - 3\sin 3x - 3\sin x]$

$= -\frac{3}{4}\sin 3x$

= ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \cos^3 x + \cos^3(120^\circ + x) + \cos^3(240^\circ + x)$

$= \frac{\cos 3x + 3\cos x}{4} + \frac{\cos 3(120^\circ + x) + 3\cos(120^\circ + x)}{4} + \frac{\cos 3(240^\circ + x) + 3\cos(240^\circ + x)}{4}$

$= \frac{\cos 3x + 3\cos x + \cos(360^\circ + 3x) + 3\cos(120^\circ + x) + \cos(720^\circ + 3x) + 3\cos(240^\circ + x)}{4}$

$= \frac{\cos 3x + 3\cos x + \cos 3x + 3\cos(120^\circ + x) + \cos 3x + 3\cos(240^\circ + x)}{4}$

$= \frac{3\cos 3x + 3\cos x + 3\{\cos(120^\circ + x) + \cos(240^\circ + x)\}}{4}$

$= \frac{3\cos 3x + 3\cos x + 3.2\cos(180^\circ + x)\cos 60^\circ}{4}$

$= \frac{3\cos 3x + 3\cos x - 6\cos x \,.\, \frac{1}{2}}{4}$

$= \frac{3\cos 3x + 3\cos x - 3\cos x}{4}$

$= \frac{3}{4}\cos 3x$

= ডানপক্ষ (প্রমাণিত)

12. বামপক্ষ $= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + \frac{8}{\tan 8\theta}$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + \frac{8}{\tan(2.4\theta)}$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + \frac{8(1 - \tan^2 4\theta)}{2\tan 4\theta}$

$= \tan\theta + 2\tan 2\theta + \frac{8\tan^2 4\theta + 8 - 8\tan^2 4\theta}{2\tan 4\theta}$

$= \tan\theta + 2\tan 2\theta + \frac{4}{\tan(2.2\theta)}$

$= \tan\theta + 2\tan 2\theta + \frac{4(1 - \tan^2 2\theta)}{2\tan 2\theta}$

$= \tan\theta + \frac{4\tan^2 2\theta + 4 - 4\tan^2 2\theta}{2\tan 2\theta}$

$= \tan\theta + \frac{2}{\tan 2\theta}$

$= \tan\theta + \frac{2(1 - \tan^2\theta)}{2\tan\theta}$

$= \frac{2\tan^2\theta + 2 - 2\tan^2\theta}{2\tan\theta}$

$= \frac{2}{2\tan\theta} = \cot\theta =$ ডানপক্ষ (প্রমাণিত)

13. আমরা জানি, $(2\cos\theta + 1)(2\cos\theta - 1)$

$= 4\cos^2\theta - 1$

$= 2.2\cos^2\theta - 1$

$= 2(1 + \cos 2\theta) - 1$

$= 2\cos 2\theta + 1$ ... ... (i)

এবং $(2\cos 2\theta + 1)(2\cos 2\theta - 1)$

$= 4\cos^2 2\theta - 1$

$= 2(1 + \cos 4\theta) - 1$

$= 2\cos 2^2\theta + 1$ ... ... (ii)

অনুরূপভাবে, $(2\cos 2^2\theta + 1)(2\cos 2^2\theta - 1)$

$= 2\cos 2^3\theta + 1$ ... ... (iii)

... ...

$(2\cos 2^{n-1}\theta + 1)(2\cos 2^{n-1}\theta - 1) = 2\cos 2^n\theta + 1$ ... (n)

এখন (i) থেকে (n) পর্যন্ত উভয়পক্ষকে পরস্পর গুণ করে পাই,

$(2\cos\theta + 1)(2\cos\theta - 1)(2\cos2\theta + 1)(2\cos2\theta - 1)$
$\ldots\ldots (2\cos2^{n-1}\theta + 1)(2\cos2^{n-1}\theta - 1)$
$= (2\cos2\theta + 1)(2\cos2^2\theta + 1)(2\cos2^3\theta + 1)\ldots$
$(2\cos2^n\theta + 1).$

বা, $(2\cos\theta + 1)(2\cos\theta - 1)(2\cos2^2\theta - 1)\ldots\ldots$
$\ldots (2\cos2^{n-1}\theta - 1) = 2\cos2^n\theta + 1$

$\therefore\ (2\cos\theta - 1)(2\cos2\theta - 1)\ldots\ldots(2\cos2^{n-1}\theta - 1)$
$= \dfrac{2\cos2^n\theta + 1}{2\cos\theta + 1}$ (প্রমাণিত)

14. (i) বামপক্ষ $= \cos2\theta$

$= 2\cos^2\theta - 1 = 2\left\{\frac{1}{2}\left(x + \frac{1}{x}\right)\right\}^2 - 1$ [মান বসিয়ে]

$= 2.\frac{1}{4}\left(x^2 + \frac{1}{x^2} + 2\right) - 1$

$= \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) + \frac{1}{2}.2 - 1$

$= \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$

= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$= 4\left\{\frac{1}{2}\left(x + \frac{1}{x}\right)\right\}^3 - 3.\frac{1}{2}\left(x + \frac{1}{x}\right)$ [মান বসিয়ে]

$= 4.\frac{1}{8}\left(x + \frac{1}{x}\right)^3 - \frac{3}{2}\left(x + \frac{1}{x}\right)$

$= \frac{1}{2}\left\{x^3 + \frac{1}{x^3} + 3x.\frac{1}{x}\left(x + \frac{1}{x}\right)\right\} - \frac{3}{2}\left(x + \frac{1}{x}\right)$

$= \frac{1}{2}\left(x^3 + \frac{1}{x^3}\right) + \frac{3}{2}\left(x + \frac{1}{x}\right) - \frac{3}{2}\left(x + \frac{1}{x}\right)$

$= \frac{1}{2}\left(x^3 + \frac{1}{x^3}\right)$

= ডানপক্ষ (প্রমাণিত)

15. দেওয়া আছে, $\tan\theta = \frac{1}{3}$ এবং $\tan\varphi = \frac{1}{7}$

বামপক্ষ $= \sin 4\theta = 2\sin 2\theta \cos 2\theta$

$= 2.\dfrac{2\tan\theta}{1 + \tan^2\theta}.\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= 2.\dfrac{2.\frac{1}{3}}{1 + \frac{1}{9}}.\dfrac{1 - \frac{1}{9}}{1 + \frac{1}{9}}$

$= 2.\frac{3}{5}.\frac{4}{5} = \frac{24}{25}$

ডানপক্ষ $= \cos 2\varphi = \dfrac{1 - \tan^2\varphi}{1 + \tan^2\varphi} = \dfrac{1 - \frac{1}{49}}{1 + \frac{1}{49}} = \dfrac{24}{25}$

$\therefore$ বামপক্ষ = ডানপক্ষ (দেখানো হলো)

16. দেওয়া আছে, $\tan^2\theta = 1 + 2\tan^2\varphi$

$\therefore\ \tan^2\varphi = \dfrac{\tan^2\theta - 1}{2} \Rightarrow \dfrac{1}{\tan^2\varphi} = \dfrac{2}{\tan^2\theta - 1}$

$\Rightarrow \dfrac{1 - \tan^2\varphi}{1 + \tan^2\varphi} = \dfrac{2 - (\tan^2\theta - 1)}{2 + (\tan^2\theta - 1)}$

[বিয়োজন-যোজন করে]

$\Rightarrow \cos2\varphi = \dfrac{3 - \tan^2\theta}{1 + \tan^2\theta}$

$= \dfrac{1 + \tan^2\theta}{1 + \tan^2\theta} + \dfrac{2(1 - \tan^2\theta)}{1 + \tan^2\theta}$

$\Rightarrow \cos2\varphi = 1 + 2\cos2\theta$ (প্রমাণিত)

17.(i) দেওয়া আছে,

$2\tan\alpha = 3\tan\beta$

$\therefore \tan\alpha = \dfrac{3\tan\beta}{2}$

$\therefore \tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \dfrac{\frac{3\tan\beta}{2} - \tan\beta}{1 + \frac{3\tan\beta}{2}.\tan\beta}$

$= \dfrac{\tan\beta}{2 + 3\tan^2\beta}$

$= \dfrac{\frac{\sin\beta}{\cos\beta}}{2 + 3.\frac{\sin^2\beta}{\cos^2\beta}}$

$= \dfrac{\sin\beta\cos\beta}{2\cos^2\beta + 3\sin^2\beta}$

$= \dfrac{2\sin\beta\cos\beta}{4\cos^2\beta + 6\sin^2\beta}$

$= \dfrac{\sin2\beta}{4 - 4\sin^2\beta + 6\sin^2\beta}$

$= \dfrac{\sin2\beta}{4 + 2\sin^2\beta}$

$= \dfrac{\sin2\beta}{5 - (1 - 2\sin^2\beta)}$

$= \dfrac{\sin2\beta}{5 - \cos2\beta}$

$\therefore \tan(\alpha - \beta) = \dfrac{\sin2\beta}{5 - \cos2\beta}$ (প্রমাণিত)

(ii) দেওয়া আছে,

$\tan\alpha = 2\tan\beta \quad \therefore \tan\beta = \frac{\tan\alpha}{2}$

$\therefore \tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\tan\alpha + \frac{\tan\alpha}{2}}{1 - \tan\alpha.\left(\frac{\tan\alpha}{2}\right)}$

$= \frac{\frac{3\tan\alpha}{2}}{1 - \frac{\tan^2\alpha}{2}}$

$= \frac{3\tan\alpha}{2 - \tan^2\alpha}$

$= \frac{\frac{3\sin\alpha}{\cos\alpha}}{2 - \frac{\sin^2\alpha}{\cos^2\alpha}}$

$= \frac{3\sin\alpha\cos\alpha}{2\cos^2\alpha - \sin^2\alpha}$

$= \frac{3 \times 2\sin\alpha\cos\alpha}{4\cos^2\alpha - 2\sin^2\alpha}$

$= \frac{3\sin2\alpha}{4 - 4\sin^2\alpha - 2\sin^2\alpha}$

$= \frac{3\sin2\alpha}{4 - 6\sin^2\alpha}$

$= \frac{3\sin2\alpha}{1 + 3(1 - 2\sin^2\alpha)}$

$= \frac{3\sin2\alpha}{1 + 3\cos2\alpha}$

$\therefore \tan(\alpha+\beta) = \frac{3\sin2\alpha}{1 + 3\cos2\alpha}$ **(দেখানো হলো)**

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-7.2.8 | পৃষ্ঠা-২৫৪

(i) $\sin54° = \sin(90° - 36°) = \cos36°$
$\cos36°$ এর মান অনুচ্ছেদ 7.2.8 Text Book পৃষ্ঠা-২৫৩ দেখ।
$\cos54° = \cos(90° - 36°) = \sin36°$
$\sin36°$ এর মান অনুচ্ছেদ 7.2.8 Text Book পৃষ্ঠা-২৫৩ দেখ।
$\sin72° = \sin(90° - 18°) = \cos18°$
$\cos18°$ এর মান অনুচ্ছেদ 7.2.8 Text Book পৃষ্ঠা-২৫৩ দেখ।
$\cos72° = \cos(90° - 18°) = \sin18°$
$\sin18°$ এর মান অনুচ্ছেদ 7.2.8 Text Book পৃষ্ঠা-২৫৩ দেখ।

(ii) বামপক্ষ $= \sin 67\frac{1}{2}°$

$= \sin\frac{135°}{2} = \sqrt{\frac{1}{2}.2\sin^2\frac{135°}{2}}$

$= \sqrt{\frac{1}{2}(1 - \cos 135°)} = \sqrt{\frac{1}{2} - \frac{1}{2}\cos135°}$

$= \sqrt{\frac{1}{2} - \frac{1}{2}\cos(90° + 45°)} = \sqrt{\frac{1}{2} - \frac{1}{2}(-\sin45°)}$

$= \sqrt{\frac{1}{2} + \frac{1}{2}.\frac{1}{\sqrt{2}}} = \sqrt{\frac{1}{2} + \frac{1}{2}.\frac{1}{\sqrt{2}}}$

$= \sqrt{\frac{1}{2}\left(1 + \frac{1}{\sqrt{2}}\right)} = \sqrt{\frac{1}{2}.\frac{\sqrt{2}+1}{\sqrt{2}}}$

$= \sqrt{\frac{2}{4}.\frac{(\sqrt{2}+1)}{\sqrt{2}}}$

$= \frac{1}{2}\sqrt{\sqrt{2}(\sqrt{2}+1)}$

$= \frac{1}{2}\sqrt{2+\sqrt{2}} =$ ডানপক্ষ

$\therefore \sin 67\frac{1}{2}° = \frac{1}{2}\sqrt{2+\sqrt{2}}$ **(দেখানো হলো)**

(iii) বামপক্ষ $= 2\sin\frac{\pi}{24} = \sqrt{4\sin^2\frac{\pi}{24}}$

$= \sqrt{2.2\sin^2\frac{\pi}{24}} = \sqrt{2.\left(1 - \cos 2.\frac{\pi}{24}\right)}$

$= \sqrt{2 - 2\cos\frac{\pi}{12}} = \sqrt{2 - \sqrt{4.\cos^2\frac{\pi}{12}}}$

$= \sqrt{2 - \sqrt{2\left(1 + \cos\frac{\pi}{6}\right)}}$

$= \sqrt{2 - \sqrt{2 + 2.\frac{\sqrt{3}}{2}}}$

$= \sqrt{2 - \sqrt{2+\sqrt{3}}} =$ ডানপক্ষ

$\therefore 2\sin\frac{\pi}{24} = \sqrt{2 - \sqrt{2+\sqrt{3}}}$ **(প্রমাণিত)**

(iv) বামপক্ষ $= 2\sin\frac{\pi}{32} = \sqrt{4\sin^2\frac{\pi}{32}}$

$= \sqrt{2.2\sin^2\frac{\pi}{32}} = \sqrt{2\left(1 - \cos\frac{\pi}{16}\right)}$

$= \sqrt{2 - 2\cos\frac{\pi}{16}} = \sqrt{2 - \sqrt{4\cos^2\frac{\pi}{16}}}$

$= \sqrt{2 - \sqrt{2.2\cos^2\frac{\pi}{16}}}$

$$=\sqrt{2-\sqrt{2\left(1+\cos\frac{\pi}{8}\right)}}$$
$$=\sqrt{2-\sqrt{2+2\cos\frac{\pi}{8}}}$$
$$=\sqrt{2-\sqrt{2+\sqrt{4\cos^2\frac{\pi}{8}}}}$$
$$=\sqrt{2-\sqrt{2+\sqrt{2.2\cos^2\frac{\pi}{8}}}}$$
$$=\sqrt{2-\sqrt{2+\sqrt{2\left(1+\cos\frac{\pi}{4}\right)}}}$$
$$=\sqrt{2-\sqrt{2+\sqrt{2+2\cos\frac{\pi}{4}}}}$$
$$=\sqrt{2-\sqrt{2+\sqrt{2+2.\frac{1}{\sqrt{2}}}}}$$
$$=\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}= \text{ডানপক্ষ}$$
$$\therefore\ 2\sin\frac{\pi}{32}=\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2}}}}$$ (দেখানো হলো)

অনুশীলনী-7(E) এর সমাধান

1. বামপক্ষ $=\dfrac{1+\sin x-\cos x}{1+\sin x+\cos x}=\dfrac{1-\cos x+\sin x}{1+\cos x+\sin x}$
$$=\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}$$
$$=\frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\cos\frac{x}{2}\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}$$
$$=\frac{\sin\frac{x}{2}}{\cos\frac{x}{2}}$$
$=\tan\frac{x}{2}=$ ডানপক্ষ (প্রমাণিত)

2. বামপক্ষ $=\dfrac{1+\sin\theta}{1-\sin\theta}=\dfrac{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}-2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$
$$=\left(\frac{\cos\frac{\theta}{2}+\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}-\sin\frac{\theta}{2}}\right)^2=\left(\frac{1+\tan\frac{\theta}{2}}{1-\tan\frac{\theta}{2}}\right)^2$$
$$=\left(\frac{\tan\frac{\pi}{4}+\tan\frac{\theta}{2}}{1-\tan\frac{\pi}{4}\tan\frac{\theta}{2}}\right)^2=\tan^2\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$$
= ডানপক্ষ (প্রমাণিত)

3. বামপক্ষ $=\sec x+\tan x$
$$=\frac{1}{\cos x}+\frac{\sin x}{\cos x}=\frac{1+\sin x}{\cos x}$$
$$=\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}$$
$$=\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}$$
$$=\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}=\frac{1+\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}{1-\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}}$$
$$=\frac{\tan\frac{\pi}{4}+\tan\frac{x}{2}}{1-\tan\frac{\pi}{4}\tan\frac{x}{2}}=\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$
= ডানপক্ষ (প্রমাণিত)

4. বামপক্ষ $=\cot\theta=\dfrac{\cos\theta}{\sin\theta}=\dfrac{\cos^2\frac{\theta}{2}-\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$
$$=\frac{1}{2}\left(\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}-\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)=\frac{1}{2}\left(\cot\frac{\theta}{2}-\tan\frac{\theta}{2}\right)$$
= ডানপক্ষ (প্রমাণিত)

5. বামপক্ষ $=\dfrac{\cos\frac{\theta}{2}-\sqrt{1+\sin\theta}}{\sin\frac{\theta}{2}-\sqrt{1+\sin\theta}}$
$$=\frac{\cos\frac{\theta}{2}-\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}{\sin\frac{\theta}{2}-\sqrt{\cos^2\frac{\theta}{2}+\sin^2\frac{\theta}{2}+2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}}$$

$$= \frac{\cos\frac{\theta}{2} - \sqrt{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2}}{\sin\frac{\theta}{2} - \sqrt{\left(\cos\frac{\theta}{2} + \sin\frac{\theta}{2}\right)^2}}$$

$$= \frac{\cos\frac{\theta}{2} - \cos\frac{\theta}{2} - \sin\frac{\theta}{2}}{\sin\frac{\theta}{2} - \cos\frac{\theta}{2} - \sin\frac{\theta}{2}}$$

$= \frac{-\sin\frac{\theta}{2}}{-\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$ = ডানপক্ষ (প্রমাণিত)

6. বামপক্ষ $= \cos^4\frac{\theta}{2} + \sin^4\frac{\theta}{2}$

$= \left(\cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2}\right)^2 - 2\cos^2\frac{\theta}{2}\sin^2\frac{\theta}{2}$

$= 1 - 2.\frac{1}{4}\left(2\cos\frac{\theta}{2}\sin\frac{\theta}{2}\right)^2$

$= 1 - \frac{1}{4}.2\sin^2\theta$

$= 1 - \frac{1}{4}(1 - \cos2\theta)$

$= \frac{4 - 1 + \cos2\theta}{4}$

$= \frac{1}{4}(3 + \cos2\theta)$

= ডানপক্ষ (প্রমাণিত)

7. (i) বামপক্ষ $= \cos^2\frac{\theta}{2} + \cos^2\left(\frac{\theta}{2} + 60°\right) + \cos^2\left(\frac{\theta}{2} - 60°\right)$

$= \frac{1}{2}[1 + \cos\theta + 1 + \cos(\theta + 120°) + 1 + \cos(\theta - 120°)]$

$= \frac{1}{2}[3 + \cos\theta + \cos(\theta + 120°) + \cos(\theta - 120°)]$

$= \frac{1}{2}[3 + \cos\theta + 2\cos120° \cos\theta]$

$= \frac{1}{2}[3 + \cos\theta - 2.\frac{1}{2}.\cos\theta)$ [$\because \cos120° = -\frac{1}{2}$]

$= \frac{1}{2}(3 + \cos\theta - \cos\theta) = \frac{3}{2}$

= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin^2\frac{\theta}{2} + \sin^2\left(\frac{\theta}{2} + 60°\right) + \sin^2\left(\frac{\theta}{2} - 60°\right)$

$= \frac{1}{2}[1 - \cos\theta + 1 - \cos(\theta + 120°) + 1 - \cos(\theta - 120°)]$

$= \frac{1}{2}[3 - \cos\theta - \{\cos(\theta + 120°) + \cos(\theta - 120°)\}]$

$= \frac{1}{2}[3 - \cos\theta - 2\cos\theta \cos120°]$

$= \frac{1}{2}[3 - \cos\theta - 2\cos\theta . \left(-\frac{1}{2}\right)]$ [$\because \cos120° = -\frac{1}{2}$]

$= \frac{1}{2}(3 - \cos\theta + \cos\theta) = \frac{3}{2}$

= ডানপক্ষ (প্রমাণিত)

8. (i) বামপক্ষ $= \cos^4\frac{\pi}{8} + \cos^4\frac{3\pi}{8} + \cos^4\frac{5\pi}{8} + \cos^4\frac{7\pi}{8}$

$= \frac{1}{4}\left\{\left(2\cos^2\frac{\pi}{8}\right)^2 + \left(2\cos^2\frac{3\pi}{8}\right)^2 + \left(2\cos^2\frac{5\pi}{8}\right)^2 + \left(2\cos^2\frac{7\pi}{8}\right)^2\right\}$

$= \frac{1}{4}\left\{\left(1 + \cos\frac{\pi}{4}\right)^2 + \left(1 + \cos\frac{3\pi}{4}\right)^2 + \left(1 + \cos\frac{5\pi}{4}\right)^2 + \left(1 + \cos\frac{7\pi}{4}\right)^2\right\}$

$= \frac{1}{4}\left[\left(1 + \cos\frac{\pi}{4}\right)^2 + \left\{1 + \cos\left(\pi - \frac{\pi}{4}\right)\right\}^2 + \left\{1 + \cos\left(\pi + \frac{\pi}{4}\right)\right\}^2 + \left\{1 + \cos\left(2\pi - \frac{\pi}{4}\right)\right\}^2\right]$

$= \frac{1}{4}\left\{\left(1 + \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 + \cos\frac{\pi}{4}\right)^2\right\}$

$= \frac{1}{2}\left\{\left(1 + \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{\pi}{4}\right)^2\right\}$

$= \frac{1}{2}.2\left(1 + \cos^2\frac{\pi}{4}\right)$

$= 1 + \frac{1}{2} = \frac{3}{2}$ = ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin^4\frac{\pi}{8} + \sin^4\frac{3\pi}{8} + \sin^4\frac{5\pi}{8} + \sin^4\frac{7\pi}{8}$

$= \frac{1}{4}\left\{\left(2\sin^2\frac{\pi}{8}\right)^2 + \left(2\sin^2\frac{3\pi}{8}\right)^2 + \left(2\sin^2\frac{5\pi}{8}\right)^2 + \left(2\sin^2\frac{7\pi}{8}\right)^2\right\}$

$= \frac{1}{4}\left\{\left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{3\pi}{4}\right)^2 + \left(1 - \cos\frac{5\pi}{4}\right)^2 + \left(1 - \cos\frac{7\pi}{4}\right)^2\right\}$

$= \frac{1}{4}\left\{\left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 + \cos\frac{\pi}{4}\right)^2 + \left(1 + \cos\frac{\pi}{4}\right)^2 + \left(1 - \cos\frac{\pi}{4}\right)^2\right\}$

$= \frac{1}{2}\left\{\left(1 - \cos\frac{\pi}{4}\right)^2 + \left(1 + \cos\frac{\pi}{4}\right)^2\right\}$

$= \cos^2\frac{\pi}{4} + 1 = \frac{1}{2} + 1 = \frac{3}{2}$

= ডানপক্ষ (প্রমাণিত)

9. (i) বামপক্ষ $= 2\cos\frac{\pi}{16}$

$= \sqrt{4\cos^2\frac{\pi}{16}} = \sqrt{2.2\cos^2\frac{\pi}{16}}$

$= \sqrt{2\left(1+\cos\frac{\pi}{8}\right)} = \sqrt{2+2\cos\frac{\pi}{8}}$

$= \sqrt{2+\sqrt{4\cos^2\frac{\pi}{8}}}$

$= \sqrt{2+\sqrt{2.2\cos^2\frac{\pi}{8}}}$

$= \sqrt{2+\sqrt{2\left(1+\cos\frac{\pi}{4}\right)}}$

$= \sqrt{2+\sqrt{2\left(1+\cos\frac{1}{\sqrt{2}}\right)}}$

$= \sqrt{2+\sqrt{2+\sqrt{2}}}$

= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= 2\cos 7\frac{1°}{2}$

$= \sqrt{4\cos^2 7\frac{1°}{2}}$

$= \sqrt{2\times 2\cos^2 7\frac{1°}{2}}$

$= \sqrt{2(1+\cos 15°)}$

$= \sqrt{2+2\cos 15°}$

$= \sqrt{2+\sqrt{4\cos^2 15°}}$

$= \sqrt{2+\sqrt{2\times 2\cos^2 15°}}$

$= \sqrt{2+\sqrt{2(1+\cos 30°)}}$

$= \sqrt{2+\sqrt{2\left(1+\frac{\sqrt{3}}{2}\right)}}$

$= \sqrt{2+\sqrt{2\times\frac{2+\sqrt{3}}{2}}}$

$= \sqrt{2+\sqrt{2+\sqrt{3}}}$

= ডানপক্ষ (প্রমাণিত)

10. দেওয়া আছে, $\sin A + \sin B = 2\sin(A+B)$

বা, $2\sin\frac{A+B}{2}\cos\frac{A-B}{2} = 4\sin\frac{A+B}{2}\cos\frac{A+B}{2}$

বা, $\sin\frac{A+B}{2}\left(\cos\frac{A-B}{2} - 2\cos\frac{A+B}{2}\right) = 0$

যেহেতু $A + B \neq 0$

$\therefore \sin\left(\frac{A+B}{2}\right) \neq 0$

$\therefore \cos\frac{A-B}{2} - 2\cos\frac{A+B}{2} = 0$

বা, $\cos\frac{A-B}{2} = 2\cos\frac{A+B}{2}$

বা, $\cos\left(\frac{A}{2}-\frac{B}{2}\right) = 2\cos\left(\frac{A}{2}+\frac{B}{2}\right)$

বা, $\cos\frac{A}{2}\cos\frac{B}{2} + \sin\frac{A}{2}\sin\frac{B}{2} = 2\left(\cos\frac{A}{2}\cos\frac{B}{2} - \sin\frac{A}{2}\sin\frac{B}{2}\right)$

বা, $3\sin\frac{A}{2}\sin\frac{B}{2} = \cos\frac{A}{2}\cos\frac{B}{2}$

বা, $3\tan\frac{A}{2}\tan\frac{B}{2} = 1$

$\therefore \tan\frac{A}{2}\tan\frac{B}{2} = \frac{1}{3}$ (প্রমাণিত)

11. (i) দেওয়া আছে, $\sin\theta + \sin\varphi = a$

এবং $\cos\theta + \cos\varphi = b$

ডানপক্ষ $= \frac{b^2 - a^2}{b^2 + a^2}$

$= \frac{(\cos\theta + \cos\varphi)^2 - (\sin\theta + \sin\varphi)^2}{(\cos\theta + \cos\varphi)^2 + (\sin\theta + \sin\varphi)^2}$

$= \frac{\cos^2\theta + \cos^2\varphi - \sin^2\theta - \sin^2\varphi + 2(\cos\theta\cos\varphi - \sin\theta\sin\varphi)}{\cos^2\theta + \cos^2\varphi + \sin^2\theta + \sin^2\varphi + 2(\cos\theta\cos\varphi + \sin\theta\sin\varphi)}$

$= \frac{\cos 2\theta + \cos 2\varphi + 2\cos(\theta+\varphi)}{2 + 2\cos(\theta-\varphi)}$

$= \frac{2\cos(\theta+\varphi)\cos(\theta-\varphi) + 2\cos(\theta+\varphi)}{2\{1+\cos(\theta-\varphi)\}}$

$= \frac{2\cos(\theta+\varphi)\{1+\cos(\theta-\varphi)\}}{2\{1+\cos(\theta-\varphi)\}}$

$= \cos(\theta+\varphi) =$ বামপক্ষ (প্রমাণিত)

(ii) এখানে, $a^2 + b^2 = (\sin\theta + \sin\varphi)^2 + (\cos\theta + \cos\varphi)^2$

$= \sin^2\theta + \sin^2\varphi + \cos^2\theta + \cos^2\varphi + 2(\cos\theta\cos\varphi + \sin\theta\sin\varphi)$

$= 2 + 2\cos(\theta-\varphi) = 2\{1+\cos(\theta-\varphi)\}$

$= 2.2\cos^2\frac{\theta-\varphi}{2}$

$\therefore a^2 + b^2 = 4\cos^2\frac{\theta-\varphi}{2}$

বা, $\cos^2\frac{\theta-\varphi}{2} = \frac{a^2+b^2}{4}$

$\therefore \cos\frac{1}{2}(\theta-\varphi) = \pm\frac{1}{2}\sqrt{a^2+b^2}$ (প্রমাণিত)

(iii) দেওয়া আছে, $\sin\theta + \sin\varphi = a$ ... ... (i)

এবং $\cos\theta + \cos\varphi = b$ ... ... (ii)

(i) ও (ii) কে বর্গ করে যোগ করে পাই,

$2 + 2(\cos\theta\cos\varphi + \sin\theta\sin\varphi) = a^2 + b^2$

(ii) এখানে, $(b+c)\sin\frac{A}{2}$

$= (2R\sin B + 2R\sin C)\sin\frac{A}{2}$

$[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R]$

$= 2R(\sin B + \sin C)\sin\frac{A}{2}$

$= 2R.\ 2\sin\frac{B+C}{2}\cos\frac{B-C}{2}\sin\frac{A}{2}$

$= 2R.\ 2\sin\left(\frac{\pi}{2}-\frac{A}{2}\right)\cos\frac{B-C}{2}\sin\frac{A}{2}$

$\left[\because A+B+C=\pi,\ \therefore \frac{B+C}{2} = \frac{\pi}{2}-\frac{A}{2}\right]$

$= 2R.\ 2\cos\frac{A}{2}\cos\frac{B-C}{2}\sin\frac{A}{2}$

$= 2R.\ 2\sin\frac{A}{2}\cos\frac{A}{2}\cos\frac{B-C}{2}$

$= 2R.\sin A\cos\frac{B-C}{2}$

$= a\cos\frac{B-C}{2}$ $[\because a = 2R\sin A]$

$\therefore \cos\frac{B-C}{2} = \frac{b+c}{a}\sin\frac{A}{2}$ (প্রমাণিত)

(iii) এখানে, $\frac{b+c}{a} = \frac{2R\sin B + 2R\sin C}{2R\sin A}$

$\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= \frac{\sin B + \sin C}{\sin A} = \frac{\sin B + \sin\{\pi-(A+B)\}}{\sin A}$

$[\because A+B+C=\pi]$

$= \frac{\sin B + \sin(A+B)}{\sin A}$

$= \frac{2\sin\frac{A+2B}{2}\cos\frac{A}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}} = \frac{\sin\left(\frac{A}{2}+B\right)}{\sin\frac{A}{2}}$

$\therefore a\sin\left(\frac{A}{2}+B\right) = (b+c)\sin\frac{A}{2}$ (প্রমাণিত)

2. বামপক্ষ $= \frac{b-c}{a}\cos^2\frac{A}{2} + \frac{c-a}{b}\cos^2\frac{B}{2} + \frac{a-b}{c}\cos^2\frac{C}{2}$

$= \frac{2R\sin B - 2R\sin C}{2R\sin A}.\cos^2\frac{A}{2} + \frac{2R\sin C - 2R\sin A}{2R\sin B}.\cos^2\frac{B}{2} + \frac{2R\sin A - 2R\sin B}{2R\sin C}.\cos^2\frac{C}{2}$

$= \frac{\sin B - \sin C}{\sin A}.\cos^2\frac{A}{2} + \frac{\sin C - \sin A}{\sin B}.\cos^2\frac{B}{2} + \frac{\sin A - \sin B}{\sin C}.\cos^2\frac{C}{2}$

$= \frac{2.\sin\frac{B-C}{2}.\cos\frac{B+C}{2}}{2\sin\frac{A}{2}\cos\frac{A}{2}}.\cos^2\frac{A}{2} + \frac{2.\sin\frac{C-A}{2}.\cos\frac{C+A}{2}}{2\sin\frac{B}{2}\cos\frac{B}{2}}.\cos^2\frac{B}{2} + \frac{2.\sin\frac{A-B}{2}.\cos\frac{A+B}{2}}{2\sin\frac{C}{2}\cos\frac{C}{2}}.\cos^2\frac{C}{2}$

$= \frac{\sin\frac{B-C}{2}.\cos\left(\frac{\pi}{2}-\frac{A}{2}\right)}{\sin\frac{A}{2}}.\cos\frac{A}{2} + \frac{\sin\frac{C-A}{2}.\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)}{\sin\frac{B}{2}}.\cos\frac{B}{2} + \frac{\sin\frac{A-B}{2}.\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)}{\sin\frac{C}{2}}.\cos\frac{C}{2}$

$= \frac{\sin\frac{B-C}{2}\sin\frac{A}{2}\cos\left(\frac{\pi}{2}-\frac{B+C}{2}\right)}{\sin\frac{A}{2}} + \frac{\sin\frac{C-A}{2}.\sin\frac{B}{2}\cos\left(\frac{\pi}{2}-\frac{C+A}{2}\right)}{\sin\frac{B}{2}} + \frac{\sin\frac{A-B}{2}\sin\frac{C}{2}\cos\left(\frac{\pi}{2}-\frac{A+B}{2}\right)}{\sin\frac{C}{2}}$

$= \sin\frac{B-C}{2}\sin\frac{B+C}{2} + \sin\frac{C-A}{2}\sin\frac{C+A}{2} + \sin\frac{A-B}{2}\sin\frac{A+B}{2}$

$= \frac{1}{2}(\sin B - \sin C) + \frac{1}{2}(\sin C - \sin A) + \frac{1}{2}(\sin A - \sin B)$

$= \frac{1}{2}(\sin B - \sin C + \sin C - \sin A + \sin A - \sin B)$

$= \frac{1}{2}.0 = 0$

= ডানপক্ষ (প্রমাণিত)

3. (i) বামপক্ষ $= (b-c)\sin A + (c-a)\sin B + (a-b)\sin C$

$= (b-c)\frac{a}{2R} + (c-a)\frac{b}{2R} + (a-b)\frac{c}{2R}$ $\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= \frac{ab}{2R} - \frac{ac}{2R} + \frac{bc}{2R} - \frac{ab}{2R} + \frac{ac}{2R} - \frac{bc}{2R}$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= (b+c)\cos A + (c+a)\cos B + (a+b)\cos C$

$= b\cos A + c\cos A + c\cos B + a\cos B + a\cos C + b\cos C$

$= (b\cos C + c\cos B) + (c\cos A + a\cos C) + (a\cos B + b\cos A)$

$= a + b + c$

$=$ ডানপক্ষ (প্রমাণিত)

4. বামপক্ষ $= a(\sin B - \sin C) + b(\sin C - \sin A) + c(\sin A - \sin B)$

$= a\left(\frac{b}{2R} - \frac{c}{2R}\right) + b\left(\frac{c}{2R} - \frac{a}{2R}\right) + c\left(\frac{a}{2R} - \frac{b}{2R}\right)$ $\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= \frac{ab}{2R} - \frac{ac}{2R} + \frac{bc}{2R} - \frac{ab}{2R} + \frac{ac}{2R} - \frac{bc}{2R}$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

5. (i) বামপক্ষ $= a(\cos B + \cos C)$

$= a\cos B + a\cos C$

$= c - b\cos A + b - c\cos A$

$[\because c = a\cos B + b\cos A,\ b = c\cos A + a\cos C]$

$= (b+c) - (b+c)\cos A$

$= (b+c)(1-\cos A)$

$= (b+c).\ 2\sin^2\frac{A}{2}$

$= 2(b+c)\sin^2\frac{A}{2} =$ ডানপক্ষ (প্রমাণিত)

(ii) এখানে, $a(\cos C - \cos B) = a\cos C - a\cos B$

$= b - c\cos A - c + b\cos A$

$[\because b = c\cos A + a\cos C,\ c = b\cos A + a\cos B]$

$= (b-c) + (b-c)\cos A$

$= (b-c)(1+\cos A) = 2(b-c).\cos^2\frac{A}{2}$

$\therefore \cos C - \cos B = \frac{2(b-c)}{a}\cos^2\frac{A}{2}$ (প্রমাণিত)

6. (i) বামপক্ষ $= a^2(\sin^2 B - \sin^2 C) + b^2(\sin^2 C - \sin^2 A) + c^2(\sin^2 A - \sin^2 B)$

$= a^2\left(\frac{b^2}{4R^2} - \frac{c^2}{4R^2}\right) + b^2\left(\frac{c^2}{4R^2} - \frac{a^2}{4R^2}\right) + c^2\left(\frac{a^2}{4R^2} - \frac{b^2}{4R^2}\right)$ $\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{C}{\sin} = 2R\right]$

$= \frac{1}{4R^2}(a^2b^2 - c^2a^2 + b^2c^2 - a^2b^2 + a^2c^2 - b^2c^2)$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= a^2(\cos^2 B - \cos^2 C) + b^2(\cos^2 C - \cos^2 A) + c^2(\cos^2 A - \cos^2 B)$

$= a^2(\sin^2 C - \sin^2 B) + b^2(\sin^2 A - \sin^2 C) + c^2(\sin^2 B - \sin^2 A)$

$= a^2\left(\frac{c^2}{4R^2} - \frac{b^2}{4R^2}\right) + b^2\left(\frac{a^2}{4R^2} - \frac{c^2}{4R^2}\right) + c^2\left(\frac{b^2}{4R^2} - \frac{a^2}{4R^2}\right)$

$= \frac{1}{4R^2}(a^2c^2 - a^2b^2 + a^2b^2 - b^2c^2 + b^2c^2 - a^2c^2)$

$= \frac{1}{4R^2} \times 0$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= (b^2 - c^2)\sin^2 A + (c^2 - a^2)\sin^2 B + (a^2 - b^2)\sin^2 C$

$= (b^2 - c^2)\frac{a^2}{4R^2} + (c^2 - a^2)\frac{b^2}{4R^2} + (a^2 - b^2)\frac{c^2}{4R^2}$ $\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= \frac{1}{4R^2}(a^2b^2 - a^2c^2) + \frac{1}{4R^2}(b^2c^2 - a^2b^2) + \frac{1}{4R^2}(a^2c^2 - b^2c^2)$

$= \frac{1}{4R^2}(a^2b^2 - a^2c^2 + b^2c^2 - a^2b^2 + a^2c^2 - b^2c^2)$

$= \frac{1}{4R^2}.0 = 0 =$ ডানপক্ষ (প্রমাণিত)

7. (i) বামপক্ষের ১ম অংশ $= a\sin(B - C)$

$= a(\sin B\cos C - \cos B\sin C)$

$= 2R\sin A(\sin B\cos C - \cos B\sin C)$ $\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= 2R(\sin A\sin B\cos C - \sin A\cos B\sin C)$

অনুরূপভাবে, বামপক্ষের ২য় অংশ, $b\sin(C - A)$

$= 2R(\cos A\sin B\sin C - \sin A\sin B\cos C)$

এবং বামপক্ষের ৩য় অংশ, $c\sin(A - B)$

$= 2R(\sin A\cos B\sin C - \cos A\sin B\sin C)$

এখন,

বামপক্ষ $= a\sin(B - C) + b\sin(C - A) + c\sin(A - B)$

$= 2R(\sin A\sin B\cos C - \sin A\cos B\sin C + \cos A\sin B\sin C - \sin A\sin B\cos C + \sin A\cos B\sin C - \cos A\sin B\sin C)$

$= 2R \times 0$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষের ১ম অংশ $= a^3\sin(B - C)$

$= a^3(\sin B\cos C - \cos B\sin C)$

$= a^3\left(\frac{b}{2R}\cos C - \cos B.\frac{c}{2R}\right)$

$= a^2 . a \frac{1}{2R} (b \cos C - c \cos B)$

$= \frac{a^2}{2R} (b \cos C + c \cos B)(b \cos C - c \cos B)$

$= \frac{1}{2R} (a^2b^2 \cos^2 C - c^2a^2 \cos^2 B)$

অনুরূপভাবে,

২য় অংশ $= \frac{1}{2R} (b^2c^2 \cos^2 A - a^2b^2 \cos^2 C)$

৩য় অংশ $= \frac{1}{2R} (c^2a^2 \cos^2 B - b^2c^2 \cos^2 A)$

$\therefore$ বামপক্ষ $= a^3 \sin (B - C) + b^3 \sin(C - A) + c^3 \sin (A - B)$

$= \frac{1}{2R} (a^2b^2 \cos^2 C - c^2a^2 \cos^2 B) + \frac{1}{2R} (b^2c^2 \cos^2 A - a^2b^2 \cos^2 C) + \frac{1}{2R} (c^2a^2 \cos^2 B - b^2c^2 \cos^2 A)$

$= \frac{1}{2R} (a^2b^2 \cos^2 C - a^2c^2 \cos^2 B + b^2c^2 \cos^2 A - a^2b^2 \cos^2 C + a^2c^2 \cos^2 B - b^2c^2 \cos^2 A)$

$= \frac{1}{2R} .0 = 0 =$ ডানপক্ষ **(প্রমাণিত)**

(iii) বামপক্ষের ১ম অংশ $= a^3 \cos (B - C)$

$= a^3 (\cos B \cos C + \sin B \sin C)$

$= a(a^2 \cos B \cos C + a^2 \sin B \sin C)$

$= a(a \cos B. a \cos C + a \sin B. a \sin C)$

$= a \left\{ (c - b \cos A)(b - c \cos A) + 2R \sin A . \frac{b}{2R} . 2R \sin A . \frac{c}{2R} \right\}$

$= a\{bc - (b^2 + c^2) \cos A + bc \cos^2 A + bc \sin^2 A\}$

$= a\{bc - (b^2 + c^2) \cos A + bc\}$

$= a\{2bc - (b^2 + c^2) \cos A\}$

$= 2abc - a(b^2 + c^2) \cos A$

অনুরূপভাবে, ২য় অংশ $= 2abc - b(a^2 + c^2) \cos B$

৩য় অংশ $= 2abc - c(a^2 + b^2) \cos C$

$\therefore$ বামপক্ষ $= a^3 \cos(B - C) + b^3 \cos(C - A) + c^3 \cos(A - B)$

$= 2abc - a(b^2 + c^2) \cos A + 2abc - b(a^2 + c^2) \cos B + 2abc - c(a^2 + b^2) \cos C$

$= 6abc - \{ab^2 \cos A + ac^2 \cos A + ba^2 \cos B + bc^2 \cos B + ca^2 \cos C + cb^2 \cos C\}$

$= 6abc - \{ab (b \cos A + a \cos B) + bc (c \cos B + b \cos C) + ac(c \cos A + a \cos C)\}$

$= 6abc - (abc + abc + abc) = 6abc - 3abc$

$= 3abc =$ ডানপক্ষ **(প্রমাণিত)**

8. এখানে, ১ম অংশ $= \frac{a \sin(B - C)}{b^2 - c^2}$

$= \frac{2R \sin A \sin(B - C)}{4R^2 \sin^2 B - 4R^2 \sin^2 C}$

$= \frac{2R \sin\{\pi - (B + C)\} \sin(B - C)}{4R^2 (\sin^2 B - \sin^2 C)}$

$= \frac{1}{2R} \cdot \frac{\sin(B + C) \sin(B - C)}{\sin(B + C) \sin(B - C)}$

$= \frac{1}{2R}$

অনুরূপভাবে, ২য় অংশ $= \frac{1}{2R}$, ৩য় অংশ $= \frac{1}{2R}$

$\therefore \frac{a \sin(B - C)}{b^2 - c^2} = \frac{b \sin(C - A)}{c^2 - a^2} = \frac{c \sin(A - B)}{a^2 - b^2}$

(প্রমাণিত)

9. বামপক্ষ $= a \sin \frac{A}{2} \sin \frac{B - C}{2} + b \sin \frac{B}{2} \sin \frac{C - A}{2} + c \sin \frac{C}{2} \sin \frac{A - B}{2}$

এখন, $a \sin \frac{A}{2} \sin \frac{B - C}{2}$

$= a \sin \left\{ \frac{\pi}{2} - \left(\frac{B + C}{2} \right) \right\} \sin \frac{B - C}{2}$ $[\because A + B + C = \pi]$

$= a \cos \left(\frac{B}{2} + \frac{C}{2} \right) \sin \left(\frac{B}{2} - \frac{C}{2} \right)$

$= \frac{1}{2} a. 2\cos \left(\frac{B}{2} + \frac{C}{2} \right) \sin \left(\frac{B}{2} - \frac{C}{2} \right)$

$= \frac{1}{2} a. \left\{ \sin \left(\frac{B}{2} + \frac{C}{2} + \frac{B}{2} - \frac{C}{2} \right) - \sin \left(\frac{B}{2} + \frac{C}{2} - \frac{B}{2} + \frac{C}{2} \right) \right\}$

$= \frac{1}{2} a (\sin B - \sin C) = \frac{1}{2} a. \left(\frac{b}{2R} - \frac{c}{2R} \right) = \frac{1}{4R} (ab - ac)$

অনুরূপভাবে, $b \sin \frac{B}{2} \sin \frac{C - A}{2} = \frac{1}{4R} (bc - ab)$

এবং $c \sin \frac{C}{2} \sin \frac{A - B}{2} = \frac{1}{4R} (ca - bc)$

$\therefore$ বামপক্ষ $= \frac{1}{4R} (ab - ac + bc - ab + ca - bc)$

$= \frac{1}{4R} \times 0 = 0$

= ডানপক্ষ **(প্রমাণিত)**

10. (i) বামপক্ষের ১ম অংশ $= \frac{a^2 \sin(B - C)}{\sin A}$

$= \frac{4R^2 \sin^2 A \sin(B - C)}{\sin A}$

$= \frac{4R^2 . \sin A . \sin A . \sin(B - C)}{\sin A}$

$= 4R^2 \sin\{\pi - (B + C)\} \sin(B - C)$ $[\because A + B + C = \pi]$

$= 4R^2 \sin(B + C) \sin(B - C)$

$= 4R^2 (\sin^2 B - \sin^2 C)$

অনুরূপভাবে, ২য় অংশ $= 4R^2 (\sin^2 C - \sin^2 A)$

৩য় অংশ $= 4R^2 (\sin^2 A - \sin^2 B)$

$\therefore$ বামপক্ষ $= \frac{a^2 \sin(B - C)}{\sin A} + \frac{b^2 \sin(C - A)}{\sin B} + \frac{c^2 \sin(A - B)}{\sin C}$

$= 4R^2 (\sin^2 B - \sin^2 C + \sin^2 C - \sin^2 A + \sin^2 A - \sin^2 B)$

$= 4R^2 . 0 = 0 =$ ডানপক্ষ **(প্রমাণিত)**

(ii) বামপক্ষের ১ম অংশ $= \frac{a^2\sin(B-C)}{\sin B + \sin C}$

$= \frac{a.a\sin(B-C)}{\sin B+\sin C} = \frac{a.2R\sin A.\sin(B-C)}{\sin B+\sin C}$

$= \frac{2a.R\sin\{\pi-(B+C)\}\sin(B-C)}{\sin B+\sin C}$

$= \frac{2aR.\sin(B+C)\sin(B-C)}{\sin B+\sin C}$

$= \frac{2aR(\sin^2B-\sin^2C)}{\sin B+\sin C}$

$= 2aR(\sin B-\sin C)$

$= 4R^2\sin A(\sin B-\sin C)$

অনুরূপভাবে, ২য় অংশ $= 4R^2\sin B(\sin C-\sin A)$

৩য় অংশ $= 4R^2\sin C(\sin A-\sin B)$

$\therefore$ বামপক্ষ $= \frac{a^2\sin(B-C)}{\sin B+\sin C} + \frac{b^2\sin(C-A)}{\sin C+\sin A} + \frac{c^2\sin(A-B)}{\sin A+\sin B}$

$= 4R^2\sin A(\sin B-\sin C) + 4R^2\sin B(\sin C-\sin A) + 4R^2\sin C(\sin A-\sin B)$

$= 4R^2(\sin A\sin B - \sin A\sin C + \sin B\sin C - \sin B\sin A + \sin C\sin A - \sin C\sin B)$

$= 4R^2.0$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষের ১ম অংশ $= \frac{b^2-c^2}{\cos B+\cos C}$

$= \frac{4R^2(\sin^2B-\sin^2C)}{\cos B+\cos C}$ $\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= \frac{4R^2(\cos^2C-\cos^2B)}{\cos B+\cos C}$ $[\because \sin^2\theta = 1-\cos^2\theta]$

$= \frac{4R^2(\cos C+\cos B)(\cos C-\cos B)}{(\cos B+\cos C)}$

$= 4R^2(\cos C-\cos B)$

অনুরূপভাবে,

বামপক্ষের ২য় অংশ $= \frac{c^2-a^2}{\cos C+\cos A} = 4R^2(\cos A-\cos C)$

এবং বামপক্ষের ৩য় অংশ $= \frac{a^2-b^2}{\cos A+\cos B}$

$= 4R^2(\cos B-\cos A)$

$\therefore$ বামপক্ষ $= \frac{b^2-c^2}{\cos B+\cos C} + \frac{c^2-a^2}{\cos C+\cos A} + \frac{a^2-b^2}{\cos A+\cos B}$

$= 4R^2(\cos C-\cos B) + 4R^2(\cos A-\cos C) + 4R^2(\cos B-\cos A)$

$= 4R^2(\cos C-\cos B+\cos A-\cos C+\cos B-\cos A)$

$= 4R^2\times 0$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

11. বামপক্ষের ১ম অংশ $= \frac{b^2-c^2}{a^2}\sin 2A$

$= \frac{4R^2\sin^2B - 4R^2\sin^2C}{4R^2\sin^2A}.2\sin A\cos A$

$= \frac{\sin^2B-\sin^2C}{\sin A}.2\cos A$

$= \frac{\sin(B+C)\sin(B-C)}{\sin A}.2\cos A$

$= \frac{\sin(\pi-A)\sin(B-C)}{\sin A}.2\cos A$ $[\because A+B+C=\pi]$

$= \sin(B-C).2\cos\{\pi-(B+C)\}$

$= -2\sin(B-C).\cos(B+C)$

$= -(\sin 2B-\sin 2C)$

$= \sin 2C-\sin 2B$

অনুরূপভাবে, ২য় অংশ $= \sin 2A-\sin 2C$

এবং ৩য় অংশ $= \sin 2B-\sin 2A$

$\therefore$ বামপক্ষ $= \frac{b^2-c^2}{a^2}\sin 2A + \frac{c^2-a^2}{b^2}\sin 2B + \frac{a^2-b^2}{c^2}\sin 2C$

$= \sin 2C-\sin 2B+\sin 2A-\sin 2C+\sin 2B-\sin 2A$

$= 0 =$ ডানপক্ষ (প্রমাণিত)

12. ডানপক্ষ $= (a-b)^2\cos^2\frac{C}{2} + (a+b)^2\sin^2\frac{C}{2}$

$= \frac{1}{2}(a-b)^2(1+\cos C) + \frac{1}{2}(a+b)^2(1-\cos C)$

$= \frac{1}{2}\{(a-b)^2+(a+b)^2\} + \frac{1}{2}\{(a-b)^2 - (a+b)^2\}\cos C$

$= \frac{1}{2}(2a^2+2b^2) - \frac{1}{2}.4ab\cos C$

$= a^2+b^2-2ab\cos C$

$= c^2 =$ বামপক্ষ (প্রমাণিত)

13. বামপক্ষ $= b^2\sin 2C + c^2\sin 2B$

$= 4R^2\sin^2B\sin 2C + 4R^2\sin^2C\sin 2B$

$= 4R^2\sin^2B.2\sin C\cos C + 4R^2\sin^2C.2\sin B\cos B$

$= 8R^2\sin B\sin C(\sin B\cos C+\cos B\sin C)$

$= 8R^2\sin B\sin C\sin(B+C)$

$= 2.2R\sin B.2R\sin C.\sin A$

$[\because \sin(B+C) = \sin(\pi-A)]$

$= 2b.c.\sin A$

$= 2bc\sin A$

$=$ ডানপক্ষ (প্রমাণিত)

14. বামপক্ষ $= bc\cos^2\frac{A}{2} + ca\cos^2\frac{B}{2} + ab\cos^2\frac{C}{2}$

$= bc.\frac{s(s-a)}{bc} + ca.\frac{s(s-b)}{ca} + ab.\frac{s(s-c)}{ab}$

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$= s(s-a) + s(s-b) + s(s-c)$
$= s^2 - sa + s^2 - sb + s^2 - sc$
$= 3s^2 - s(a+b+c)$
$= 3s^2 - s.2s \quad \left[\because s = \frac{a+b+c}{2}\right]$
$= 3s^2 - 2s^2$
$= s^2$
= ডানপক্ষ (প্রমাণিত)

15. বামপক্ষ $= \frac{a^2-b^2}{2} \cdot \frac{\sin A \sin B}{\sin(A-B)}$
$= \frac{4R^2\sin^2 A - 4R^2 \sin^2 B}{2} \cdot \frac{\sin A \sin B}{\sin(A-B)}$
$= \frac{4R^2(\sin^2 A - \sin^2 B)}{2} \cdot \frac{\sin A \sin B}{\sin(A-B)}$
$= 2R^2 \sin(A+B)\sin(A-B) \cdot \frac{\sin A \sin B}{\sin(A-B)}$
$= 2R^2 \sin(\pi - C).\sin A \sin B \quad [\because A+B+C=\pi]$
$= 2R^2.\sin A \sin B \sin C$
$= 2R^2.\frac{a}{2R} \cdot \frac{b}{2R} \sin C \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$
$= \frac{ab}{2}\sin C$
$= \Delta$
= ডানপক্ষ (প্রমাণিত)

16. বামপক্ষ $= \sin A + \sin B + \sin C$
$= \frac{a}{2R} + \frac{b}{2R} + \frac{c}{2R} \left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$
$= \frac{a+b+c}{2R}$
$= \frac{2s}{2R} \qquad [\because a+b+c = 2s]$
$= \frac{s}{R}$
= ডানপক্ষ (প্রমাণিত)

17. বামপক্ষের ১ম অংশ $= 4\Delta \cot A$
$= 4.\frac{1}{2} bc \sin A.\frac{\cos A}{\sin A} = 2bc \cos A$
$= 2bc\left(\frac{b^2+c^2-a^2}{2bc}\right) = b^2+c^2-a^2$
অনুরূপভাবে, বামপক্ষের ২য় অংশ $= 4\Delta \cot B$
$= c^2 + a^2 - b^2$
এবং বামপক্ষের ৩য় অংশ $= 4\Delta \cot C = a^2 + b^2 - c^2$
$\therefore$ বামপক্ষ $= 4\Delta(\cot A + \cot B + \cot C)$
$= 4\Delta \cot A + 4\Delta \cot B + 4\Delta \cot C$
$= b^2 + c^2 - a^2 + c^2 + a^2 - b^2 + a^2 + b^2 - c^2$
$= a^2 + b^2 + c^2$
= ডানপক্ষ (প্রমাণিত)

18. বামপক্ষ $= b + c$
$= 2R(\sin B + \sin C)$
$= 2R.2\sin\frac{B+C}{2}\cos\frac{B-C}{2}$
$\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$
$= 2R.2\sin 60° \cos\frac{B-C}{2}$
$[\because A = 60°, \therefore B + C = 120°]$
$= 2R.\ 2\sin A \cos\frac{B-C}{2}$
$= 2.\ 2R \sin A \cos\frac{B-C}{2}$
$= 2a\cos\frac{B-C}{2}$
= ডানপক্ষ (দেখানো হলো)

19. (i) মনে করি, ত্রিভুজটির অর্ধপরিসীমা $= s$
তাহলে $2s = 2\left(\frac{y}{z} + \frac{z}{x} + \frac{x}{y}\right)$
$\therefore s = \frac{y}{z} + \frac{z}{x} + \frac{x}{y}$
ধরি, $a = \frac{y}{z} + \frac{z}{x}$, $b = \frac{z}{x} + \frac{x}{y}$ এবং $c = \frac{x}{y} + \frac{y}{z}$
$\therefore s - a = \frac{y}{z} + \frac{z}{x} + \frac{x}{y} - \frac{y}{z} - \frac{z}{x} = \frac{x}{y}$
অনুরূপভাবে, $s - b = \frac{y}{z}$, $s - c = \frac{z}{x}$
$\therefore$ ত্রিভুজের ক্ষেত্রফল $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{\left(\frac{y}{z} + \frac{z}{x} + \frac{x}{y}\right) \cdot \frac{x}{y} \cdot \frac{y}{z} \cdot \frac{z}{x}}$
$= \sqrt{\frac{y}{z} + \frac{z}{x} + \frac{x}{y}}$ **(Ans.)**

(ii) দেওয়া আছে, $a = 13$ একক, $b = 14$ একক
এবং $c = 15$ একক
তাহলে, $2s = a + b + c$
$= (13 + 14 + 15)$ একক
$= 42$ একক
$\therefore s = 21$ একক
এখন, ক্ষেত্রফল $= \sqrt{s(s-a)(s-b)(s-c)}$
$= \sqrt{21(21-13)(21-14)(21-15)}$
$= \sqrt{21.8.7.6}$
$= \sqrt{7056}$
$= 84$ বর্গ একক **(Ans.)**

20. আমরা জানি, যে কোনো ত্রিভুজে $\frac{a}{\sin A} = \frac{b}{\sin B}$
বা, $\frac{2b}{\sin A} = \frac{b}{\sin B} \quad [\because a = 2b]$

বা, $\frac{2}{\sin A} = \frac{1}{\sin B}$

বা, $\frac{2}{\sin 3B} = \frac{1}{\sin B}$ [$\because A = 3B$]

বা, $\sin 3B = 2\sin B$

বা, $3\sin B - 4\sin^3 B = 2\sin B$

বা, $-4\sin^3 B = 2\sin B - 3\sin B$

বা, $-4\sin^3 B = -\sin B$

বা, $4\sin^2 B = 1$ [$\because \sin B \neq 0$]

বা, $\sin^2 B = \frac{1}{4}$

বা, $\sin B = \frac{1}{2}$

বা, $\sin B = \sin 30°$

$\therefore B = 30°$

$\therefore A = 3B = 3 \times 30° = 90°$ [$\because B = 30°$]

আমরা জানি, $A + B + C = 180°$

বা, $C = 180° - (A + B)$

বা, $C = 180° - (90° + 30°)$

বা, $C = 180° - 120°$ $\therefore C = 60°$

$\therefore$ নির্ণেয় $A = 90°, B = 30°, C = 60°$ **(Ans.)**

21. (i) দেওয়া আছে, $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

বা, $\frac{b+c+a+c}{(a+c)(b+c)} = \frac{3}{a+b+c}$

বা, $(a+b+c)(a+b+c+c) = 3(a+c)(b+c)$

বা, $(a+b+c)^2 + ac + bc + c^2 = 3(ab + ac + bc + c^2)$

বা, $(a+b+c)^2 - 3ab - 2ac - 2bc - 2c^2 = 0$

বা, $a^2 + b^2 + c^2 + 2ab + 2bc + 2ac - 3ab - 2ac - 2bc - 2c^2 = 0$

বা, $a^2 + b^2 - c^2 - ab = 0$

বা, $a^2 + b^2 - c^2 = ab$

বা, $\frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}$

বা, $\cos C = \frac{1}{2} = \cos 60°$

$\therefore C = 60°$ **(দেখানো হলো)**

(ii) দেওয়া আছে, $C = 60°$

বা, $\cos C = \cos 60°$

বা, $\frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}$ $\left[\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}\right]$

বা, $\frac{a^2 + b^2 - c^2}{ab} = 1$

বা, $a^2 + b^2 - c^2 = ab$

বা, $a^2 + b^2 - c^2 - ab = 0$

বা, $(a+b+c)^2 - 3ab - 2bc - 2ca - 2c^2 = 0$

বা, $(a+b+c)^2 + ac + bc + c^2 = 3ab + 3bc + 3ac + 3c^2$

বা, $(a+b+c)^2 + c(a+b+c) = 3b(a+c) + 3c(a+c)$

বা, $(a+b+c)(a+b+c+c) = (a+c)(3b+3c)$

বা, $(a+b+c)\{(a+c)+(b+c)\} = 3(a+c)(b+c)$

বা, $\frac{(a+c)+(b+c)}{(a+c)(b+c)} = \frac{3}{a+b+c}$

$\therefore \frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ **(দেখানো হলো)**

22. দেওয়া আছে, $(a+b+c)(b+c-a) = 3bc$

বা, $ab + b^2 + bc + ac + bc + c^2 - a^2 - ab - ac = 3bc$

বা, $b^2 + c^2 - a^2 = 3bc - bc - bc$

বা, $b^2 + c^2 - a^2 = bc$

বা, $\frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2}$

বা, $\cos A = \frac{1}{2} = \cos 60°$

$\therefore A = 60°$ **(Ans.)**

23. এখানে, বৃহত্তম বাহু = 7

মনে করি, বৃহত্তম কোণ = A

$\therefore \cos A = \frac{3^2 + 5^2 - 7^2}{2.3.5}$

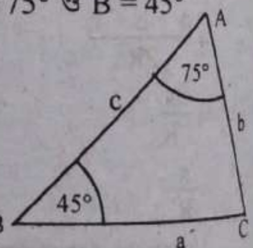
$= \frac{9 + 25 - 49}{30}$

$= -\frac{1}{2} = \cos 120°$

অর্থাৎ, $A = 120°$ সুতরাং ত্রিভুজটি স্থূলকোণী।

এবং স্থূলকোণটির মান $120°$ **(দেখানো হলো)**

24. দেওয়া আছে, ΔABC-এ $A = 75°$ ও $B = 45°$



তাহলে, $C = 180° - (A + B)$

$= 180° - (75° + 45°)$

$= 180° - 120°$

$= 60°$

আবার, ত্রিভুজ সূত্র হতে পাই,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$\therefore b = 2R \sin B$

$c = 2R \sin C$

এখন, $\frac{b}{c} = \frac{2R\sin B}{2R\sin C} = \frac{\sin 45°}{\sin 60°} = \frac{\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{2}}{\sqrt{3}}$

$\therefore b : c = \sqrt{2} : \sqrt{3}$ **(দেখানো হলো)**

25. $a = \sqrt{3} + 1$, $b = \sqrt{3} - 1$ এবং $C = 60°$

আমরা জানি, $c^2 = a^2 + b^2 - 2ab\cos C$

$= (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - 2(\sqrt{3}+1)(\sqrt{3}-1)\cos 60°$

$= 2(3+1) - 2(3-1)\frac{1}{2} = 8 - 2 = 6$

$\therefore c = \sqrt{6}$ **(Ans.)**

আবার ট্যানজেন্ট সূত্র হতে আমরা পাই,

$\tan\frac{A-B}{2}=\frac{a-b}{a+b}\cot\frac{C}{2}$

বা, $\tan\frac{A-B}{2}=\frac{\sqrt{3}+1-\sqrt{3}+1}{\sqrt{3}+1+\sqrt{3}-1}\cot\frac{60^\circ}{2}$

বা, $\tan\frac{A-B}{2}=\frac{2}{2\sqrt{3}}.\cot30^\circ$

বা, $\tan\frac{A-B}{2}=\frac{1}{\sqrt{3}}.\sqrt{3}$

বা, $\tan\frac{A-B}{2}=1$ বা, $\tan\frac{A-B}{2}=\tan45^\circ$

বা, $\frac{A-B}{2}=45^\circ$ বা, $A-B=90^\circ$ ... ... (i)

$A+B+C=180^\circ$ বা, $A+B+60^\circ=180^\circ$

বা, $A+B=120^\circ$ ... ... (ii)

(i) ও (ii) যোগ করে, $A-B+A+B=90^\circ+120^\circ$

বা, $2A=210^\circ$ বা, $A=105^\circ$**(Ans.)**

(ii) থেকে (i) বিয়োগ করে,

$A+B-A+B=210^\circ-90^\circ$

বা, $2B=30^\circ$ $\therefore B=15^\circ$

বহুনির্বাচনি প্রশ্নের উত্তর ও ব্যাখ্যা

1. ক; **ব্যাখ্যা:** $\tan15^\circ=\tan(45^\circ-30^\circ)$

$=\frac{\tan45^\circ-\tan30^\circ}{1+\tan45^\circ\tan30^\circ}=\frac{1-\frac{1}{\sqrt{3}}}{1+\frac{1}{\sqrt{3}}}=\frac{\sqrt{3}-1}{\sqrt{3}+1}$

2. ঘ; **ব্যাখ্যা:** $\tan\left(6\times\pi+\frac{\pi}{3}\right)=\tan\frac{\pi}{3}=\sqrt{3}$

3. খ; **ব্যাখ্যা:** $\cos(11\times90^\circ-30^\circ)=-\sin30^\circ=-\frac{1}{2}$

4. গ; **ব্যাখ্যা:** $\sin\left\{-\pi+(-1)^{-1}\frac{\pi}{6}\right\}=\sin\left(-\pi-\frac{\pi}{6}\right)$

$=\sin\left\{-\left(2\times\frac{\pi}{2}+\frac{\pi}{6}\right)\right\}=+\sin\frac{\pi}{6}=\frac{1}{2}$

5. ক; **ব্যাখ্যা:** $\text{cosec}\,(3\times90^\circ-\theta)=-\sec\theta$

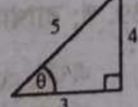
6. গ; **ব্যাখ্যা:** $\tan45^\circ=1$ তাহলে, $\tan(36^\circ+9^\circ)=1$

বা, $\frac{\tan36^\circ+\tan9^\circ}{1-\tan36^\circ\tan9^\circ}=1$

$\therefore \tan36^\circ+\tan9^\circ+\tan36^\circ.\tan9^\circ=1$

7. ঘ; 8. ক;

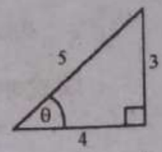
9. খ; **ব্যাখ্যা:** $\frac{\tan\theta+\sec(-\theta)}{\cot\theta+\text{cosec}(-\theta)}=\frac{-\frac{4}{3}-\frac{5}{3}}{-\frac{3}{4}-\frac{5}{4}}$



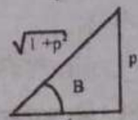
$\left[\because \frac{\pi}{2}<\theta<2\right]$

$=\frac{3}{2}$

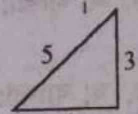
10. ক; **ব্যাখ্যা:** $\tan\theta=\frac{\sin\theta}{\cos\theta}=\frac{\frac{3}{5}}{-\frac{4}{5}}=-\frac{3}{4}$



11. গ; **ব্যাখ্যা:** $\sin B=\frac{\text{লম্ব}}{\text{অতিভুজ}}=\frac{p}{\sqrt{1+p^2}}$



12. ঘ; **ব্যাখ্যা:** $\frac{1+\left(\frac{3}{4}\right)^2}{1-\left(\frac{3}{4}\right)^2}=\frac{16+9}{16-9}=\frac{25}{7}$



13. খ; **ব্যাখ্যা:** যেহেতু θ কোন চতুর্ভাগে তা উল্লেখ নেই তাই $\tan\theta$ এর মান ধনাত্মক বা ঋণাত্মক দুটিই হতে পারে।

14. ক; **ব্যাখ্যা:** $\cos A=\frac{1}{2}$ এবং $\sin B=\frac{1}{\sqrt{2}}$

$\therefore \sin A=\sqrt{1-\cos^2A}=\sqrt{1-\left(\frac{1}{2}\right)^2}=\frac{\sqrt{3}}{2}$

$\therefore \sin(A+B)\sin(A-B)=\sin^2A-\sin^2B$

$=\left(\frac{\sqrt{3}}{2}\right)^2-\left(\frac{1}{\sqrt{2}}\right)^2=\frac{1}{4}$

15. খ; **ব্যাখ্যা:** $\frac{\tan45^\circ+\tan25^\circ}{1-\tan45^\circ.\tan25^\circ}=\tan(45^\circ+25^\circ)=\tan70^\circ$

16. খ; **ব্যাখ্যা:** $\cos A+2\cos120^\circ.\cos A$

$=\cos A+2(-\frac{1}{2}\cos A)=0$

17. ঘ; **ব্যাখ্যা:** $\frac{2\cos45^\circ\cos A}{2\sin45^\circ\sin A}=\cot A$

18. ঘ; **ব্যাখ্যা:** $\sin(45^\circ+\theta+45^\circ-\theta)=\sin90^\circ=1$

19. খ; **ব্যাখ্যা:** শুধুমাত্র cos ও sec এর θ ঋণাত্মক হলে, তা ধনাত্মক হয়।

অর্থাৎ $\cos(-\theta)=\cos\theta$, $\sec(-\theta)=\sec\theta$

20. ঘ; **ব্যাখ্যা:** (i) সঠিক, $\frac{b}{\sin B}=\frac{c}{\sin C}$

বা, $\frac{2c}{\sin3C}=\frac{c}{\sin C}$ $\therefore \sin3C=2\sin C$

(ii) সঠিক; $\sin3C=2\sin C$

বা, $3\sin C-4\sin^3C=2\sin C$

বা, $4\sin^3c=\sin C$ বা, $\sin^2C=\frac{1}{4}$

বা, $\sin C=\frac{1}{2}=\sin30^\circ$ $\therefore C=30^\circ$

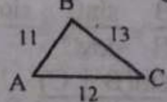
(iii) সঠিক, $B=3C=3\times30^\circ=90^\circ$

21. ঘ;

22. গ; **ব্যাখ্যা:** i. সঠিক নয়, $\cos A=\frac{11^2+12^2-13^2}{2\times11\times12}$

$=0.3636$

$\therefore A=68.68^\circ$



বৃহত্তম কোণ সূক্ষ্মকোণ বলে ত্রিভুজটি সূক্ষ্মকোণী।

ii. $S = \dfrac{11 + 12 + 13}{2} = 18$

$\therefore$ ত্রিভুজের ক্ষেত্রফল

$= \sqrt{18(18-11)(18-12)(18-13)}$

$= 61.48$ বর্গ একক (প্রায়)

iii. পরিব্যাসার্ধ, $R = \dfrac{13}{2 \times \sin 68.68^\circ} = 7$ একক (প্রায়)

23. **খ; ব্যাখ্যা:** ii. সঠিক নয়; কারণ $\tan 105^\circ . \tan 165^\circ = 1$ [Calculator ব্যবহার করে]

24. **ঘ; ব্যাখ্যা:** $\tan(B + C) = \tan\left(\dfrac{\pi}{2} - A\right) = \cot A$

বা, $\dfrac{\tan B + \tan C}{1 - \tan B . \tan C} = \dfrac{1}{\tan A}$

$\therefore \tan A . \tan B + \tan B . \tan C + \tan C . \tan A = 1$

আবার, $\tan(A + B - C) = \tan\left(\dfrac{\pi}{2} - C - C\right)$

$= \tan\left(\dfrac{\pi}{2} - 2C\right) = \cot 2C$

25. **ঘ; ব্যাখ্যা:** $\cot x = -\sqrt{3} \therefore \tan x = -\dfrac{1}{\sqrt{3}}$

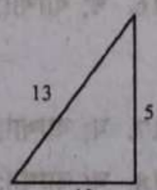
আবার, $\dfrac{\cos x}{\sin x} = -\sqrt{3}$ বা, $\cos x = -\dfrac{\sqrt{3}}{2}$

26. **ক; ব্যাখ্যা:** $\tan\theta = \dfrac{5}{12} \therefore \cos\theta = \dfrac{12}{13}$

কিন্তু $\sin\theta$ ঋণাত্মক এবং $\tan\theta$ ধনাত্মক।

সুতরাং $\cos\theta$ অবশ্যই ঋণাত্মক হবে।

$\therefore \cos\theta = \dfrac{-12}{13}$



27. **ঘ; ব্যাখ্যা:** $\sin\theta = -\dfrac{5}{13}$, $\cos\theta = -\dfrac{12}{13}$,

$\sec\theta = -\dfrac{13}{12}$, $\tan\theta = \dfrac{5}{12}$

$= \dfrac{\sin\theta + \cos(-\theta)}{\sec(-\theta) + \tan\theta} = \dfrac{\sin\theta + \cos\theta}{\sec\theta + \tan\theta}$

$= \dfrac{-\frac{5}{13} - \frac{12}{13}}{-\frac{13}{12} + \frac{5}{12}} = \dfrac{-\frac{17}{13}}{\frac{-8}{12}} = -\dfrac{17}{13} \times \dfrac{12}{-8} = \dfrac{51}{26}$

28. **গ;** **29.** **ঘ;**

30. **খ; ব্যাখ্যা:** $\dfrac{1}{\tan\alpha} + \dfrac{1}{\tan\beta} = a$ বা, $\dfrac{\tan\alpha + \tan\beta}{\tan\alpha . \tan\beta} = a$

$\therefore \tan\alpha . \tan\beta = \dfrac{b}{a}$

31. **খ; ব্যাখ্যা:** $\dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha . \tan\beta} = \dfrac{b}{1 - \frac{b}{a}} = \dfrac{ab}{a - b}$

32. **গ; ব্যাখ্যা:** $\dfrac{\sin B}{\cos B} + \dfrac{\sin C}{\cos C} = \dfrac{\sin B . \cos C + \cos B . \sin C}{\cos B \cos C}$

$= \dfrac{\sin(B + C)}{\cos A} = \dfrac{\sin(\pi - A)}{\cos A} = \tan A$

33. **ঘ; ব্যাখ্যা:** $\tan(B + C) = \tan(\pi - A)$

বা, $\dfrac{\tan B + \tan C}{1 - \tan B . \tan C} = -\tan A$

বা, $\tan A = -\tan A + \tan A . \tan B . \tan C$

বা, $2\tan A = \tan A . \tan B . \tan C \therefore \tan B . \tan C = 2$

34. **খ; ব্যাখ্যা:** $\cos\theta \cos 2\theta \cos 4\theta$

$= \cos 20^\circ \cos 40^\circ \cos 80^\circ \quad \left[\because \theta = \dfrac{\pi}{9} = 20^\circ\right]$

$= \dfrac{1}{2} \cos 20^\circ (\cos 120^\circ + \cos 40^\circ)$

$= \dfrac{1}{2} \cos 20^\circ \left(-\dfrac{1}{2} + \cos 40^\circ\right)$

$= -\dfrac{1}{4} \cos 20^\circ + \dfrac{1}{2} \cos 40^\circ \cos 20^\circ$

$= -\dfrac{1}{4} \cos 20^\circ + \dfrac{1}{4} (\cos 60^\circ + \cos 20^\circ)$

$= \dfrac{1}{4} \cos 60^\circ = \dfrac{1}{4} . \dfrac{1}{2} = \dfrac{1}{8}$

35. **খ; ব্যাখ্যা:** $\tan\dfrac{\theta}{2} = \dfrac{3}{4}$

$\therefore \cos\theta = \dfrac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} = \dfrac{1 - \frac{9}{16}}{1 + \frac{9}{16}} = \dfrac{7}{25}$

36. **খ; ব্যাখ্যা:** $\sin(A - 30^\circ) + \sin(150^\circ + A)$

$= \sin(A - 30^\circ) + \sin\{(180^\circ + (A - 30^\circ)\}$

$= \sin(A - 30^\circ) - \sin(A - 30^\circ) = 0$

37. **খ; ব্যাখ্যা:** $\cos(\theta + 150^\circ)$

$= \cos\{180^\circ - (30^\circ - \theta)\} = -\cos(30^\circ - \theta)$

$= -(\cos 30^\circ \cos\theta + \sin 30^\circ \sin\theta)$

$= -\left(\dfrac{\sqrt{3}}{2} \cos\theta + \dfrac{1}{2} \sin\theta\right) = -\dfrac{1}{2} (\sin\theta + \sqrt{3}\cos\theta)$

38. **ঘ; ব্যাখ্যা:** ধরি, $a = 3x$, $b = 7x$, $c = 5x$

$\therefore \cos B = \dfrac{(3x)^2 + (5x)^2 - (7x)^2}{2 . 3x . 5x}$

$= \dfrac{9x^2 + 25x^2 - 49x^2}{30x^2} = \dfrac{-15x^2}{30x^2} = -\dfrac{1}{2}$

$\therefore \cos B = -\dfrac{1}{2} = \cos 120^\circ$

$\therefore B = 120^\circ$

39. **খ; ব্যাখ্যা:** ধরি, $\angle C = \dfrac{\pi}{2} \quad \therefore A + B = \dfrac{\pi}{2}$

এখন, $\cos^2 A + \cos^2 B + \cos^2 C = \cos^2 A + \cos^2 B + 0$

$= \cos^2 A + 1 - \sin^2 B$

$= \cos^2 A + 1 - \left\{\sin\left(\dfrac{\pi}{2} - A\right)\right\}^2$

$= \cos^2 A + 1 - \cos^2 A = 1$

40. গ; ব্যাখ্যা: $\cos A + \cos C = \sin B$

$\Rightarrow 2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = \sin B$

$\Rightarrow 2\cos\left(\frac{\pi}{2}-\frac{B}{2}\right)\cos\frac{A-C}{2} = \sin B$

$\Rightarrow 2\sin\frac{B}{2}\cos\frac{A-C}{2} = 2\sin\frac{B}{2}\cos\frac{B}{2}$

$\Rightarrow \cos\frac{A-C}{2} = \cos\frac{B}{2} \Rightarrow \frac{A-C}{2} = \frac{B}{2}$

$\Rightarrow \frac{A}{2} = \frac{B}{2} + \frac{C}{2} \Rightarrow \frac{A}{2} = \frac{\pi}{2} - \frac{A}{2} \Rightarrow A = \frac{\pi}{2}$

41. গ; ব্যাখ্যা: বৃত্তে অন্তর্লিখিত সমবাহু ত্রিভুজের পরিব্যাসার্ধ ঐ বৃত্তের ব্যাসার্ধের সমান। সমবাহু ত্রিভুজের বাহুর দৈর্ঘ্য x হলে, $\frac{x}{\sin 60°} = 2.R$

বা, $\frac{x}{\frac{\sqrt{3}}{2}} = 2.1$ [পরিব্যাসার্ধ = 1]

$\therefore x = 2.\frac{\sqrt{3}}{2} = \sqrt{3}$

42. গ; ব্যাখ্যা: $\sin A + \cos A = \sin B + \cos B$

বা, $\sin A - \sin B = \cos B - \cos A$

বা, $2\cos\frac{A+B}{2}.\sin\frac{A-B}{2} = 2\sin\frac{A+B}{2}.\sin\frac{A-B}{2}$

বা, $\cos\frac{A+B}{2} = \sin\frac{A+B}{2}$

বা, $\tan\frac{A+B}{2} = \tan\frac{\pi}{4}$ বা, $\frac{A+B}{2} = \frac{\pi}{4}$

$\therefore A + B = \frac{\pi}{2}$

43. ক; **44.** ক; **45.** ঘ;

46. ঘ; ব্যাখ্যা: $3\sec^4\theta + 8 = 10\sec^2\theta$

বা, $3\sec^4\theta - 10\sec^2\theta + 8 = 0$

বা, $3\sec^4\theta - 6\sec^2\theta - 4\sec^2\theta + 8 = 0$

বা, $3\sec^2\theta(\sec^2\theta - 2) - 4(\sec^2\theta - 2) = 0$

বা, $(\sec^2\theta - 2)(3\sec^2\theta - 4) = 0$

বা, $(\sec^2\theta - 1 - 1)(3\sec^2\theta - 3 - 1) = 0$

$\therefore (\tan^2\theta - 1)(3\tan^2\theta - 1) = 0$

হয়, $\tan^2\theta - 1 = 0$ অথবা, $3\tan^2\theta - 1 = 0$

বা, $\tan^2\theta = 1$ বা, $\tan^2\theta = \frac{1}{3}$

$\therefore \tan\theta = \pm 1$ $\therefore \tan\theta = \pm\frac{1}{\sqrt{3}}$

47. ঘ; ব্যাখ্যা: $\sin 18° + \cos 18° = \cos 72° + \cos 18°$

$= 2\cos\frac{72° + 18°}{2}.\cos\frac{72° - 18°}{2}$

$= 2.\cos 45°.\cos 27°$

$= 2.\frac{1}{\sqrt{2}}\cos 27° = \sqrt{2}\cos 27°$

48. গ; ব্যাখ্যা: $2(\sin\theta.\cos\theta + \sqrt{3}) = \sqrt{3}\cos\theta + 4\sin\theta$

বা, $2\sin\theta.\cos\theta - 4\sin\theta = \sqrt{3}\cos\theta - 2\sqrt{3}$

বা, $2\sin\theta.(\cos\theta - 2) = \sqrt{3}(\cos\theta - 2)$

বা, $2\sin\theta = \sqrt{3}$ $\therefore \sin\theta = \frac{\sqrt{3}}{2}$ $\therefore \theta = 60°, 120°$

49. গ;

50. খ; ব্যাখ্যা: $\cos^2 A - \cos^2 B = \sin(B + A)\sin(B - A)$

$= \sin\frac{\pi}{2}\sin(B - A)$

$= \sin(B - A)$

51. খ; ব্যাখ্যা: $\cot A + \cot B + \cot C$

$= \cot A + \frac{\cos B}{\sin B} + \frac{\cos C}{\sin C}$

$= \cot A + \frac{\cos B\sin C + \sin B\cos C}{\sin B\sin C}$

$= \cot A + \frac{\sin(B + C)}{-\sin A} = \cot A - \frac{\sin\left(\frac{\pi}{2} - A\right)}{\sin A}$

$= \cot A - \frac{\cos A}{\sin A} = \cot A - \cot A = 0$

52. গ; ব্যাখ্যা: $(a + b + c)(b + c - a) = 3bc$

বা, $(b + c)^2 - a^2 = 3bc$

বা, $b^2 + c^2 + 2bc - a^2 = 3bc$

বা, $b^2 + c^2 - a^2 = bc$

বা, $\frac{b^2 + c^2 - a^2}{2bc} = \frac{1}{2}.1$

বা, $\cos A = \frac{1}{2}$

$\therefore A = 60°$

53. খ;

54. ঘ; ব্যাখ্যা: $g(\theta) = \frac{1 - \tan\theta}{1 + \tan\theta}$

$$g\left(\frac{\pi}{4} - \theta\right) = \frac{1 - \tan\left(\frac{\pi}{4} - \theta\right)}{1 + \tan\left(\frac{\pi}{4} - \theta\right)} = \frac{1 - \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}}{1 + \frac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}.\tan\theta}}$$

$$= \frac{1 - \frac{1 - \tan\theta}{1 + \tan\theta}}{1 + \frac{1 - \tan\theta}{1 + \tan\theta}} = \frac{\frac{1 + \tan\theta - 1 + \tan\theta}{1 + \tan\theta}}{\frac{1 + \tan\theta + 1 - \tan\theta}{1 + \tan\theta}}$$

$$= \frac{2\tan\theta}{2} = \tan\theta$$

55. ঘ; ব্যাখ্যা: $\sec\theta = \frac{13}{12}$ বা, $\sec^2\theta = \left(\frac{13}{12}\right)^2$

বা, $1 + \tan^2\theta = \frac{169}{144}$ বা, $\tan^2\theta = \frac{25}{144}$

বা, $\tan\theta = \pm\frac{5}{12}$ $\therefore \cot\theta = \pm\frac{12}{5}$

56. গ; ব্যাখ্যা: $\cot x = -\sqrt{3}$

$\therefore \tan x = \frac{1}{\cot x} = \frac{1}{-\sqrt{3}} = \frac{-\sqrt{3}}{3}$

57. গ; 58. গ; 59. ঘ;

60. গ; ব্যাখ্যা: $5^2 = 25,\ 12^2 = 144$

$13^2 = 169 = 144 + 25 = 12^2 + 5^2$

$\therefore$ ত্রিভুজের বাহুগুলির একটির বর্গ অপর দুটির বর্গের সমষ্টির সমান।

$\therefore$ পীথাগোরাসের উপপাদ্য অনুসারে ত্রিভুজটি সমকোণী।

61. গ; ব্যাখ্যা: ত্রিভুজের বৃহত্তম বাহুর বিপরীত কোণ বৃহত্তম।

$\therefore$ স্থূলকোণটি θ হলে,

$\cos\theta = \frac{3^2 + 5^2 - 7^2}{2.3.5} = \frac{9 + 25 - 49}{30} = \frac{-15}{30}$

$\therefore \cos\theta = -\frac{1}{2}$ $\therefore \theta = 120°$

62. খ; ব্যাখ্যা: $\cos 2\theta = 2\cos^2\theta - 1$

$= 2.\frac{1}{4}\left(x + \frac{1}{x}\right)^2 - 1$

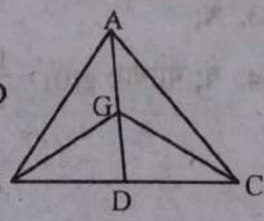
$= \frac{1}{2}\left(x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2}\right) - 1 = \frac{1}{2}\left(x^2 + 2 + \frac{1}{x^2}\right) - 1$

$= \frac{1}{2}x^2 + 1 + \frac{1}{2}\frac{1}{x^2} - 1 = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$

63. ঘ; ব্যাখ্যা: $\pi r_1^2 : \pi r_2^2 : \pi r_3^2 = 1 : 2 : 4$

$\Rightarrow r_1^2 : r_2^2 : r_3^2 = 1 : 2 : 4$

$\Rightarrow r_1 : r_2 : r_3 = 1 : \sqrt{2} : 2$



64. ঘ; ব্যাখ্যা: $\Delta ABC = \frac{1}{2}.BC.AD$

$\Delta BGC = \frac{1}{2}.BC.GD$

$\therefore \frac{\Delta BGC}{\Delta ABC} = \frac{\frac{1}{2}.BC.GD}{\frac{1}{2}BC.AD} = \frac{GD}{AD} = \frac{GD}{3GD} = \frac{1}{3}$

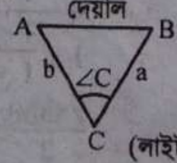
▶ সৃজনশীল প্রশ্নের সমাধান

1. ক ত্রিভুজ আকৃতির ক্ষেত্রটির ক্ষেত্রফল

$= \frac{1}{2} \times AC \times BC \times \sin\angle ACB$

$= \frac{1}{2}.b.a.\sin C$

$= \frac{1}{2}$ ab sinC (Ans.)



(লাইট হাউস)

খ ΔABC এর তিনটি কোণ যথাক্রমে A, B ও C

$\therefore A + B + C = \pi$

অতঃপর অনুশীলনী 7(F) এর 4(iii) নং সমাধান দ্রষ্টব্য। পৃষ্ঠা-৩০৭

গ সার্চ লাইটটিকে যেহেতু $60°$ কোণে ঘুরাতে হবে সেহেতু $\angle C = 60°$

অতঃপর অনুশীলনী 7(G) এর 21(ii) নং সমাধান দ্রষ্টব্য। পৃষ্ঠা-৩১৮

2. ক অনুশীলনী 7(B) এর 10(ii) এর সমাধান দ্রষ্টব্য। পৃষ্ঠা-২৮৭

খ এখানে, আপতন কোণ $= 2\alpha$

প্রতিসরণ কোণ $= 2\theta$

শর্তমতে, $\frac{\sin 2\alpha}{\sin 2\theta} = k$

অতঃপর অনুশীলনী 7(C) এর 13(iii) এর সমাধান দ্রষ্টব্য। পৃষ্ঠা-২৯৫

গ এখানে, $\alpha = \frac{\theta}{3} = 9°$

আবার শর্তমতে, $k = \frac{\sin 2\alpha}{\sin 2\theta} = \frac{\sin(2 \times 9°)}{\sin(2 \times 9° \times 3)}$

$= \frac{\sin 18°}{\sin 54°}$

মনে করি, $x = 18°$

$\therefore 5x = 90° = \frac{\pi}{2}$

আবার, $2x = 5x - 3x$ $\therefore 2x = \frac{\pi}{2} - 3x$

বা, $\sin(2x) = \sin\left(\frac{\pi}{2} - 3x\right)$

বা, $2\sin x.\cos x = \cos 3x$

বা, $2\sin x.\cos x = 4\cos^3 x - 3\cos x$

বা, $2\sin x = 4\cos^2 x - 3$

[cosx দ্বারা ভাগ করে, যেহেতু $\cos 18° \neq 0$]

বা, $2\sin x = 4 - 4\sin^2 x - 3$

বা, $4\sin^2 x + 2\sin x - 1 = 0$

$\therefore \sin x = \frac{-2 \pm \sqrt{4 + 16}}{2.4} = \frac{-2 \pm 2\sqrt{5}}{8} = \frac{\pm\sqrt{5} - 1}{4}$

যেহেতু $x = 18°$ একটি সূক্ষ্মকোণ

$\therefore \sin 18° = \frac{\sqrt{5} - 1}{4}$

আবার, $\sin 54° = \cos(90° - 54°) = \cos 36°$

$= \cos(2 \times 18°) = 1 - 2\sin^2 18°$

$= 1 - 2\left(\frac{\sqrt{5} - 1}{4}\right)^2 = 1 - 2\frac{5 - 2\sqrt{5} + 1}{16}$

$= \frac{8 - 5 + 2\sqrt{5} - 1}{8} = \frac{2 + 2\sqrt{5}}{8} = \frac{\sqrt{5} + 1}{4}$

$\therefore k = \frac{\sin 18°}{\sin 54°} = \frac{\left(\frac{\sqrt{5} - 1}{4}\right)}{\left(\frac{\sqrt{5} + 1}{4}\right)} = \frac{\sqrt{5} - 1}{\sqrt{5} + 1}$ (প্রমাণিত)

3. ক দেওয়া আছে, $\theta = \cot^{-1}3$ বা, $\cot\theta = 3$

বা, $\frac{1}{\tan\theta} = 3 \therefore \tan\theta = \frac{1}{3}$

প্রদত্ত রাশি $= \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\left(\frac{1}{3}\right)^2}{1+\left(\frac{1}{3}\right)^2}$

$= \frac{1-\frac{1}{9}}{1+\frac{1}{9}} = \frac{8}{10} = \frac{4}{5}$ **(Ans.)**

খ দেওয়া আছে, $\frac{1}{\operatorname{cosec}\alpha} + \frac{1}{\operatorname{cosec}\beta} = \sqrt{5}\left(\frac{1}{\sec\beta} - \frac{1}{\sec\alpha}\right)$

বা, $\sin\alpha + \sin\beta = \sqrt{5}(\cos\beta - \cos\alpha)$

বা, $2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2} = \sqrt{5}.2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$

বা, $\frac{\sin\frac{\alpha-\beta}{2}}{\cos\frac{\alpha-\beta}{2}} = \frac{1}{\sqrt{5}}$ বা, $\tan\frac{\alpha-\beta}{2} = \frac{1}{\sqrt{5}}$

বা, $\tan^2\frac{\alpha-\beta}{2} = \frac{1}{5}$ বা, $\sec^2\frac{\alpha-\beta}{2} - 1 = \frac{1}{5}$

বা, $\sec^2\frac{\alpha-\beta}{2} = \frac{6}{5}$ বা, $\frac{1}{\cos^2\frac{\alpha-\beta}{2}} = \frac{6}{5}$

বা, $\cos^2\frac{\alpha-\beta}{2} = \frac{5}{6}$ বা, $\cos\frac{\alpha-\beta}{2} = \sqrt{\frac{5}{6}}$

বা, $\frac{\alpha-\beta}{2} = \cos^{-1}\sqrt{\frac{5}{6}}$

$\therefore \alpha - \beta = 2\cos^{-1}\sqrt{\frac{5}{6}}$ (প্রমাণিত)

গ দেওয়া আছে, $P + Q + R = n\pi$

$\therefore P + Q + R = 2\pi \quad [\because n = 2]$

বামপক্ষ $= \cos^2P + \cos^2Q + \cos^2R$

$= \frac{1}{2}(2\cos^2P + 2\cos^2Q) + \cos^2R$

$= \frac{1}{2}(1 + \cos 2P + 1 + \cos 2Q) + \cos^2R$

$= 1 + \frac{1}{2}(\cos 2P + \cos 2Q) + \cos^2R$

$= 1 + \frac{1}{2}.2\cos\frac{2P+2Q}{2}\cos\frac{2P-2Q}{2} + \cos^2R$

$= 1 + \cos(P+Q)\cos(P-Q) + \cos^2R$

$= 1 + \cos(2\pi - R)\cos(P-Q) + \cos^2R$

$= 1 + \cos R\cos(P-Q) + \cos^2R$

$= 1 + \cos R\{\cos(P-Q) + \cos R\}$

$= 1 + \cos R\{\cos(P-Q) + \cos(2\pi - P - Q)\}$

$= 1 + \cos R\{\cos(P-Q) + \cos(P+Q)\}$

$= 1 + \cos R.2\cos P\cos Q$

$= 2\cos P\cos Q\cos R + 1 =$ ডানপক্ষ

$\therefore \cos^2P + \cos^2Q + \cos^2R = 2\cos P\cos Q\cos R + 1$

(প্রমাণিত)

4. ক দেওয়া আছে, $D = \frac{\pi}{12} = 15°$

$\therefore \frac{1+\cot D}{1-\cot D} = \frac{1+\cot 15°}{1-\cot 15°} = \frac{1+\frac{1}{\tan 15°}}{1-\frac{1}{\tan 15°}}$

$= \frac{\tan 15° + 1}{\tan 15° - 1} = -\frac{1+\tan 15°}{1-\tan 15°}$

$= -\frac{\tan 45° + \tan 15°}{1 - \tan 45°.\tan 15°}$

$= -\tan(45° + 15°)$

$= -\tan 60° = -\sqrt{3}$

$\therefore \frac{1+\cot D}{1-\cot D} = -\sqrt{3}$ **(Ans.)**

খ আমরা জানি, $\frac{b}{\sin B} = \frac{c}{\sin C}$

বা, $\frac{2c}{\sin B} = \frac{c}{\sin C} \quad [\because b = 2c]$

বা, $\frac{2}{\sin B} = \frac{1}{\sin C}$ বা, $\frac{2}{\sin 3C} = \frac{1}{\sin C} \quad [\because B = 3C]$

বা, $\sin 3C = 2\sin C$ বা, $3\sin C - 4\sin^3C = 2\sin C$

বা, $-4\sin^3C = 2\sin C - 3\sin C$

বা, $-4\sin^3C = -\sin C$

বা, $4\sin^2C = 1 \quad [\because \sin C \neq 0]$

বা, $\sin^2C = \frac{1}{4}$ বা, $\sin C = \frac{1}{2}$

বা, $\sin C = \sin 30° \quad \therefore C = 30°$

$\therefore B = 3C = 3 \times 30° = 90° \quad [\because C = 30°]$

আমরা জানি, $A + B + C = 180°$

বা, $A = 180° - (B + C)$

বা, $A = 180° - (90° + 30°)$

$\therefore A = 60°$

$\therefore$ নির্ণেয় $A = 60°, B = 90°, C = 30°$

গ দেওয়া আছে, $D = \frac{\pi}{12} = \left(\frac{\pi}{12} \times \frac{180}{\pi}\right)^\circ = 15°$

দেখাতে হবে যে,

$\tan\left(\frac{2\pi}{9} + D\right) = \tan\left(\frac{\pi}{9} + D\right) + 2\tan\left(D + \frac{\pi}{36}\right)$

বা, $\tan(40° + 15°) = \tan(20° + 15°) + 2\tan(15° + 5°)$

বা, $\tan 55° = \tan 35° + 2\tan 20°$

এখন, $\tan 20° = \tan(55° - 35°)$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1 + \tan 55°.\tan 35°}$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1 + \tan(90° - 35°)\tan 35°}$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1 + \cot 35°.\dfrac{1}{\cot 35°}}$

বা, $\tan 20° = \dfrac{\tan 55° - \tan 35°}{1+1}$

বা, $\tan 55° - \tan 35° = 2\tan 20°$

বা, $\tan 55° = \tan 35° + 2\tan 20°$

$\therefore \tan\left(\dfrac{2\pi}{9} + D\right) = \tan\left(\dfrac{\pi}{9} + D\right) + 2\tan\left(D + \dfrac{\pi}{36}\right)$

(দেখানো হলো)

5. **ক** দেওয়া আছে, $B = 60°$

আমরা জানি, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R$

তাহলে, $\dfrac{b}{\sin 60°} = 2R$ বা, $R = \dfrac{b}{\dfrac{\sqrt{3}}{2}.2}$ $\therefore R = \dfrac{b}{\sqrt{3}}$

সুতরাং ত্রিভুজটির পরিব্যাসার্ধ $\dfrac{b}{\sqrt{3}}$ **(Ans.)**

খ দেওয়া আছে, $B = 60°$

বা, $\cos B = \cos 60°$

বা, $\dfrac{c^2 + a^2 - b^2}{2ca} = \dfrac{1}{2}$ $\left[\because \cos B = \dfrac{c^2 + a^2 - b^2}{2ca}\right]$

বা, $\dfrac{c^2 + a^2 - b^2}{ca} = 1$

বা, $c^2 + a^2 - b^2 = ca$

বা, $c^2 + a^2 - b^2 - ca = 0$

বা, $a^2 + b^2 + c^2 - ac - 2b^2 = 0$

বা, $(a + b + c)^2 - 2ab - 2ac - 2bc - ac - 2b^2 = 0$

বা, $(a+b+c)^2 + ab + bc + b^2 = 3ac + 3ab + 3bc + 3b^2$

বা, $(a+b+c)^2 + b(a+b+c) = 3a(b+c) + 3b(b+c)$

বা, $(a+b+c)(a+b+c+b) = 3(a+b)(b+c)$

বা, $(a+b+c)\{(a+b)+(b+c)\} = 3(a+b)(b+c)$

বা, $\dfrac{(a+b)+(b+c)}{(a+b)(b+c)} = \dfrac{3}{(a+b+c)}$

$\therefore \dfrac{1}{a+b} + \dfrac{1}{b+c} = \dfrac{3}{a+b+c}$ (দেখানো হলো)

গ দেওয়া আছে, $\angle A = 45°, \angle B = 60°$

$\therefore \angle C = 180° - (\angle A + \angle B)$

$= 180° - (45° + 60°) = 75°$

$\therefore \cos C = \cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \dfrac{1}{\sqrt{2}}.\dfrac{\sqrt{3}}{2} - \dfrac{1}{\sqrt{2}}.\dfrac{1}{2} = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$

এখন, $b = c\cos A + a\cos C$

$= c\cos 45° + a\cos 75°$

$= \dfrac{c}{\sqrt{2}} + \dfrac{a(\sqrt{3}-1)}{2\sqrt{2}}$

বা, $2b = \dfrac{2c}{\sqrt{2}} + \dfrac{2a(\sqrt{3}-1)}{2\sqrt{2}}$

$\therefore \left(\dfrac{\sqrt{3}-1}{\sqrt{2}}\right)a + c\sqrt{2} = 2b$ (দেখানো হলো)

6. **ক** $\cos 3B = \cos(2B + B)$

$= \cos 2B \cos B - \sin 2B \sin B$

$= (2\cos^2 B - 1)\cos B - 2\sin B \cos B.\sin B$

$[\because 2\cos^2\theta = 1 + \cos 2\theta$ এবং $2\sin\theta\cos\theta = \sin 2\theta]$

$= 2\cos^3 B - \cos B - 2\cos B(1 - \cos^2 B)$

$[\because \sin^2\theta = 1 - \cos^2\theta]$

$= 2\cos^3 B - \cos B - 2\cos B + 2\cos^3 B$

$= 4\cos^3 B - 3\cos B$ **(Ans.)**

খ দেওয়া আছে, $E = \alpha + \dfrac{\pi}{3}, F = \alpha - \dfrac{\pi}{3}$

$\therefore \tan E + \tan F = \tan\left(\alpha + \dfrac{\pi}{3}\right) + \tan\left(\alpha - \dfrac{\pi}{3}\right)$

$= \dfrac{\sin\left(\alpha + \frac{\pi}{3}\right)}{\cos\left(\alpha + \frac{\pi}{3}\right)} + \dfrac{\sin\left(\alpha - \frac{\pi}{3}\right)}{\cos\left(\alpha - \frac{\pi}{3}\right)}$

$= \dfrac{\sin\left(\alpha + \frac{\pi}{3}\right)\cos\left(\alpha - \frac{\pi}{3}\right) + \cos\left(\alpha + \frac{\pi}{3}\right)\sin\left(\alpha - \frac{\pi}{3}\right)}{\cos\left(\alpha + \frac{\pi}{3}\right)\cos\left(\alpha - \frac{\pi}{3}\right)}$

$= \dfrac{\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{\frac{1}{2}.2\cos\left(\alpha + \frac{\pi}{3}\right)\cos\left(\alpha - \frac{\pi}{3}\right)}$

$= \dfrac{2\sin 2\alpha}{\cos\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right) + \cos\left(\alpha + \frac{\pi}{3} - \alpha + \frac{\pi}{3}\right)}$

$= \dfrac{2\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{\cos 2\alpha + \cos\frac{2\pi}{3}} = \dfrac{2\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{\cos 2\alpha - \frac{1}{2}}$

$= \dfrac{4\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{2\cos 2\alpha - 1} = \dfrac{4\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{2(1 - 2\sin^2\alpha) - 1}$

$$= \frac{4\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{2 - 4\sin^2\alpha - 1} = \frac{4\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{1 - 4\sin^2\alpha}$$

$$= \frac{4\sin\left(\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}\right)}{1 - 4\sin^2\left(\frac{\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{2}}{2}\right)}$$

$$= \frac{4\sin(E + F)}{1 - 4\sin^2\left(\frac{E+F}{2}\right)}$$ (প্রমাণিত)

গ $\cos^3\left(\frac{E+F}{2}\right) + \cos^3\left\{(E-F) + \left(\frac{E+F}{2}\right)\right\} + \cos^3\left\{2(E-F) + \left(\frac{E+F}{2}\right)\right\}$

$$= \cos^3\left(\frac{\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}}{2}\right) + \cos^3\left\{\left(\alpha + \frac{\pi}{3} - \alpha + \frac{\pi}{3}\right) + \left(\frac{\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}}{2}\right)\right\} + \cos^3\left\{2\left(\alpha + \frac{\pi}{3} - \alpha + \frac{\pi}{3}\right) + \left(\frac{\alpha + \frac{\pi}{3} + \alpha - \frac{\pi}{3}}{2}\right)\right\}$$

$\left[\because E = \alpha + \frac{\pi}{3}\right.$ এবং $\left.F = \alpha - \frac{\pi}{3}\right]$

$$= \cos^3\alpha + \cos^3\left(\frac{2\pi}{3} + \alpha\right) + \cos^3\left(\frac{4\pi}{3} + \alpha\right)$$

$$= \cos^3\alpha + \cos^3(120° + \alpha) + \cos^3(240° + \alpha)$$

$$= \frac{\cos 3\alpha + 3\cos\alpha}{4} + \frac{\cos 3(120° + \alpha) + 3\cos(120° + \alpha)}{4} + \frac{\cos 3(240° + \alpha) + 3\cos(240° + \alpha)}{4}$$

$$= \frac{\cos 3\alpha + 3\cos\alpha + \cos(360° + 3\alpha) + 3\cos(120° + \alpha) + \cos(720° + 3\alpha) + 3\cos(240° + \alpha)}{4}$$

$$= \frac{\cos 3\alpha + 3\cos\alpha + \cos 3\alpha + 3\cos(120° + \alpha) + \cos 3\alpha + 3\cos(240° + \alpha)}{4}$$

$$= \frac{3\cos 3\alpha + 3\cos\alpha + 3\{\cos(120° + \alpha) + \cos(240° + \alpha)\}}{4}$$

$$= \frac{3\cos 3\alpha + 3\cos\alpha + 3.2\cos(180° + \alpha)\cos 60°}{4}$$

$$= \frac{3\cos 3\alpha + 3\cos\alpha - 6\cos\alpha.\frac{1}{2}}{4}$$

$$= \frac{3\cos 3\alpha + 3\cos\alpha - 3\cos\alpha}{4} = \frac{3}{4}\cos 3\alpha$$ **(Ans.)**

7. **ক** $\sin 420° \cos 390° + \sin(-300°)\cos(-330°)$

$= \sin 420° \cos 390° - \sin 300° \cos 330°$

$= \sin(4 \times 90° + 60°)\cos(4 \times 90° + 30°) - \sin(4 \times 90° - 60°)\cos(4 \times 90° - 30°)$

$= \sin 60°.\cos 30° - (-\sin 60°).\cos 30°$

$= \sin 60°.\cos 30° + \sin 60°.\cos 30°$

$= 2\sin 60°\cos 30° = 2.\frac{\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = \frac{3}{2}$ **(Ans.)**

খ দেওয়া আছে, $n\sin 2\alpha = 2(1 - n\sin^2\alpha)\tan\beta$

বা, $n\, 2\sin\alpha\cos\alpha = 2(1 - n\sin^2\alpha)\tan\beta$

বা, $\tan\beta = \frac{n\sin\alpha\cos\alpha}{1 - n\sin^2\alpha} = \frac{\frac{n\sin\alpha\cos\alpha}{\cos^2\alpha}}{\frac{1}{\cos^2\alpha} - \frac{n\sin^2\alpha}{\cos^2\alpha}}$

[লব ও হরকে $\cos^2\alpha$ দ্বারা ভাগ করে]

$\therefore \tan\beta = \frac{n\tan\alpha}{\sec^2\alpha - n\tan^2\alpha}$

$= \frac{n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}$ ……… (i)

ডানপক্ষ $= \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$= \frac{\tan\alpha - \frac{n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}{1 + \tan\alpha.\frac{n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}$ [(i) ব্যবহার করে]

$= \frac{\frac{\tan\alpha + \tan^3\alpha - n\tan^3\alpha - n\tan\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}{\frac{1 + \tan^2\alpha - n\tan^2\alpha + n\tan^2\alpha}{1 + \tan^2\alpha - n\tan^2\alpha}}$

$= \frac{\tan\alpha(1 + \tan^2\alpha) - n\tan\alpha(1 + \tan^2\alpha)}{1 + \tan^2\alpha}$

$= \frac{(1 + \tan^2\alpha)(\tan\alpha - n\tan\alpha)}{1 + \tan^2\alpha}$

$= (1 - n)\tan\alpha =$ বামপক্ষ

$\therefore (1 - n)\tan\alpha = \tan(\alpha - \beta)$ **(দেখানো হলো)**

গ দেওয়া আছে, $x = 4\theta$

$\therefore \quad \tan\frac{x}{4} + 2\tan\frac{x}{2} + 4\tan x + 8\cot 2x$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + 8\cot 8\theta$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + \frac{8}{\tan 8\theta}$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + \frac{8}{\tan(2.4\theta)}$

$= \tan\theta + 2\tan 2\theta + 4\tan 4\theta + \frac{8(1 - \tan^2 4\theta)}{2\tan 4\theta}$

$= \tan\theta + 2\tan 2\theta + \frac{8\tan^2 4\theta + 8 - 8\tan^2 4\theta}{2\tan 4\theta}$

$= \tan\theta + 2\tan 2\theta + \dfrac{4}{\tan(2.2\theta)}$

$= \tan\theta + 2\tan 2\theta + \dfrac{4(1-\tan^2 2\theta)}{2\tan 2\theta}$

$= \tan\theta + \dfrac{4\tan^2 2\theta + 4 - 4\tan^2 2\theta}{2\tan 2\theta}$

$= \tan\theta + \dfrac{2}{\tan 2\theta} = \tan\theta + \dfrac{2(1-\tan^2\theta)}{2\tan\theta}$

$= \dfrac{2\tan^2\theta + 2 - 2\tan^2\theta}{2\tan\theta}$

$= \dfrac{2}{2\tan\theta} = \cot\theta = \cot\dfrac{x}{4}$

$\therefore \tan\dfrac{x}{4} + 2\tan\dfrac{x}{2} + 4\tan x + 8\cot 2x = \cot\dfrac{x}{4}$

(প্রমাণিত)

8. **ক** $\tan A = \dfrac{1}{\sqrt{5}}$

$\cos 2A = \dfrac{1-\tan^2 A}{1+\tan^2 A} = \dfrac{1-\left(\frac{1}{\sqrt{5}}\right)^2}{1+\left(\frac{1}{\sqrt{5}}\right)^2} = \dfrac{1-\frac{1}{5}}{1+\frac{1}{5}} = \dfrac{4}{6} = \dfrac{2}{3}$

(Ans.)

খ বামপক্ষ $= \sqrt{3}\,\mathrm{cosec}X - \sec X$

$= \sqrt{3}\,\mathrm{cosec}\,20° - \sec 20°$ $[\because x = 180° - L - M]$

$= \dfrac{\sqrt{3}}{\sin 20°} - \dfrac{1}{\cos 20°} = 2\left(\dfrac{\frac{\sqrt{3}}{2}}{\sin 20°} - \dfrac{\frac{1}{2}}{\cos 20°}\right)$

$= 2\left(\dfrac{\sin 60°}{\sin 20°} - \dfrac{\cos 60°}{\cos 20°}\right)$

$= 2\left(\dfrac{\sin 60°\cos 20° - \cos 60°\sin 20°}{\sin 20°\cos 20°}\right)$

$= \dfrac{2\sin(60°-20°)}{\sin 20°\cos 20°} = \dfrac{2\sin 40°}{\sin 20°\cos 20°}$

$= \dfrac{4\sin 40°}{2\sin 20°\cos 20°} = \dfrac{4\sin 40°}{\sin 40°} = 4$

= ডানপক্ষ (দেখানো হলো)

গ বামপক্ষ $= \sin^2(\theta + A) + \sin^2 A + \sin^2(\theta - A)$

$= \sin^2(60° + A) + \sin^2 A + \sin^2(60° - A)$

$= \dfrac{1}{2}[2\sin^2(60°+A) + 2\sin^2 A + 2\sin^2(60°-A)]$

$= \dfrac{1}{2}[1 - \cos(120°+2A) + 1 - \cos 2A + 1 - \cos(120° - 2A)]$

$= \dfrac{1}{2}[3 - \cos 2A - \{\cos(120°+2A) + \cos(120° - 2A)\}]$

$= \dfrac{1}{2}[3 - \cos 2A - 2\cos 120°\cos 2A]$

$= \dfrac{1}{2}\left[3 - \cos 2A - 2\left(-\dfrac{1}{2}\right)\cos 2A\right]$

$= \dfrac{1}{2}[3 - \cos 2A + \cos 2A]$

$= \dfrac{3}{2}$ = ডানপক্ষ (দেখানো হলো)

9. **ক** বামপক্ষ $= \dfrac{1-\tan D}{1+\tan D}$

$= \dfrac{1-\tan 15°}{1+\tan 15°} = \dfrac{\tan 45° - \tan 15°}{1 + \tan 45°\tan 15°}$

$= \tan(45° - 15°) = \tan 30°$

$= \dfrac{1}{\sqrt{3}}$ = ডানপক্ষ

$\therefore \dfrac{1-\tan D}{1+\tan D} = \dfrac{1}{\sqrt{3}}$ (দেখানো হলো)

খ বামপক্ষ $= \sin\dfrac{D}{2} = \sin\dfrac{15°}{2}$

$= \sqrt{\sin^2\dfrac{15°}{2}} = \sqrt{\dfrac{1}{2}(1-\cos 15°)}$

$= \sqrt{\dfrac{1}{2}\{1 - \sqrt{\cos^2 15°}\}}$

$= \sqrt{\dfrac{1}{2}\left[1 - \sqrt{\dfrac{1}{2}(2\cos^2 15°)}\right]}$

$= \sqrt{\dfrac{1}{2}\left[1 - \sqrt{\dfrac{1}{2}(1+\cos 30°)}\right]}$

$= \sqrt{\dfrac{1}{2}\left[1 - \sqrt{\dfrac{1}{2}\left(1+\dfrac{\sqrt{3}}{2}\right)}\right]} = \sqrt{\dfrac{1}{2}\left[1 - \sqrt{\dfrac{1}{2}+\dfrac{\sqrt{3}}{4}}\right]}$

$= \sqrt{\dfrac{1}{2}\left[1 - \sqrt{\dfrac{2+\sqrt{3}}{4}}\right]} = \sqrt{\dfrac{1}{2}\left[1 - \dfrac{\sqrt{2+\sqrt{3}}}{2}\right]}$

$= \sqrt{\dfrac{1}{2}\times\dfrac{2-\sqrt{2+\sqrt{3}}}{2}} = \dfrac{1}{2}\sqrt{2-\sqrt{2+\sqrt{3}}}$

= ডানপক্ষ (প্রমাণিত)

গ দেওয়া আছে, $A + B + C = \pi$

বা, $\dfrac{A}{2} + \dfrac{B}{2} + \dfrac{C}{2} = \dfrac{\pi}{2}$ বা, $\dfrac{A}{2} + \dfrac{B}{2} = \dfrac{\pi}{2} - \dfrac{C}{2}$

বা, $\tan\left(\dfrac{A}{2} + \dfrac{B}{2}\right) = \tan\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right)$

বা, $\dfrac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}.\tan\frac{B}{2}} = \cot\dfrac{C}{2}$

বা, $\dfrac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \dfrac{1}{\tan\frac{C}{2}}$

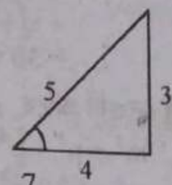
বা, $\tan\frac{C}{2}\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)=1-\tan\frac{A}{2}\tan\frac{B}{2}$

$\therefore\ \tan\frac{B}{2}\tan\frac{C}{2}+\tan\frac{C}{2}\tan\frac{A}{2}+\tan\frac{A}{2}\tan\frac{B}{2}=1$

(দেখানো হলো)

10. **ক** দেওয়া আছে, $\cos\theta=\frac{4}{5}$

পাশের চিত্র হতে,



$\tan\theta=\frac{3}{4}$

$\therefore \frac{1-\tan^2\theta}{1+\tan^2\theta}=\frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2}=\frac{1-\frac{9}{16}}{1+\frac{9}{16}}=\frac{\frac{7}{16}}{\frac{25}{16}}=\frac{7}{25}$ **(Ans.)**

খ দেয়া আছে, $\varphi=\frac{\pi}{32}$

এখন, $\cos\varphi=\cos\frac{\pi}{32}=\frac{1}{2}2\cos\frac{\pi}{32}$

$=\frac{1}{2}\sqrt{2.2\cos^2\frac{\pi}{32}}=\frac{1}{2}\sqrt{2\left(1+\cos\frac{\pi}{16}\right)}$

$=\frac{1}{2}\sqrt{2+2\cos\frac{\pi}{16}}=\frac{1}{2}\sqrt{2+\sqrt{2.2\cos^2\frac{\pi}{16}}}$

$=\frac{1}{2}\sqrt{2+\sqrt{2\left(1+\cos\frac{\pi}{8}\right)}}$

$=\frac{1}{2}\sqrt{2+\sqrt{2+2\cos\frac{\pi}{8}}}$

$=\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2.2\cos^2\frac{\pi}{8}}}}$

$=\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2\left(1+\cos\frac{\pi}{4}\right)}}}$

$=\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+2.\frac{1}{\sqrt{2}}}}}$

$=\frac{1}{2}\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}$ **(Ans.)**

গ ত্রিভুজ হতে, $A+B+C=\pi$

বা, $B+C=\pi-A$ বা, $\cot(B+C)=\cot(\pi-A)$

বা, $\frac{\cot B\cot C-1}{\cot B+\cot C}=-\cot A$

বা, $\cot A\cot B+\cot B\cot C+\cot C\cot A=1$ ... ... (i)

দেওয়া আছে, $\tan\left(\frac{\pi}{2}-A\right)+\tan\left(\frac{\pi}{2}-B\right)+\tan\left(\frac{\pi}{2}-C\right)=\frac{3}{\sqrt{3}}$

বা, $\cot A+\cot B+\cot C=\sqrt{3}.1$

বা, $(\cot A+\cot B+\cot C)^2=(\sqrt{3})^2.(\cot A\cot B+\cot B\cot C+\cot C\cot A)$ [(i) এর সাহায্যে]

বা, $\cot^2A+\cot^2B+\cot^2C+2\cot A\cot B+2\cot B\cot C+2\cot C\cot A=3(\cot A\cot B+\cot B\cot C+\cot C\cot A)$

বা, $\frac{1}{2}\{(\cot A-\cot B)^2+(\cot B-\cot C)^2+(\cot C-\cot A)^2\}=0$

$\therefore \cot A-\cot B=0$ | এবং $\cot B-\cot C=0$

বা, $\cot A=\cot B$ | বা, $\cot B=\cot C$

$\therefore A=B$ | $\therefore B=C$

$\therefore A=B=C$

$\therefore$ ত্রিভুজটি সমবাহু। **(দেখানো হলো)**

11. **ক** আমরা জানি, $\pi^c=180°$ বা, $1^c=\left(\frac{180}{\pi}\right)^\circ$

$\therefore\ R^c=\left(\frac{180}{\pi}\times R\right)^\circ$

আবার যেহেতু একই কোণের ডিগ্রী ও রেডিয়ানে প্রকাশ

সুতরাং, $D=\frac{180}{\pi}\times R$

$\therefore \frac{D}{180}=\frac{R}{\pi}$. **(দেখানো হলো)**

খ দেওয়া আছে, $\alpha=40°$, $\beta=80°$ এবং $\gamma=60°$

$\therefore$ বামপক্ষ $=\cos20°\cos\alpha\cos\beta\cos\gamma$

$=\cos20°\cos40°\cos80°\cos60°$

$=\frac{1}{2}\cos80°\cos20°\cos40°$ $\quad[\because \cos60°=\frac{1}{2}]$

$=\frac{1}{4}\times2\cos80°\cos20°\cos40°$

$=\frac{1}{4}(\cos100°+\cos60°)\cos40°$

$=\frac{1}{4}(\cos100°\cos40°+\frac{1}{2}\cos40°)$

$=\frac{1}{8}\cos40°+\frac{1}{4}\cos100°\cos40°$

$=\frac{1}{8}\cos40°+\frac{1}{8}\times2\cos100°\cos40°$

$=\frac{1}{8}\cos40°+\frac{1}{8}\times(\cos140°+\cos60°)$

$=\frac{1}{8}\cos40°+\frac{1}{8}\cos(180°-40°)+\frac{1}{8}\times\frac{1}{2}$

$=\frac{1}{8}\cos40°-\frac{1}{8}\cos40°+\frac{1}{16}=\frac{1}{16}=$ ডানপক্ষ

$\therefore \cos20°\cos\alpha\cos\beta\cos\gamma=\frac{1}{16}$ **(প্রমাণিত)**

গ দেওয়া আছে, $\frac{1}{c+a}+\frac{1}{b+c}=\frac{3}{a+b+c}$

বা, $\frac{b+c+c+a}{(c+a)(b+c)}=\frac{3}{a+b+c}$

বা, $(a+b+c)(a+b+c+c)=3(c+a)(b+c)$

বা, $(a+b+c)^2+ac+bc+c^2=3(bc+ab+c^2+ac)$

বা, $(a+b+c)^2-3ab-2bc-2ac-2c^2=0$

বা, $a^2+b^2+c^2+2ab+2bc+2ca-3ab-2bc-2ac-2c^2=0$

বা, $a^2+b^2-c^2-ab=0$ বা, $a^2+b^2-c^2=ab$

বা, $\frac{a^2+b^2-c^2}{2ab}=\frac{1}{2}$ বা, $\cos\angle C=\cos60°$

$\therefore \angle C=60°$ **(প্রমাণিত)**

12. **ক** $\sin 36°+\cos36°=\sin36°+\cos(90°-54°)$

$=\sin36°+\sin54°$

$=2\sin\frac{54°+36°}{2}\cos\frac{54°-36°}{2}$

$=2\sin45°\cos9°=2.\frac{1}{\sqrt{2}}.\cos9°$

$=\sqrt{2}\cos9°$ **(Ans.)**

খ বামপক্ষ $=2\sin\frac{\pi}{24}=\sqrt{4\sin^2\frac{\pi}{24}}$

$=\sqrt{2.2\sin^2\frac{\pi}{24}}=\sqrt{2.\left(1-\cos2.\frac{\pi}{24}\right)}$

$=\sqrt{2-2\cos\frac{\pi}{12}}=\sqrt{2-\sqrt{4.\cos^2\frac{\pi}{12}}}$

$=\sqrt{2-\sqrt{2\left(1+\cos\frac{\pi}{6}\right)}}$

$=\sqrt{2-\sqrt{2+2.\frac{\sqrt{3}}{2}}}$

$=\sqrt{2-\sqrt{2+\sqrt{3}}}=$ ডানপক্ষ

$\therefore 2\sin\frac{\pi}{24}=\sqrt{2-\sqrt{2+\sqrt{3}}}$ **(দেখানো হলো)**

গ দেওয়া আছে, $\sin B+\sin C=AB$ এবং $\cos B+\cos C=AC$

ডানপক্ষ $=\frac{AC^2-AB^2}{AC^2+AB^2}$

$=\frac{(\cos B+\cos C)^2-(\sin B+\sin C)^2}{(\cos B+\cos C)^2+(\sin B+\sin C)^2}$

$=\frac{\cos^2B+\cos^2C-\sin^2B-\sin^2C+2(\cos B\cos C-\sin B\sin C)}{\cos^2B+\cos^2C+\sin^2B+\sin^2C+2(\cos B\cos C+\sin B\sin C)}$

$=\frac{\cos2B+\cos2C+2\cos(B+C)}{2+2\cos(B-C)}$

$=\frac{2\cos(B+C)\cos(B-C)+2\cos(B+C)}{2\{1+\cos(B-C)\}}$

$=\frac{2\cos(B+C)\{1+\cos(B-C)\}}{2\{1+\cos(B-C)\}}$

$=\cos(B+C)=$ বামপক্ষ

$\therefore \cos(B+C)=\frac{AC^2-AB^2}{AC^2+AB^2}$ **(দেখানো হলো)**

13. **ক** এখানে, অর্ধপরিসীমা, $s=\frac{m+n+p}{2}$

$=\frac{13+12+5}{2}=15$

$\therefore \Delta MNP$ এর ক্ষেত্রফল $=\sqrt{s(s-m)(s-n)(s-p)}$

$=\sqrt{15\times(15-13)\times(15-12)\times(15-5)}$

$=30\ cm^2$ **(Ans.)**

খ দেওয়া আছে, $m^4+n^4+p^4=2p^2(m^2+n^2)$

বা, $m^4+n^4+p^4-2p^2m^2-2n^2p^2=0$

বা, $m^4+n^4+p^4-2p^2m^2-2n^2p^2+2m^2n^2=2m^2n^2$

বা, $(m^2+n^2-p^2)^2=2m^2n^2$

বা, $m^2+n^2-p^2=\pm\sqrt{2}\,mn$

বা, $\frac{m^2+n^2-p^2}{2mn}=\pm\frac{1}{\sqrt{2}}$ সুতরাং $\cos P=\pm\frac{1}{\sqrt{2}}$

এখন, $\cos P=\frac{1}{\sqrt{2}}\Rightarrow\cos P=\cos45°\ \therefore P=45°$

আবার, $\cos P=-\frac{1}{\sqrt{2}}$

বা, $\cos P=-\cos45°=\cos(180°-45°)=\cos135°$

অর্থাৎ, $P=135°$

সুতরাং $\angle P=45°$ অথবা, $135°$ **(দেখানো হলো)**

গ দেওয়া আছে, $\cot A+\cot B=x$

বা, $\frac{1}{\tan A}+\frac{1}{\tan B}=x$ বা, $\frac{\tan B+\tan A}{\tan A\tan B}=x$

বা, $\frac{y}{\tan A\tan B}=x$ $[\because \tan A+\tan B=y]$

বা, $x\tan A\tan B=y$ বা, $\tan A\tan B=\frac{y}{x}$

আবার, দেওয়া আছে, $A+B=\theta$

বা, $\tan(A+B)=\tan\theta$

বা, $\frac{\tan A+\tan B}{1-\tan A\tan B}=\tan\theta$

বা, $\frac{y}{1-\frac{y}{x}}=\tan\theta$

বা, $\frac{xy}{x-y}=\tan\theta\ \therefore \tan\theta=\frac{xy}{x-y}$ **(প্রমাণিত)**

14. **ক** এখানে, $a=\sqrt{3}+1, b=\sqrt{3}-1, R=\sqrt{2}$

$\sin A+\sin B=\frac{a}{2R}+\frac{b}{2R}\left[\because\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R\right]$

$=\frac{\sqrt{3}+1}{2.\sqrt{2}}+\frac{\sqrt{3}-1}{2\sqrt{2}}=\frac{\sqrt{3}+1+\sqrt{3}-1}{2\sqrt{2}}$

$=\frac{2\sqrt{3}}{2\sqrt{2}}=\frac{\sqrt{3}}{\sqrt{2}}$ **(Ans.)**

খ $a = \sqrt{3} + 1, b = \sqrt{3} - 1$ এবং $C = 60°$
আমরা জানি,
$c^2 = a^2 + b^2 - 2ab\cos C$
$= (\sqrt{3} + 1)^2 + (\sqrt{3} - 1)^2 - 2(\sqrt{3} + 1)(\sqrt{3} - 1)\cos 60°$
$= 2(3 + 1) - 2(3 - 1).\frac{1}{2} = 6$
$\therefore c = \sqrt{6}$
ত্রিভুজের ক্ষেত্রফল,
$\Delta = \frac{abc}{4R} = \frac{(\sqrt{3} + 1)(\sqrt{3} - 1) \times \sqrt{6}}{4.\sqrt{2}}$
$= \frac{\{(\sqrt{3})^2 - 1\}\sqrt{6}}{4\sqrt{2}} = \frac{2\sqrt{6}}{4\sqrt{2}} = \frac{\sqrt{3}}{2}$ বর্গ একক **(Ans.)**

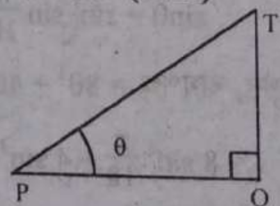
গ ΔABC-এ, $A + B + C = 180°$
বা, $B = 180° - (A + C) = 180° - (75° + 60°)$
$\therefore B = 45°$
ত্রিভুজ সূত্র পতে পাই, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$
$\therefore b = 2R \sin B = 2\sqrt{2} . \sin 45° = 2\sqrt{2} . \frac{1}{\sqrt{2}} = 2$
এবং $c = 2R \sin C = 2\sqrt{2} . \sin 60°$
$= 2\sqrt{2} . \frac{\sqrt{3}}{2} = \sqrt{2} . \sqrt{3}$
$\therefore b : c = 2 : \sqrt{2} . \sqrt{3} = \sqrt{2} : \sqrt{3}$ **(Ans.)**

15. **ক** $23.2'' = \frac{23.2'}{60} = \frac{29'}{75}$
$12' 23.2'' = 12\frac{29'}{75} = \frac{929'}{75} = \left(\frac{929}{75 \times 60}\right)^\circ = \left(\frac{929}{4500}\right)^\circ$
$45°12'23.2'' = 45\frac{929°}{4500} = \frac{203429°}{4500}$
$= \frac{203429}{4500} \times \frac{\pi}{180}$ রেডিয়ান
$= 0.25115\pi$ রেডিয়ান **(Ans.)**

খ বামপক্ষ $= \sin^2 P - \sin^2 Q + \sin^2 R$
$= \frac{1}{2}\{2\sin^2 P + 2\sin^2 R\} - \sin^2 Q$
$= \frac{1}{2}(1 - \cos 2P + 1 - \cos 2R) - \sin^2 Q$
$= \frac{1}{2}(2 - \cos 2P - \cos 2R) - \sin^2 Q$
$= 1 - \frac{1}{2}(\cos 2P + \cos 2R) - \sin^2 Q$
$= 1 - \frac{1}{2}.2\cos(P+R)\cos(P-R) - \sin^2 Q$
$= 1 - \cos(\pi - Q)\cos(P - R) - \sin^2 Q$ $[\because P + Q + R = \pi]$
$= 1 - \sin^2 Q + \cos Q \cos(P - R)$
$= \cos^2 Q + \cos Q \cos(P - R)$
$= \cos Q\{\cos Q + \cos(P - R)\}$
$= \cos Q[\cos\{\pi - (P + R)\} + \cos(P - R)]$
$= \cos Q\{-\cos(P + R) + \cos(P - R)\}$
$= \cos Q\{\cos(P - R) - \cos(P + R)\}$
$= \cos Q. 2\sin P \sin R$
$= 2\sin P \cos Q \sin R$
= ডানপক্ষ
$\therefore \sin^2 P - \sin^2 Q + \sin^2 R = 2\sin P.\cos Q.\sin R$ (প্রমাণিত)

গ দেওয়া আছে, বৃত্তটির ব্যাসার্ধ, $CQ = CP = 5$ cm
$\therefore PQ = 5 \times 2 = 10$ cm
$\therefore$ PRQ অর্ধবৃত্তের ক্ষেত্রফল $= \frac{1}{2}.\pi.5^2 = 12.5\pi$ cm^2
$\angle PRQ$ অর্ধবৃত্তস্থ কোণ।
$\therefore \angle PRQ = 90°$
সাইন সূত্রানুসারে পাই, $\frac{PQ}{\sin R} = \frac{QR}{\sin P}$
বা, $\frac{10}{\sin 90°} = \frac{6}{\sin P}$ বা, $\sin P = \frac{6}{10} = \frac{3}{5} = 0.6$
$\therefore P = \sin^{-1}(0.6) = 36.87°$
$\therefore Q = 180° - (90° + 36.87°) = 180° - 126.87° = 53.13°$
$\therefore \Delta PQR$ এর ক্ষেত্রফল $= \frac{1}{2} \times PQ \times QR \times \sin Q$
$= \frac{1}{2} \times 10 \times 6 \times \sin 53.13°$
$= 30 \times 0.8 = 24$ cm^2
$\therefore$ ছায়াকৃত অংশের ক্ষেত্রফল $= (12.5\pi - 24)$cm^2
$= 15.27$ cm^2 **(Ans.)**

16. **ক** ΔPQT-এ



$\frac{PT}{QT} = \frac{\text{অতিভুজ}}{\text{লম্ব}} = \text{cosec}\theta$
এবং $\frac{PQ}{QT} = \frac{\text{ভূমি}}{\text{লম্ব}} = \cot\theta$
এখন, $\left(\frac{PT^2}{QT^2} + \frac{PQ^2}{QT^2}\right)^2 = (\text{cosec}^2\theta + \cot^2\theta)^2$
$= (1 + \cot^2\theta + \cot^2\theta)^2 = (1 + 2\cot^2\theta)^2$
$= 1 + 4\cot^2\theta + 4\cot^4\theta = 1 + 4\cot^2\theta(1 + \cot^2\theta)$
$= 1 + 4\cot^2\theta . \text{cosec}^2\theta = 1 + 4 . \frac{PQ^2}{QT^2} . \frac{PT^2}{QT^2}$
$\therefore 1 + 4.\frac{PT^2}{QT^2}.\frac{PQ^2}{QT^2} = \left(\frac{PT^2}{QT^2} + \frac{PQ^2}{QT^2}\right)^2$ (দেখানো হলো)

খ চিত্রে, $PQ = QR = 12$ সে.মি. [বর্গের বাহু]
$QM = QN = 13$ সে.মি. [বৃত্তের ব্যাসার্ধ]
এখন, সমকোণী ΔPQM-এ
$PM = \sqrt{QM^2 - PQ^2} = \sqrt{13^2 - 12^2} = 5$ সে. মি. $= RN$

আবার, $\tan\beta = \frac{5}{12}$

$\therefore \beta = \tan^{-1}\left(\frac{5}{12}\right) = 0.39479$ রেডিয়ান

$\therefore \angle MQN = \left(\frac{\pi}{2} - 2 \times 0.39479\right)$ রেডিয়ান

$= (1.57079 - 0.78958)$ রেডিয়ান

$= 0.78121$ রেডিয়ান

ΔPQM এর ক্ষেত্রফল $= \frac{1}{2} \times PQ \times PM$ বর্গ একক

$= \frac{1}{2} \times 12 \times 5$ বর্গ সে.মি. $= 30$ বর্গ সে.মি.

$= \Delta QNR$ এর ক্ষেত্রফল

বৃত্তকলা MQN এর ক্ষেত্রফল $= \frac{1}{2} . QM^2 : \angle MQN$ বর্গ একক

$= \frac{1}{2} \times 13^2 \times 0.78121$ বর্গ সে.মি.

$= 66.012245$ বর্গ সে.মি.

বর্গক্ষেত্রের ক্ষেত্রফল $= PQ^2$ বর্গ একক

$= 12^2$ বর্গ সে.মি. $= 144$ বর্গ সে.মি.

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল = বর্গক্ষেত্রের ক্ষেত্রফল

$-$ ($2 \times \Delta PQM$ এর ক্ষেত্রফল + বৃত্তকলার ক্ষেত্রফল)

$= 144 - (2 \times 30 + 66.012245)$ বর্গ সে.মি.

$= 144 - 126.012245$ বর্গ সে.মি.

$= 17.987755$ বর্গ সে.মি.$=18$ বর্গ সে.মি. (প্রায়) **(Ans.)**

গ দেওয়া আছে, $\frac{QT}{PT} = \sin\left(\sin\frac{\pi}{18}\right)$

ΔPQT এ $\frac{QT}{PT} = \sin\theta$

$\therefore \sin\theta = \sin\left(\sin\frac{\pi}{18}\right) \therefore \theta = \sin\frac{\pi}{18}$

এখন, বামপক্ষ $= 8\theta^4 + 4\theta^3 - 6\theta^2 - 2\theta + \frac{1}{2}$

$= 8\sin^4\frac{\pi}{18} + 4\sin^3\frac{\pi}{18} - 6\sin^2\frac{\pi}{18} - 2\sin\frac{\pi}{18} + \frac{1}{2}$

$= 2.4\sin^4 x + 4\sin^3 x - 6\sin^2 x - 2\sin x + \frac{1}{2}$

$[\frac{\pi}{18} = x$ ধরে$]$

$= 2.(2\sin^2 x)^2 + 4\sin^3 x - 3.2\sin^2 x - 2\sin x + \frac{1}{2}$

$= 2(1-\cos 2x)^2 + 4\sin^3 x - 3(1-\cos 2x)$

$- 2\sin x + \frac{1}{2}$

$= 2(1 - 2\cos 2x + \cos^2 2x) + 3\sin x - \sin 3x$

$- 3 + 3\cos 2x - 2\sin x + \frac{1}{2}$

$= 2 - 4\cos 2x + 2\cos^2 2x + \sin x - \sin 3x - 3$

$+ 3\cos 2x + \frac{1}{2}$

$= -\frac{1}{2} - \cos 2x + 1 + \cos 4x + \sin x - \sin\left(3.\frac{\pi}{18}\right)$

$= \frac{1}{2} + 2\sin\frac{2x+4x}{2}\sin\frac{2x-4x}{2} + \sin x - \sin\frac{\pi}{6}$

$= \frac{1}{2} + 2\sin 3x.\sin(-x) + \sin x - \frac{1}{2}$

$= \sin x - 2\sin\left(3.\frac{\pi}{18}\right)\sin x$

$= \sin x - 2\sin\left(\frac{\pi}{6}\right).\sin x = \sin x - 2.\frac{1}{2}.\sin x$

$= \sin x - \sin x = 0 =$ ডানপক্ষ

$\therefore 8\theta^4 + 4\theta^3 - 6\theta^2 - 2\theta + \frac{1}{2} = 0$ (প্রমাণিত)

17. **ক** দেওয়া আছে, ব্যাসার্ধ, $AC = r = 3m$

কেন্দ্রে উৎপন্ন কোণ, $\angle BAC = \theta = 60°$

$= \frac{60\pi}{180} = \frac{\pi}{3}\left[\because 1° = \frac{\pi}{180}\right]$

$\therefore$ বৃত্তচাপের দৈর্ঘ্য, $BC = s = r\theta = 3 \times \frac{\pi}{3}$

$= 3.1416m$ (প্রায়) **(Ans.)**

খ বামপক্ষ $= b + c = 2R(\sin B + \sin C)$

$\left[\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R\right]$

$= 2R.2\sin\frac{B+C}{2}\cos\frac{B-C}{2}$

$= 2R.2\sin 60°\cos\frac{B-C}{2}$

$[\because A = 60°, \therefore B + C = 120°]$

$= 2R.\ 2\sin A\ \cos\frac{B-C}{2} = 2.\ 2R\sin A\ \cos\frac{B-C}{2}$

$= 2a\cos\frac{B-C}{2} =$ ডানপক্ষ

$\therefore b + c = 2a\cos\frac{B-C}{2}$ (প্রমাণিত)

গ চিত্রে A কেন্দ্রবিশিষ্ট বৃত্তের বৃত্তচাপ BC.

দেওয়া আছে, AB = 3 মিটার

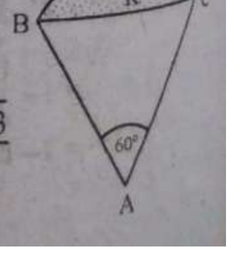
সুতরাং ব্যাসার্ধ, r = AB = AC = 3 মিটার

এবং $\theta = \angle BAC = 60°$

$\therefore$ ABC বৃত্তকলার ক্ষেত্রফল $= \frac{\pi r^2\theta}{360}$

$= \frac{3.1416 \times 3^2 \times 60}{360} = 4.7124$ বর্গ মিটার

ΔABC এর ক্ষেত্রফল



$= \frac{1}{2} \times AB \times AC \times \sin\angle BAC$

$= \frac{1}{2} \times 3 \times 3 \times \sin 60° = \frac{9}{2} \times \frac{\sqrt{3}}{2}$

$= 3.8971$ বর্গ মিটার

$\therefore$ ছায়াঘেরা অর্থাৎ R এলাকার ক্ষেত্রফল

$= 4.7124 - 3.8971$

$= 0.8153$ বর্গ মিটার

প্রতি বর্গমিটার টাইলস করতে খরচ হয় 1000 টাকা।

$\therefore$ R এলাকা টাইলস করতে খরচ হবে

$= (1000 \times 0.8153)$ টাকা

$= 815.3$ টাকা **(Ans.)**

18. ক এখানে, $\theta = 30°$

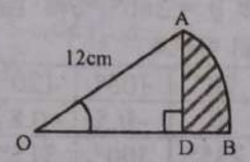
$\therefore \frac{\theta}{2} = \frac{30°}{2} = \frac{30}{2} \cdot \frac{\pi}{180}$ রেডিয়ান

$= \frac{\pi}{12}$ রেডিয়ান $= \frac{3.1416}{12}$ রেডিয়ান

≈ 0.262 রেডিয়ান (প্রায়) **(Ans.)**

খ এখানে, $\theta = 30° = \frac{\pi}{6}$

এবং $OA = 12cm$



$\therefore$ AOB বৃত্তকলার ক্ষেত্রফল $= \frac{1}{2}(OA)^2.\theta$

$= \frac{1}{2}(12)^2\frac{\pi}{6}\text{ cm}^2 = 12\pi\text{ cm}^2$

এখন, ΔAOD এর ক্ষেত্রফল $= \frac{1}{2} \times OD \times AD$

$= \frac{1}{2} \times OA\cos 30° \times OA \sin 30°$

$= \frac{1}{2} \times 12 \times \frac{\sqrt{3}}{2} \times 12 \times \frac{1}{2} = 18\sqrt{3}\text{ cm}^2$

সুতরাং ছায়াঘেরা অংশের ক্ষেত্রফল = ABD অংশের ক্ষেত্রফল = AOB বৃত্তকলার ক্ষেত্রফল – ΔAOD এর ক্ষেত্রফল

$= (12\pi - 18\sqrt{3})\text{ cm}^2$

$= 6(2\pi - 3\sqrt{3})\text{ cm}^2$ **(Ans.)**

গ এখানে, $\theta = 30°$

$\sin\frac{30°}{4} = \sin\frac{15°}{2} = \sqrt{\sin^2\frac{15°}{2}} = \sqrt{\frac{1}{2} \cdot 2\sin^2\frac{15°}{2}}$

$= \sqrt{\frac{1}{2}\left(1 - \cos 2 \cdot \frac{15°}{2}\right)} = \sqrt{\frac{1}{2}(1 - \cos 15°)}$

$= \sqrt{\frac{1}{2} - \frac{1}{2}\cos 15°} = \sqrt{\frac{1}{2} - \sqrt{\frac{1}{4}\cos^2 15°}}$

$= \sqrt{\frac{1}{2} - \sqrt{\frac{1}{8}(1 + \cos 30°)}}$

$= \sqrt{\frac{1}{2} - \sqrt{\frac{1}{8}\left(1 + \frac{\sqrt{3}}{2}\right)}} = \sqrt{\frac{1}{2} - \sqrt{\frac{2+\sqrt{3}}{16}}}$

$= \sqrt{\frac{1}{2} - \frac{1}{4}\sqrt{2+\sqrt{3}}} = \frac{1}{2}\sqrt{2 - \sqrt{2+\sqrt{3}}}$

$\therefore \sin\frac{\theta}{4} = \frac{1}{2}\sqrt{2 - \sqrt{2+\sqrt{3}}}$ **(প্রমাণিত)**

19. ক এখানে, বৃত্তের ব্যাসার্ধ,

$r = 6.4$ km ও $\angle CBD = 8.5°$

$\therefore$ বহিঃস্থ $\angle CBD = 360° - 8.5° = 351.5°$

$= 351.5 \times \frac{\pi}{180}$ রেডিয়ান

$= 6.1348$ রেডিয়ান

$\therefore$ CD অধিচাপটির দৈর্ঘ্য $= 6.4 \times 6.1348$ km

$= 39.26272$ km

$= 39.26272 \times 1000$ m

$= 39262.72$ m **(Ans.)**

খ চিত্রানুসারে, $BC = a = 6.4$ km

$AB = 3$ km

$\angle CBD = \angle CBA = 8.5° = 8.5 \times \frac{\pi}{180}$ রেডিয়ান

$= 0.14835$ রেডিয়ান

বৃত্তচাপ CBD এর ক্ষেত্রফল $= \frac{1}{2}.a^2\theta$ বর্গ একক

$= \frac{1}{2} \times (6.4)^2 \times 0.14835\text{ km}^2 = 3.0382\text{ km}^2$

আবার, ΔABC এর ক্ষেত্রফল $= \frac{1}{2}.AB.BC \sin\angle CBA$

$= \frac{1}{2} \times 3 \times 6.4 \times \sin 8.5°\text{ km}^2$

$= 9.6 \times 0.14781\text{ km}^2 = 1.41898\text{ km}^2$

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল $= (3.0382 - 1.41898)\text{ km}^2$

$= 1.61922\text{ km}^2$ **(Ans.)**

গ চিত্রানুসারে, $a = 6.4$, $c = 3$ এবং $B = 8.5°$

ট্যানজেন্ট সূত্র হতে আমরা পাই,

$\tan\frac{A-C}{2} = \frac{a-c}{a+c}\cot\left(\frac{B}{2}\right)$

বা, $\tan\frac{A-C}{2} = \frac{6.4-3}{6.4+3}\cot\left(\frac{8.5°}{2}\right)$

বা, $\tan\frac{A-C}{2} = \frac{3.4}{9.4}\cot(4.25°)$

বা, $\tan\frac{A-C}{2} = 0.3617 \times 13.4566$

বা, $\tan\frac{A-C}{2} = 4.8672$

বা, $\frac{A-C}{2} = \tan^{-1}(4.8672)$

বা, $\frac{A-C}{2} = 78.39°$

$\therefore A - C = 156.78°$ ……… (i)

আবার, ΔABC-এ, $A + B + C = 180°$

বা, $A + C = 180° - B$

বা, $A + C = 180° - 8.5°$

$\therefore A + C = 171.5°$ ……… (ii)

(i) ও (ii) যোগ করে পাই, $2A = 328.28°$

$\therefore A = 164.14°$ **(Ans.)**

A এর মান (ii) নং এ বসিয়ে, $164.14° + C = 171.5°$

$\therefore C = 7.36°$ **(Ans.)**

20. **ক** এখানে, $\cos 10° \cos 30° \cos 50° \cos 70°$

$= \cos 10°. \frac{\sqrt{3}}{2} \cos 50° \cos 70°$

$= \frac{\sqrt{3}}{2}. \cos 10°. \frac{1}{2}$ $(2\cos 70° \cos 50°)$

$= \frac{\sqrt{3}}{4} \cos 10°$ $\{\cos(70° + 50°) + \cos(70° - 50°)\}$

$= \frac{\sqrt{3}}{4} \cos 10°$ $(\cos 120° + \cos 20°)$

$= \frac{\sqrt{3}}{4} \cos 10° \cos 120° + \frac{\sqrt{3}}{4} \cos 10° \cos 20°$

$= \frac{\sqrt{3}}{4} \cos 10° \left(-\frac{1}{2}\right) + \frac{\sqrt{3}}{4} . \frac{1}{2} (2\cos 20° \cos 10°)$

$= -\frac{\sqrt{3}}{8} \cos 10° + \frac{\sqrt{3}}{8} \{\cos(20° + 10°) + \cos(20° - 10°)\}$

$= -\frac{\sqrt{3}}{8} \cos 10° + \frac{\sqrt{3}}{8} \cos 30° + \frac{\sqrt{3}}{8} \cos 10°$

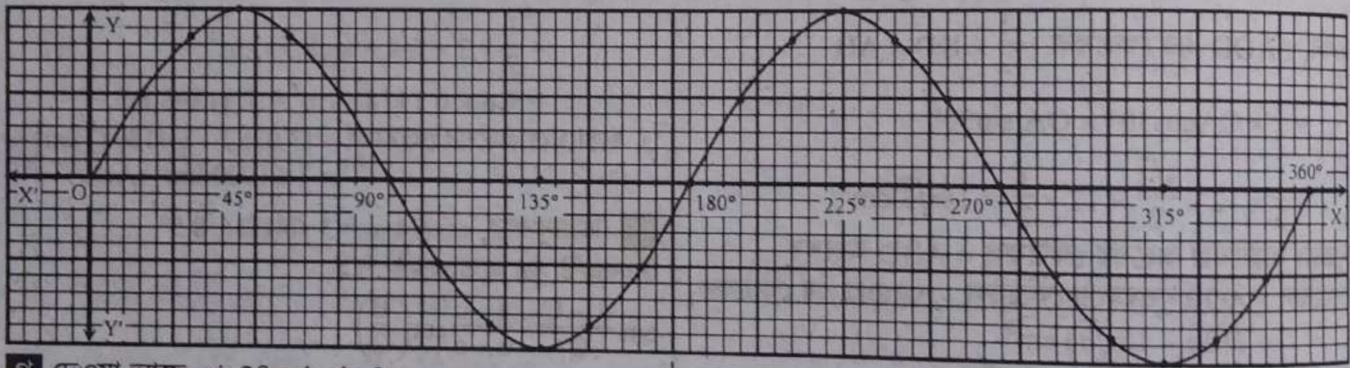
$= \frac{\sqrt{3}}{8} \cos 30° = \frac{\sqrt{3}}{8} . \frac{\sqrt{3}}{2} = \frac{3}{16}$ **(Ans.)**

খ দেওয়া আছে, $y = \sin 2\theta$, $0° \leq \theta \leq 360°$

প্রদত্ত ফাংশনটি সাইন ফাংশন। সাইন সারণি এর $\theta = 0°$ হতে $\theta = 360°$ পর্যন্ত $15°$ ব্যবধানে $\sin 2\theta$ এর মান নেওয়া হয়েছে। θ এর মানের প্রতিসঙ্গী y এর মানগুলি ছকে স্থাপন করা হয়েছে।

θ	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
$y = \sin 2\theta$	0	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0
θ	195°	210°	225°	240°	255°	270°	285°	300°	315°	330°	345°	360°	
$y = \sin 2\theta$	0.5	0.87	1	0.87	0.5	0	−0.5	−0.87	−1	−0.87	−0.5	0	

স্কেল: x-অক্ষ বরাবর ছোট বর্গক্ষেত্রের 1 বাহু = 5° এবং y-অক্ষ বরাবর ছোট বর্গক্ষেত্রের 1 বাহু = 0.1 একক ধরে বিন্দুগুলি গ্রাফ কাগজে স্থাপন করে যোগ করলে $\sin 2\theta$ এর লেখচিত্র উৎপন্ন হবে।



গ দেওয়া আছে, $\sin 2\theta = k \sin 2\alpha$

বা, $\frac{\sin 2\theta}{\sin 2\alpha} = k$

বা, $\frac{\sin 2\theta - \sin 2\alpha}{\sin 2\theta + \sin 2\alpha} = \frac{k-1}{k+1}$ [বিয়োজন-যোজন করে]

বা, $\frac{2\cos\left(\frac{2\theta + 2\alpha}{2}\right) \sin\left(\frac{2\theta - 2\alpha}{2}\right)}{2\sin\left(\frac{2\theta + 2\alpha}{2}\right) \cos\left(\frac{2\theta - 2\alpha}{2}\right)} = \frac{k-1}{k+1}$

বা, $\frac{\cos(\theta + \alpha) \sin(\theta - \alpha)}{\sin(\theta + \alpha) \cos(\theta - \alpha)} = \frac{k-1}{k+1}$

বা, $\cot(\theta + \alpha) \tan(\theta - \alpha) = \frac{k-1}{k+1}$

বা, $\frac{1}{\tan(\theta + \alpha)} . \tan(\theta - \alpha) = \frac{k-1}{k+1}$

$\therefore \tan(\theta - \alpha) = \frac{k-1}{k+1} \tan(\theta + \alpha)$ **(দেখানো হলো)**

21. **ক** বৃত্তের ব্যাসার্ধ, $r = 15$ সে.মি.

বৃত্তচাপ দ্বারা কেন্দ্রে উৎপন্ন কোণ,

$\theta = 60° = 60 \times \frac{\pi}{180}$ রেডিয়ান $= \frac{\pi}{3}$ রেডিয়ান

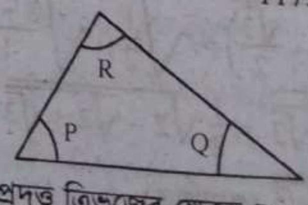
আমরা জানি, বৃত্তকলার ক্ষেত্রফল $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2} \times (15)^2 \times \frac{\pi}{3} = \frac{75\pi}{2}$

$= \frac{75 \times 3.1416}{2}$

$= 117.81$ ব. সে.মি. (প্রায়) **(Ans.)**

খ



প্রদত্ত ত্রিভুজের ক্ষেত্রে $P + Q + R = \pi$

বামপক্ষ $= \cos P + \cos Q + \cos R$

$= 2\cos\frac{P+Q}{2}\cos\frac{P-Q}{2} + \cos R$

$= 2\cos\left(\frac{\pi}{2} - \frac{R}{2}\right)\cos\frac{P-Q}{2} + \cos R$

$\left[\begin{array}{l}\because P+Q+R=\pi \\ \therefore \frac{P+Q}{2} = \frac{\pi}{2} - \frac{R}{2}\end{array}\right]$

$= 2\sin\frac{R}{2}\cos\frac{P-Q}{2} + 1 - 2\sin^2\frac{R}{2}$

$= 2\sin\frac{R}{2}\left\{\cos\frac{P-Q}{2} - \sin\frac{R}{2}\right\} + 1$

$= 2\sin\frac{R}{2}\left[\cos\frac{P-Q}{2} - \sin\left\{\frac{\pi}{2} - \frac{P+Q}{2}\right\}\right] + 1$

$= 2\sin\frac{R}{2}\left\{\cos\left(\frac{P}{2} - \frac{Q}{2}\right) - \cos\left(\frac{P}{2} + \frac{Q}{2}\right)\right\} + 1$

$= 2\sin\frac{R}{2}\, 2\sin\frac{P}{2}\sin\frac{Q}{2} + 1$

$= 1 + 4\sin\frac{R}{2}\sin\frac{P}{2}\sin\frac{Q}{2}$

$= 1 + 4\sin\left\{\frac{\pi}{2} - \frac{P+Q}{2}\right\}\sin\left\{\frac{\pi}{2} - \frac{Q+R}{2}\right\}\sin\left\{\frac{\pi}{2} - \frac{P+R}{2}\right\}$

$= 1 + 4\cos\left(\frac{P+Q}{2}\right)\cos\left(\frac{Q+R}{2}\right)\cos\left(\frac{P+R}{2}\right)$

(প্রমাণিত)

গ দেওয়া আছে, $\sin p + \sin q = m$ .... (1)

এবং $\sin\left(\frac{\pi}{2} + p\right) + \sin\left(\frac{\pi}{2} + q\right) + n = 0$

বা, $\cos p + \cos q + n = 0$

$\therefore \cos p + \cos q = -n$ .... (2)

এবং $p + q = r$

(2) নং কে (1) নং দ্বারা ভাগ করে পাই,

$\frac{\cos p + \cos q}{\sin p + \sin q} = -\frac{n}{m}$

বা, $\frac{(\cos p + \cos q)^2}{(\sin p + \sin q)^2} = \frac{n^2}{m^2}$ [বর্গ করে]

বা, $\frac{\cos^2 p + \cos^2 q + 2\cos p\cos q}{\sin^2 p + \sin^2 q + 2\sin p\sin q} = \frac{n^2}{m^2}$

বা, $\frac{\cos^2 p + \cos^2 q + 2\cos p\cos q - \sin^2 p - \sin^2 q - 2\sin p\sin q}{\cos^2 p + \cos^2 q + 2\cos p\cos q + \sin^2 p + \sin^2 q + 2\sin p\sin q}$

$= \frac{n^2 - m^2}{n^2 + m^2}$ [বিয়োজন-যোজন করে]

বা, $\frac{\cos 2p + \cos 2q + 2(\cos p\cos q - \sin p\sin q)}{1 + 1 + 2(\cos p\cos q + \sin p\sin q)} = \frac{n^2 - m^2}{n^2 + m^2}$

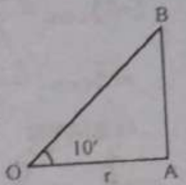
বা, $\frac{2\cos(p+q)\cos(p-q) + 2\cos(p+q)}{2 + 2\cos(p-q)} = \frac{n^2 - m^2}{n^2 + m^2}$

বা, $\frac{2\cos(p+q)\{\cos(p-q) + 1\}}{2\{1 + \cos(p-q)\}} = \frac{n^2 - m^2}{n^2 + m^2}$

বা, $\cos(p+q) = \frac{n^2 - m^2}{n^2 + m^2}$

$\therefore \cos r = \frac{n^2 - m^2}{n^2 + m^2}$ $[\because p + q = r]$ (প্রমাণিত)

22. **ক** মনে করি, AB স্তম্ভটির উচ্চতা। AB স্তম্ভ O বিন্দুতে $10'$ কোণ উৎপন্ন করে।



এখানে, $\theta = 10' = \left(\frac{10}{60} \times \frac{\pi}{180}\right)$ রেডিয়ান

এবং $r = 6$ কি. মি. $= 6000$ মিটার

AB এর উচ্চতা s হলে,

$s = r\theta = 6000 \times \frac{10}{60} \times \frac{\pi}{180} = 17.46$ (প্রায়)

$\therefore$ স্তম্ভটির উচ্চতা 17.46 মিটার (প্রায়) **(Ans.)**

খ ΔPQR-এর পরিসীমা s. সুতরাং $s = a + b + c$

দেওয়া আছে, $R = 60°$

বা, $\cos R = \cos 60°$

বা, $\frac{a^2 + b^2 - c^2}{2ab} = \frac{1}{2}$ $\left[\because \cos C = \frac{a^2 + b^2 - c^2}{2ab}\right]$

বা, $\frac{a^2 + b^2 - c^2}{ab} = 1$

বা, $a^2 + b^2 - c^2 = ab$

বা, $a^2 + b^2 - c^2 - ab = 0$

বা, $(a + b + c)^2 - 3ab - 2bc - 2ac - 2c^2 = 0$

বা, $(a+b+c)^2 + ac + bc + c^2 = 3ab + 3bc + 3ac + 3c^2$

বা, $(a+b+c)^2 + c(a+b+c) = 3b(a+c) + 3c(a+c)$

বা, $(a+b+c)(a+b+c+c) = (a+c)(3b+3c)$

বা, $(a+b+c)\{(a+c)+(b+c)\} = 3(a+c)(b+c)$

বা, $\frac{(a+c)+(b+c)}{(a+c)(b+c)} = \frac{3}{a+b+c}$

বা, $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

বা, $\frac{1}{a+b+c-b} + \frac{1}{a+b+c-a} = \frac{3}{a+b+c}$

$\therefore \frac{1}{s-b} + \frac{1}{s-a} = \frac{3}{s}$ **(দেখানো হলো)**

গ বামপক্ষ $= \cos\frac{P}{2} + \cos\frac{Q}{2} + \cos\frac{R}{2}$

$= 2\cos\frac{P+Q}{4}.\cos\frac{P-Q}{4} + \cos\frac{R}{2}$

$= 2\cos\left(\frac{\pi - R}{4}\right)\cos\left(\frac{P-Q}{4}\right) + \cos\frac{\pi}{2} + \cos\frac{R}{2}$

$= 2\cos\left(\frac{\pi - R}{4}\right)\cos\left(\frac{P-Q}{4}\right)$

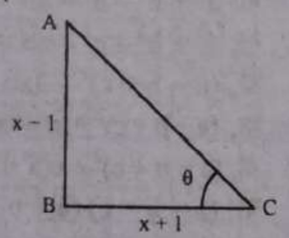
$+ 2.\cos\left(\frac{\pi + R}{4}\right).\cos\left(\frac{\pi - R}{4}\right)$

$$= 2\cos\left(\frac{\pi - R}{4}\right)\left[\cos\left(\frac{P-Q}{4}\right) + \cos\left(\frac{\pi + R}{4}\right)\right]$$
$$= 2\cos\left(\frac{\pi - R}{4}\right)\left[\cos\left(\frac{P-Q}{4}\right) + \cos\frac{2\pi - (P+Q)}{4}\right]$$
$$= 2\cos\left(\frac{\pi - R}{4}\right)\left[2\cos\left(\frac{\pi - Q}{4}\right).\cos\left(\frac{\pi - P}{4}\right)\right]$$
$$= 4\cos\left(\frac{\pi - P}{4}\right)\cos\left(\frac{\pi - Q}{4}\right)\cos\left(\frac{\pi - R}{4}\right)$$
= ডানপক্ষ

$$\therefore \cos\frac{P}{2} + \cos\frac{Q}{2} + \cos\frac{R}{2} = 4\cos\frac{\pi - P}{4}\cos\frac{\pi - Q}{4}\cos\frac{\pi - R}{4}$$
(প্রমাণিত)

23. **ক** আমরা জানি,
$\tan 45° = 1$
বা, $\tan(25° + 20°) = 1$
বা, $\frac{\tan 25° + \tan 20°}{1 - \tan 25° \tan 20°} = 1$
$\left[\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right]$
বা, $\tan 25° + \tan 20° = 1 - \tan 25° \tan 20°$
$\therefore \tan 25° + \tan 20° + \tan 25° \tan 20° = 1$ **(প্রমাণিত)**

খ চিত্রানুসারে, $\angle ABC$ অর্ধবৃত্তস্থ কোণ।
সুতরাং $\angle ABC = 90°$
$\therefore$ সমকোণী ΔABC – এ,



$\tan\theta = \frac{AB}{BC} = \frac{x-1}{x+1}$
এখন, $\sec^2 2\theta + \tan^2 2\theta = 1 + \tan^2 2\theta + \tan^2 2\theta$
$$= 1 + 2\tan^2 2\theta = 1 + 2.\left(\frac{2\tan\theta}{1 - \tan^2\theta}\right)^2$$
$$= 1 + 8.\frac{\left(\frac{x-1}{x+1}\right)^2}{\left\{1 - \left(\frac{x-1}{x+1}\right)^2\right\}^2}$$
$$= 1 + 8\frac{\left(\frac{x-1}{x+1}\right)^2}{\left\{\frac{(x+1)^2 - (x-1)^2}{(x+1)^2}\right\}^2}$$
$$= 1 + 8.\frac{(x-1)^2}{(x+1)^2}.\frac{(x+1)^4}{(4x)^2}$$
$$= 1 + \frac{(x-1)^2(x+1)^2}{2x^2} = 1 + \frac{(x^2-1)^2}{2x^2}$$
$$= \frac{2x^2 + x^4 - 2x^2 + 1}{2x^2} = \frac{x^4 + 1}{2x^2}$$
$$= \frac{1}{2}\left(\frac{x^4}{x^2} + \frac{1}{x^2}\right) = \frac{1}{2}\left(x^2 + \frac{1}{x^2}\right)$$ (প্রমাণিত)

গ $x = 3$ মিটার হলে পাই,
$AB = 3 - 1 = 2$ মিটার এবং $BC = 3 + 1 = 4$ মিটার।
চিত্রানুসারে ΔABC সমকোণী।
$\therefore AC = \sqrt{AB^2 + BC^2} = \sqrt{4^2 + 2^2}$
$= \sqrt{16 + 4} = \sqrt{20} = 2\sqrt{5}$
$\therefore \Delta ABC$ এর ক্ষেত্রফল $= \frac{1}{2} \times AB \times BC$ বর্গ একক
$= \frac{1}{2} \times 2 \times 4$ বর্গমিটার
$= 4$ বর্গ মিটার।
বৃত্তের ব্যাসার্ধ $= \frac{AC}{2} = \frac{2\sqrt{5}}{2} = \sqrt{5}$ মিটার
$\therefore$ বৃত্তের ক্ষেত্রফল $= \pi(\sqrt{5})^2$ বর্গমিটার
$= 5\pi$ বর্গমিটার।
$\therefore$ অর্ধবৃত্তের ক্ষেত্রফল $= \frac{5\pi}{2}$ বর্গমিটার
$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল
= অর্ধবৃত্তের ক্ষেত্রফল – ত্রিভুজের ক্ষেত্রফল
$= \left(\frac{5\pi}{2} - 4\right)$ বর্গমিটার $= 3.854$ বর্গমিটার
$\therefore$ ঘাস লাগানোর খরচ $= (3.854 \times 500)$ টাকা
$= 1927$ টাকা **(Ans.)**

24. **ক** আমরা জানি, $\tan 45° = 1$
বা, $\tan(36° + 9°) = 1$
বা, $\frac{\tan 36° + \tan 9°}{1 - \tan 36° \tan 9°} = 1$
বা, $\tan 36° + \tan 9° = 1 - \tan 36° \tan 9°$
$\therefore \tan 36° + \tan 9° + \tan 36° \tan 9° = 1$ [দেখানো হলো]

খ এখানে, $OA = 6$ cm
$AC = 3$ cm
AOB বৃত্তকলার ক্ষেত্রফল $= 12\text{ cm}^2$
ধরি, $\angle AOB = \theta$
প্রশ্নমতে, $\frac{1}{2}OA^2 \times \theta = 12$
বা, $\theta = \frac{12 \times 2}{6^2}$ $\therefore \theta = \frac{24}{36} = \frac{2}{3}$ rad
$\therefore$ AB চাপের দৈর্ঘ্য $= OA \times \theta\text{ cm} = 6 \times \frac{2}{3}\text{ cm} = 4\text{ cm}$
এবং CD চাপের দৈর্ঘ্য $= OC \times \theta = (6 + 3) \times \frac{2}{3}$
$= 9 \times \frac{2}{3} = 6$ cm
$\therefore$ ACDB এর পরিসীমা = (AC + চাপ CD + BD + চাপ AB)cm
$= (3 + 6 + 3 + 4)$
$= 16$ cm **(Ans.)**

আবার, OCD বৃত্তকলার ক্ষেত্রফল $= \frac{1}{2} \times OC^2 \times \theta$

$= \frac{1}{2} \times 9^2 \times \frac{2}{3} = 27\ cm^2$

$\therefore$ ACDB চাপের ক্ষেত্রফল $= (27 - 12)\ cm^2$

$= 15\ cm^2$ **(Ans.)**

গ দেওয়া আছে, $\frac{1}{p+r} + \frac{1}{q+r} = \frac{3}{p+r+q}$

বা, $\frac{q+r+p+r}{(p+r)(q+r)} = \frac{3}{p+q+r}$

বা, $(p+q+r)(p+q+r+r) = 3(p+r)(q+r)$

বা, $(p+q+r)^2 + pr + rq + r^2 = 3(pq + pr + qr + r^2)$

বা, $(p+q+r)^2 - 3pq - 2pr - 2qr - 2r^2 = 0$

বা, $p^2 + q^2 + r^2 + 2pq + 2qr + 2pr - 3pq - 2pr - 2qr - 2r^2 = 0$

বা, $p^2 + q^2 - r^2 - pq = 0$

বা, $p^2 + q^2 - r^2 = pq$

বা, $\frac{p^2 + q^2 - r^2}{2pq} = \frac{1}{2}$

বা, $\cos\theta = \frac{1}{2} = \cos 60^\circ$

$\therefore \theta = 60^\circ$

25. **ক** $\sin 25^\circ + \cos 25^\circ = \sin 25^\circ + \cos(90^\circ - 65^\circ)$

$= \sin 25^\circ + \sin 65^\circ$

$= 2\sin\frac{65^\circ + 25^\circ}{2}\cos\frac{65^\circ - 25^\circ}{2}$

$= 2\sin 45^\circ \cos 20^\circ$

$= 2 \times \frac{1}{\sqrt{2}} \times \cos 20^\circ$

$= \sqrt{2}\cos 20^\circ$ **(Ans.)**

খ ΔABC-এ, $\theta + 2\theta + 120^\circ = 180^\circ$

বা, $3\theta = 60^\circ$

$\therefore \theta = 20^\circ$

$\therefore A = \theta = 20^\circ$ এবং $B = 2\theta = 2 \times 20^\circ = 40^\circ$

ধরি, $BC = a = 2$

$\therefore$ ত্রিভুজের সাইন সূত্রানুসারে,

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

বা, $\frac{2}{\sin 20^\circ} = \frac{b}{\sin 40^\circ} = \frac{c}{\sin 120^\circ}$

$\therefore \frac{b}{\sin 40^\circ} = \frac{2}{\sin 20^\circ}$

বা, $b = \frac{2\sin 40^\circ}{\sin 20^\circ}$

$\therefore b = 3.76$ (প্রায়) **(Ans.)**

এবং $\frac{c}{\sin 120^\circ} = \frac{2}{\sin 20^\circ}$

বা, $c = \frac{2\sin 120^\circ}{\sin 20^\circ}$

$\therefore c = 5.06$ (প্রায়) **(Ans.)**

গ চিত্রানুসারে, $\theta + \alpha + 80^\circ = 180^\circ$

বা, $20^\circ + \alpha + 80^\circ = 180^\circ$

$\therefore \alpha = 80^\circ$

বামপক্ষ $= p$

$= \tan\theta \tan 2\theta \tan\alpha$

$= \tan 20^\circ \tan 40^\circ \tan 80^\circ$

$= \tan 20^\circ \tan(60^\circ - 20^\circ)\tan(60^\circ + 20^\circ)$

$= \tan 20^\circ . \frac{\tan 60^\circ - \tan 20^\circ}{1 + \tan 60^\circ . \tan 20^\circ} . \frac{\tan 60^\circ + \tan 20^\circ}{1 - \tan 60^\circ . \tan 20^\circ}$

$= \tan 20^\circ . \frac{\sqrt{3} - \tan 20^\circ}{1 + \sqrt{3}\tan 20^\circ} . \frac{\sqrt{3} + \tan 20^\circ}{1 - \sqrt{3}\tan 20^\circ}$

$= \tan 20^\circ . \frac{(\sqrt{3})^2 - \tan^2 20^\circ}{1 - (\sqrt{3}\tan 20^\circ)^2}$

$= \tan 20^\circ . \frac{3 - \tan^2 20^\circ}{1 - 3\tan^2 20^\circ} = \frac{3\tan 20^\circ - \tan^3 20^\circ}{1 - 3\tan^2 20^\circ}$

$= \tan(3.20^\circ) = \tan 60^\circ = \sqrt{3}$

= ডানপক্ষ

$\therefore p = \sqrt{3}$ (দেখানো হলো)

26. **ক** দেওয়া আছে, $\cos\theta = \frac{3}{\sqrt{13}}$

বা, $\cos^2\theta = \left(\frac{3}{\sqrt{13}}\right)^2 = \frac{9}{13}$

বা, $1 - \sin^2\theta = \frac{9}{13}$

বা, $1 - \frac{9}{13} = \sin^2\theta$

$\therefore \sin^2\theta = \frac{4}{13}$

$\therefore \cot^2\theta = \frac{\cos^2\theta}{\sin^2\theta} = \frac{9}{13} \times \frac{13}{4} = \frac{9}{4}$

প্রদত্ত রাশি $= \sqrt{\frac{2 - \cot^2\theta}{2 + \cot^2\theta}}$

$= \sqrt{\frac{2 - \frac{9}{4}}{2 + \frac{9}{4}}} = \sqrt{\frac{\frac{8-9}{4}}{\frac{8+9}{4}}}$

$= \sqrt{\frac{-1}{4} \times \frac{4}{17}} = \sqrt{\frac{-1}{17}}$

খ দেওয়া আছে, $f(x) + f(y) = p$

$\therefore \sin x + \sin y = p \ldots\ldots\ldots \text{(i)}$

এবং $g(x) + g(y) = q$

$\therefore \cos x + \cos y = q \ldots\ldots\ldots \text{(ii)}$

এখানে, $\sin x + \sin y = p$

$\therefore 2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = p \ldots\ldots\ldots \text{(iii)}$

এবং $\cos x + \cos y = q$

$\therefore 2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = q \ldots\ldots\ldots \text{(iv)}$

এখন, (iii) নং ও (iv) নং কে বর্গ করে যোগ করে পাই,

$$4\cos^2\left(\frac{x-y}{2}\right)\sin^2\left(\frac{x+y}{2}\right) + 4\cos^2\left(\frac{x-y}{2}\right)\cos^2\left(\frac{x+y}{2}\right) = p^2 + q^2$$

বা, $4\cos^2\left(\frac{x-y}{2}\right)\left\{\sin^2\left(\frac{x+y}{2}\right) + \cos^2\left(\frac{x+y}{2}\right)\right\} = p^2 + q^2$

বা, $4\cos^2\left(\frac{x-y}{2}\right) = p^2 + q^2$

বা, $4\left\{1 - \sin^2\left(\frac{x-y}{2}\right)\right\} = p^2 + q^2$

বা, $4\sin^2\left(\frac{x-y}{2}\right) = 4 - p^2 - q^2$

বা, $\sin^2\left(\frac{x-y}{2}\right) = \frac{1}{4}(4 - p^2 - q^2)$

বা, $\sin\frac{1}{2}(x-y) = \pm\frac{1}{2}\sqrt{4 - p^2 - q^2}$

$\therefore f\left(\frac{x-y}{2}\right) = \pm\frac{1}{2}\sqrt{4 - p^2 - q^2}$ (প্রমাণিত)

গ দেওয়া আছে, $f(x) = \sin x$

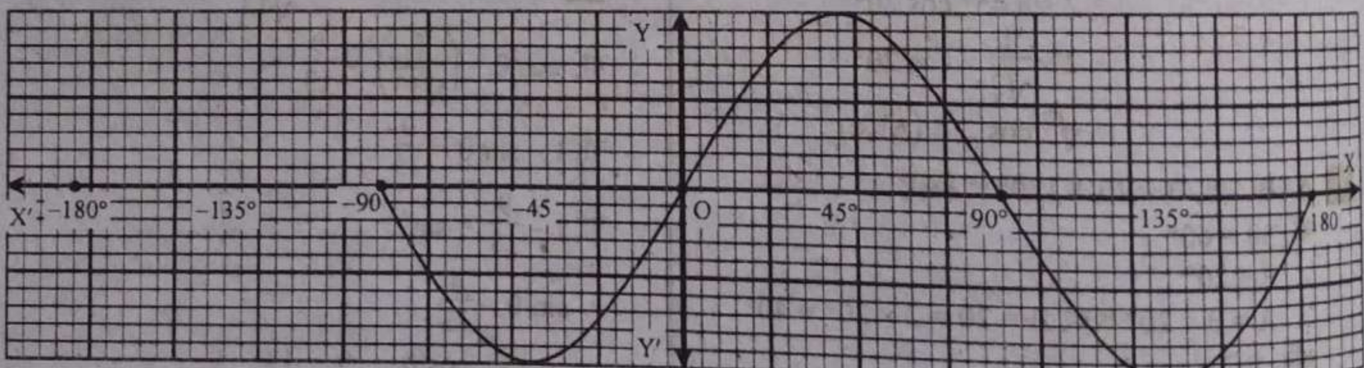
$\therefore f(2x) = \sin 2x; -\frac{\pi}{2} \le x \le \pi$

প্রদত্ত ফাংশনটি সাইন ফাংশন। প্রদত্ত রেডিয়ান মানকে ডিগ্রীতে রূপান্তর করে সাইন সারণি হতে $x = -\frac{\pi}{2}$ বা $-90°$ হতে $x = \pi$ বা $180°$ পর্যন্ত $15°$ ব্যবধানে $\sin 2x$ এর মান নেওয়া হয়েছে। x এর মানের প্রতিসঙ্গী y এর মানগুলি ছকে স্থাপন করা হয়েছে।

x	0°	±15°	±30°	±45°	±60°	±75°	±90°	105°	120°	135°	150°	165°	180°
y = sin2x	0	±0.5	±0.87	±1	±0.87	±0.5	0	− 0.5	− 0.87	− 1	−0.87	− 0.5	0

স্কেল নির্ধারণ : x অক্ষ বরাবর ছোট বর্গের এক বাহু = 5° এবং y-অক্ষ বরাবর ছোট বর্গের 1 বাহু = 0.1 একক।

এখন নির্ধারিত স্কেলে বিন্দুগুলি (ছক হতে প্রাপ্ত) গ্রাফ কাগজে বসিয়ে সুষমভাবে যোগ করে $\sin 2x$ এর লেখচিত্র অঙ্কন করি।



বৈশিষ্ট্য : লেখচিত্রটি অবিচ্ছিন্ন, ঢেউয়ের মত এবং মূলবিন্দুগামী, $y = \sin 2x$ একটি পর্যায়বৃত্ত ফাংশন যার পর্যায়কাল π.

27. **ক** বামপক্ষ $= \cos 2p = \cos^2 p - \sin^2 p$

$= \cos^2 p\left(1 - \frac{\sin^2 p}{\cos^2 p}\right)$

$= \frac{1}{\sec^2 p}(1 - \tan^2 p)$

$= \frac{1 - \tan^2 p}{1 + \tan^2 p} =$ ডানপক্ষ

$\therefore \cos 2p = \frac{1 - \tan^2 p}{1 + \tan^2 p}$ (প্রমাণিত)

খ দেওয়া আছে, $A = \frac{2\pi}{15}$

বামপক্ষ $= 16\cos A\cos 2A\cos 4A\cos 7A$

$= 16\cos\frac{2\pi}{15}\cos\frac{4\pi}{15}\cos\frac{8\pi}{15}\cos\frac{14\pi}{15}$

$= 16\cos\theta\cos 2\theta\cos 4\theta\cos 7\theta \quad \left[\frac{2\pi}{15} = \theta \text{ ধরি}\right]$

$= \frac{8}{\sin\theta}(2\sin\theta\cos\theta)\cos 2\theta\cos 4\theta\cos 7\theta$

$= \frac{4}{\sin\theta}(2\sin 2\theta \cos 2\theta)\cos 4\theta \cos 7\theta$

$= \frac{2}{\sin\theta}(2\sin 4\theta \cos 4\theta)\cos 7\theta$

$= \frac{1}{\sin\theta}(2\sin 8\theta \cos 7\theta) = \frac{1}{\sin\theta}(\sin 15\theta + \sin\theta)$

$= \frac{1}{\sin\theta}(\sin 2\pi + \sin\theta)$ $[\because 2\pi = 15\theta]$

$= \frac{1}{\sin\theta}(0 + \sin\theta) = 1 =$ ডানপক্ষ

$\therefore 16\cos A \cos 2A \cos 4A \cos 7A = 1$ (প্রমাণিত)

গ দেওয়া আছে, $\alpha + \beta + \gamma = \pi \Rightarrow \beta + \gamma = \pi - \alpha$

এবং $\cos\alpha = \cos\beta\cos\gamma$

ডানপক্ষ $= \tan\beta + \tan\gamma = \frac{\sin\beta}{\cos\beta} + \frac{\sin\gamma}{\cos\gamma}$

$= \frac{\sin\beta\cos\gamma + \cos\beta\sin\gamma}{\cos\beta\cos\gamma} = \frac{\sin(\beta+\gamma)}{\cos\alpha}$

$= \frac{\sin(\pi - \alpha)}{\cos\alpha} = \frac{\sin\alpha}{\cos\alpha} = \tan\alpha =$ বামপক্ষ

$\therefore \tan\alpha = \tan\beta + \tan\gamma$ (প্রমাণিত)

28. **ক** দেওয়া আছে, $p^2 + q^2 - r^2 = \sqrt{2}pq$

বা, $\frac{p^2 + q^2 - r^2}{2pq} = \frac{\sqrt{2}pq}{2pq}$

বা, $\cos R = \frac{1}{\sqrt{2}}$

বা, $\cos R = \cos 45°$

$\therefore R = 45°$ **(Ans.)**

খ দেওয়া আছে, $\sin x + \sin y = a$

বা, $2\sin\frac{x+y}{2}\cos\frac{x-y}{2} = a$ ... ... (i)

এবং $\cos x + \cos y = b$

বা, $2\cos\frac{x+y}{2}\cos\frac{x-y}{2} = b$ ... ... (ii)

(i) ÷ (ii) করে পাই,

$\tan\frac{x+y}{2} = \frac{a}{b}$

বা, $\tan^2\frac{x+y}{2} = \frac{a^2}{b^2}$

বা, $\sec^2\frac{x+y}{2} - 1 = \frac{a^2}{b^2}$

বা, $\sec^2\frac{x+y}{2} = 1 + \frac{a^2}{b^2}$

বা, $\sec^2\frac{x+y}{2} = \frac{a^2+b^2}{b^2}$

বা, $\cos^2\frac{x+y}{2} = \frac{b^2}{a^2+b^2}$

বা, $\frac{1}{2}\{1 + \cos(x+y)\} = \frac{b^2}{a^2+b^2}$

বা, $\cos(x+y) = \frac{2b^2}{a^2+b^2} - 1 = \frac{2b^2 - a^2 - b^2}{a^2+b^2}$

$\therefore \cos(x+y) = \frac{b^2 - a^2}{b^2 + a^2}$ **(Ans.)**

গ বামপক্ষ $= \sin^2 A - \sin^2 B + \sin^2 C$

$= \frac{1}{2}\{2\sin^2 A + 2\sin^2 C\} - \sin^2 B$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2C) - \sin^2 B$

$= \frac{1}{2}(2 - \cos 2A - \cos 2C) - \sin^2 B$

$= 1 - \frac{1}{2}(\cos 2A + \cos 2C) - \sin^2 B$

$= 1 - \frac{1}{2}.2\cos(A+C)\cos(A-C) - \sin^2 B$

$= 1 - \cos(\pi - B)\cos(A - C) - \sin^2 B$

$[\because A + B + C = \pi]$

$= 1 - \sin^2 B + \cos B\cos(A - C)$

$= \cos^2 B + \cos B\cos(A - C)$

$= \cos B\{\cos B + \cos(A - C)\}$

$= \cos B[\cos\{\pi - (A + C)\} + \cos(A - C)]$

$= \cos B\{-\cos(A + C) + \cos(A - C)\}$

$= \cos B\{\cos(A - C) - \cos(A + C)\}$

$= \cos B.\ 2\sin A\sin C$

$= 2\sin A\cos B\sin C$

= ডানপক্ষ

$\therefore \sin^2 A - \sin^2 B + \sin^2 C = 2\sin A\cos B\sin C$ (প্রমাণিত)

29. **ক** $\sin(A - B + C) = \sin(A + C - B)$

$= \sin(\pi - B - B)$ $[\because A + B + C = \pi]$

$= \sin(\pi - 2B) = \sin 2B$ **(Ans.)**

খ দেওয়া আছে, $\frac{1}{\sec A} = \frac{1}{\operatorname{cosec} C} - \frac{1}{\sec B}$

বা, $\cos A = \sin C - \cos B$

বা, $\cos A + \cos B = \sin C$

বা, $2\cos\frac{A+B}{2}\cos\frac{A-B}{2} = \sin 2.\frac{C}{2}$

বা, $2\cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\frac{A-B}{2} = 2\sin\frac{C}{2}\cos\frac{C}{2}$

বা, $2\sin\frac{C}{2}\cos\frac{A-B}{2} = 2\sin\frac{C}{2}\cos\frac{C}{2}$

বা, $\cos\frac{A-B}{2} = \cos\frac{C}{2}$

বা, $\frac{A-B}{2} = \frac{C}{2}$ বা, $A - B = C$

বা, $A = B + C$ বা, $A + A = A + B + C$

বা, $2A = \pi$ $\therefore A = \frac{\pi}{2}$

$\therefore \Delta ABC$ এর A কোণটি সমকোণ। **(Ans.)**

গ চিত্রে, PE = PF = 4 cm

PQ = 4 + 4 = 8 cm

RQ = 8 cm এবং PR = 4 + 4 = 8 cm

∴ PQR সমবাহু ত্রিভুজ। যার প্রত্যেকটি কোণের পরিমাণ 60°.

∴ ΔPQR এর ক্ষেত্রফল $= \frac{\sqrt{3}}{4}(PQ)^2$

$= \frac{\sqrt{3}}{4} \times 8^2 = 16\sqrt{3}$

$= 27.71$ বর্গ সে. মি. (প্রায়)

PEF বৃত্তকলার ক্ষেত্রফল $= \frac{\pi r^2 \theta}{360}$

$= \frac{3.1416 \times 4^2 \times 60}{360}$ [$r = PE = PF = 4$]

$= 8.38$ বর্গ সে.মি. (প্রায়)

∴ EQRF এলাকার ক্ষেত্রফল $= (27.71 - 8.38)$

$= 19.33$ বর্গ সে.মি. (প্রায়) **(Ans.)**

30. ক $\frac{\cos 75° + \cos 15°}{\cos 75° - \cos 15°}$

$$= \frac{2\cos\frac{75° + 15°}{2}\cos\frac{75° - 15°}{2}}{2\sin\frac{75° + 15°}{2}\sin\frac{15° - 75°}{2}}$$

$$= \frac{\cos 45° \cos 30°}{\sin 45° \sin(-30°)} = \cot 45° (-\cot 30°)$$

$$= 1 \times (-\sqrt{3}) = -\sqrt{3} \text{ (Ans.)}$$

খ দেওয়া আছে, $\sec B = \sec C \sec A$

বা, $\frac{1}{\cos B} = \frac{1}{\cos C} \cdot \frac{1}{\cos A}$

∴ $\cos B = \cos A \cos C$

আমরা জানি, $A + B + C = \pi$

বা, $B = \pi - (A + C)$

বা, $\cos B = \cos\{\pi - (A + C)\}$

বা, $\cos B = -\cos(A + C)$

বা, $\cos A \cos C = -(\cos A \cos C - \sin A \sin C)$

বা, $2\cos A \cos C = \sin A \sin C$

বা, $\frac{2\cos C}{\sin C} = \frac{\sin A}{\cos A}$ [$\cos A \sin C$ দ্বারা যোগ করে]

∴ $\tan A = 2\cot C$ **(দেখানো হলো)**

গ দেওয়া আছে,

$4s(s - b) = 3ca$

বা, $2s(2s - 2b) = 3ca$

বা, $(a + b + c)(a + b + c - 2b) = 3ca$ [$\because 2s = a + b + c$]

বা, $(a + c + b)(a + c - b) = 3ca$

বা, $(a + c)^2 - b^2 = 3ca$

বা, $a^2 + c^2 + 2ac - b^2 = 3ca$

বা, $a^2 + c^2 - b^2 = ac$

বা, $\frac{a^2 + c^2 - b^2}{2ac} = \frac{ac}{2ac}$

বা, $\cos B = \frac{1}{2} = \cos 60°$

∴ $B = 60°$ **(Ans.)**

31. ক $\cos 74°33' \cos 14°33' + \cos 75°27' \cos 15°27'$

$= \cos(90° - 15°27')\cos 14°33' + \cos(90° - 14°33')\cos 15°27'$

$= \sin 15°27' \cos 14°33' + \sin 14°33' \cos 15°27'$

$= \sin(15°27' + 14°33')$

$= \sin 30° = \frac{1}{2}$ **(Ans.)**

খ দেওয়া আছে, $\cos X = \sin Y - \cos Z$

বা, $\cos X + \cos Z = \sin Y$

বা, $2\cos\frac{X + Z}{2}\cos\frac{X - Z}{2} = \sin 2.\frac{Y}{2}$

বা, $2\sin\frac{Y}{2}\cos\frac{X - Z}{2} = 2\sin\frac{Y}{2}\cos\frac{Y}{2}$

বা, $\cos\frac{X - Z}{2} = \cos\frac{Y}{2}$ [$\because \sin\frac{Y}{2} \neq 0$]

বা, $\sin\left(\frac{\pi}{2} - \frac{X - Z}{2}\right) = \sin\left(\frac{\pi}{2} + \frac{Y}{2}\right)$

বা, $\left(\frac{\pi}{2} - \frac{X - Z}{2}\right) = \left(\frac{\pi}{2} + \frac{Y}{2}\right)$

বা, $\frac{Z - X}{2} = \frac{Y}{2}$

বা, $Z - X = Y$

∴ $\angle X + \angle Y = \angle Z$ **(প্রমাণিত)**

গ $f(x) = \frac{1}{2}\sin\frac{x}{2}$

∴ $f(2\pi - 4\theta) = \frac{1}{2}\sin\frac{2\pi - 4\theta}{2}$

$= \frac{1}{2}\sin(\pi - 2\theta)$

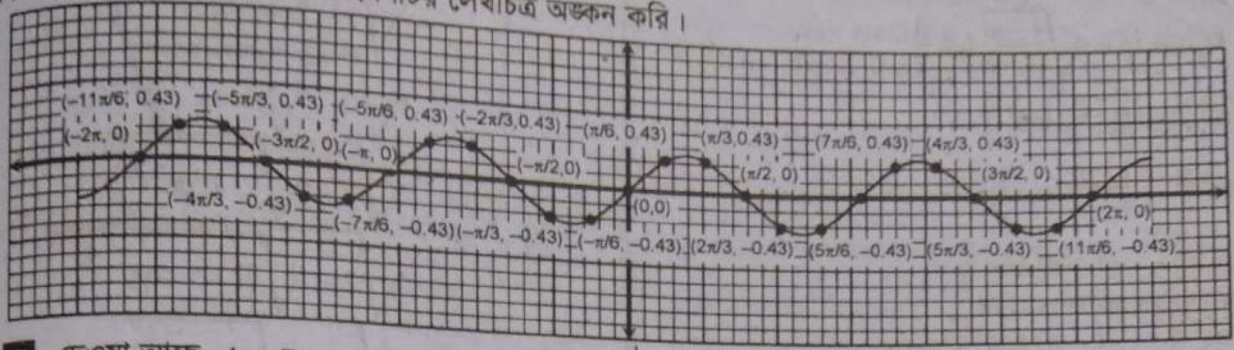
$= \frac{1}{2}\sin 2\theta$

ধরি, $y = f(2\pi - 4\theta) = \frac{1}{2}\sin 2\theta$

$\theta = -2\pi$ হতে $\theta = 2\pi$ সীমার মধ্যে $\frac{\pi}{6}$ ব্যবধানে বিভিন্ন বিন্দুতে $y = \frac{1}{2}\sin 2\theta$ এর মান নির্ণয় করি:

θ	-2π	$\frac{-11\pi}{6}$	$-\frac{10\pi}{6}$	$-\frac{9\pi}{6}$	$\frac{-8\pi}{6}$	$\frac{-7\pi}{6}$	$-\pi$	$\frac{-5\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
$y = \frac{1}{2}\sin 2\theta$	0	0.43	0.43	0	−0.43	−0.43	0	0.43	0.43	0	−0.43	−0.43	0	0.43	0.43	0	−0.43	−0.43	0	0.43	0.43	0	−0.43	−0.43	0

x-অক্ষ বরাবর ক্ষুদ্রতম বর্গের 1 বাহু $= \frac{\pi}{18}$ এবং y-অক্ষ বরাবর 5 বাহু = 1 একক ধরে প্রদত্ত বিন্দুগুলি ছক কাগজে স্থাপন করি। বিন্দুগুলি যোগ করে প্রদত্ত ফাংশনটির লেখচিত্র অঙ্কন করি।



32. **ক** দেওয়া আছে, $A + B = 105°$

বা, $180° - C = 105°$ [$\because A + B + C = 180°$]

বা, $C = 75°$

বা, $\sin C = \sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2} = \frac{\sqrt{3}+1}{2\sqrt{2}}$ **(Ans.)**

খ $a^2 + b^2 + c^2$

$= (2R \sin A)^2 + (2R \sin B)^2 + (2R \sin C)^2$

$= 4R^2 (\sin^2 A + \sin^2 B + \sin^2 C)$

এখন, $\sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1}{2}(2\sin^2 A + 2\sin^2 B) + \sin^2 C$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2B) + \sin^2 C$

$= \frac{1}{2}(2 - \cos 2A - \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2}(\cos 2A + \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2} \cdot 2\cos(A + B)\cos(A - B) + 1 - \cos^2 C$

$= 2 - \cos(\pi - C)\cos(A - B) - \cos^2 C$

$= 2 + \cos C \cos(A - B) - \cos^2 C$

$= 2 + \cos C[\cos(A - B) - \cos C]$

$= 2 + \cos C[\cos(A - B) + \cos(A + B)]$ [$\because A + B + C = \pi$]

$= 2 + \cos C . 2 . \cos A . \cos B$

$= 2 + 2\cos A \cos B \cos C$

$\therefore a^2 + b^2 + c^2 = 4R^2(2 + 2\cos A \cos B \cos C)$

$\therefore a^2 + b^2 + c^2 = 8R^2(1 + \cos A \cos B \cos C)$ (প্রমাণিত)

গ ΔPQS এর $PQ = s$, $QS = p$ এবং $PS = q$ ধরি।

প্রশ্নমতে, $\frac{1}{PQ + PS} = \frac{3}{PS + PQ + QS} - \frac{1}{PS + QS}$

বা, $\frac{1}{s + q} = \frac{3}{q + s + p} - \frac{1}{q + p}$

বা, $\frac{1}{s + q} + \frac{1}{q + p} = \frac{3}{q + s + p}$

বা, $\frac{q + p + s + q}{(s + q)(q + p)} = \frac{3}{p + q + s}$

বা, $\frac{2q + p + s}{sq + sp + q^2 + pq} = \frac{3}{p + q + s}$

বা, $2pq + 2q^2 + 2qs + p^2 + pq + ps + ps + qs + s^2$
$= 3sq + 3sp + 3q^2 + 3pq$

বা, $p^2 + s^2 - q^2 = sp$ বা, $\frac{p^2 + s^2 - q^2}{2sp} = \frac{1}{2}$

বা, $\cos Q = \cos 60°$ $\therefore Q = 60°$ **(Ans.)**

33. **ক** $\tan 75° = \tan(45° + 30°) = \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} = \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$= \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)} = \frac{3 + \sqrt{3} + \sqrt{3} + 1}{(\sqrt{3})^2 - 1^2}$

$= \frac{4 + 2\sqrt{3}}{3 - 1} = \frac{2(2 + \sqrt{3})}{2}$

$\therefore \tan 75° = 2 + \sqrt{3}$ (প্রমাণিত)

খ দেওয়া আছে, $\cos X = \sin Y - \cos Z$

$\Rightarrow \cos X + \cos Z = \sin Y$

$\Rightarrow \cos X + \cos\{\pi - (X + Y)\} = \sin Y$ [$\because X + Y + Z = \pi$]

$\Rightarrow \cos X - \cos(X + Y) = \sin Y$

$\Rightarrow 2\sin\frac{2X + Y}{2} \sin\frac{Y}{2} = 2\sin\frac{Y}{2}\cos\frac{Y}{2}$

$\Rightarrow \sin\left(X + \frac{Y}{2}\right) = \cos\frac{Y}{2}$

$\Rightarrow \sin\left(X + \frac{Y}{2}\right) = \sin\left(\frac{\pi}{2} - \frac{Y}{2}\right)$

$\therefore X + \frac{Y}{2} = \frac{\pi}{2} - \frac{Y}{2} \Rightarrow X + Y = \frac{\pi}{2}$ $\therefore Z = \frac{\pi}{2}$

অতএব, ΔXYZ সমকোণী। (দেখানো হলো)

গ দেওয়া আছে, $\sqrt{1+n}\tan\frac{\alpha}{2}=\sqrt{1-n}\,.\tan\frac{\beta}{2}$

বা, $\tan\frac{\alpha}{2}=\sqrt{\frac{1-n}{1+n}}\tan\frac{\beta}{2}$

$\Rightarrow \tan^2\frac{\alpha}{2}=\frac{1-n}{1+n}\tan^2\frac{\beta}{2}$

$\Rightarrow \tan^2\frac{\beta}{2}=\frac{1+n}{1-n}\tan^2\frac{\alpha}{2}$

$$\Rightarrow \frac{\sin^2\frac{\beta}{2}}{\cos^2\frac{\beta}{2}}=\frac{(1+n)\sin^2\frac{\alpha}{2}}{(1-n)\cos^2\frac{\alpha}{2}}$$

$$\Rightarrow \frac{\cos^2\frac{\beta}{2}}{\sin^2\frac{\beta}{2}}=\frac{(1-n)\cos^2\frac{\alpha}{2}}{(1+n)\sin^2\frac{\alpha}{2}}$$

$$\Rightarrow \frac{\cos^2\frac{\beta}{2}-\sin^2\frac{\beta}{2}}{\cos^2\frac{\beta}{2}+\sin^2\frac{\beta}{2}}=\frac{(1-n)\cos^2\frac{\alpha}{2}-(1+n)\sin^2\frac{\alpha}{2}}{(1-n)\cos^2\frac{\alpha}{2}+(1+n)\sin^2\frac{\alpha}{2}}$$

[বিয়োজন-যোজন করে]

$$\Rightarrow \frac{\cos\beta}{1}=\frac{\left(\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\right)-n\left(\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}\right)}{\left(\cos^2\frac{\alpha}{2}+\sin^2\frac{\alpha}{2}\right)-n\left(\cos^2\frac{\alpha}{2}-\sin^2\frac{\alpha}{2}\right)}$$

$\therefore \cos\beta=\frac{\cos\alpha-n}{1-n\cos\alpha}$ **(দেখানো হলো)**

34. ক দেওয়া আছে, $\tan\beta=\frac{1}{3}$

$$\therefore \sin2\beta=\frac{2\tan\beta}{1+\tan^2\beta}=\frac{2\times\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}$$

$$=\frac{2}{3}\times\frac{9}{10}=\frac{3}{5}\ \textbf{(Ans.)}$$

খ $\angle E+\angle F=65°$ ... ... (i)

$\angle F-\angle E=25°$ ... ... (ii)

(i) ও (ii) নং যোগ করে পাই,

$2\angle F=90°\ \therefore \angle F=45°$

বামপক্ষ $=2\sin\left(\pi+\frac{F}{4}\right)=-2\sin\frac{F}{4}$

$$=-2\sin\frac{45°}{4}=-\sqrt{4\sin^2\frac{45°}{4}}$$

$$=-\sqrt{2.\,2\sin^2\frac{45°}{4}}$$

$$=-\sqrt{2\left(1-\cos2.\,\frac{45°}{4}\right)}$$

$$=-\sqrt{2-2\cos\frac{45°}{2}}$$

$$=-\sqrt{2-\sqrt{4\cos^2\frac{45°}{2}}}$$

$$=-\sqrt{2-\sqrt{2.\,2\cos^2\frac{45°}{2}}}$$

$$=-\sqrt{2-\sqrt{2\left(1+\cos2.\,\frac{45°}{2}\right)}}$$

$$=-\sqrt{2-\sqrt{2+2.\,\frac{1}{\sqrt{2}}}}$$

$=-\sqrt{2-\sqrt{2+\sqrt{2}}}=$ ডানপক্ষ

$\therefore 2\sin\left(\pi+\frac{F}{4}\right)=-\sqrt{2-\sqrt{2+\sqrt{2}}}$ (দেখানো হলো)

গ $\angle E=65°-\angle F=65°-45°=20°$

বামপক্ষ $=\tan\angle E\tan2\angle E\tan3\angle E\tan4\angle E$

$=\tan20°\tan40°\tan60°\tan80°$

$=\sqrt{3}\tan20°\tan40°\tan80°$

$=\sqrt{3}\tan20°\tan(60°-20°)\tan(60°+20°)$

$$=\sqrt{3}\tan20°.\frac{\tan60°-\tan20°}{1+\tan60°.\tan20°}.\frac{\tan60°+\tan20°}{1-\tan60°.\tan20°}$$

$$=\sqrt{3}\tan20°.\frac{\sqrt{3}-\tan20°}{1+\sqrt{3}\tan20°}.\frac{\sqrt{3}+\tan20°}{1-\sqrt{3}\tan20°}$$

$$=\sqrt{3}\tan20°.\frac{(\sqrt{3})^2-\tan^220°}{1-(\sqrt{3}\tan20°)^2}$$

$$=\sqrt{3}\tan20°.\frac{3-\tan^220°}{1-3\tan^220°}$$

$$=\sqrt{3}\times\frac{3\tan20°-\tan^320°}{1-3\tan^220°}$$

$=\sqrt{3}\tan(3.20°)=\sqrt{3}\tan60°$

$=\sqrt{3}\times\sqrt{3}=3=$ ডানপক্ষ

$\therefore \tan\angle E.\tan2\angle E.\tan3\angle E.\tan4\angle E=3$ (দেখানো হলো)

35. ক দেওয়া আছে, $\tan\theta=\frac{b}{a}$

বা, $\frac{\sin\theta}{\cos\theta}=\frac{b}{a}$

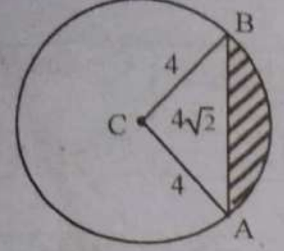
বা, $\frac{\cos\theta}{\sin\theta}=\frac{a}{b}$

বা, $\frac{a\cos\theta}{b\sin\theta}=\frac{a^2}{b^2}$

বা, $\frac{a\cos\theta+b\sin\theta}{a\cos\theta-b\sin\theta}=\frac{a^2+b^2}{a^2-b^2}$ [যোজন-বিয়োজন করে]

$\therefore \frac{a\cos\theta+b\sin\theta}{a\cos\theta-b\sin\theta}=\frac{a^2+b^2}{a^2-b^2}$ **(Ans.)**

খ



চিত্র থেকে পাই, ABC বৃত্তকলার ব্যাসার্ধ 4

এখন, $\angle ACB = \cos^{-1}\left(\frac{4^2 + 4^2 - (4\sqrt{2})^2}{2 \times 4 \times 4}\right)$

$= \cos^{-1}\left(\frac{16 + 16 - 32}{32}\right)$

$= \cos^{-1}0 = \frac{\pi}{2}$

$\therefore$ ABC বৃত্তকলার ক্ষেত্রফল $= \frac{1}{2} \times 4^2 \times \frac{\pi}{2}$ বর্গ একক

$= 4\pi$ বর্গ একক

এখানে, $AB^2 = AC^2 + BC^2$

$\therefore$ ABC সমকোণী সমদ্বিবাহু ত্রিভুজের ক্ষেত্রফল

$= \frac{1}{2} \times 4 \times 4$ বর্গ একক

$= 8$ বর্গ একক

$\therefore$ ছায়াঘেরা অংশের ক্ষেত্রফল = ABC বৃত্তকলার ক্ষেত্রফল – ABC সমকোণী সমদ্বিবাহু ত্রিভুজের ক্ষেত্রফল

$= (4\pi - 8)$ বর্গ একক

$= 4.57$ বর্গ একক **(Ans.)**

গ দেওয়া আছে, $\sin C + \sin D = p$

$\cos C + \cos D = q$

এখন, $\sin C + \sin D = p$

$\therefore 2\sin\frac{C+D}{2}\cos\frac{C-D}{2} = p$ ……… (i)

এবং $\cos C + \cos D = q$

$\therefore 2\cos\frac{C+D}{2}\cos\frac{C-D}{2} = q$ ……… (ii)

এখন, (i) ও (ii) কে বর্গ করে যোগ করে পাই,

$4\sin^2\frac{C+D}{2}\cos^2\frac{C-D}{2} + 4\cos^2\frac{C+D}{2}\cos^2\frac{C-D}{2} = p^2 + q^2$

বা, $4\cos^2\frac{C-D}{2}\left(\sin^2\frac{C+D}{2} + \cos^2\frac{C+D}{2}\right) = p^2 + q^2$

বা, $4\cos^2\frac{C-D}{2} = p^2 + q^2$

বা, $4\left(1 - \sin^2\frac{C-D}{2}\right) = p^2 + q^2$

বা, $4 - 4\sin^2\frac{C-D}{2} = p^2 + q^2$

বা, $4\sin^2\frac{C-D}{2} = 4 - p^2 - q^2$

বা, $\sin^2\frac{C-D}{2} = \frac{1}{4}(4 - p^2 - q^2)$

$\therefore \sin\frac{C-D}{2} = \pm\frac{1}{2}\sqrt{4 - p^2 - q^2}$ (প্রমাণিত)

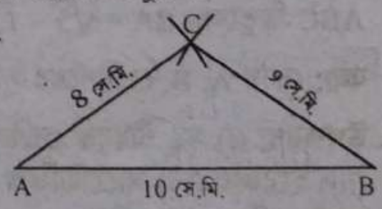
পাঠ্যবইয়ের ব্যবহারিকের সমাধান

অনুচ্ছেদ-7.5.1 | পৃষ্ঠা-২৮৩

সমস্যা নং 7.5.1	কোনো ত্রিভুজের বাহুগুলি যথাক্রমে 8 সে.মি., 9 সে.মি. এবং 10 সে.মি. হলে ত্রিভুজের বৃহত্তম কোণটি নির্ণয় কর।	তারিখ : ………

সমস্যা: কোনো ত্রিভুজের বাহুগুলি যথাক্রমে 8 সে.মি., 9 সে.মি. এবং 10 সে.মি. হলে ত্রিভুজের বৃহত্তম কোণটি নির্ণয় কর।

সমাধান: মনেকরি, ABC একটি ত্রিভুজ যার $a = 9$ সে.মি., $b = 8$ সে.মি. ও $c = 10$ সে.মি.। ABC ত্রিভুজের $CA = 8$ সে.মি., $BC = 9$ সে.মি., $AB = 10$ সে.মি.। তাহলে ΔABC এর বৃহত্তম কোণ C.



বৃহত্তম কোণ নির্ণয়:

তত্ত্ব: বৃহত্তম কোণ C নির্ণয়ের সূত্র: $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

উপকরণ: (i) সরু শীষযুক্ত পেন্সিল (ii) স্কেল (iii) ইরেজার (iv) সায়েন্টিফিক ক্যালকুলেটর।

কার্যপদ্ধতি: 1. উপরিউক্ত সূত্রে a, b, c এর মান বসিয়ে cosC এর মান নির্ণয় করি।
2. ক্যালকুলেটর ব্যবহার করে cosC এর মান হতে C কোণের পরিমাণ নির্ণয় করি।

$\cos C = \frac{9^2 + 8^2 - 10^2}{2.9.8} = \frac{81 + 64 - 100}{144}$

$\therefore \cos C = 0.3125$

$\therefore C = \cos^{-1}(0.3125) = 71.790043 = 71°47'24.16''$

বা, $2 + 2\cos(\theta - \varphi) = a^2 + b^2$

বা, $2\{1 + \cos(\theta - \varphi)\} = a^2 + b^2$

বা, $4\cos^2\frac{1}{2}(\theta - \varphi) = a^2 + b^2$

বা, $\sec^2\frac{1}{2}(\theta - \varphi) = \frac{4}{a^2 + b^2}$

বা, $\tan^2\frac{1}{2}(\theta - \varphi) = \frac{4}{a^2 + b^2} - 1$

বা, $\tan^2\frac{1}{2}(\theta - \varphi) = \frac{4 - a^2 - b^2}{a^2 + b^2}$

$\therefore \tan\frac{1}{2}(\theta - \varphi) = \pm\sqrt{\frac{4 - a^2 - b^2}{a^2 + b^2}}$ (প্রমাণিত)

12. (i) দেওয়া আছে, $a\sin\alpha + b\sin\beta = c$

$\therefore a^2\sin^2\alpha + b^2\sin\beta + 2ab\sin\alpha\sin\beta = c^2$ ... ... (i)

এবং $a\cos\alpha + b\cos\beta = c$

$\therefore a^2\cos^2\alpha + b^2\cos^2\beta + 2ab\cos\alpha\cos\beta = c^2$ ... ... (ii)

(i) নং ও (ii) নং যোগ করে,

$a^2 + b^2 + 2ab(\cos\alpha\cos\beta + \sin\alpha\sin\beta) = 2c^2$

বা, $a^2 + b^2 + 2ab\cos(\alpha - \beta) = 2c^2$

বা, $a^2 + b^2 + 2ab\left\{1 - 2\sin^2\left(\frac{\alpha - \beta}{2}\right)\right\} = 2c^2$

বা, $a^2 + b^2 + 2ab - 4ab\sin^2\left(\frac{\alpha - \beta}{2}\right) = 2c^2$

বা, $(a + b)^2 - 2c^2 = 4ab\sin^2\left(\frac{\alpha - \beta}{2}\right)$

বা, $\sin^2\left(\frac{\alpha - \beta}{2}\right) = \frac{(a + b)^2 - 2c^2}{4ab}$

$\therefore \sin\left(\frac{\alpha - \beta}{2}\right) = \pm\frac{1}{2}\sqrt{\frac{(a + b)^2 - 2c^2}{ab}}$ (প্রমাণিত)

(ii) দেওয়া আছে, $a\sin\alpha + b\sin\beta = c$

$\therefore a^2\sin^2\alpha + \sin^2\beta + 2ab\sin\alpha\sin\beta = c^2$ ... ... (i)

এবং $a\cos\alpha + b\cos\beta = c$

$\therefore a^2\cos^2\alpha + b^2\cos^2\beta + 2ab\cos\alpha\cos\beta = c^2$ ... ... (ii)

(i) এবং (ii) যোগ করে,

$a^2 + b^2 + 2ab\cos(\alpha - \beta) = 2c^2$

বা, $\cos(\alpha - \beta) = \frac{2c^2 - a^2 - b^2}{2ab}$

বা, $2\cos^2\frac{\alpha - \beta}{2} - 1 = \frac{2c^2 - a^2 - b^2}{2ab}$

বা, $2\cos^2\frac{\alpha - \beta}{2} = \frac{2c^2 - a^2 - b^2 + 2ab}{2ab}$

বা, $2\cos^2\frac{\alpha - \beta}{2} = \frac{2c^2 - (a - b)^2}{2ab}$

বা, $\cos^2\frac{\alpha - \beta}{2} = \frac{2c^2 - (a - b)^2}{4ab}$

$\therefore \cos\frac{1}{2}(\alpha - \beta) = \pm\frac{1}{2}\sqrt{\frac{2c^2 - (a - b)^2}{ab}}$ (প্রমাণিত)

13. বামপক্ষ $= (\cos\alpha + \cos\beta)^2 + (\sin\alpha - \sin\beta)^2$

$= \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta - 2\sin\alpha\sin\beta$

$= (\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2\cos\alpha\cos\beta - 2\sin\alpha\sin\beta$

$= 1 + 1 + 2(\cos\alpha\cos\beta - \sin\alpha\sin\beta)$

$= 2 + 2.\cos(\alpha + \beta) = 2\{1 + \cos(\alpha + \beta)\}$

$= 2.2.\cos^2\left(\frac{\alpha + \beta}{2}\right)$

$= 4\cos^2\left(\frac{\alpha + \beta}{2}\right)$

= ডানপক্ষ (প্রমাণিত)

পাঠ্যবইয়ের কাজের সমাধান

অনুচ্ছেদ-7.2.9 | পৃষ্ঠা-২৫৭

(i) বামপক্ষ $= \sin A + \sin B + \sin C$

$= 2\sin\frac{A + B}{2}.\cos\frac{A - B}{2} + \sin C$

$= 2\sin\left\{\frac{\pi}{2} - \frac{C}{2}\right\}.\cos\frac{A - B}{2} + 2.\sin\frac{C}{2}.\cos\frac{C}{2}$

$\left[\because A + B + C = \pi \therefore \frac{C}{2} = \frac{\pi}{2} - \frac{A + B}{2}\right]$

$= 2.\cos\frac{C}{2}.\cos\frac{A - B}{2} + 2\sin\frac{C}{2}.\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\left(\cos\frac{A - B}{2} + \sin\frac{C}{2}\right)$

$= 2\cos\frac{C}{2}\left[\cos\frac{A - B}{2} + \sin\left\{\frac{\pi}{2} - \frac{A + B}{2}\right\}\right]$

$= 2\cos\frac{C}{2}\left[\cos\frac{A - B}{2} + \cos\frac{A + B}{2}\right]$

$= 2\cos\frac{C}{2}.2.\cos\frac{A}{2}.\cos\frac{B}{2}$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ = ডানপক্ষ

$\therefore \sin A + \sin B + \sin C = 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$ (প্রমাণিত)

(ii) বামপক্ষ

$= \cos^2A + \cos^2B + \cos^2C - 2\cos A\cos B\cos C$

$= \frac{1}{2}(\cos^2A + 2\cos^2B) + \cos^2C - 2\cos A\cos B\cos C$

$= \frac{1}{2}(1 + \cos2A + 1 + \cos2B) + \cos^2C - 2\cos A\cos B\cos C$

$= \frac{1}{2}(2 + \cos2A + \cos2B) + \cos^2C - 2\cos A\cos B\cos C$

$= 1 + \frac{1}{2}(\cos^2 A + \cos^2 B) + \cos^2 C - 2\cos A \cos B \cos C$

$= 1 + \frac{1}{2}. 2\cos(A+B).\cos(A-B) + \cos^2 C - 2\cos A \cos B \cos C$

$= 1 + \cos(A+B)\cos(A-B) + \cos^2 C - 2\cos A \cos B \cos C$

$= 1 + \cos(2\pi - C)\cos(A-B) + \cos^2 C - 2\cos A \cos B \cos C$

$[\because A + B + C = 2\pi \quad \therefore A + B = 2\pi - C]$

$= 1 + \cos C.\cos(A-B) + \cos^2 C - 2\cos A\cos B\cos C$

$= 1 + \cos C\{\cos(A-B) + \cos C\} - 2\cos A\cos B\cos C$

$= 1 + \cos C[\cos(A-B) + \cos\{2\pi - (A+B)\}] - 2\cos A\cos B\cos C$

$= 1 + \cos C\{\cos(A-B) + \cos(A+B)\} - 2\cos A\cos B\cos C$

$= 1 + 2\cos C.\cos A.\cos B - 2\cos A\cos B\cos C$

$= 1 =$ ডানপক্ষ (প্রমাণিত)

অনুশীলনী-7(F) এর সমাধান

1. (i) দেওয়া আছে, $A + B + C = \pi$

বা, $A + B = \pi - C$

বা, $\cot(A+B) = \cot(\pi - C)$

বা, $\frac{\cot A \cot B - 1}{\cot B + \cot A} = -\cot C$

বা, $\cot A \cot B - 1 = -\cot C \cot B - \cot C \cot A$

$\therefore \cot A \cot B + \cot B \cot C + \cot C \cot A = 1$

(প্রমাণিত)

(ii) দেওয়া আছে, $A + B + C = \pi$

বা, $\frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}$

বা, $\frac{A}{2} + \frac{B}{2} = \frac{\pi}{2} - \frac{C}{2}$

বা, $\tan\left(\frac{A}{2} + \frac{B}{2}\right) = \tan\left(\frac{\pi}{2} - \frac{C}{2}\right)$

বা, $\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \cot\frac{C}{2}$

বা, $\frac{\tan\frac{A}{2} + \tan\frac{B}{2}}{1 - \tan\frac{A}{2}\tan\frac{B}{2}} = \frac{1}{\tan\frac{C}{2}}$

বা, $\tan\frac{C}{2}\left(\tan\frac{A}{2} + \tan\frac{B}{2}\right) = 1 - \tan\frac{A}{2}\tan\frac{B}{2}$

$\therefore \tan\frac{B}{2}\tan\frac{C}{2} + \tan\frac{C}{2}\tan\frac{A}{2} + \tan\frac{A}{2}\tan\frac{B}{2} = 1$

(প্রমাণিত)

2.(i) বামপক্ষ $= \sin 2A - \sin 2B + \sin 2C$

$= 2\cos(A+B)\sin(A-B) + \sin 2C$

$= 2\cos(\pi - C)\sin(A-B) + \sin 2C$

$[\because A + B + C = \pi]$

$= -2\cos C\sin(A-B) + 2\sin C\cos C$

$= 2\cos C\{\sin C - \sin(A-B)\}$

$= 2\cos C[\sin\{\pi - (A+B)\} - \sin(A-B)]$

$= 2\cos C\{\sin(A+B) - \sin(A-B)\}$

$= 2\cos C.\ 2\cos A\sin B$

$= 4\cos A\sin B\cos C$

$=$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin(A+B)\cos(A-B) + \sin 2C$

$= 2\sin(\pi - C)\cos(A-B) + \sin 2C$

$[\because A + B + C = \pi \therefore B + C = \pi - A]$

$= 2\sin C\cos(A-B) + 2\sin C\cos C$

$= 2\sin C\{\cos(A-B) + \cos C\}$

$= 2\sin C\{\cos(A-B) + \cos[\pi - (A+B)]\}$

$[\because A + B + C = \pi]$

$= 2\sin C\{\cos(A-B) - \cos(A+B)\}$

$= 2\sin C.\ 2\sin A\sin B = 4\sin A\sin B\sin C$

$=$ ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \cos 2A + \cos 2B + \cos 2C$

$= (\cos 2A + \cos 2B) + \cos 2C$

$= 2\cos(A+B)\cos(A-B) + 2\cos^2 C - 1$

$= 2\cos(\pi - C)\cos(A-B) + 2\cos^2 C - 1$

$[\because A + B + C = \pi]$

$= -2\cos C\cos(A-B) + 2\cos^2 C - 1$

$= -2\cos C[\cos(A-B) - \cos C] - 1$

$= -2\cos C[\cos(A-B) - \cos\{\pi - (A+B)\}] - 1$

$= -2\cos C[\cos(A-B) + \cos(A+B)] - 1$

$= -2\cos C \times 2\cos A\cos B - 1$

$= -4\cos A\cos B\cos C - 1$

$=$ ডানপক্ষ (প্রমাণিত)

3. (i) বামপক্ষ $= \sin A + \sin B - \sin C$

$= 2\sin\frac{A+B}{2}.\cos\frac{A-B}{2} - \sin 2.\frac{C}{2}$

$= 2\sin\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\frac{A-B}{2} - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\cos\frac{A-B}{2} - 2\sin\frac{C}{2}\cos\frac{C}{2}$

$= 2\cos\frac{C}{2}\left[\cos\frac{A-B}{2} - \sin\frac{C}{2}\right]$

$= 2\cos\frac{C}{2}\left[\cos\frac{A-B}{2} - \sin\left\{\frac{\pi}{2} - \left(\frac{A+B}{2}\right)\right\}\right]$

$= 2\cos\frac{C}{2}\left[\cos\frac{A-B}{2} - \cos\frac{A+B}{2}\right]$

$= 2\cos\frac{C}{2}.\ 2\sin\frac{A}{2}.\sin\frac{B}{2}$

$= 4\sin\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$

= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \cos A - \cos B + \cos C$

$= \cos A + \cos C - \cos B$

$= 2\cos\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) + 1 - 1 - \cos B$

$= 2\cos\left(\frac{\pi - B}{2}\right)\cos\left(\frac{A-C}{2}\right) + 2\sin^2\frac{B}{2} - 1$

$= 2\sin\frac{B}{2}\cos\left(\frac{A-C}{2}\right) + 2\sin^2\frac{B}{2} - 1$

$= 2\sin\frac{B}{2}\left[\cos\left(\frac{A-C}{2}\right) + \sin\frac{B}{2}\right] - 1$

$= 2\sin\frac{B}{2}\left[\cos\left(\frac{A-C}{2}\right) + \cos\left(\frac{A+C}{2}\right)\right] - 1$

$= 2\sin\frac{B}{2} . 2\cos\frac{A}{2}\cos\frac{C}{2} - 1$

$= 4\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2} - 1$

= ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \cos A + \cos B - \cos C$

$= (\cos A + \cos B) - \cos C$

$= 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 1 - \cos C - 1$

$= 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} - \left(1 - 2\sin^2\frac{C}{2}\right)$

$= 2\cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\frac{A-B}{2} + 2\sin^2\frac{C}{2} - 1$

$= 2\sin\frac{C}{2}\cos\frac{A-B}{2} + 2\sin^2\frac{C}{2} - 1$

$= 2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2} + \sin\frac{C}{2}\right\} - 1$

$= 2\sin\frac{C}{2}\left\{\cos\frac{A-B}{2} + \sin\left(\frac{\pi}{2} - \frac{A+B}{2}\right)\right\} - 1$

$= 2\sin\frac{C}{2}\left\{\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \cos\left(\frac{A}{2} + \frac{B}{2}\right)\right\} - 1$

$= 2\sin\frac{C}{2} . 2\cos\frac{A}{2} . \cos\frac{B}{2} - 1$

$= 4\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} - 1$

= ডানপক্ষ (প্রমাণিত)

4. (i) বামপক্ষ $= \cos^2 2A + \cos^2 2B + \cos^2 2C$

$= \frac{1}{2} . 2(\cos^2 2A + \cos^2 2B) + \cos^2 2C$

$= \frac{1}{2}(1 + \cos 4A + 1 + \cos 4B) + \cos^2 2C$

$= \frac{1}{2}(2 + \cos 4A + \cos 4B) + \cos^2 2C$

$= 1 + \frac{1}{2}(\cos 4A + \cos 4B) + \cos^2 2C$

$= 1 + \frac{1}{2} . 2 . \cos(2A + 2B)\cos(2A - 2B) + \cos^2 2C$

$= 1 + \cos 2(A + B) . \cos 2(A - B) + \cos^2 2C$

$= 1 + \cos(2\pi - 2C)\cos 2(A - B) + \cos^2 2C$

$= 1 + \cos 2C \cos 2(A - B) + \cos^2 2C$

$= 1 + \cos 2C [\cos 2(A - B) + \cos 2C]$

$= 1 + \cos 2C [\cos 2(A - B) + \cos 2(A + B)]$

$= 1 + \cos 2C \times 2 . \cos 2A \cos 2B$

$= 1 + 2\cos 2A \cos 2B \cos 2C$

= ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1}{2}(2\sin^2 A + 2\sin^2 B) + \sin^2 C$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2B) + \sin^2 C$

$= \frac{1}{2}(2 - \cos 2A - \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2}(\cos 2A + \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2} . 2\cos(A + B)\cos(A - B) + 1 - \cos^2 C$

$= 2 - \cos(\pi - C)\cos(A - B) - \cos^2 C$

$= 2 + \cos C \cos(A - B) - \cos^2 C$

$= 2 + \cos C [\cos(A - B) - \cos C]$

$= 2 + \cos C [\cos(A - B) + \cos(A + B)]$

$[\because A + B + C = \pi]$

$= 2 + \cos C . 2 . \cos A . \cos B$

$= 2 + 2\cos A \cos B \cos C$

= ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \sin^2 A - \sin^2 B + \sin^2 C$

$= \frac{1}{2}\{2\sin^2 A + 2\sin^2 C\} - \sin^2 B$

$= \frac{1}{2}(1 - \cos 2A + 1 - \cos 2C) - \sin^2 B$

$= \frac{1}{2}(2 - \cos 2A - \cos 2C) - \sin^2 B$

$= 1 - \frac{1}{2}(\cos 2A + \cos 2C) - \sin^2 B$

$= 1 - \frac{1}{2} . 2\cos(A+C)\cos(A-C) - \sin^2 B$

$= 1 - \cos(\pi - B)\cos(A - C) - \sin^2 B$

$[\because A + B + C = \pi]$

$= 1 - \sin^2 B + \cos B \cos(A - C)$

$= \cos^2 B + \cos B \cos(A - C)$

$= \cos B\{\cos B + \cos(A - C)\}$

$= \cos B[\cos\{\pi - (A + C)\} + \cos(A - C)]$

$= \cos B\{-\cos(A + C) + \cos(A - C)\}$

$= \cos B\{\cos(A - C) - \cos(A + C)\}$

$= \cos B. 2\sin A \sin C$
$= 2\sin A \cos B \sin C$
= ডানপক্ষ (প্রমাণিত)

(iv) বামপক্ষ $= \sin^2\frac{A}{2} + \sin^2\frac{B}{2} + \sin^2\frac{C}{2}$

$= \frac{1}{2}\left(2\sin^2\frac{A}{2} + 2\sin^2\frac{B}{2}\right) + \sin^2\frac{C}{2}$

$= \frac{1}{2}(1 - \cos A + 1 - \cos B) + \sin^2\frac{C}{2}$

$= 1 - \frac{1}{2}(\cos A + \cos B) + \sin^2\frac{C}{2}$

$= 1 - \frac{1}{2}. 2\cos\left(\frac{A}{2} + \frac{B}{2}\right)\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \sin^2\frac{C}{2}$

$= 1 - \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \sin^2\frac{C}{2}$

$[\because A + B + C = \pi \therefore \frac{A}{2} + \frac{B}{2} + \frac{C}{2} = \frac{\pi}{2}]$

$= 1 - \sin\frac{C}{2}\cos\left(\frac{A}{2} - \frac{B}{2}\right) + \sin^2\frac{C}{2}$

$= 1 - \sin\frac{C}{2}\left\{\cos\left(\frac{A}{2} - \frac{B}{2}\right) - \sin\left[\frac{\pi}{2} - \left(\frac{A}{2} + \frac{B}{2}\right)\right]\right\}$

$= 1 - \sin\frac{C}{2}\left\{\cos\left(\frac{A}{2} - \frac{B}{2}\right) - \cos\left(\frac{A}{2} + \frac{B}{2}\right)\right\}$

$= 1 - \sin\frac{C}{2}. 2\sin\frac{A}{2}\sin\frac{B}{2}$

$= 1 - 2\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$ = ডানপক্ষ (প্রমাণিত)

(v) বামপক্ষ $= \cos^2 A + \cos^2 B - \cos^2 C$

$= \frac{1}{2}(2\cos^2 A + 2\cos^2 B) - \cos^2 C$

$= \frac{1}{2}(1 + \cos 2A + 1 + \cos 2B) - \cos^2 C$

$= \frac{1}{2}(2 + \cos 2A + \cos 2B) - \cos^2 C$

$= 1 + \frac{1}{2}(\cos 2A + \cos 2B) - \cos^2 C$

$= 1 + \frac{1}{2}.2\cos(A+B)\cos(A - B) - \cos C. \cos C$

$= 1 + \cos(\pi - C)\cos(A - B) - \cos C.\cos\{\pi - (A + B)\}$

$= 1 - \cos C\cos(A-B) + \cos C\cos(A+B)$
$= 1 - \cos C\{\cos(A-B) - \cos(A + B)\}$
$= 1 - \cos C . 2\sin A \sin B$
$= 1 - 2\sin A \sin B \cos C$
= ডানপক্ষ (প্রমাণিত)

(vi) বামপক্ষ $= \cos^2\frac{A}{2} + \cos^2\frac{B}{2} + \cos^2\frac{C}{2}$

$= \frac{1}{2}\left(2\cos^2\frac{A}{2} + 2\cos^2\frac{B}{2}\right) + \cos^2\frac{C}{2}$

$= \frac{1}{2}(1 + \cos A + 1 + \cos B) + \cos^2\frac{C}{2}$

$= \frac{1}{2}(2 + \cos A + \cos B) + \cos^2\frac{C}{2}$

$= 1 + \frac{1}{2}(\cos A + \cos B) + \cos^2\frac{C}{2}$

$= 1 + \frac{1}{2}. 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + 1 - \sin^2\frac{C}{2}$

$= 2 + \cos\left(\frac{\pi}{2} - \frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) - 2\sin^2\frac{C}{2}$

$[\because A + B + C = \pi$ বা, $\frac{A+B}{2} = \frac{\pi}{2} - \frac{C}{2}]$

$= 2 + \sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) - \sin^2\frac{C}{2}$

$= 2 + \sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \sin\frac{C}{2}\right]$

$= 2 + \sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right) - \cos\left(\frac{A+B}{2}\right)\right]$

$= 2 + \sin\frac{C}{2}. 2\sin\frac{A}{2}.\sin\frac{B}{2}$

$= 2 + 2\sin\frac{A}{2}\sin\frac{B}{2}.\sin\frac{C}{2}$

= ডানপক্ষ (প্রমাণিত)

5. বামপক্ষ $= \sin(B + C - A) + \sin(C + A - B) + \sin(A + B - C)$

$= \sin(\pi - 2A) + \sin(\pi - 2B) + \sin(\pi - 2C)$

$\left[\begin{array}{l}\because A + B + C = \pi \\ \therefore B + C = \pi - A\end{array}\right]$

$= \sin 2A + \sin 2B + \sin 2C$

$= 2\sin\frac{2A + 2B}{2}\cos\frac{2A - 2B}{2} + \sin 2C$

$= 2\sin(A + B)\cos(A - B) + \sin 2C$
$= 2\sin(\pi - C)\cos(A - B) + \sin 2C$
$= 2\sin C\cos(A - B) + 2\sin C\cos C$
$= 2\sin C[\cos(A - B) + \cos C]$
$= 2\sin C[\cos(A - B) + \cos\{\pi - (A + B)\}]$
$= 2\sin C[\cos(A - B) - \cos(A + B)]$

$= 2\sin C.2\sin\frac{A - B + A + B}{2}\sin\frac{A + B - A + B}{2}$

$= 4\sin C\sin A\sin B$
$= 4\sin A\sin B\sin C$
= ডানপক্ষ (প্রমাণিত)

6. (i) বামপক্ষ $= \frac{\cos A}{\sin B\sin C} + \frac{\cos B}{\sin C\sin A} + \frac{\cos C}{\sin A\sin B}$

$= \frac{\sin A\cos A + \sin B\cos B + \sin C\cos C}{\sin B\sin C\sin A}$

$= \frac{2\sin A\cos A + 2\sin B\cos B + 2\sin C\cos C}{2\sin B\sin C\sin A}$

$= \frac{\sin 2A + \sin 2B + \sin 2C}{2\sin B\sin C\sin A}$

$= \frac{2\sin(A+B)\cos(A-B) + 2\sin C\cos C}{2\sin A\sin B\sin C}$

$= \frac{2\sin C\cos(A-B) + 2\sin C\cos C}{2\sin A\sin B\sin C}$ $[\because A+B+C=\pi]$

$= \frac{2\sin C[\cos(A-B) + \cos C]}{2\sin A\ \sin B\ \sin C}$

$= \frac{2\sin C[\cos(A-B) - \cos(A+B)]}{2\sin A\ \sin B\ \sin C}$

$= \frac{2\sin C\ .\ 2\sin A\ \sin B}{2\sin A\ \sin B\ \sin C}$

$= 2 =$ ডানপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \frac{\cot B + \cot C}{\tan B + \tan C} + \frac{\cot C + \cot A}{\tan C + \tan A} + \frac{\cot A + \cot B}{\tan A + \tan B}$

$= \frac{\frac{1}{\tan B} + \frac{1}{\tan C}}{\tan B + \tan C} + \frac{\frac{1}{\tan C} + \frac{1}{\tan A}}{\tan C + \tan A} + \frac{\frac{1}{\tan A} + \frac{1}{\tan B}}{\tan A + \tan B}$

$= \frac{\frac{\tan B + \tan C}{\tan B\tan C}}{\tan B + \tan C} + \frac{\frac{\tan C + \tan A}{\tan C\tan A}}{\tan C + \tan A} + \frac{\frac{\tan A + \tan B}{\tan A\tan B}}{\tan A + \tan B}$

$= \frac{1}{\tan B\tan C} + \frac{1}{\tan C\tan A} + \frac{1}{\tan A\tan B}$

$= \frac{\tan A + \tan B + \tan C}{\tan B\tan C\tan A}$ ...... (i)

আবার, দেওয়া আছে, $A + B + C = \pi$

বা, $A + B = \pi - C$

বা, $\tan(A+B) = \tan(\pi - C)$

বা, $\frac{\tan A + \tan B}{1 - \tan A\tan B} = -\tan C$

বা, $\tan A + \tan B + \tan C = \tan A\tan B\tan C$ ...... (ii)

(i) নং সমীকরণে (ii) এর মান বসিয়ে,

$\frac{\tan A + \tan B + \tan C}{\tan A\tan B\tan C}$

$= \frac{\tan A\tan B\tan C}{\tan A\tan B\tan C}$

$= 1 =$ ডানপক্ষ

$\therefore \frac{\cot B + \cot C}{\tan B + \tan C} + \frac{\cot C + \cot A}{\tan C + \tan A} + \frac{\cot A + \cot B}{\tan A + \tan B} = 1$ (প্রমাণিত)

7. (i) ডানপক্ষ $= 1 + 4\sin\frac{B+C}{4}\sin\frac{C+A}{4}\sin\frac{A+B}{4}$

$= 1 + 2\left(2\sin\frac{B+C}{4}\sin\frac{C+A}{4}\right)\sin\frac{A+B}{4}$

$= 1 + 2\left\{\cos\left(\frac{B-A}{4}\right) - \cos\left(\frac{A+B+2C}{4}\right)\right\}\sin\left(\frac{A+B}{4}\right)$

$= 1 + 2\cos\frac{B-A}{4}\sin\frac{A+B}{4} - 2\cos\frac{A+B+2C}{4}\sin\frac{A+B}{4}$

$= 1 + \sin\left(\frac{A+B}{4} + \frac{B-A}{4}\right) + \sin\left(\frac{A+B}{4} - \frac{B-A}{4}\right)$
$- \left\{\sin\left(\frac{A+B}{4} + \frac{A+B+2C}{4}\right) + \sin\left(\frac{A+B}{4} - \frac{A+B+2C}{4}\right)\right\}$

$= 1 + \sin\frac{B}{2} + \sin\frac{A}{2} - \left\{\sin\frac{A+B+C}{2} + \sin\left(-\frac{C}{2}\right)\right\}$

$= 1 + \sin\frac{B}{2} + \sin\frac{A}{2} - \left\{\sin\frac{A+B+C}{2} - \sin\left(\frac{C}{2}\right)\right\}$ $[\because A+B+C=\pi]$

$= 1 + \sin\frac{B}{2} + \sin\frac{A}{2} - \sin\frac{\pi}{2} + \sin\frac{C}{2}$

$= 1 + \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} - 1$

$= \sin\frac{A}{2} + \sin\frac{B}{2} + \sin\frac{C}{2} =$ বামপক্ষ (প্রমাণিত)

(ii) বামপক্ষ $= \cos\frac{A}{2} + \cos\frac{B}{2} + \cos\frac{C}{2}$

$= 2\cos\frac{A+B}{4}.\cos\frac{A-B}{4} + \cos\frac{C}{2}$

$= 2\cos\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A-B}{4}\right) + \cos\frac{\pi}{2} + \cos\frac{C}{2}$

$= 2\cos\left(\frac{\pi - C}{4}\right)\cos\left(\frac{A-B}{4}\right) + 2.\cos\left(\frac{\pi + C}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$

$= 2\cos\left(\frac{\pi - C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right) + \cos\left(\frac{\pi + C}{4}\right)\right]$

$= 2\cos\left(\frac{\pi - C}{4}\right)\left[\cos\left(\frac{A-B}{4}\right) + \cos\frac{2\pi - (A+B)}{4}\right]$

$= 2\cos\left(\frac{\pi - C}{4}\right)\left[2\cos\left(\frac{\pi - B}{4}\right).\cos\left(\frac{\pi - A}{4}\right)\right]$

$= 4\cos\left(\frac{\pi - A}{4}\right)\cos\left(\frac{\pi - B}{4}\right)\cos\left(\frac{\pi - C}{4}\right)$

$=$ ডানপক্ষ (প্রমাণিত)

8. (i) দেওয়া আছে, $A + B + C = \frac{\pi}{2}$

বা, $A + B = \frac{\pi}{2} - C$

বা, $\tan(A+B) = \tan\left(\frac{\pi}{2} - C\right)$

বা, $\frac{\tan A + \tan B}{1 - \tan A\tan B} = \cot C$

বা, $\frac{\tan A + \tan B}{1 - \tan A \tan B} = \frac{1}{\tan C}$

বা, $\tan C(\tan A + \tan B) = 1 - \tan A \tan B$

বা, $\tan C \tan A + \tan C \tan B = 1 - \tan A \tan B$

$\therefore$ $\tan B \tan C + \tan C \tan A + \tan A \tan B = 1$ (প্রমাণিত)

(ii) দেওয়া আছে, $A + B + C = \frac{\pi}{2}$

বা, $A + B = \frac{\pi}{2} - C$

বা, $\cot(A + B) = \cot\left(\frac{\pi}{2} - C\right)$

বা, $\frac{\cot A \cot B - 1}{\cot B + \cot A} = \tan C$

বা, $\frac{\cot A \cot B - 1}{\cot B + \cot A} = \frac{1}{\cot C}$

বা, $\cot A \cot B \cot C - \cot C = \cot A + \cot B$

$\therefore$ $\cot A + \cot B + \cot C = \cot A \cot B \cot C$ (প্রমাণিত)

9. (i) এখানে, $\sin^2 A + \sin^2 B + \sin^2 C$

$= \frac{1}{2}\{2 \sin^2 A + 2 \sin^2 B\} + \sin^2 C$

$= \frac{1}{2}\{1 - \cos 2A + 1 - \cos 2B\} + \sin^2 C$

$= \frac{1}{2}(2 - \cos 2A - \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2}(\cos 2A + \cos 2B) + \sin^2 C$

$= 1 - \frac{1}{2} . 2 \cos(A + B) \cos(A - B) + \sin^2 C$

$= 1 - \cos\left(\frac{\pi}{2} - C\right) \cos(A - B) + \sin^2 C$

$= 1 - \sin C \cos(A - B) + \sin^2 C$

$= 1 - \sin C [\cos(A - B) - \sin C]$

$= 1 - \sin C\left[\cos(A - B) - \sin\left\{\frac{\pi}{2} - (A + B)\right\}\right]$

$= 1 - \sin C [\cos(A - B) - \cos(A + B)]$

$= 1 - \sin C . 2 \sin A . \sin B$

$= 1 - 2 \sin A \sin B \sin C$

$\therefore \sin^2 A + \sin^2 B + \sin^2 C + 2\sin A \sin B \sin C = 1$ (প্রমাণিত)

(ii) বামপক্ষ $= \sin A + \sin B + \sin C$

$= \sin A + \sin B + \sin C - \sin\frac{\pi}{2} + 1$

$= \sin A + \sin B + \sin C - \sin(A + B + C) + 1$

$= 2\sin\left(\frac{A + B}{2}\right)\cos\left(\frac{A - B}{2}\right)$

$+ 2 \cos\left(\frac{A + B + 2C}{2}\right) \sin\left\{-\frac{1}{2}(A + B)\right\} + 1$

$= 2 \sin\left(\frac{A + B}{2}\right)$

$\left\{\cos\left(\frac{A - B}{2}\right) - \cos\left(\frac{A + B + 2C}{2}\right)\right\} + 1$

$= 4 \sin\left(\frac{A + B}{2}\right) . \sin\left(\frac{C + A}{2}\right) \sin\left(\frac{B + C}{2}\right) + 1$

$= 4\sin\frac{1}{2}\left(\frac{\pi}{2} - C\right) \sin\frac{1}{2}\left(\frac{\pi}{2} - B\right) \sin\frac{1}{2}\left(\frac{\pi}{2} - A\right) + 1$

$= 1 + 4\sin\frac{\pi - 2A}{4} \sin\frac{\pi - 2B}{4} \sin\frac{\pi - 2C}{4}$

= ডানপক্ষ (প্রমাণিত)

10. (i) বামপক্ষ $= \cos^2 A + \cos^2 B + \cos^2 C$

$= \frac{1}{2}(2\cos^2 A + 2\cos^2 B) + \cos^2 C$

$= \frac{1}{2}(1 + \cos 2A + 1 + \cos 2B) + \cos^2 C$

$= 1 + \frac{1}{2}(\cos 2A + \cos 2B) + \cos^2 C$

$= 1 + \cos(A + B) \cos(A - B) + \cos^2 C$

$= 1 + \cos C \cos(A - B) + \cos^2 C$ [$\because A + B = C$]

$= 1 + \cos C [\cos(A - B) + \cos C]$

$= 1 + \cos C [\cos(A - B) + \cos(A + B)]$

$= 1 + \cos C . 2\cos A \cos B$

$= 1 + 2\cos A \cos B \cos C$

= ডানপক্ষ (প্রমাণিত)

(ii) এখানে, $A + B + C = 0$ বা, $\frac{A + B}{2} = -\frac{C}{2}$

বামপক্ষ $= \cos A + \cos B + \cos C$

$= 2\cos\frac{A + B}{2} \cos\frac{A - B}{2} + \cos C$

$= 2\cos\left(-\frac{C}{2}\right) \cos\frac{A - B}{2} + 2\cos^2\frac{C}{2} - 1$

$= 2\cos\frac{C}{2}\left\{\cos\frac{A - B}{2} + \cos\frac{C}{2}\right\} - 1$

$= 2\cos\frac{C}{2}\left\{\cos\frac{A - B}{2} + \cos\left(-\frac{A + B}{2}\right)\right\} - 1$

$= 2\cos\frac{C}{2}\left\{\cos\frac{A - B}{2} + \cos\left(\frac{A + B}{2}\right)\right\} - 1$

$= 2\cos\frac{C}{2} 2\cos\frac{A}{2} \cos\frac{B}{2} - 1$

$= 4 \cos\frac{A}{2} \cos\frac{B}{2} \cos\frac{C}{2} - 1$

= ডানপক্ষ (প্রমাণিত)

(iii) বামপক্ষ $= \cos^2(\beta - \gamma) + \cos^2(\gamma - \alpha) + \cos^2(\alpha - \beta)$

ধরি, $\beta - \gamma = A$, $\gamma - \alpha = B$ এবং $\alpha - \beta = C$

$\therefore A + B + C = \beta - \gamma + \gamma - \alpha + \alpha - \beta = 0$

$\therefore$ বামপক্ষ $= \cos^2 A + \cos^2 B + \cos^2 C$

$= \frac{1}{2}(2\cos^2 A + 2\cos^2 B) + \cos^2 C$

$=\frac{1}{2}(1+\cos 2A+1+\cos 2B)+\cos^2 C$

$=\frac{1}{2}(2+\cos 2A+\cos 2B)+\cos^2 C$

$=1+\frac{1}{2}(\cos 2A+\cos 2B)+\cos^2 C$

$=1+\frac{1}{2}.2\cos\frac{2A+2B}{2}\cos\frac{2A-2B}{2}+\cos^2 C$

$=1+\cos(A+B)\cos(A-B)+\cos^2 C$

$=1+\cos(-C)\cos(A-B)+\cos^2 C$

[$\because A+B+C=0$ বা, $A+B=-C$]

$=1+\cos C\cos(A-B)+\cos^2 C$

$=1+\cos C\{\cos(A-B)+\cos C\}$

$=1+\cos C[\cos(A-B)+\cos\{-(A+B)\}]$

$=1+\cos C.[\cos(A-B)+\cos(A+B)]$

$=1+\cos C\ 2\cos A\cos B=1+2\cos A\cos B\cos C$

$=1+2\cos(\beta-\gamma)\cos(\gamma-\alpha)\cos(\alpha-\beta)$

= ডানপক্ষ

∴ বামপক্ষ = ডানপক্ষ (প্রমাণিত)

11. (i) ধরি, $x=\tan A$, $y=\tan B$, $z=\tan C$

দেওয়া আছে, $x+y+z=xyz$

বা, $\tan A+\tan B+\tan C=\tan A\tan B\tan C$

বা, $\tan B+\tan C=-\tan A+\tan A\tan B\tan C$

বা, $\tan B+\tan C=-\tan A(1-\tan B\tan C)$

বা, $\frac{\tan B+\tan C}{1-\tan B\tan C}=-\tan A$

বা, $\tan(B+C)=-\tan A$

বা, $\tan(B+C)=\tan(\pi-A)$

বা, $B+C=\pi-A$

বা, $A+B+C=\pi$

বা, $2A+2B+2C=2\pi$

বা, $\tan(2A+2B+2C)=\tan 2\pi$

বা, $\frac{\tan 2A+\tan 2B+\tan 2C-\tan 2A\tan 2B\tan 2C}{1-\tan 2B\tan 2C-\tan 2C\tan 2A-\tan 2A\tan 2B}=0$

বা, $\tan 2A+\tan 2B+\tan 2C-\tan 2A\tan 2B\tan 2C=0$

বা, $\tan 2A+\tan 2B+\tan 2C=\tan 2A\tan 2B\tan 2C$

বা, $\frac{2\tan A}{1-\tan^2 A}+\frac{2\tan B}{1-\tan^2 B}+\frac{2\tan C}{1-\tan^2 C}=\frac{2\tan A}{1-\tan^2 A}.\frac{2\tan B}{1-\tan^2 B}.\frac{2\tan C}{1-\tan^2 C}$

অর্থাৎ, $\frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}.\frac{2y}{1-y^2}.\frac{2z}{1-z^2}$

[tanA, tanB, tanC এর মান বসিয়ে]

$\therefore \frac{2x}{1-x^2}+\frac{2y}{1-y^2}+\frac{2z}{1-z^2}=\frac{2x}{1-x^2}.\frac{2y}{1-y^2}.\frac{2z}{1-z^2}$

(প্রমাণিত)

(ii) ধরি, $x=\cot A$, $y=\cot B$, $z=\cot C$

দেওয়া আছে, $yz+zx+xy=1$

বা, $\cot B\cot C+\cot C\cot A+\cot A\cot B=1$

বা, $\cot B\cot C-1=-\cot A(\cot C+\cot B)$

বা, $\frac{\cot B\cot C-1}{\cot C+\cot B}=-\cot A$

বা, $\cot(B+C)=\cot(\pi-A)$

বা, $B+C=\pi-A$

বা, $2B+2C=2\pi-2A$

বা, $\cot(2B+2C)=\cot(2\pi-2A)$

বা, $\frac{\cot 2B\cot 2C-1}{\cot 2C+\cot 2B}=-\cot 2A$

বা, $\cot 2B\cot 2C+\cot 2C\cot 2A+\cot 2A\cot 2B=1$

বা, $\frac{\cot^2 B-1}{2\cot B}.\frac{\cot^2 C-1}{2\cot C}+\frac{\cot^2 C-1}{2\cot C}.\frac{\cot^2 A-1}{2\cot A}+\frac{\cot^2 A-1}{2\cot A}.\frac{\cot^2 B-1}{2\cot B}=1$

বা, $\frac{y^2-1}{2y}.\frac{z^2-1}{2z}+\frac{z^2-1}{2z}.\frac{x^2-1}{2x}+\frac{x^2-1}{2x}.\frac{y^2-1}{2y}=1$

$\therefore \frac{(x^2-1)(y^2-1)}{xy}+\frac{(y^2-1)(z^2-1)}{yz}+\frac{(z^2-1)(x^2-1)}{xz}=4$

(প্রমাণিত)

পাঠ্যবইয়ের কাজের সমাধান

▶ অনুচ্ছেদ-7.3.1 | পৃষ্ঠা-২৬১

আমরা জানি, $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=2R$

$\therefore \frac{c}{\sin C}=\frac{a}{\sin A}$ বা, $\frac{c}{a}=\frac{\sin C}{\sin A}$

বা, $\frac{c-a}{c+a}=\frac{\sin C-\sin A}{\sin C+\sin A}$ [যোজন-বিয়োজন করে]

$=\frac{2\cos\left(\frac{C+A}{2}\right)\sin\left(\frac{C-A}{2}\right)}{2\sin\left(\frac{C+A}{2}\right)\cos\left(\frac{C-A}{2}\right)}$

$=\cot\left(\frac{C+A}{2}\right)\tan\left(\frac{C-A}{2}\right)$

$=\cot\left\{\frac{\pi}{2}-\frac{B}{2}\right\}.\tan\left(\frac{C-A}{2}\right)$

[$\because A+B+C=\pi$ বা, $C+A=\pi-B$ বা, $\frac{C+A}{2}=\frac{\pi}{2}-\frac{B}{2}$]

$=\tan\frac{B}{2}.\tan\left(\frac{C-A}{2}\right)$

বা, $\tan\frac{C-A}{2}.\tan\frac{B}{2}=\frac{c-a}{c+a}$

$\therefore \tan\frac{C-A}{2}=\frac{c-a}{c+a}.\cot\frac{B}{2}$ (প্রমাণিত)

অনুচ্ছেদ-7.4 | পৃষ্ঠা-২৬১

আমরা জানি, $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$

বা, $\cos 30° = \dfrac{(3\sqrt{3})^2 + c^2 - (3)^2}{2.3\sqrt{3}.c}$ [মান বসিয়ে]

বা, $\dfrac{\sqrt{3}}{2} = \dfrac{27 + c^2 - 9}{6\sqrt{3}\,c} \Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{18 + c^2}{6\sqrt{3}c}$

বা, $2c^2 + 36 = 18c$

বা, $2c^2 - 18c + 36 = 0$

বা, $c^2 - 9c + 18 = 0$

বা, $c^2 - 6c - 3c + 18 = 0$

$\therefore c = 6, c = 3$

এখন, $c = 6$ হলে, $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$= \dfrac{(3)^2 + (6)^2 - (3\sqrt{3})^2}{2.3.6} = \dfrac{9 + 36 - 27}{36} = \dfrac{18}{36} = \dfrac{1}{2}$

$\therefore \cos B = \dfrac{1}{2} = \cos 60° \quad \therefore B = 60°$

$\cos C = \dfrac{a^2 + b^2 - c^2}{2ab} = \dfrac{(3)^2 + (3\sqrt{3})^2 - (6)^2}{2.3.3\sqrt{3}}$

$= \dfrac{9 + 27 - 36}{18\sqrt{3}} = 0$

বা, $\cos C = 0 = \cos 90° \quad \therefore C = 90°$

এবং $c = 3$ হলে, $\cos B = \dfrac{(3)^2 + (3)^2 - (3\sqrt{3})^2}{2.3.3}$

$= \dfrac{9 + 9 - 27}{18} = -\dfrac{9}{18} = -\dfrac{1}{2}$

বা, $\cos B = -\dfrac{1}{2} = \cos 120° \quad \therefore B = 120°$

$\cos C = \dfrac{(3)^2 + (3\sqrt{3})^2 - (3)^2}{2.3.3\sqrt{3}}$

$= \dfrac{9 + 27 - 9}{18\sqrt{3}} = \dfrac{27}{18\sqrt{3}} = \dfrac{\sqrt{3}}{2}$

বা, $\cos C = \dfrac{\sqrt{3}}{2} = \cos 30° \quad \therefore C = 30°$

$\therefore B = 120°$ বা $60°$ এবং $C = 90°$ বা, $30°$

অনুচ্ছেদ-7.4 | পৃষ্ঠা-২৬৩

$\cot A = \dfrac{\cos A}{\sin A}$

$= \dfrac{\dfrac{b^2+c^2-a^2}{2bc}}{\dfrac{a}{2R}}$ $\left[\because \cos A = \dfrac{b^2+c^2-a^2}{2bc}; \sin A = \dfrac{a}{2R}\right]$

$= \dfrac{2R(b^2 + c^2 - a^2)}{2abc}$

অনুরূপভাবে, $\cot B = \dfrac{2R(c^2 + a^2 - b^2)}{2abc}$

এবং $\cot C = \dfrac{2R(a^2 + b^2 - c^2)}{2abc}$

বামপক্ষ $= \dfrac{2\cot A + \cot B + \cot C}{\cot A - \cot B + 2\cot C}$

$= \dfrac{\dfrac{2.2R(b^2+c^2-a^2)}{2abc} + \dfrac{2R(c^2+a^2-b^2)}{2abc} + \dfrac{2R(a^2+b^2-c^2)}{2abc}}{\dfrac{2R(b^2+c^2-a^2)}{2abc} - \dfrac{2R(c^2+a^2-b^2)}{2abc} + \dfrac{2.2R(a^2+b^2-c^2)}{2abc}}$

$= \dfrac{\dfrac{2(b^2+c^2-a^2) + (c^2+a^2-b^2) + (a^2+b^2-c^2)}{abc}}{\dfrac{(b^2+c^2-a^2) - (c^2+a^2-b^2) + 2(a^2+b^2-c^2)}{abc}}$

$= \dfrac{\dfrac{2b^2+2c^2-2a^2+c^2+a^2-b^2+a^2+b^2-c^2}{abc}}{\dfrac{b^2+c^2-a^2-c^2-a^2+b^2+2a^2+2b^2-2c^2}{abc}}$

$= \dfrac{2b^2 + 2c^2}{4b^2 - 2c^2} = \dfrac{b^2 + c^2}{2b^2 - c^2}$

= ডানপক্ষ

$\therefore \dfrac{2\cot A + \cot B + \cot C}{\cot A - \cot B + 2\cot C} = \dfrac{b^2 + c^2}{2b^2 - c^2}$ (দেখানো হলো)

অনুশীলনী-7(G) এর সমাধান

1. (i) ডানপক্ষ $= \dfrac{a - b}{a + b}\cot\dfrac{C}{2}$

$= \dfrac{2R\sin A - 2R\sin B}{2R\sin A + 2R\sin B}.\cot\dfrac{C}{2}$

$\left[\because \dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C} = 2R\right]$

$= \dfrac{\sin A - \sin B}{\sin A + \sin B}\cot\dfrac{C}{2}$

$= \dfrac{2\cos\dfrac{A+B}{2}\sin\dfrac{A-B}{2}}{2\sin\dfrac{A+B}{2}\cos\dfrac{A-B}{2}}\cot\dfrac{C}{2}$

$= \cot\dfrac{A+B}{2}\tan\dfrac{A-B}{2}.\dfrac{1}{\tan\dfrac{C}{2}}$

$= \cot\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right)\tan\dfrac{A-B}{2}.\dfrac{1}{\tan\dfrac{C}{2}}$ $[\because A + B + C = \pi]$

$= \tan\dfrac{C}{2}.\tan\dfrac{A-B}{2}.\dfrac{1}{\tan\dfrac{C}{2}} = \tan\dfrac{A-B}{2}$

= বামপক্ষ (প্রমাণিত)