

1. Rolle's Theorem

If f is continuous on $[a, b]$, differentiable on (a, b) and $f(a) = f(b)$, then \exists such that $f'(c) = 0$.

2. Lagrange's Mean Value Theorem

$\exists c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

3. Cauchy's Mean Value Theorem

$$\frac{f(b) - f(a)}{g(b) - g(a)} = \frac{f'(c)}{g'(c)}$$

4. Taylor series of $\sin x$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$$

5. Taylor series of $\cos x$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$$

6. Taylor series of e^x

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

7. Taylor series of e^{-x}

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \dots$$

8. Is $f(x) = 1/x$ continuous on $[-1, 1]$?

✗ No

**9. Rolle's theorem for $f(x) = x^2$ on $[-1, 1]$
 $c = 0$**

**10. Lagrange's theorem for $f(x) = x^2$ on $[-1, 1]$
 $c = 0$**

**11. Rolle's theorem for $f(x) = \sin x$ on $[0, \pi]$
 $c = \pi/2$**

**12. Rolle's theorem for $f(x) = \cos x$ on $[-\pi, \pi]$
 $c = 0$**

13. Geometrical meaning of Rolle's theorem
Tangent parallel to x-axis

14. Geometrical meaning of Lagrange's theorem
Tangent parallel to chord joining endpoints

15. Is $x^{1/3}$ differentiable on $[-1, 1]$?
✗ No

UNIT – IV : Questions & Answers

1. $\frac{\partial}{\partial x}(x^y) = yx^{y-1}$
2. $\frac{\partial}{\partial y}(x^y) = x^y \ln x$
3. $\frac{\partial}{\partial x}(y^x) = y^x \ln y$
4. Degree of $(x^5 + y^5)/(x^6 + y^6) = -1$
5. Degree of $(x + y)/(x - y) = \mathbf{0}$
6. Degree of $(x^9 + y^9)/(x^3 y^3) = 3$
7. Degree of $(x^9 + y^9)/(x^3 - y^3) = 6$
8. **Euler's theorem:** $xf_x + yf_y = nf$
9. $\partial(x, y)/\partial(u, v) = \mathbf{u + v}$
10. $\partial(u, v)/\partial(x, y) = e^{2x}$
11. Functional dependence \Rightarrow Jacobian = 0
12. Functional independence \Rightarrow Jacobian $\neq 0$
13. Neither max nor min \Rightarrow **Saddle point**
14. Max or min \Rightarrow **Extreme point**
15. $\partial(u, v)/\partial(x, y) = 4xy$

UNIT – V : Questions & Answers

1. $\int_0^1 \int_0^1 dx dy = \mathbf{1}$
2. $\int_1^2 \int_1^2 dx dy = 1$
3. $\int_0^1 \int_0^1 xy, dx dy = 1/4$
4. $\int_1^2 \int_1^2 xy, dx dy = 9/4$
5. Area (Cartesian): $\iint dx dy$
6. Area (Polar): $\iint r, dr, d\theta$
7. Volume (Cartesian): $\iiint dx dy dz$
8. $\int_1^2 \int_1^2 \int_1^2 dx dy dz = 1$
9. $\int_0^1 \int_0^1 \int_0^1 xyz, dx dy dz = 1/8$
10. Polar conversion: $x = r \cos \theta,; y = r \sin \theta$
11. Change of order = interchanging limits
12. Area of circle = πa^2
13. Area of ellipse = πab
14. Volume of sphere = $\frac{4}{3}\pi a^3$
15. Volume of cube $0 \leq x, y, z \leq 1 = 1$