

main

November 21, 2025

0.1 MegaMart Customer Segmentation Analysis

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```
[1]: import pandas as pd
import numpy as np
import plotly.express as px
from plotly.subplots import make_subplots
import plotly.graph_objects as go
import plotly.subplots as sp
from plotly.offline import init_notebook_mode
from sklearn.preprocessing import StandardScaler
from sklearn.cluster import KMeans
from sklearn.decomposition import PCA
from sklearn.metrics import silhouette_score, silhouette_samples
from scipy.cluster.hierarchy import dendrogram, linkage, fcluster, cophenet
import matplotlib.pyplot as plt
from scipy.spatial.distance import pdist

import plotly.io as pio
pio.renderers.default = "png"
pio.templates.default = "plotly_white"

# Load the dataset
df = pd.read_csv('retail_customer_data.csv')
```

```
[2]: df.shape
```

```
[2]: (3000, 10)
```

```
[3]: df.head()
```

```
[3]:   customer_id  monthly_transactions  avg_basket_size  total_spend \
0    CUST_0001                  1.8            3.0      527.04
1    CUST_0002                  5.4            4.5     1137.72
2    CUST_0003                  2.9            4.4      670.28
3    CUST_0004                 12.0           22.8     6359.35
```

```

4    CUST_0005           2.2          17.6      4058.48
                                              avg_session_duration  email_open_rate  product_views_per_visit \
0                      38.1            0.95             17.3
1                      20.0            0.00             16.1
2                      64.3            0.52             36.9
3                      37.9            0.17             42.2
4                      22.5            0.53             18.4

  return_rate  customer_tenure_months  recency_days
0      0.00                  23            33
1      0.41                  1              8
2      0.11                  32            39
3      0.00                  21              5
4      0.15                  20            18

```

[4]: df.info()

```

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 3000 entries, 0 to 2999
Data columns (total 10 columns):
 #   Column           Non-Null Count  Dtype  
--- 
 0   customer_id      3000 non-null   object 
 1   monthly_transactions  3000 non-null   float64
 2   avg_basket_size   3000 non-null   float64
 3   total_spend       3000 non-null   float64
 4   avg_session_duration  3000 non-null   float64
 5   email_open_rate   3000 non-null   float64
 6   product_views_per_visit  3000 non-null   float64
 7   return_rate        3000 non-null   float64
 8   customer_tenure_months  3000 non-null   int64  
 9   recency_days       3000 non-null   int64  
dtypes: float64(7), int64(2), object(1)
memory usage: 234.5+ KB

```

[5]: df.describe()

```

[5]:    monthly_transactions  avg_basket_size  total_spend \
count      3000.000000      3000.000000  3000.000000
mean       6.011300        9.485133  2367.375387
std        4.776564        7.961280  2248.271454
min        0.200000        1.000000  50.000000
25%        2.100000        3.300000  511.115000
50%        5.000000        5.600000  1456.105000
75%        8.800000        17.325000 3772.227500
max       22.300000       31.600000  8746.290000

```

	avg_session_duration	email_open_rate	product_views_per_visit	\
count	3000.000000	3000.000000	3000.000000	
mean	38.441533	0.443720	31.359467	
std	14.334159	0.427384	9.964088	
min	3.400000	0.000000	3.000000	
25%	27.000000	0.000000	24.900000	
50%	36.550000	0.340000	31.600000	
75%	49.400000	0.950000	37.800000	
max	87.300000	0.950000	62.400000	
	return_rate	customer_tenure_months	recency_days	
count	3000.000000	3000.000000	3000.000000	
mean	0.185890	17.912667	20.680667	
std	0.162283	11.242294	12.055419	
min	0.000000	1.000000	1.000000	
25%	0.040000	9.000000	11.000000	
50%	0.150000	18.000000	18.000000	
75%	0.290000	26.000000	29.250000	
max	0.500000	59.000000	67.000000	

```
[6]: missing_summary = df.isnull().sum()
missing_summary
```

```
[6]: customer_id          0
monthly_transactions     0
avg_basket_size         0
total_spend              0
avg_session_duration    0
email_open_rate          0
product_views_per_visit 0
return_rate               0
customer_tenure_months   0
recency_days              0
dtype: int64
```

Dataset Size: 3,000 customers with 10 variables Available: - customer_id - monthly_transactions - avg_basket_size - total_spend - avg_session_duration - email_open_rate - product_views_per_visit - return_rate - customer_tenure_months - recency_days

Data Quality: No missing values detected in any column and all 3,000 records are complete

Data Types: - 7 numerical (float64) behavioral variables - 2 numerical (int64) variables - 1 categorical (object) identifier

0.2 EDA

```
[7]: #histograms for all behavioral variables (excluding customer_id)
behavioral_vars = df.columns.drop('customer_id')

fig = sp.make_subplots(
    rows=3, cols=3,
    subplot_titles=behavioral_vars,
    vertical_spacing=0.08,
    horizontal_spacing=0.05
)

for i, column in enumerate(behavioral_vars):
    row = i // 3 + 1
    col = i % 3 + 1

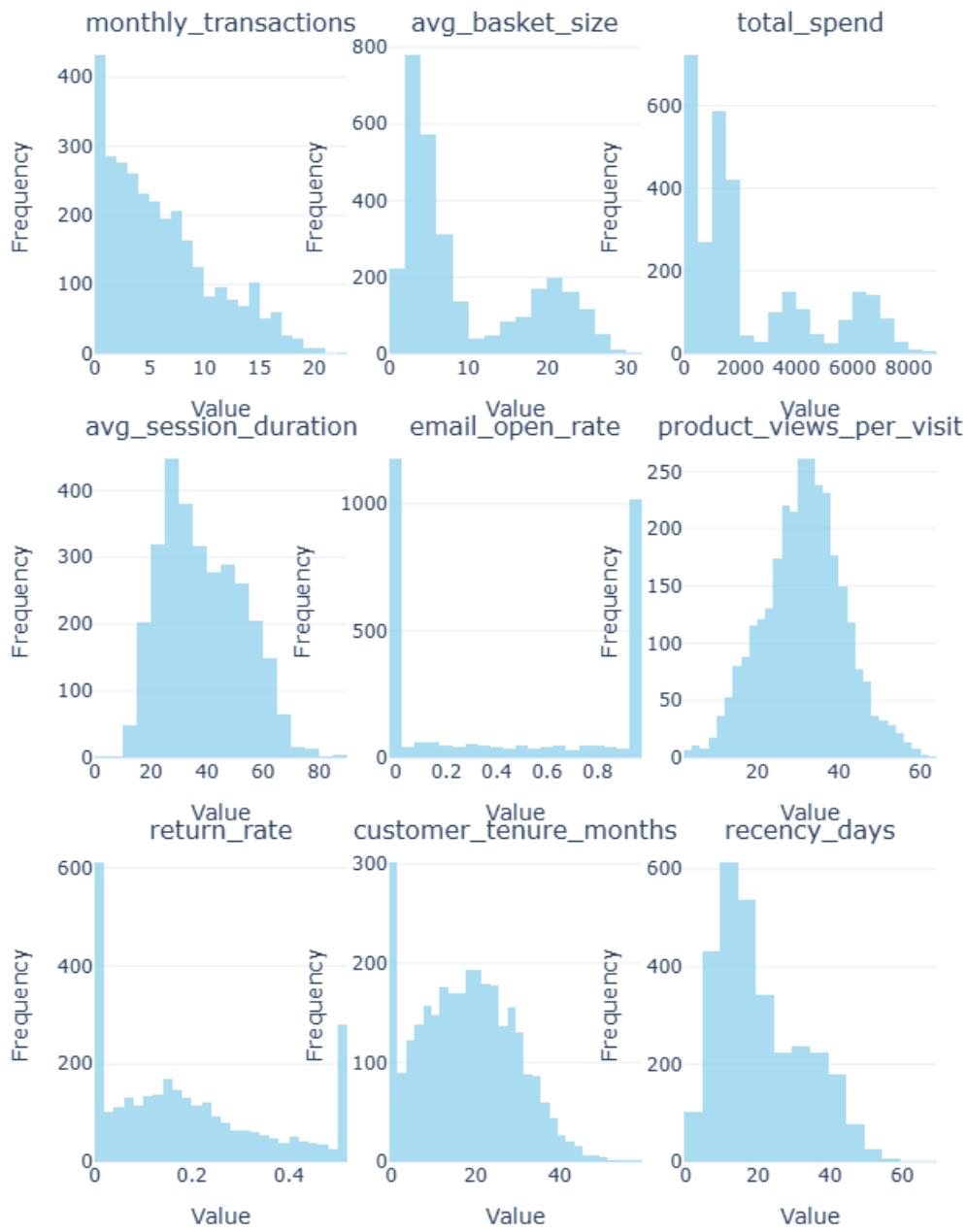
    fig.add_trace(
        go.Histogram(
            x=df[column],
            name=column,
            nbinsx=30,
            marker_color='skyblue',
            opacity=0.7
        ),
        row=row, col=col
    )

fig.update_layout(
    height=900,
    title_text="Distribution of Behavioral Variables",
    title_x=0.5,
    showlegend=False,
    template="plotly_white"
)

for i in range(1, 10):
    fig.update_xaxes(title_text="Value", row=(i-1)//3 + 1, col=(i-1)%3 + 1)
    fig.update_yaxes(title_text="Frequency", row=(i-1)//3 + 1, col=(i-1)%3 + 1)

fig.show()
```

Distribution of Behavioral Variables



```
[8]: #correlation matrix (excluding customer_id)
corr_matrix = df[behavioral_vars].corr()

fig = go.Figure(data=go.Heatmap(
```

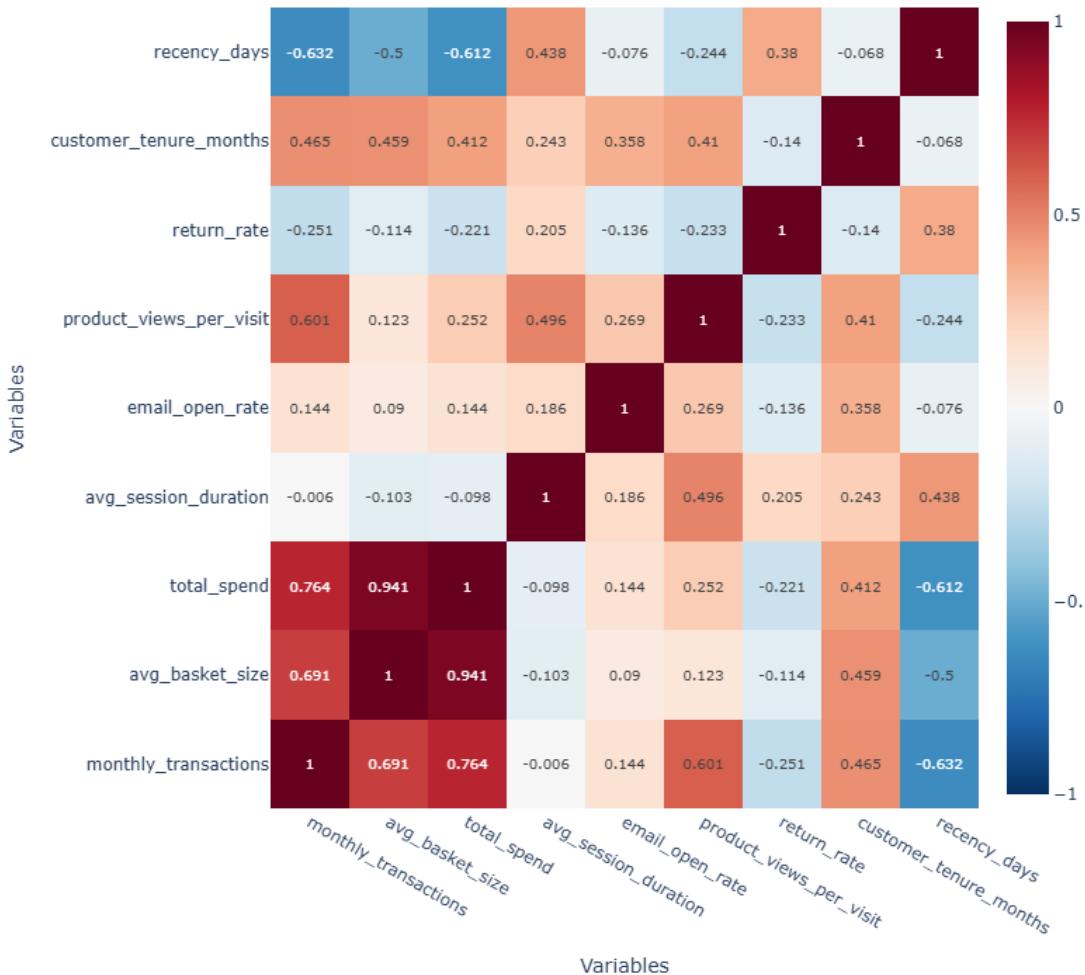
```

z=corr_matrix.values,
x=corr_matrix.columns,
y=corr_matrix.columns,
colorscale='RdBu_r',
zmin=-1,
zmax=1,
hoverongaps=False,
text=corr_matrix.round(3),
texttemplate="%{text}",
textfont={"size": 10}
))

fig.update_layout(
    title="Correlation Matrix Heatmap - Customer Behavioral Variables",
    width=800,
    height=800,
    xaxis_title="Variables",
    yaxis_title="Variables",
    template="plotly_white"
)
fig.show()

```

Correlation Matrix Heatmap - Customer Behavioral Variables



```
[9]: #print highly correlated pairs
print("Highly Correlated Variable Pairs (|r| > 0.5):")
high_corr_pairs = []
for i in range(len(corr_matrix.columns)):
    for j in range(i+1, len(corr_matrix.columns)):
        if abs(corr_matrix.iloc[i, j]) > 0.5:
            high_corr_pairs.append((
                corr_matrix.columns[i],
                corr_matrix.columns[j],
                corr_matrix.iloc[i, j]
            ))
```

```

for var1, var2, corr in sorted(high_corr_pairs, key=lambda x: abs(x[2]), reverse=True):
    print(f'{var1} vs {var2}:{corr:.3f}')

```

Highly Correlated Variable Pairs ($|r| > 0.5$):

avg_basket_size vs total_spend:0.941
monthly_transactions vs total_spend:0.764
monthly_transactions vs avg_basket_size:0.691
monthly_transactions vs recency_days:-0.632
total_spend vs recency_days:-0.612
monthly_transactions vs product_views_per_visit:0.601

```

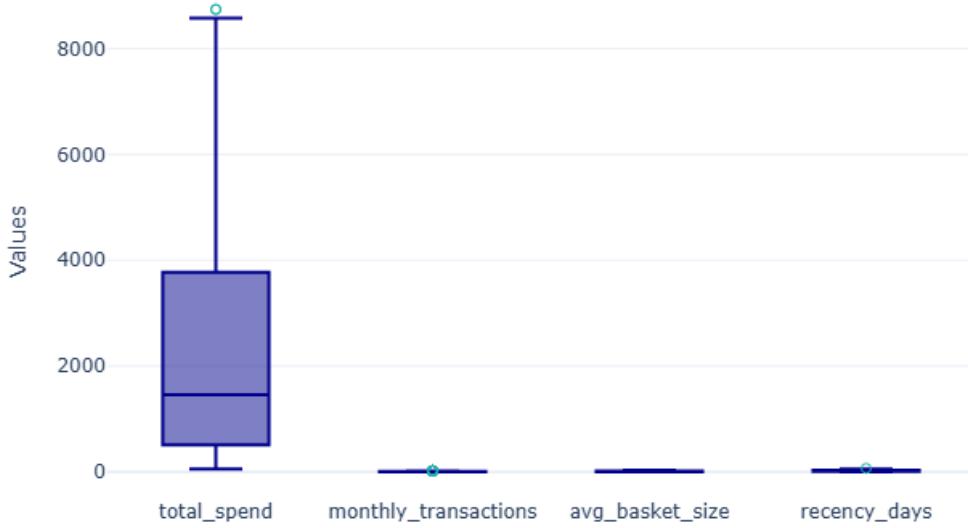
[10]: #box plots for key variables to detect outliers
key_Variables = ['total_spend', 'monthly_transactions', 'avg_basket_size', 'recency_days']

fig = go.Figure()
for var in key_Variables:
    fig.add_trace(go.Box(
        y=df[var],
        name=var,
        boxpoints='suspectedoutliers',
        marker_color='lightseagreen',
        line_color='darkblue'
    ))

fig.update_layout(
    title="Outlier Detection - Box Plots for Key Variables",
    yaxis_title="Values",
    template="plotly_white",
    height=500,
    showlegend=False
)
fig.show()

```

Outlier Detection - Box Plots for Key Variables



```
[11]: #quantitative outlier analysis using IQR method
print("Outlier Analysis (IQR Method):")
for var in key_Variables:
    q1 = df[var].quantile(0.25)
    q3 = df[var].quantile(0.75)
    iqr = q3 - q1
    lower_bound = q1 - 1.5 * iqr
    upper_bound = q3 + 1.5 * iqr
    outliers = df[(df[var] < lower_bound) | (df[var] > upper_bound)]
    print(f"{var}: {len(outliers)} outliers ({len(outliers)/len(df)*100:.1f}%)")
```

```
Outlier Analysis (IQR Method):
total_spend: 2 outliers (0.1%)
monthly_transactions: 18 outliers (0.6%)
avg_basket_size: 0 outliers (0.0%)
recency_days: 3 outliers (0.1%)
```

```
[12]: #total Spend vs Monthly Transactions
fig1 = px.scatter(
    df,
    x='monthly_transactions',
    y='total_spend',
```

```

        color='avg_basket_size',
        size='customer_tenure_months',
        hover_data=['recency_days', 'email_open_rate'],
        title="Total Spend vs Monthly Transactions  
Color: Avg Basket Size, Size: Customer Tenure",
        labels={
            'monthly_transactions': 'Monthly Transactions',
            'total_spend': 'Total Spend ($)',
            'avg_basket_size': 'Avg Basket Size'
        },
        color_continuous_scale='viridis'
    )

fig1.update_layout(template="plotly_white")
fig1.show()

```

Total Spend vs Monthly Transactions
Color: Avg Basket Size, Size: Customer Tenure



```
[13]: #recency vs Frequency behavior
fig2 = px.scatter(
    df,
    x='recency_days',
    y='monthly_transactions',
```

```

color='total_spend',
size='avg_basket_size',
hover_data=['email_open_rate', 'return_rate'],
title="Recency vs Monthly Transactions  
Color: Total Spend, Size: Avg Basket Size",
labels={
    'recency_days': 'Recency (Days since last purchase)',
    'monthly_transactions': 'Monthly Transactions',
    'total_spend': 'Total Spend ($)'
},
color_continuous_scale='plasma'
)

fig2.update_layout(template="plotly_white")
fig2.show()

```

Recency vs Monthly Transactions
Color: Total Spend, Size: Avg Basket Size



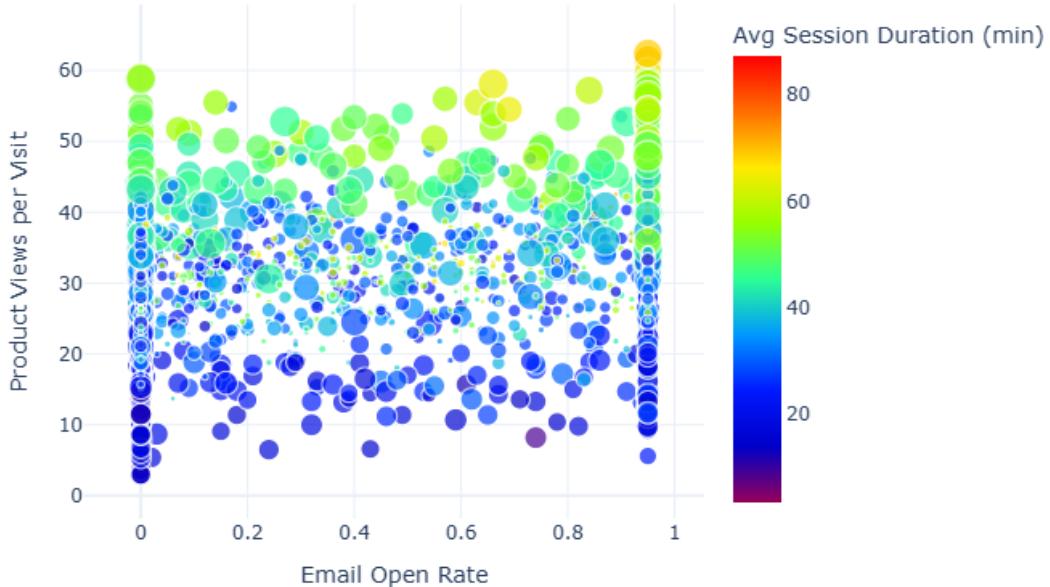
```
[14]: #engagement metrics
fig3 = px.scatter(
    df,
    x='email_open_rate',
    y='product_views_per_visit',
```

```

        color='avg_session_duration',
        size='total_spend',
        hover_data=['return_rate', 'customer_tenure_months'],
        title="Email Engagement vs Product Views  
Color: Session Duration,  
Size: Total Spend",
        labels={
            'email_open_rate': 'Email Open Rate',
            'product_views_per_visit': 'Product Views per Visit',
            'avg_session_duration': 'Avg Session Duration (min)'
        },
        color_continuous_scale='rainbow'
)
fig3.update_layout(template="plotly_white")
fig3.show()

```

Email Engagement vs Product Views
Color: Session Duration, Size: Total Spend



0.3 Data Preprocessing

[15]: missing_summary

[15]: customer_id	0
monthly_transactions	0

```
avg_basket_size          0
total_spend              0
avg_session_duration     0
email_open_rate           0
product_views_per_visit   0
return_rate               0
customer_tenure_months    0
recency_days              0
dtype: int64
```

```
[16]: #prepare data for clustering (exclude customer_id)
z = df[behavioral_vars].copy()

print("Original Data Statistics:")
z.head()
```

Original Data Statistics:

```
[16]: monthly_transactions  avg_basket_size  total_spend  avg_session_duration \
0                  1.8            3.0      527.04          38.1
1                  5.4            4.5     1137.72          20.0
2                  2.9            4.4      670.28          64.3
3                 12.0            22.8     6359.35          37.9
4                  2.2            17.6     4058.48          22.5

email_open_rate  product_views_per_visit  return_rate \
0             0.95                  17.3        0.00
1             0.00                  16.1        0.41
2             0.52                  36.9        0.11
3             0.17                  42.2        0.00
4             0.53                  18.4        0.15

customer_tenure_months  recency_days
0                      23            33
1                      1              8
2                      32            39
3                      21            5
4                      20            18
```

```
[17]: print("Original Data Ranges (Min-Max):")
original_ranges = pd.DataFrame({
    'Variable': z.columns,
    'Min': z.min().values,
    'Max': z.max().values,
    'Range': (z.max() - z.min()).values,
    'Std': z.std().values
})
original_ranges.round(2)
```

Original Data Ranges (Min-Max):

```
[17]:      Variable    Min     Max   Range     Std
0   monthly_transactions  0.2   22.30  22.10    4.78
1       avg_basket_size  1.0   31.60  30.60    7.96
2           total_spend 50.0  8746.29 8696.29  2248.27
3   avg_session_duration  3.4   87.30  83.90   14.33
4   email_open_rate     0.0    0.95   0.95    0.43
5 product_views_per_visit  3.0   62.40  59.40    9.96
6         return_rate    0.0    0.50   0.50    0.16
7 customer_tenure_months  1.0   59.00  58.00   11.24
8        recency_days    1.0   67.00  66.00   12.06
```

```
[18]: scaler = StandardScaler()
z_scaled = scaler.fit_transform(z)

X_scaled_df = pd.DataFrame(z_scaled, columns=z.columns, index=z.index)

print("Standardized Data Statistics:")
X_scaled_df.head().round(3)
```

Standardized Data Statistics:

```
[18]:   monthly_transactions  avg_basket_size  total_spend  avg_session_duration \
0            -0.882          -0.815       -0.819          -0.024
1            -0.128          -0.626       -0.547         -1.287
2            -0.651          -0.639       -0.755          1.804
3             1.254          1.673       1.776         -0.038
4            -0.798          1.019       0.752         -1.112

   email_open_rate  product_views_per_visit  return_rate \
0            1.185          -1.411       -1.146
1            -1.038          -1.532        1.381
2             0.179           0.556       -0.468
3            -0.641           1.088       -1.146
4             0.202          -1.301       -0.221

customer_tenure_months  recency_days
0            0.453          1.022
1            -1.505         -1.052
2            1.253          1.520
3            0.275          -1.301
4            0.186          -0.222
```

```
[19]: print("Standardized Data Ranges (Min-Max):")
scaled_ranges = pd.DataFrame({
    'Variable': X_scaled_df.columns,
    'Min': X_scaled_df.min().values,
```

```

        'Max': X_scaled_df.max().values,
        'Mean': X_scaled_df.mean().values,
        'Std': X_scaled_df.std().values
    })
scaled_ranges.round(3)

```

Standardized Data Ranges (Min-Max):

```
[19]:      Variable   Min   Max   Mean   Std
0   monthly_transactions -1.217  3.411  0.0  1.0
1       avg_basket_size -1.066  2.778 -0.0  1.0
2          total_spend -1.031  2.838  0.0  1.0
3  avg_session_duration -2.445  3.409  0.0  1.0
4      email_open_rate -1.038  1.185 -0.0  1.0
5  product_views_per_visit -2.847  3.116  0.0  1.0
6          return_rate -1.146  1.936 -0.0  1.0
7  customer_tenure_months -1.505  3.655  0.0  1.0
8      recency_days -1.633  3.843 -0.0  1.0
```

0.4 Why Standardization is Necessary for Clustering

0.4.1 The Problem:

Without standardization, variables with larger numerical ranges (like `total_spend` ranging from \$50 to \$8,746) would dominate the distance calculations in clustering algorithms, while variables with smaller ranges (like `email_open_rate` ranging from 0 to 0.95) would have minimal impact.

0.4.2 Impact on Our Data:

- **Before:** Variables had vastly different scales and units
- **After:** All variables are on comparable scales, ensuring equal importance in distance calculations

0.4.3 Why This Matters for Clustering:

1. **Distance-based algorithms** (K-Means, Hierarchical) rely on Euclidean distance
2. **Equal contribution** from all variables in cluster formation
3. **Better cluster quality** and more meaningful customer segments

0.5 Hierarchical Clustering Analysis

```
[20]: #compute linkage matrices for all four methods (deepseek intervention)
linkage_methods = ['single', 'complete', 'average', 'ward']
linkage_matrices = {}

for method in linkage_methods:
    linkage_matrices[method] = linkage(X_scaled_df, method=method)
    print(f"{method.capitalize()} linkage computed, Shape:{linkage_matrices[method].shape}")
```

```

linkage_matrices['ward'][:5].round(3)

Single linkage computed, Shape: (2999, 4)
Complete linkage computed, Shape: (2999, 4)
Average linkage computed, Shape: (2999, 4)
Ward linkage computed, Shape: (2999, 4)

[20]: array([[6.260e+02, 2.619e+03, 1.130e-01, 2.000e+00],
       [1.010e+02, 1.033e+03, 1.170e-01, 2.000e+00],
       [1.440e+02, 2.059e+03, 1.560e-01, 2.000e+00],
       [9.780e+02, 2.767e+03, 1.630e-01, 2.000e+00],
       [1.367e+03, 2.567e+03, 1.750e-01, 2.000e+00]])

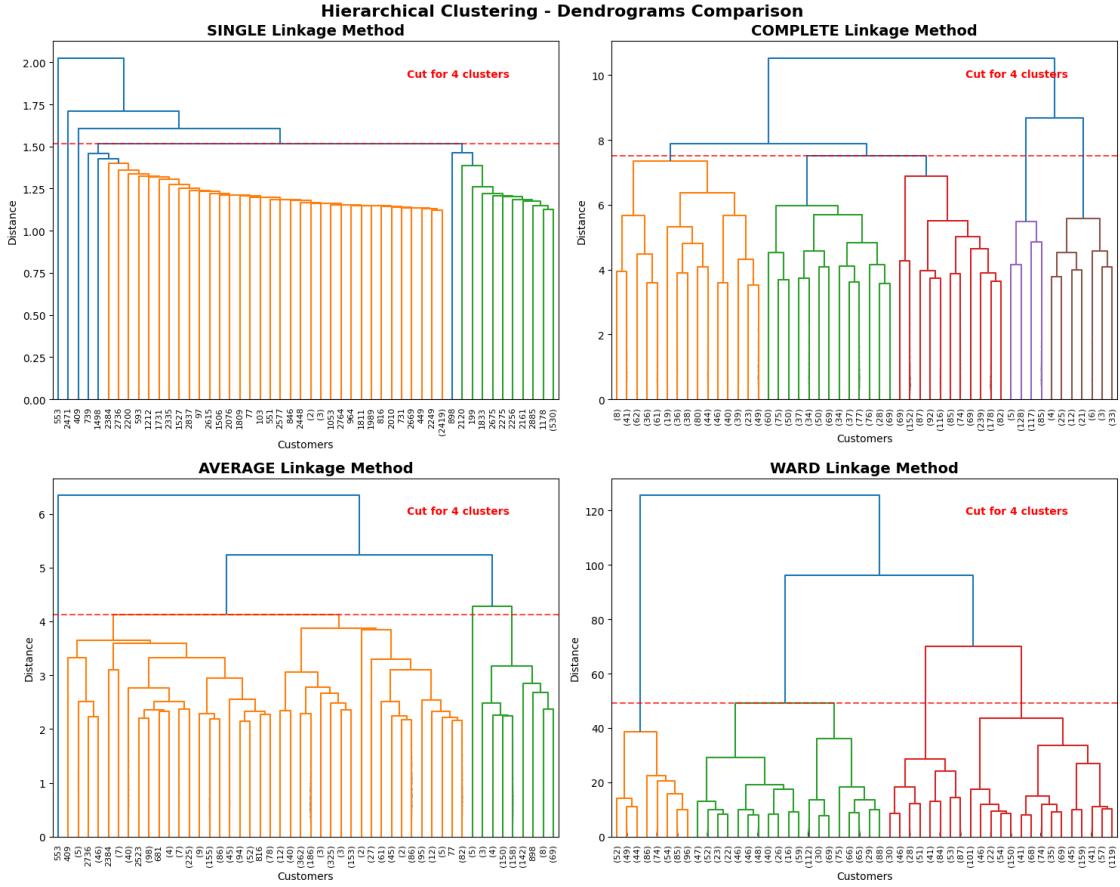
[21]: # Create dendrograms for all four linkage methods (deepseek intervention)
fig, axes = plt.subplots(2, 2, figsize=(15, 12))
axes = axes.ravel()

for i, method in enumerate(linkage_methods):
    # Create dendrogram
    dendrogram(
        linkage_matrices[method],
        ax=axes[i],
        truncate_mode='lastp',
        p=50,  # Show last 50 merges for clarity
        show_leaf_counts=True,
        leaf_rotation=90,
        leaf_font_size=8,
        show_contracted=True
    )
    axes[i].set_title(f'{method.upper()} Linkage Method', fontsize=14, fontweight='bold')
    axes[i].set_xlabel('Customers')
    axes[i].set_ylabel('Distance')

    # Add horizontal line at potential cut point
    cut_distance = linkage_matrices[method][-4, 2]  # Cut for 4 clusters
    axes[i].axhline(y=cut_distance, color='r', linestyle='--', alpha=0.7)
    axes[i].text(0.7, 0.9, f'Cut for 4 clusters',
                transform=axes[i].transAxes, color='red', fontweight='bold')

plt.suptitle('Hierarchical Clustering - Dendrograms Comparison', fontsize=16, fontweight='bold')
plt.tight_layout()
plt.show()

```



0.6 Interpretation of Dendrograms

Comparative Analysis of Linkage Methods

Single Linkage

This method tends to produce elongated, chain-like clusters due to the chaining effect, where points are merged one by one as long as there is a short distance between them. This often leads to poorly separated and less meaningful clusters, making it less suitable for customer segmentation.

Complete Linkage

Focuses on the maximum distance between points of different clusters, produces more compact and well-separated clusters compared to the Single Linkage method, but is sensible to outliers.

Average Linkage

Represents a balance between the Single and Complete methods, uses the average distance between all pairs of points in two clusters, resulting in balanced groupings that are less sensitive to outliers.

Ward Linkage

Minimizes the variance within clusters at each merge. Tends to create compact, spherical, and well-separated clusters, which makes it ideal for customer segmentation and market analysis. In

this case, the Ward method appears to be the most suitable, as it clearly defines four distinct and stable clusters.

Choosing the Cut in the Dendrogram

The cut point is selected where there is a large increase in distance between consecutive merges, this jump indicates that clusters being merged beyond that height are no longer similar, so cutting before it ensures high internal cohesion within each cluster, in the plots, the red dashed line shows the level where **4 clusters** are formed, with Ward's method showing the cleanest separation.

The Chaining Effect

Occurs when clusters form long chains of sequentially linked points that do not represent true similarity, this effect is **most pronounced in the Single Linkage method**, which makes it less effective for clearly separated groups.

0.7 Optimal number of Clusters (Hierarchical)

```
[22]: #Gemini was used to increase the quality of the plot

plt.figure(figsize=(15, 9))

ward_matrix = linkage_matrices['ward']

dendrogram(
    ward_matrix,
    truncate_mode='lastp',
    p=50,  # Show last 50 merges
    show_leaf_counts=True,
    leaf_rotation=90,
    leaf_font_size=10,
    show_contracted=True
)

plt.title("Detailed Ward's Linkage Dendrogram with Potential Cuts", fontweight='bold')
plt.xlabel('Cluster Size (or Sample Index)', fontsize=12)
plt.ylabel('Distance (Ward)', fontsize=12)

# Define the number of clusters (k) and corresponding colors
cuts = {
    3: 'green',
    4: 'orange',
    5: 'purple',
    6: 'blue'
}

xmin, xmax = plt.xlim()
```

```

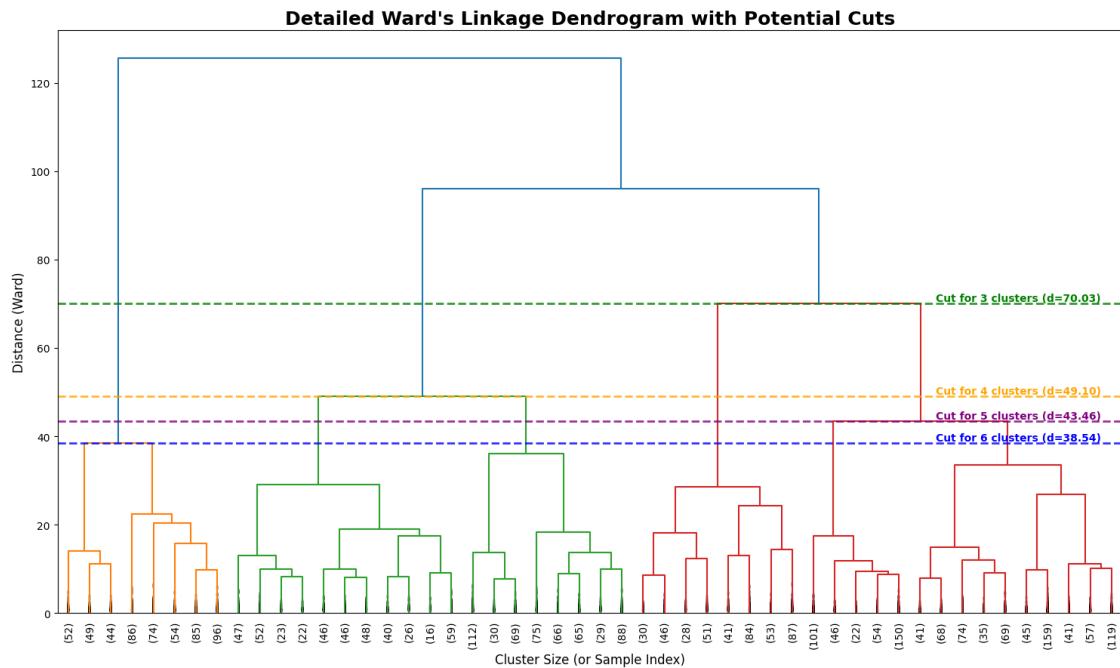
for k, color in cuts.items():
    # Get the distance for the merge that creates k clusters
    cut_distance = ward_matrix[-k, 2]

    plt.axhline(
        y=cut_distance,
        color=color,
        linestyle='--',
        alpha=0.8,
        linewidth=2
    )

    # Add a text label for the cut
    plt.text(
        xmax * 0.98,
        cut_distance,
        f'Cut for {k} clusters (d={cut_distance:.2f})',
        color=color,
        fontweight='bold',
        ha='right',
        va='bottom'
    )

plt.tight_layout()
plt.show()

```



```
[23]: #Extracting clusters for k = {3, 4, 5, 6}
k_vals = list(cuts.keys())

clusters = {}

for k in k_vals:
    assignment = fcluster(ward_matrix, t = k, criterion='maxclust') #Maxclust
    ↴ensures the k number of clusters
    clusters[k] = assignment
```

```
[24]: #Calculating silhouette scores

silhouette_scores = []

for k in k_vals:
    labels = clusters[k]
    score = silhouette_score(X_scaled_df, labels)
    silhouette_scores.append({'silhouette_score': score})

summary_scores = pd.DataFrame(silhouette_scores, index=k_vals)

summary_scores
```

```
[24]: silhouette_score
3          0.294781
4          0.315700
5          0.300267
6          0.247512
```

Given the unsupervised nature of this task, predefined class labels for customer categorization are not available. Therefore, the silhouette score was employed as the validation metric to determine the optimal number of clusters (k).

The analysis revealed that the peak silhouette score was 0.31, achieved with $k = 4$ clusters. While this score indicates a reasonable partitioning of the data, it also suggests a relatively weak cluster structure. A score in this range implies that the clusters are not sharply delineated and likely exhibit some degree of overlap, meaning the separation between groups is not entirely distinct.

0.8 K-Means Clustering - Elbow method

```
[25]: # Calculating inertia and silhouette score for each K in k means

k_values = [i for i in range(2, 11)]
inertias = []
silhouette_scores_kmeans = []
```

```

for k in k_values:
    k_means = KMeans(n_clusters=k, random_state=42, n_init='auto')
    cluster_labels = k_means.fit_predict(X_scaled_df)
    inertias.append(k_means.inertia_)
    #print(k_means.labels_)
    silhouette_scores_kmeans.append({'silhouette_score':silhouette_score(X_scaled_df, cluster_labels)})

summary_scores_kmeans = pd.DataFrame(silhouette_scores_kmeans, index=k_values)

```

[26]: # Plot of inertia vs k (gemini increased presentation of the plot)

```

fig = go.Figure()

# Add the main trace (the line and markers)
fig.add_trace(go.Scatter(
    x=k_values,
    y=inertias,
    mode='lines+markers', # Show both the line connecting points and the markers
    name='Inertia',
    marker=dict(color='blue', size=10, line=dict(width=1,color='DarkSlateGrey')),
    line=dict(color='blue', width=2)
))
fig.update_layout(
    title=dict(
        text='<b>Elbow Method for Optimal k</b>',
        x=0.5,
        font=dict(size=24, color='black')
    ),
    xaxis_title=dict(
        text='<b>Number of Clusters (k)</b>',
        font=dict(size=18, color='black')
    ),
    yaxis_title=dict(
        text='<b>Inertia (WCSS)</b>',
        font=dict(size=18, color='black')
    ),
    xaxis=dict(
        dtick=1,
        gridcolor='rgba(200, 200, 200, 0.5)',
        zeroline=False,
        showline=True,
        linewidth=1,
        linecolor='black'
    )
)

```

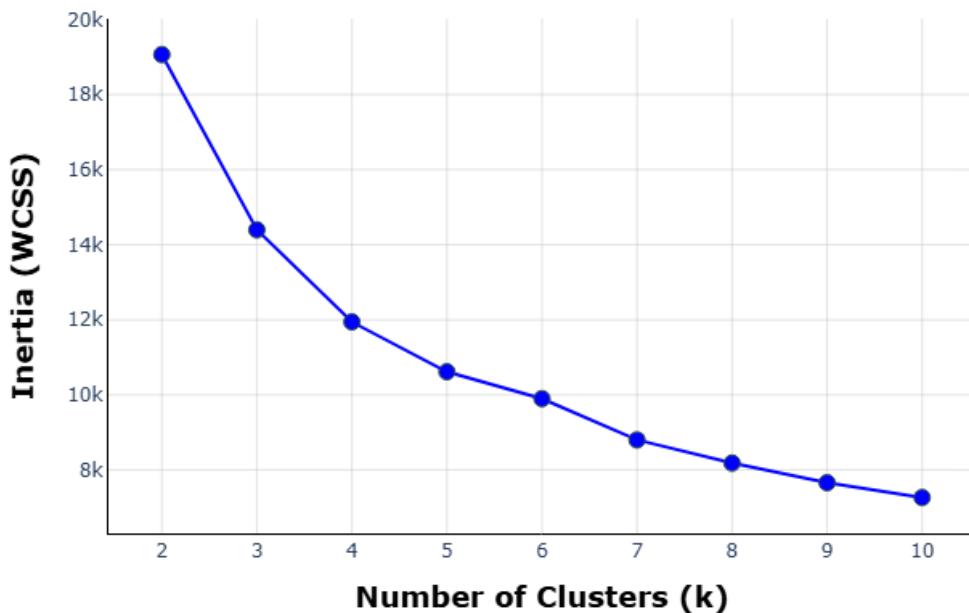
```

),
yaxis=dict(
    gridcolor='rgba(200, 200, 200, 0.5)',
    zeroline=False,
    showline=True,
    linewidth=1,
    linecolor='black'
),
plot_bgcolor='white',
paper_bgcolor='white',
hovermode='x unified'
)

fig.show()

```

Elbow Method for Optimal k



```
[27]: # Showing the silhouette scores for K Means
summary_scores_kmeans
```

```
[27]: silhouette_score
2          0.344552
3          0.297391
```

```

4          0.317311
5          0.269579
6          0.261739
7          0.268698
8          0.279358
9          0.272323
10         0.273340

```

[28]: #Gemini created the subplots

```

fig = make_subplots(
    rows=1, cols=2,
    subplot_titles=( '<b>Elbow Method (Inertia)</b>', '<b>Silhouette Scores</b>' )
)

fig.add_trace(go.Scatter(
    x=k_values,
    y=inertias,
    mode='lines+markers',
    name='Inertia (WCSS)',
    marker=dict(color='blue', size=10, line=dict(width=1, color='DarkSlateGrey')),
    line=dict(color='blue', width=2)
), row=1, col=1)

fig.add_trace(go.Scatter(
    x=summary_scores_kmeans.index,
    y=summary_scores_kmeans['silhouette_score'],
    mode='lines+markers',
    name='Silhouette Score',
    marker=dict(color='green', size=10, line=dict(width=1, color='DarkSlateGrey')),
    line=dict(color='green', width=2)
), row=1, col=2)

fig.update_layout(
    title=dict(
        text='<b>K-Means Clustering Analysis</b>',
        x=0.5,
        font=dict(size=24, color='black')
    ),
    plot_bgcolor='white',
    paper_bgcolor='white',
    showlegend=False, # The subplot titles are clear enough
    hovermode='x unified'
)

```

```

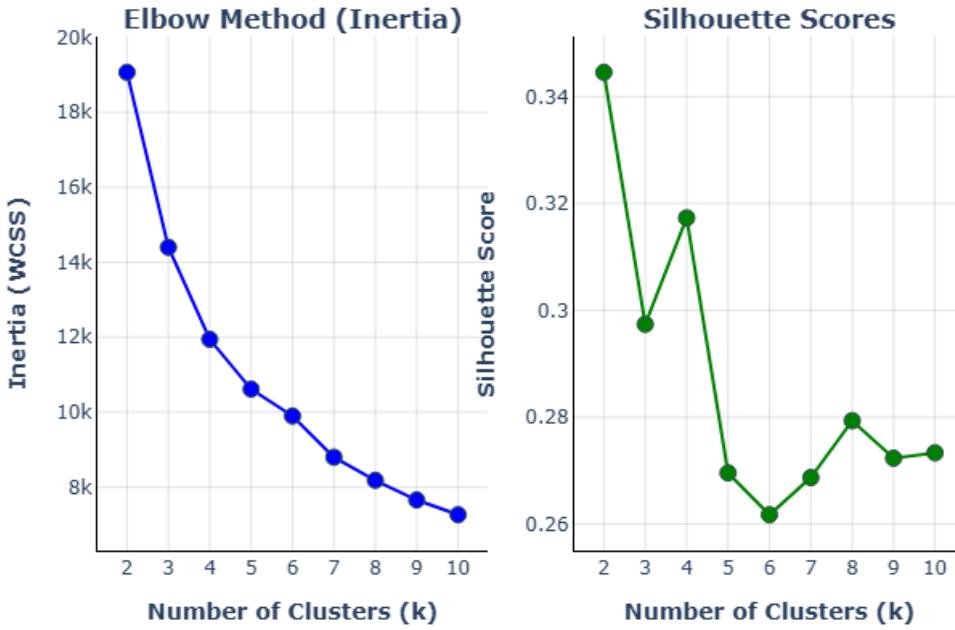
fig.update_xaxes(
    title_text='<b>Number of Clusters (k)</b>',
    dtick=1, # Tick for every k-value
    gridcolor='rgba(200, 200, 200, 0.5)',
    zeroline=False,
    showline=True,
    linewidth=1,
    linecolor='black',
    row=1, col=1
)
fig.update_yaxes(
    title_text='<b>Inertia (WCSS)</b>',
    gridcolor='rgba(200, 200, 200, 0.5)',
    zeroline=False,
    showline=True,
    linewidth=1,
    linecolor='black',
    row=1, col=1
)

# 6. Update axes for Subplot 2 (Silhouette)
fig.update_xaxes(
    title_text='<b>Number of Clusters (k)</b>',
    dtick=1,
    gridcolor='rgba(200, 200, 200, 0.5)',
    zeroline=False,
    showline=True,
    linewidth=1,
    linecolor='black',
    row=1, col=2
)
fig.update_yaxes(
    title_text='<b>Silhouette Score</b>',
    gridcolor='rgba(200, 200, 200, 0.5)',
    zeroline=False,
    showline=True,
    linewidth=1,
    linecolor='black',
    row=1, col=2
)

# Show the final combined figure
fig.show()

```

K-Means Clustering Analysis



Analysis of the elbow method plot indicates the “elbow,” or the point of diminishing returns, is located between $k = 4$ and $k = 5$. At this juncture, the rate of decrease in inertia (within-cluster sum of squares) significantly decelerates, and the slope of the curve begins to flatten.

The silhouette score analysis does not perfectly align with this finding, as the global maximum score is observed at $k = 2$. However, the score at $k = 4$ represents the second-highest peak (a strong local maximum), thereby providing support for $k = 4$ as a viable option.

0.9 Final Cluster Selection

the optimal and most justifiable number of clusters is 4, this by the following reasons:

1. Dendrogram Structure (Ward method)
 - In the “Detailed Ward’s Linkage Dendrogram,” we look for the largest vertical distance we can “cut” that represents the greatest dissimilarity between merged clusters.
 - While the single largest jump is arguably the final one (separating the data into 2 clusters) or the jump above the green line (separating into 3 clusters at $d=70.03$), the next most significant jump is cut by the orange line ($d=49.10$).
 - This cut at $d=49.10$ creates 4 distinct clusters. The vertical distance between this cut and the 3-cluster merge ($d=70.03$) is substantial, indicating that merging these four clusters into three results in a significant loss of information (i.e., we combine two groups that are quite

dissimilar). The merges below the 4-cluster line happen at much smaller distances, suggesting higher similarity within those four groups.

2. Elbow Plot Inflection Point

- The “Elbow Method (Inertia)” plot for K-Means helps identify the point of diminishing returns, where adding more clusters does not significantly reduce the within-cluster sum of squares (WCSS).
- The plot shows a sharp “bend” or “elbow” at $k=4$. The inertia drops sharply from $k=2$ to 3, and again from $k=3$ to 4.
- After $k=4$, the slope of the line flattens considerably. This indicates that the reduction in inertia gained by moving from 4 to 5 clusters (or more) is minimal and not worth the added complexity.

3. Silhouette Scores

- This is the strongest piece of evidence, as the results are consistent across both clustering methods. The silhouette score measures how similar an object is to its own cluster compared to other clusters (higher is better).
- Ward’s Linkage Scores: The silhouette score peaks at $k=4$ (0.3157). This is higher than the score for $k=3$ (0.2948) and $k=5$ (0.3003). The score drops significantly at $k=6$, indicating it is a poor choice.
- K-Means Scores: Ignoring the $k=2$ solution (which is often too general), the highest and most practical silhouette score is at $k=4$ (0.3173). After this peak, the score drops notably to 0.2696 for $k=5$, showing that 4 clusters provide the most distinct and well-separated grouping.

4. Business Considerations (Synthesis)

- For MegaMart, choosing 4 clusters provides the best strategic balance between generalization and actionability. The goal is to create distinct customer segments for targeted marketing, and the variables provided (mixing purchase value, loyalty, and digital engagement) allow for this.
- Avoids Overgeneralization ($k=2$ or $k=3$): A 3-cluster solution would likely be too simple, for example, just “Low, Medium, High Value” based on `total_spend`. This is overgeneralized because it would incorrectly lump strategically different customers together.
- Avoids Non-Actionable Complexity ($k=5+$): Both the silhouette scores and the elbow plot show that solutions with 5 or more clusters are “forced” and not statistically distinct. For a business, this translates to non-actionable segments.

0.10 Final K-Means Model

```
[29]: selected_k = 4

k_means_model = KMeans(n_clusters=selected_k, random_state=42, n_init='auto')
k_means_model.fit(X_scaled_df)

df_labels = df.copy()
df_labels['label'] = k_means_model.labels_
```

```

cluster_counts = df_labels['label'].value_counts().sort_index()
cluster_percentages = df_labels['label'].value_counts(normalize=True) .
    sort_index() * 100

cluster_summary = pd.DataFrame({
    'Cluster': cluster_counts.index,
    'Count': cluster_counts.values,
    'Percentage': cluster_percentages.values
})

print(cluster_summary.to_string())

```

	Cluster	Count	Percentage
0	0	525	17.500000
1	1	929	30.966667
2	2	433	14.433333
3	3	1113	37.100000

0.11 Cluster Profiling and Interpretation

[30]: #Calculating mean for each behavioral variabe grouped by label
`cluster_means_original = df_labels.groupby('label')[behavioral_vars].mean()`

[31]: # Heatmap (improved by Gemini)

```

scaler = StandardScaler()
scaled_means_for_color = scaler.fit_transform(cluster_means_original)

x_labels = [f'Cluster {i}' for i in cluster_means_original.index]
y_labels = cluster_means_original.columns
scaled_z_data = scaled_means_for_color.T
original_text_data = cluster_means_original.T

fig = go.Figure()

fig.add_trace(go.Heatmap(
    z=scaled_z_data,
    x=x_labels,
    y=y_labels,

    text=original_text_data,
    texttemplate="%{text:.2f}",

    colorscale='RdBu_r',
    colorbar_title='Scaled Value (Z-Score)'
))

```

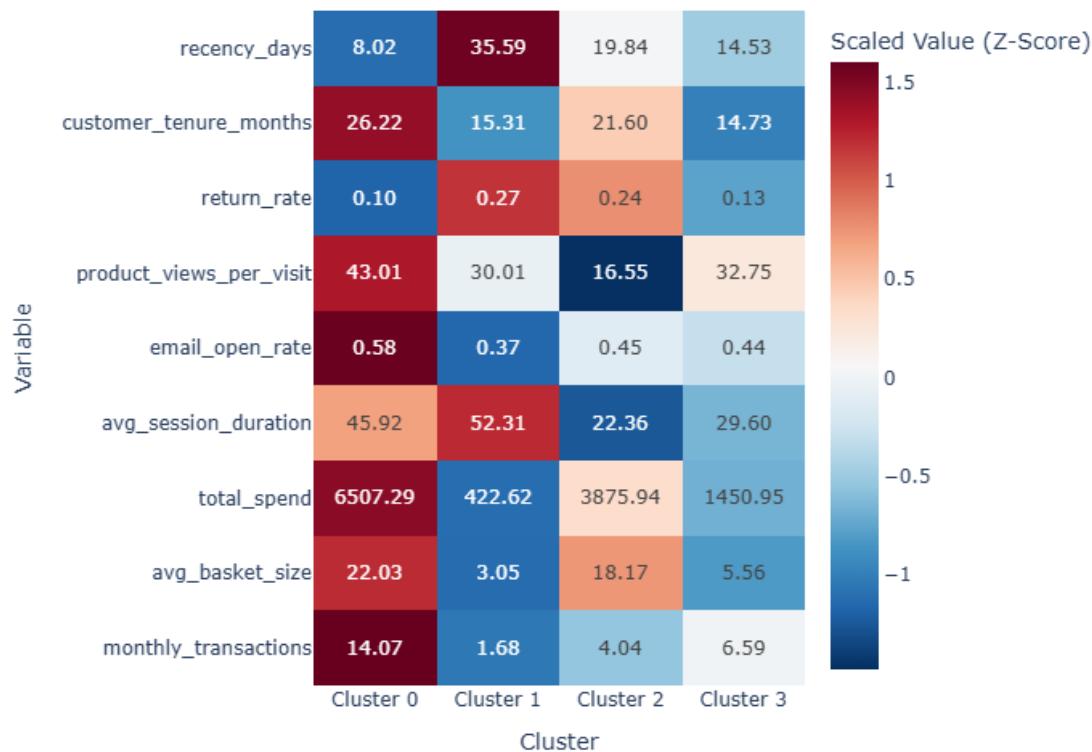
```

fig.update_layout(
    title_text='<b>Cluster Profiles (Means from Original Data, Colors Scaled per Variable)</b>',
    title_x = 0.5,
    xaxis_title='Cluster',
    yaxis_title='Variable',
    # Ensure all variable names are visible
    yaxis={'tickmode': 'array', 'tickvals': y_labels, 'ticktext': y_labels},
    height=600
)

fig.show()

```

Cluster Profiles (Means from Original Data, Colors Scaled per Variable)



[32]: behavioral_vars = list(behavioral_vars)
labels = sorted(list(df_labels['label'].unique()))

```
[33]: #Gemini helped in the creation of this plot

N_ROWS = len(labels)
N_COLS = len(behavioral_vars)

fig = make_subplots(
    rows=N_ROWS,
    cols=N_COLS,
    column_titles=list(behavioral_vars),
    shared_xaxes=False
)

for r_idx, label_val in enumerate(labels, 1):

    # Filter the DataFrame for the current label
    df_filtered_by_label = df_labels[df_labels['label'] == label_val]

    for c_idx, var_name in enumerate(behavioral_vars, 1):

        data_series = df_filtered_by_label[var_name]

        hist_trace = go.Histogram(
            x=data_series,
            name=f'L:{label_val}, V:{var_name}',
        )

        fig.add_trace(hist_trace, row=r_idx, col=c_idx)

if fig.layout.annotations:
    existing_annotations = list(fig.layout.annotations)
else:
    existing_annotations = []

row_label_annotations = [] # Renamed this list

try:
    v_spacing = fig.layout.vertical_spacing
except AttributeError:
    v_spacing = 0.02 # Default spacing

if v_spacing is None:
    v_spacing = 0.02

total_plot_height = 1.0 - (v_spacing * (N_ROWS - 1))

plot_height = total_plot_height / N_ROWS
```

```

for r_idx, label_val in enumerate(labels, 1):

    y_top = 1.0 - ( (r_idx - 1) * (plot_height + v_spacing) )

    y_bottom = y_top - plot_height

    # Midpoint of the row
    y_mid = (y_top + y_bottom) / 2

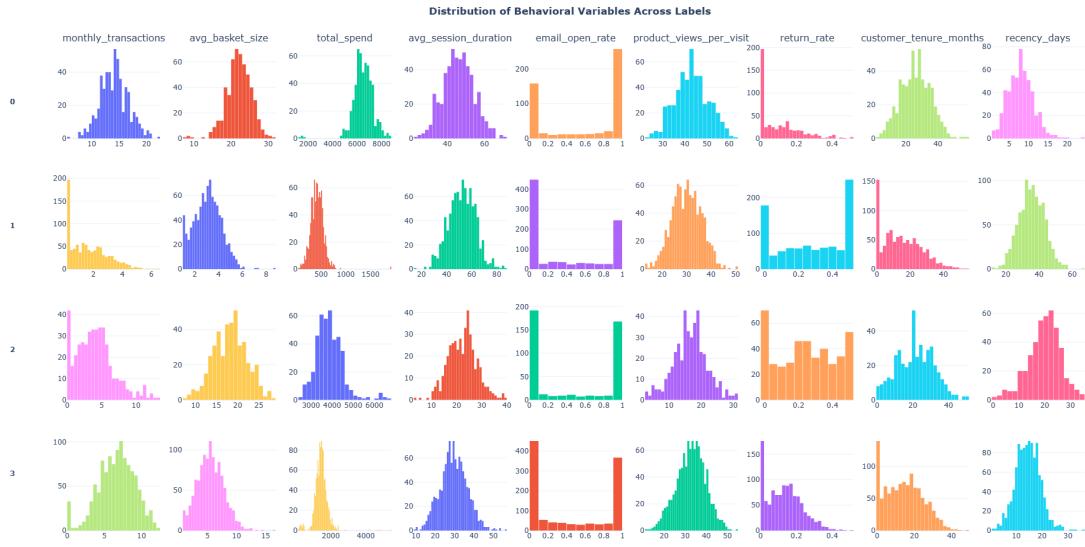
    row_label_annotations.append(
        go.layout.Annotation(
            text=f"<b>Label {label_val}</b>",
            align='right',
            showarrow=False,
            xref='paper',
            yref='paper',
            x=-0.05,
            y=y_mid,
            xanchor='right',
            yanchor='middle'
        )
    )

all_annotations = existing_annotations + row_label_annotations

fig.update_layout(
    title_text="Distribution of Behavioral Variables Across Labels",
    height=1000,
    width=1900,
    showlegend=False,
    bargap=0.05,
    annotations=all_annotations,
    margin=dict(l=100),
    title_x = 0.5
)

# Show the figure
fig.show()

```



0.11.1 Characterization

- **Cluster 0 (17.5%)**: This is MegaMart’s elite segment, defined by exceptionally high spending, large basket sizes, and frequent monthly transactions. They are also the most digitally engaged (high email open rates, session duration) and loyal, with long tenures and very low recency (they just shopped). Their low return rate makes them the most profitable and stable customer base.
- **Cluster 1 (30.9%)**: This is the least profitable group, with the lowest total spending, fewest transactions, and smallest basket sizes. They are defined by negative metrics: they have the highest recency (haven’t shopped in a long time) and a very high return rate. Despite spending a long time on the site, they are disengaged (low email open rate) and have low tenure, indicating they are likely to churn.
- **Cluster 2 (14.4%)**: This is MegaMart’s second-best spending segment, characterized by high total spend and the second-largest average basket size. They seem to be established customers (good tenure) who shop efficiently, as they have the lowest product views and shortest session durations. Their primary drawback is a high return rate, which cuts into their profitability.
- **Cluster 3 (37.1%)**: This segment consists of newer customers (low tenure) with modest total spending and small basket sizes. Their defining characteristic is frequency: they have the second-highest number of monthly transactions and low recency, suggesting they are active, casual shoppers. They are moderately engaged online and have a good low return rate, making them a high-potential group to nurture.

0.12 Cluster Validation - Silhouette Analysis

```
[34]: # Gemini helped with the code of this plot

cluster_labels = k_means_model.labels_

sample_silhouette_values = silhouette_samples(X_scaled_df, cluster_labels)

avg_silhouette_score = silhouette_score(X_scaled_df, cluster_labels)

print(f"Average Silhouette Score: {avg_silhouette_score:.4f}")


fig = go.Figure()

y_lower = 10
y_tick_positions = []

for i in range(selected_k):
    ith_cluster_silhouette_values = \
        sample_silhouette_values[cluster_labels == i]

    ith_cluster_silhouette_values.sort()

    n_samples_cluster = len(ith_cluster_silhouette_values)

    y_upper = y_lower + n_samples_cluster

    y_axis_values = np.arange(y_lower, y_upper)

    fig.add_trace(go.Bar(
        x=ith_cluster_silhouette_values,
        y=y_axis_values,
        name=f'Cluster {i}',
        orientation='h',
        marker=dict(line=dict(width=0)) # Hide lines between individual sample
        ↵bars
    ))

    y_tick_positions.append(y_lower + (n_samples_cluster / 2))

    y_lower = y_upper + 10


fig.add_vline(
    x=avg_silhouette_score,
    line=dict(color="red", width=2, dash="dash"),
```

```

annotation_text=f"Average Score: {avg_silhouette_score:.2f}",
annotation_position="bottom right"
)

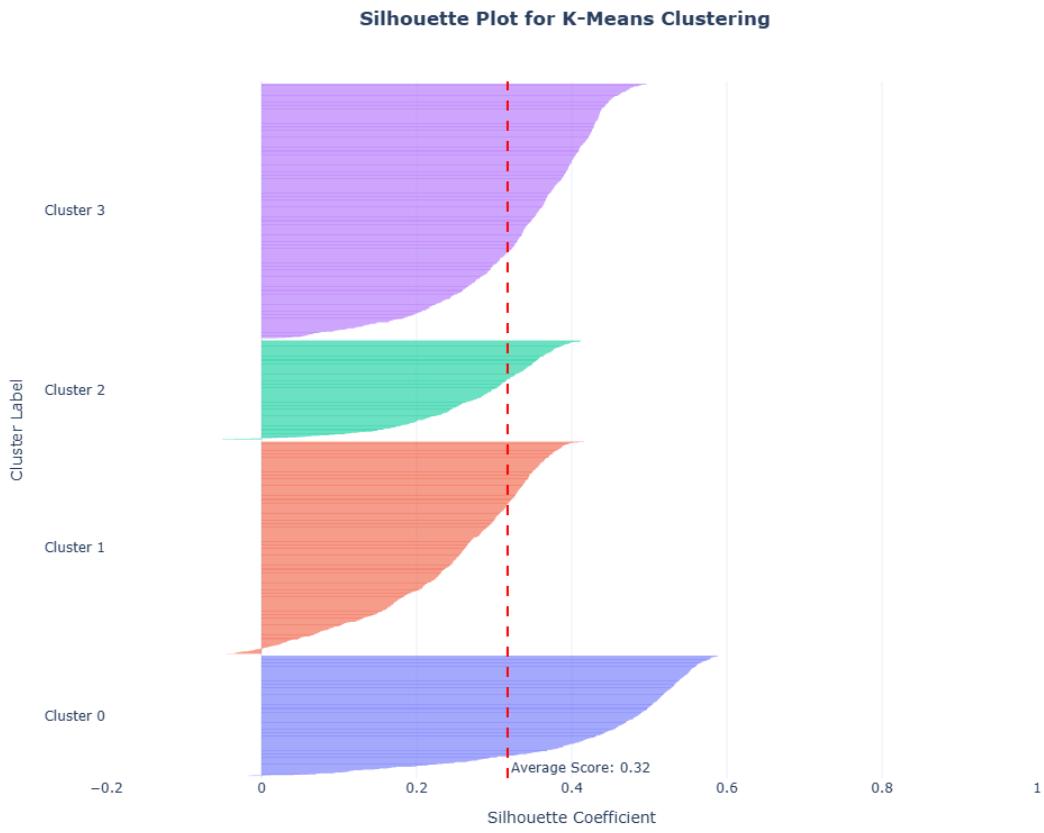
fig.update_layout(
    title_text="Silhouette Plot for K-Means Clustering",
    xaxis_title="Silhouette Coefficient",
    yaxis_title="Cluster Label",
    height=800,
    width=1000,
    title_x = 0.5,
    showlegend=False,
    xaxis=dict(range=[-0.2, 1])
)

fig.update_yaxes(
    tickvals=y_tick_positions,
    ticktext=[f"Cluster {i}" for i in range(selected_k)],
    showgrid=False,
    range=[0, y_lower]
)

fig.show()

```

Average Silhouette Score: 0.3173



Analysis of the silhouette plot for $k = 4$ reveals that Clusters 0 and 3 are relatively well-defined. These clusters are substantial in size, and a significant majority of their members possess silhouette scores exceeding the average score (0.32), indicating a reasonable fit.

Conversely, the plot also identifies a small population of poorly matched customers, represented by negative silhouette scores. These misclassified instances are present primarily within Clusters 0, 1, and 2. The existence of this small, poorly-fit group is an expected and understandable outcome, given the modest average silhouette score of 0.32, which already suggested that the cluster boundaries were not perfectly distinct and possessed some overlap.

Ultimately, this visualization helps to validate the selection of $k = 4$. It confirms that despite the known (and acceptable) limitations regarding cluster overlap, the chosen k value results in a structure where a clear majority of customers are appropriately assigned to their respective groups.

0.13 Cluster Visualization (PCA Projection)

```
[35]: pca = PCA(n_components=2)

pca_components = pca.fit_transform(X_scaled_df)
```

```
[36]: print("Eigenvalues (Explained variance):", pca.explained_variance_)
print("Eigenvectors (Principal components):\n", pca.components_)
print("Explained variance ratio (%):", pca.explained_variance_ratio_ * 100)
```

```
Eigenvalues (Explained variance): [3.6917287  1.88808201]
Eigenvectors (Principal components):
[[ 0.4724455  0.43265423  0.46747061 -0.02009984  0.1527114   0.27815479
 -0.20188386  0.30779   -0.36672899]
 [ 0.02105255 -0.13967362 -0.13359173  0.63931774  0.32477758  0.44260315
  0.12199959  0.34141798  0.3473413 ]]
Explained variance ratio (%): [41.00553471 20.97169611]
```

```
[37]: df_pca = pd.DataFrame(data=pca.components_, columns=['PC1', 'PC2'])
df_pca['label'] = df_labels['label'].astype(str)
```

```
df_pca
```

```
[37]:      PC1      PC2  label
0    -1.367173  0.319297    1
1    -1.502054 -2.390591    3
2    -0.868623  2.532739    1
3     3.144846 -0.692923    0
4     0.290450 -1.521831    2
...
2995  -1.853044  0.611707    1
2996   0.174695 -0.275909    3
2997  -1.211336 -1.699469    3
2998  -0.236893 -0.955446    3
2999  -0.447009 -0.682879    3
```

```
[3000 rows x 3 columns]
```

```
[38]: # Gemini helped with the visualization in plotly

centroids_original_space = k_means_model.cluster_centers_
centroids_pca_space = pca.transform(centroids_original_space)

df_centroids_pca = pd.DataFrame(centroids_pca_space, columns=['PC1', 'PC2'])
df_centroids_pca['label'] = [str(i) for i in range(selected_k)]

colors = px.colors.qualitative.Plotly
fig = go.Figure()
for i, label in enumerate(sorted(df_pca['label'].unique())):
    df_cluster = df_pca[df_pca['label'] == label]

    fig.add_trace(go.Scatter(
        x=df_cluster['PC1'],
```

```

y=df_cluster['PC2'],
mode='markers',
marker=dict(
    color=colors[i % len(colors)],
    size=8
),
name=f'Cluster {label}'
))

fig.add_trace(go.Scatter(
x=df_centroids_pca['PC1'],
y=df_centroids_pca['PC2'],
mode='markers',
marker=dict(
    color='black',
    size=10,
    symbol='cross',
    line=dict(width=4)
),
name='Centroids'
))

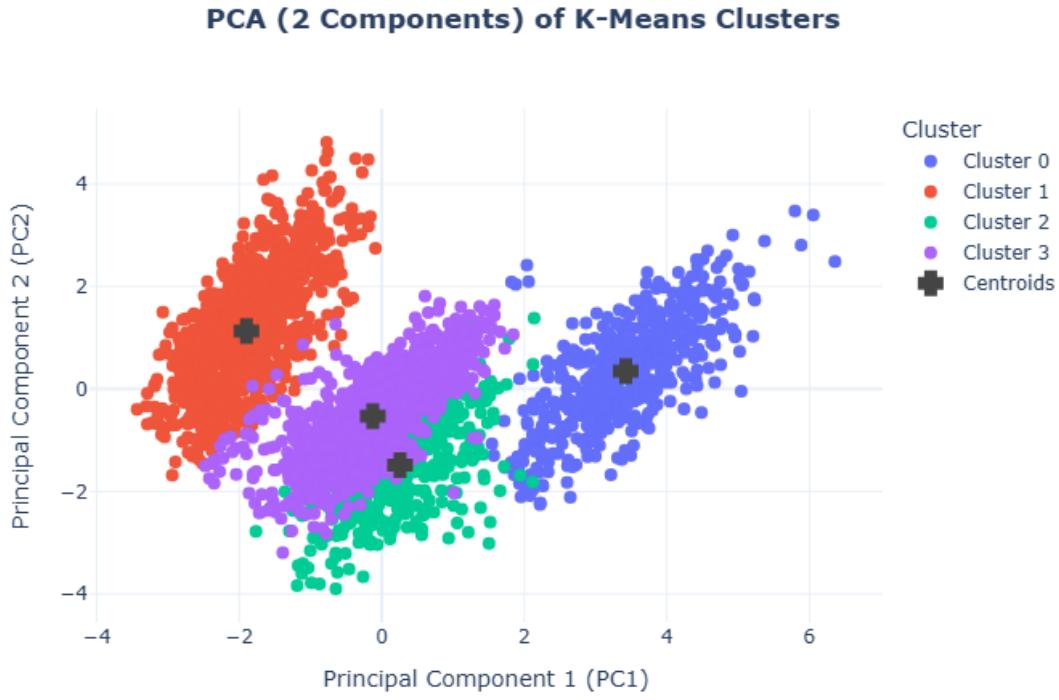
fig.update_layout(
    title='<b>PCA (2 Components) of K-Means Clusters</b>',
    xaxis_title='Principal Component 1 (PC1)',
    yaxis_title='Principal Component 2 (PC2)',
    legend_title_text='Cluster',
    hovermode="closest",
    title_x = 0.5
)

fig.show()

```

c:\Users\crdie\Documents\Tec\5Semestre\Multivariados_CienciaDatos\MegaMart-Customer-Segmentation-Analysis\.venv\Lib\site-packages\sklearn\utils\validation.py:2749: UserWarning:

X does not have valid feature names, but PCA was fitted with feature names



The visualization is a two-dimensional projection, derived from Principal Component Analysis (PCA), which accounts for only 60% of the total variance.

It is important to note that this is a simplified representation; the clusters themselves are defined within the original 9-dimensional feature space. Despite this significant reduction in dimensionality, the 2D plot serves as a practical tool for illustrating the approximate separation and relative positioning of the identified clusters.

0.14 Technical Conclusion

The clustering analysis helped uncover meaningful patterns in customer behavior. To determine the most appropriate number of groups, several validation techniques were applied, and all of them consistently indicated that the best structure is achieved with four clusters ($k = 4$).

The Ward dendrogram showed that cutting the hierarchy into four groups provides a reasonable separation between the major branches, suggesting that this number of clusters captures genuine distinctions within the data. This observation aligned with the Elbow Method, where the inertia curve displayed its most noticeable “bend” at $k = 4$, indicating that adding more clusters beyond this point does not significantly improve the compactness of the groups.

The silhouette score further supported this result. While the highest score appeared at $k = 2$, that solution is too general and does not meaningfully differentiate customer types. Among the more balanced options, $k = 4$ achieved an average silhouette score of about 0.32, which—although not very high—still reflects an acceptable level of cluster separation. The silhouette plot also showed

that most customers fit well within their assigned cluster, with only a small number displaying negative scores, which is normal in real-world data where some overlap between profiles is expected.

The cluster profiling analysis confirmed that the groups identified are not only statistically valid but also practically meaningful. Using the heatmap and cluster means, it was possible to distinguish clear behavioral patterns, such as high-value customers, occasional buyers, and users who browse frequently but convert poorly. This indicates that the segmentation provides useful insight for interpreting different types of customer behavior.

The PCA visualization offered additional support for the decision. The first two components explained over 60% of the total variance, and in this reduced space at least one cluster showed strong separation, while the others maintained reasonable boundaries. The PCA also highlighted that $k = 3$ produces stronger overlap, confirming that it does not capture the structure of the data as effectively as $k = 4$.

Overall, the technical and visual evidence leads to the same conclusion: $k = 4$ is the number of clusters that best represents the underlying patterns in this dataset. This choice provides a well-balanced, interpretable, and practically useful segmentation for future strategic decisions related to customer behavior and value.