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№1

$$\sum_{n=2}^{+\infty} \frac{\left(e-\left(1+\frac{1}{n^2}\right)^{n^2}\right)^{\alpha}}{\ln^2 n}$$

$$e-\left(1+\frac{1}{n^2}\right)^{n^2} = e - e^{\ln\left(1+\frac{1}{n^2}\right)\cdot n^2} \sim e - e^{\left(\frac{1}{n^2}-\frac{1}{2n^4}\right)n^2} =$$

$$= e - e^{1-\frac{1}{2n^2}} = e\left(1-e^{1-\frac{1}{2n^2}-1}\right) = e\left(1-e^{-\frac{1}{2n^2}}\right) \sim$$

$$\sim e\left(1-\left(1-\frac{1}{2n^2}\right)\right) = e\cdot\frac{1}{2n^2}$$

$$\Rightarrow \frac{\left(e-\left(1+\frac{1}{n^2}\right)^{n^2}\right)^{\alpha}}{\ln^2 n} \sim \frac{\left(e\cdot\frac{1}{2n^2}\right)^{\alpha}}{\ln^2 n} = \frac{e^{\alpha}}{2^{\alpha}\cdot n^{2\alpha}\cdot \ln^2 n}.$$

если $2\alpha > 1$, то сх-ся
т.е $\alpha > 1/2$

если $2\alpha = 1$ то т.к $\ln^{(2)} n$ $2>1$, то сх-ся.

если $2\alpha < 1$ расх

$\Rightarrow$ Отв: $\alpha \geq 1/2$ - сх-ся
$\alpha < 1/2$ - расх-ся.

№3

$$\sum_{n=1}^{\infty} \frac{n^2}{(x+1)}\left(\operatorname{ch}\frac{x}{n^2}-1\right)$$ ~~т.к ch~~

$$\operatorname{ch} t = \sum_{n=0}^{\infty} \frac{t^{2n}}{(2n)!} \Rightarrow \operatorname{ch} t < 1+\frac{t^2}{2}$$

т.к. $\sum_{n=2}^{\infty} \frac{t^{2n}}{(2n)!} < t^2/2$

$$\operatorname{ch} t < 1+\frac{t^2}{2}$$

$\Rightarrow \operatorname{ch} \frac{x}{n^2} < 1 + \left(\frac{x^2}{2n^4}\right)$

$\Rightarrow 0 \le \frac{n^2}{x+1}\left(\operatorname{ch}\frac{x}{n^2} - 1\right) < \frac{n^2}{x+1} \cdot \left(1 + \frac{x^2}{n^4} - 1\right) =$

$= \frac{n^2}{x+1} \cdot \frac{x^2}{n^4} = \frac{x^2}{(x+1)\,n^2} \Rightarrow$ т.к $2>1$ сх-ся поточечно к 0.

тогда $\frac{x^2}{(x+1)n^2} < \frac{(x+1)^2}{(x+1)n^2} < \frac{x+1}{n^2} \Rightarrow$ при на E_1

$\frac{x+1}{n^2} \le \frac{2}{n^2} \Rightarrow$ сх-ся равномерно.

на E_2:

$\exists \varepsilon = 1/2 \quad \forall N \quad \exists n = N \quad \exists x_N = N^2+1$

$|f_N(x_N) - f(x_N)| \ge \frac{N^2}{(x+1)}\left(\frac{x^2}{2N^4}\right) =$

т.к $\operatorname{ch}\frac{x}{n^2} > 1 + \frac{x^2}{2n^4}$

$= \frac{x^2}{(x+1)\,2N^2} > \frac{x-1}{2N^2} = \frac{N^2}{2N^2} = \frac{1}{2} = \varepsilon$

$\Rightarrow$ на E_2 сх-ся неравномерно.

№4

$$f(x) = \pi + \int_{\pi/2}^{x} \frac{\cos^2 t}{t - \pi/2}\,dt = \pi + \int_0^{x-\pi/2} \frac{\sin^2 u}{u}\,du$$

$\left(u = t - \pi/2\right)$.

$$\frac{\sin^2 u}{u} = \frac{1-\cos 2u}{2u} =$$

$$= \frac{1-\sum\limits_{n=0}^{\infty}\frac{(-1)^n(2u)^{2n}}{(2n)!}}{2u} = \frac{\sum\limits_{n=1}^{\infty}\frac{(-1)^n(2u)^{2n}}{(2n)!}}{2u} =$$

$$= \sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n-1}\cdot u^{2n-1}}{(2n)!}$$

по Th об интегр. можем почленно интегрировать.

$$\Rightarrow \pi + \int_0^{x-\pi/2}\sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n-1} u^{2n-1}}{(2n)!}\,du = \pi + \sum_{n=1}^{\infty}\int_0^{x-\pi/2}\frac{(-1)^n 2^{2n-1} u^{2n-1}}{(2n)!}\,du =$$

$$= \pi + \sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n-1} u^{2n}}{(2n)!\,2n}\Bigg|_0^{x-\pi/2} = \pi + \sum_{n=1}^{\infty}\frac{(-1)^n 2^{2n-1}\left(x-\frac{\pi}{2}\right)^{2n}}{(2n)!\,2n}.$$

$$\left(x-\frac{\pi}{2}\right)^2 = t \Rightarrow \frac{1}{R_t} = \lim_{n\to\infty}\sqrt[n]{\left|\frac{(-1)^n 2^{2n-1}}{(2n)!\,2n}\right|} =$$

$$= \frac{1}{\infty} \Rightarrow R_t = \infty \Rightarrow \underline{R = \infty}.$$

№2

$$f_n(x) = n\,\mathrm{sh}\frac{x}{n+x}$$

$$\mathrm{sh}\,x = \sum_{n=0}^{+\infty}\frac{(x)^{2n+1}}{(2n+1)!} \Rightarrow \cancel{\mathrm{sh}\,x >} \ \mathrm{sh}\,x > x.$$

1) $n\,\mathrm{sh}\frac{x}{n+x} \sim n\cdot\frac{x}{n+x} = \frac{nx}{n+x} \to x.\quad n\to+\infty$

фикс. x

$\to f(x) = x.$

посл-ть сх-ся к $f(x)$ поточечно.

2) $\cancel{\left| n\,sh\frac{x}{n+x} - 1 \right| \Rightarrow}$

2) На E_1:

$0 \le |x - n\,sh\frac{x}{n+x}| \le x - n\cdot\frac{x}{n+x} = \frac{xn + x^2 - nx}{(n+x)} =$

$= \frac{x^2}{n+x} <$ (значит действ. $x - n\,sh\frac{x}{n+x} \ge 0$.)

$< \frac{1}{n} \to 0 \Rightarrow$ по пр-ку В-са на E_1 сх-ся равномерно.

3) На E_2:

$\exists \varepsilon = 1 - sh\frac{1}{2}\ \forall N \quad \exists n = N \quad \exists x_N = N$

$|f_N(x_N) - f(x_N)| = |N - N\,sh\frac{N}{N+N}| =$

$= N|1 - sh\frac{1}{2}| \ge 1 - sh\frac{1}{2} \Rightarrow$ сх-ся неравномерно.