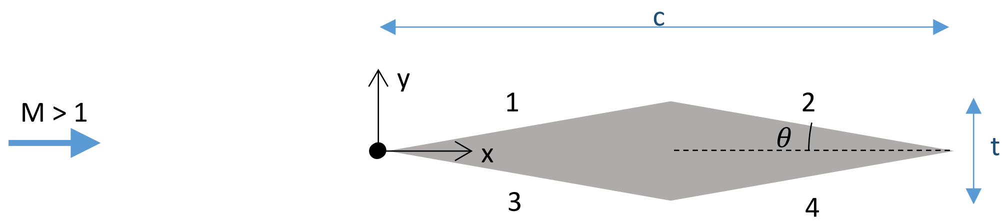
Center of Pressure Derivation



$$CP_x = \frac{\int x\, p_{net}(x)dx}{\int p_{net}(x)dx}, p_{net} = p_{top} - p_{bottom}$$

For a symmetric diamond airfoil,

$$p_{net}(x) = \begin{cases} (P_1 - P_3)\cos(\theta)\,, x < \frac{c}{2} \\ (P_2 - P_4)\cos(\theta)\,, x \geq \frac{c}{2} \end{cases}$$

Plugging this into the equation for CP_x,

$$CP_x = \frac{\int_0^{c/2} x(P_1 - P_3)\cos(\theta)\,dx + \int_{c/2}^{c} x(P_2 - P_4)\cos(\theta)\,dx}{\int_0^{c/2}(P_1 - P_3)\cos(\theta)\,dx + \int_{c/2}^{c}(P_2 - P_4)\cos(\theta)\,dx}$$

$$CP_x = \frac{\frac{1}{8}(P_1 - P_3)c^{\cancel{2}}\cancel{\cos(\theta)} + \frac{3}{8}(P_2 - P_4)c^{\cancel{2}}\cancel{\cos(\theta)}}{\frac{1}{2}(P_1 - P_3)\cancel{c\cos(\theta)} + \frac{1}{2}(P_2 - P_4)\cancel{c\cos(\theta)}}$$

$$\frac{CP_x}{c} = \frac{\frac{1}{4}(P_1 - P_3) + \frac{3}{4}(P_2 - P_4)}{P_1 - P_3 + P_2 - P_4}$$

Similarly, where

$$CP_y = \frac{\int y\, p_{net}(y)dy}{\int p_{net}(y)dy}, p_{net} = p_{front} - p_{back}$$

We can show that

$$\frac{CP_y}{t} = \frac{P_1 - P_2 - P_3 + P_4}{2(P_1 - P_2 + P_3 - P_4)}$$

For the case of a symmetric airfoil at 0° angle of attack, the net vertical pressure is zero, and the above equation for CP_x is singular. However, we can still calculate a center of pressure using only the top surface pressures:

$$\left|\frac{CP_x}{c}\right|_{\alpha=0} = \frac{\frac{1}{4}P_1 + \frac{3}{4}P_2}{P_1 + P_2}$$