Number 5 text:

The basis for this report is the Schlieren imaging technique that takes advantage of the normally imperceptible changes in the index of refraction of gases that occur due to changes in destiny. Light from a point source is reflected and made parallel by a concave parabolic mirror. This light passes through a section of air and refracted light is disturbed from its straight path. Half of this disturbed light is blocked by a razors edge before it reaches a camera. The resulting image has the disturbed light showing up as brighter spots where it converges and darker spots where it has been diverged away from. This results in the image displaying the changes in the air's density like in the shocks and expansion fans in this report.

Similarly important are the relations of shocks. If a compressible gas moves around an object, for low subsonic Mach numbers, it will simply flow around it. However, if that flow is going faster than the speed of sound and the change in properties like area are rapid, then a shock is formed. These changes in flow properties are non-isentropic as the changes in temperature and velocity are large within the shock itself. The specific relation used in this report are the oblique shock relations shown below:

$$\tan(\theta) = 2\cot(\beta)\frac{{M_1}^2 sin^2(\beta) - 1}{{M_1}^2(\gamma + \cos(2\beta)) + 2}$$

The theta-beta-Mach relation for predicting the angle "beta" the shock will form at given an incoming Mach number M_1 and turn angle the flow is making "theta".

$$M_{1N} = M_1 \sin(\beta)$$

$$ {M_{2N}}^2 = \frac{1 + [\frac{\gamma - 1}{2}]{M_{1N}}^2}{\gamma {M_{1N}}^2 - [\frac{\gamma - 1}{2}]}$$

$$M_2 = \frac{M_{2N}}{\sin(\beta - \theta)}$$

The normal shock relation is used with the assumption that the change in Mach number across the shock only occurs to the normal portion of the flow. This is used to predict the post-shock Mach number "M_2" given an incoming Mach number "M_1" across an oblique shock. The change in static pressure across the shock is modeled by:

$$P_2 = P_1[1 + \left(\frac{2\gamma}{\gamma + 1}\right)\left({M_{1N}}^2 - 1\right)]$$

Where P_1 is the freestream static pressure before the shock and P_2 being the post shock static pressure.

Following the shock on the diamond airfoils will be a Prandtl-Meyer expansion fan for the flow going over the middle section of the diamond. The flow has to turn into and expand around an angle "theta" and the relation of the pre and post fan Mach number is given by:

$$\theta = v(M_2) - v(M_1)$$

$$v = \sqrt{\frac{\gamma+1}{\gamma-1}}tan^{-1}\left(\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}\right) - tan^{-1}\left(\sqrt{M^2-1}\right)$$

Where “nu” is a function of the flow’s Mach number. The resulting Mach number can then give us the post-turn static pressure through the isentropic relations that connect a flow’s Mach number to its properties since Prandtl-Meyer fans are isentropic and thus the stagnation properties are constant through the fan.

$$P_{01} = P_1\left[1+\left(\frac{\gamma-1}{2}\right){M_1}^2\right]^{\frac{\gamma}{\gamma-1}} = P_{02}$$

$$P_2 = \frac{P_{02}}{\left[1+\left(\frac{\gamma-1}{2}\right){M_2}^2\right]^{\frac{\gamma}{\gamma-1}}}$$

The coefficients of sectional lift and drag are found from distributions of pressure across the 4 faces of the diamond to ascertain some lift and drag forces and then combined with the standard expressions of lift and drag:

$$l = \frac{1}{2}\rho_\infty {U_\infty}^2 C_l c$$

$$d = \frac{1}{2}\rho_\infty {U_\infty}^2 C_d c$$

Which then turn into:

$$C_l = \frac{1}{\gamma P_\infty {M_\infty}^2 \cos(\theta)}[(P_3-P_2)cos(\theta+\alpha)+(P_4-P_1)cos(\theta-\alpha)]$$

$$C_d = \frac{1}{\gamma P_\infty {M_\infty}^2 \cos(\theta)}[(P_1-P_4)sin(\theta-\alpha)+(P_3-P_2)sin(\theta+\alpha)]$$

Derivations of which can be found in the appendix of this report.

The center of pressure for the airfoil are determined by:

$$\frac{CP_x}{c} = \frac{\frac{1}{4}(P_1-P_3)+\frac{3}{4}(P_3-P_4)}{P_1-P_3+P_2-P_4}$$

$$if\ \alpha = 0, \qquad \frac{CP_x}{c} = \frac{\frac{1}{4}P_1+\frac{3}{4}P_2}{P_1+P_2}$$

$$\frac{CP_y}{c} = \frac{P_1-P_2-P_3+P_4}{2(P_1-P_2+P_3-P_4)}$$

Where the static pressures of the faces are: