Introduction

The synthesis of the calculus of n-variables and of n-dimensional geometry
is the basis of what Seldon once called
"my little algebra of humanity"...
(*Encyclopedia Galactica*)

(*apud I. Asimov*, Second Foundation)

Hari Seldon is a fictional character created by Isaac Asimov for his *Foundation* series of romances. A highly unlikely character, we may add: a politician with mathematical skills, who developed the discipline of psycho-history, or the mathematical description of social processes and of the evolution of history.

Asimov later tells us a bit more about the workings of psycho-history: it's basically a nonlinear dynamical system, and, in later books from the *Foundation* series, he incorporates into Seldon's creation some stuff from chaos theory.

Chaos theory? Back to the real world, then. In 1963 E. Lorenz published his famous paper on deterministic chaos, "Deterministic Non-Periodic Flow." The title itself is an understatement, or perhaps a discreet disguise: for Lorenz discusses a system of three autonomous differential equations with just two second-degree nonlinearities — which exhibits a very complex, apparently chaotic, behavior, which hinders the prediction of the time-evolution of our system.

Social systems have long been modeled by autonomous differential equations with low-degree nonlinearities, as in competition models conceived out of the Lotka–Volterra equations. Those particular equations are nonlinear but predictable, that is, with Lotka–Volterra systems at hand, we know that nonlinearities aren't sufficient for a dynamical system to exhibit chaotic behavior. (But they are a necessary condition.) So, there arose the question of an algorithmic procedure to anticipate whether a nonlinear system would exhibit chaotic behavior.

The nonexistence of such an algorithm was proved, under very general conditions, by da Costa and Doria in 1991. So, not only nonlinear systems may turn out to be unpredictable; we cannot even say (for arbitrary systems and conditions) whether they will be unpredictable or not! Moreover there will be systems which are chaotic in one model for mathematics, and perfectly predictable in another model.

Here the Gödel phenomenon enters the picture. If we could make calculations with infinite precision, we would be able to predict with all required sharpness the future behavior of chaotic systems. But there will be situations where not one of these calculations are possible.

So, we have a higher-order difficulty here — one that wasn't anticipated by our fictional sage, Hari Seldon. Chaotic systems are deterministic: therefore, given infinitely precise calculations, all future behavior can be predicted. However, when the Gödel phenomenon enters the game, no calculations might be possible because we have no algorithms to perform them; no algorithms exist (and the standard concept for mathematical proof subsumes an algorithmic construction).

A. Lewis and V. Velupillai, among others, were pioneers in the discussion of undecidability and incompleteness in mathematical economics. Now we know that, for example, equilibrium prices in efficient markets are noncomputable, in the general case (this follows from a 1995 result by Lewis). In 1998, Tsuji, da Costa, and Doria entered the fray, with their result on the incompleteness of game theory — which is valid for a wide range of axiomatizations of Nash games and analogous theories.

The reason for such a vast presence of Gödel-like phenomena in mathematical economics is simple: they follow from a Rice-like theorem, which can be proved for the language of classical analysis, which is the language used by mathematical economists when exhibiting their wares.

So, undecidability and incompleteness creep up everywhere in economics and in the social sciences, and seem to hinder the predictive character of mathematics in those theories.

Of course, one can make predictions with the help of mathematical tools. A very general theorem in that respect can be proved as a spinoff out of the Tubular Neighborhood Theorem (which asserts, roughly, that a neighborhood without singularities of a smooth vectorfield 'looks' like an n-dimensional tube). So, chaotic deterministic systems may have nicely behaved neighborhoods.

Also, undecidability doesn't mean total ignorance about a mathematical object: it only forbids the existence of a general algorithm — a general recipe — for specific problems; *ad hoc* solutions may be possible, of course. But science is made of general laws, of general computational procedures, and our question is: are such general laws possible in the (mathematical) social sciences? If so, how useful are these general procedures? Do they tell us something really interesting about the social world, as much as a system's Lagrangian tells us a lot about the system's physical behavior?

The chapters collected in the present volume attempt to evaluate the impact of Gödel incompleteness and algorithmic undecidability in the mathematics of economics and of the social sciences.

The opening chapter is a long text by F. A. Doria, which gives an overview of the consequences of the already mentioned extension of Rice's theorem in the language of analysis to phenomena described with the help of that same language.

The next chapter is a wonderful piece by G. J. Chaitin. Chaitin, like a masterful magician, cradles the development of mathematics in a metaphysical bed, while explaining to us how undecidability — uncomputability — causes mathematics to be Gödel-incomplete.

Now keep in mind that mathematics in the social sciences has one task: to compute the future, to predict future events out of data from the present. Can we do it? The remaining chapters in the book tackle this issue.

First we have a chapter by G. Becchio. Becchio presents to us a few ideas of Karl Menger, which relate his own theoretical vision with the (then recent) development of logic, by Peano, and then Russell and the Polish school, which ventures into nonclassical logics. Becchio is almost exhaustive in her treatment of the subject, which is quite unexpected, as most of orthodox economics today supports itself in a quite strict classical logical language.

Then R. Koppl enters the field. Koppl examines crashes and turmoil in the economic landscape of today and discusses their predictability in the light of the Gödel phenomenon. The interesting aspect of Koppl's chapter is the fact that he boldly relates our concrete economic scenario with the rarefied vistas of metamathematics that bear on the language of theoretical economics. And I invite the reader to enjoy the beautiful conclusion of his chapter, an elegant pastiche of a tale by Jorge Luis Borges.

D. J. Dean and E. Elliott have contributed two chapters to this volume. Both are detailed surveys of the usages of mathematics in the social sciences. The first one looks at our main question from the viewpoint of complex systems, a "traditional" way of dealing with it. The second one considers our main problem, that is, the predictability of events in the social sciences given the Gödel phenomenon. (I won't say more because there is an easter egg in his texts, and I don't want to advance it.)

Finally, S. Al-Suwailem closes our book with the question: Is Economics a Science? In addressing this question, he explores an interesting link between logical consistency and financial instability via conservation laws. He argues that ignoring the meta-mathematics of economic models might lead to misuse and, perhaps, invalidation of these models. For the models to be reliable, and economics to become a proper science, economic theory must circumvent the logical paradoxes arising from Neoclassical assumptions. Gödel's Theorem — surprisingly? — then would pave the way for real science. I must also thank Dr. S. Al-Suwailen for his prompt and efficient help with the editorial chores of this book.

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