Shadowgraph

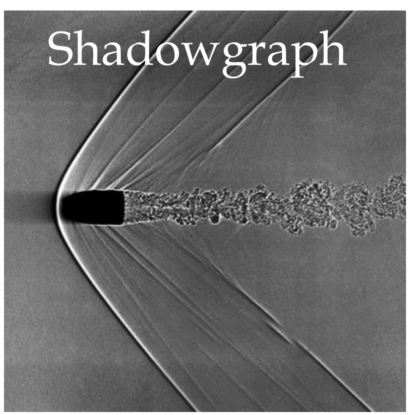
Administrative

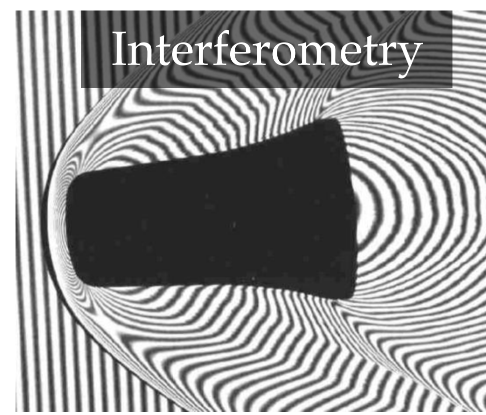
- Lab 1 is due two weeks from your lab time.

Interferometric Techniques

- Broad class of imaging techniques used for flow visualization
  - Rely on the wave properties of light to cause optical interference (hence the name)
  - Light is bent by density gradients in the flow, providing a visualization of the flow features
  - Some common interferometric techniques:
    - Interferometry
    - Schlieren
    - Shadowgraph

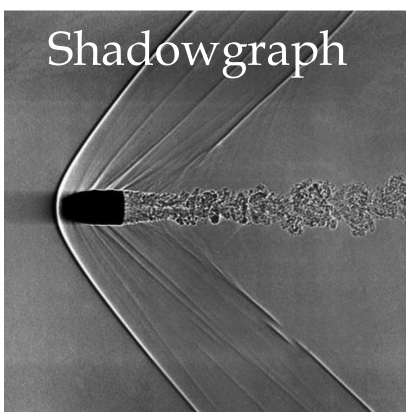
Information Sensitivity Complexity

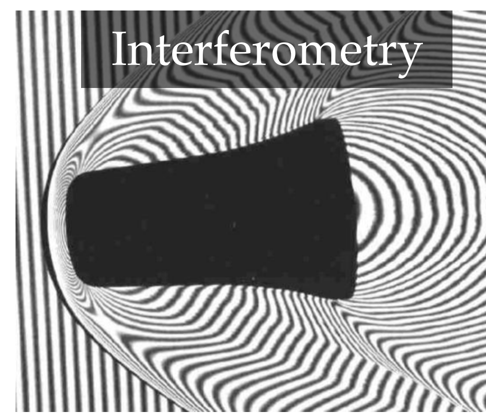




Interferometric Techniques

- A word of caution about experimental results:
  - Always make sure you understand the technique you are using/citing!
  - Experimental data is often used for validating simulations and to improve understanding of flow physics
  - If the experimental technique is not understood, the data may be misinterpreted



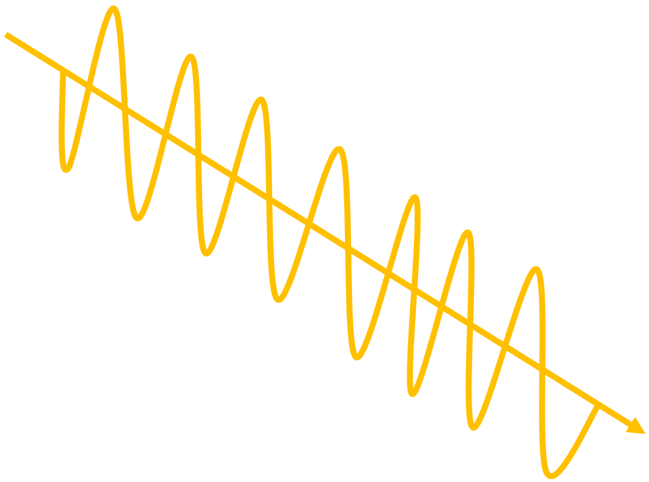


Optics

- Before we can discuss the shadowgraph technique for flow visualization, it is important to have a comfort level with some basic concepts on the nature of light

Optics

- On the macroscopic scale, light can be treated as a propagating wave
  - Amplitude = A
  - Wavelength = λ
  - Polarization = P
  - Speed = c
  - Frequency = ν



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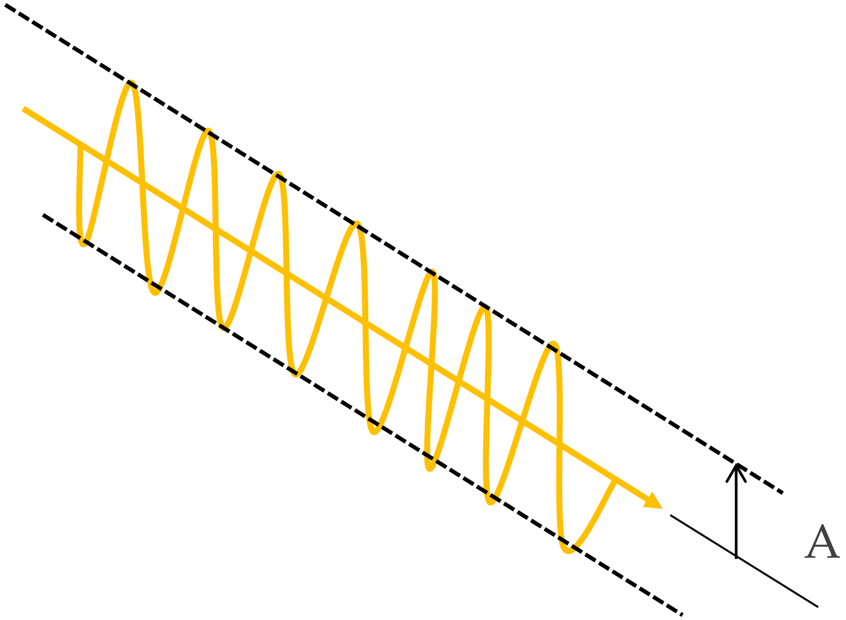

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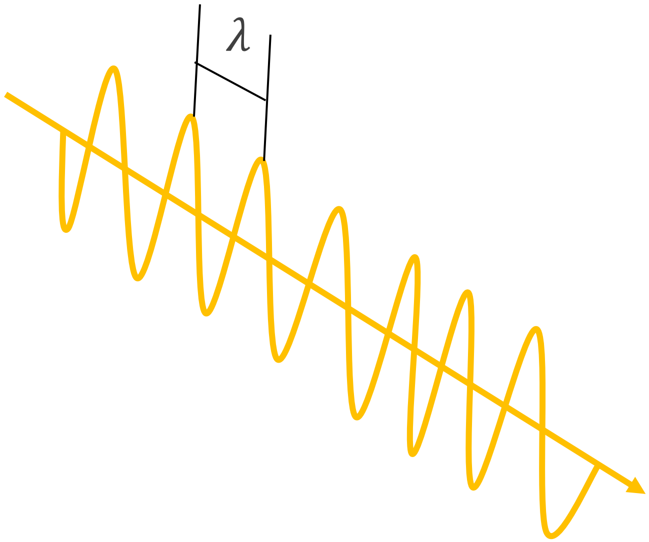
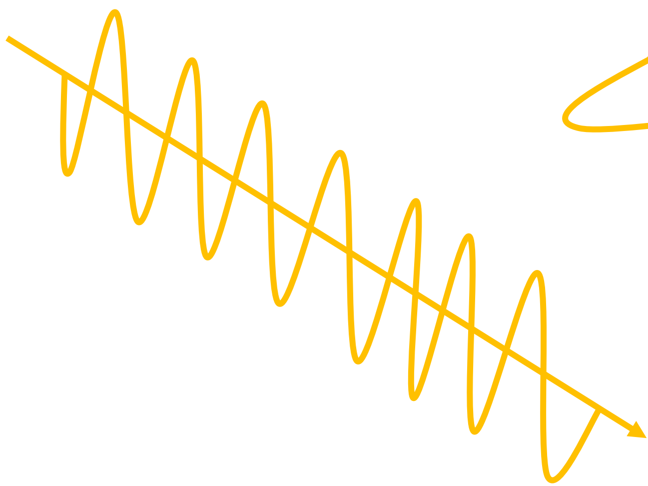
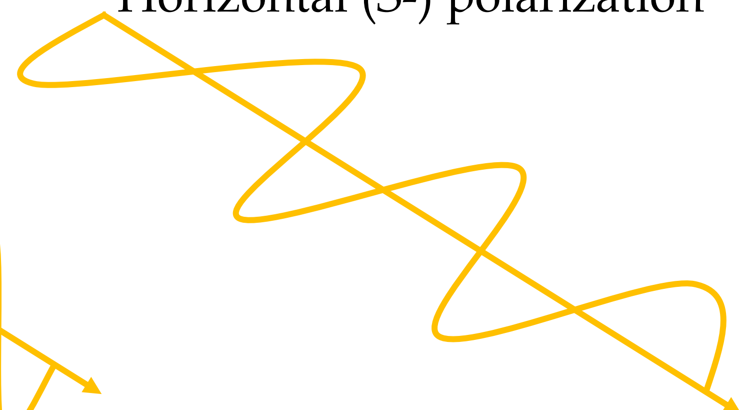


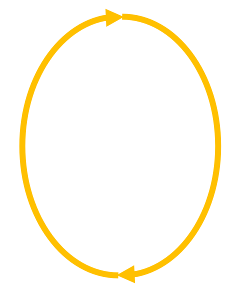
Optics

- On the macroscopic scale, light can be treated as a propagating wave
    - Amplitude = A
    - Wavelength = λ
    - **Polarization = P**
    - Speed = c
    - Frequency = ν

$P\rightarrow$
Horizontal (S-) polarization





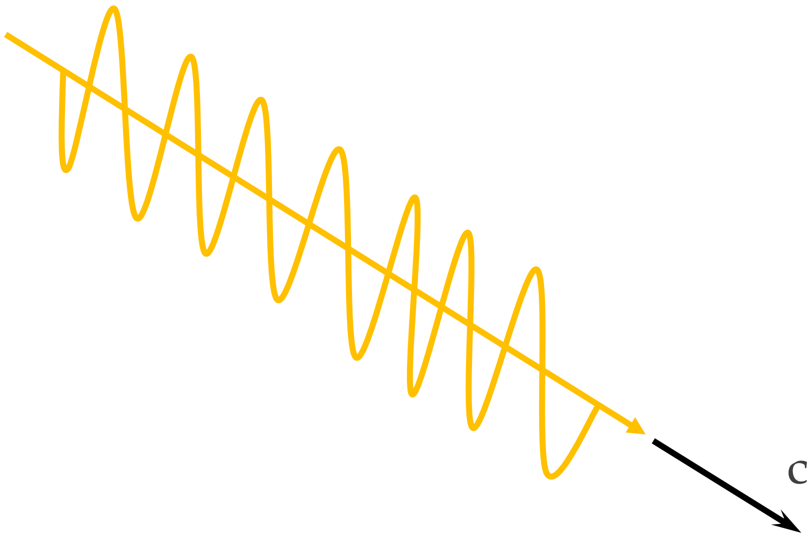


$P\uparrow$
Vertical (P-) polarization

$aP\uparrow + bP\rightarrow$
Elliptic polarization

Optics

- On the macroscopic scale, light can be treated as a propagating wave
  - Amplitude = A
  - Wavelength = λ
  - Polarization = P
  - **Speed = c**
  - Frequency = ν



Optics

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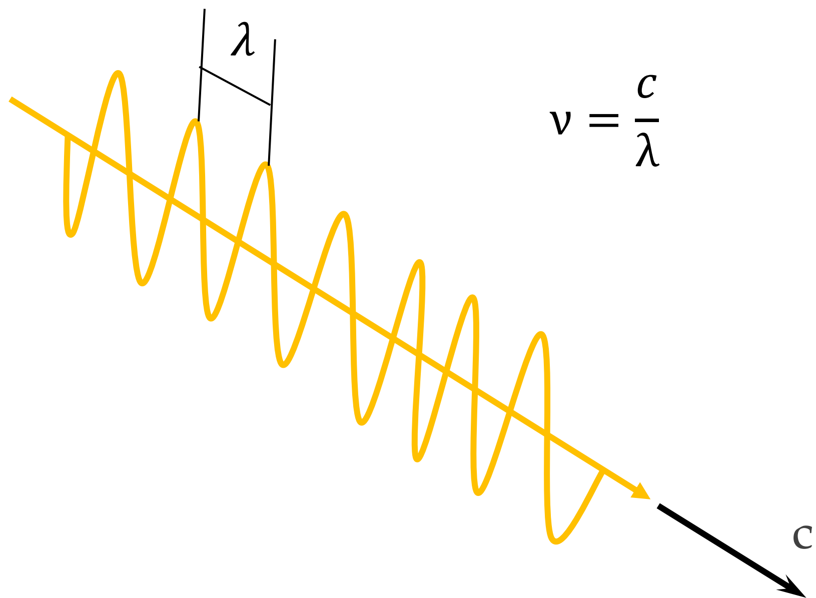
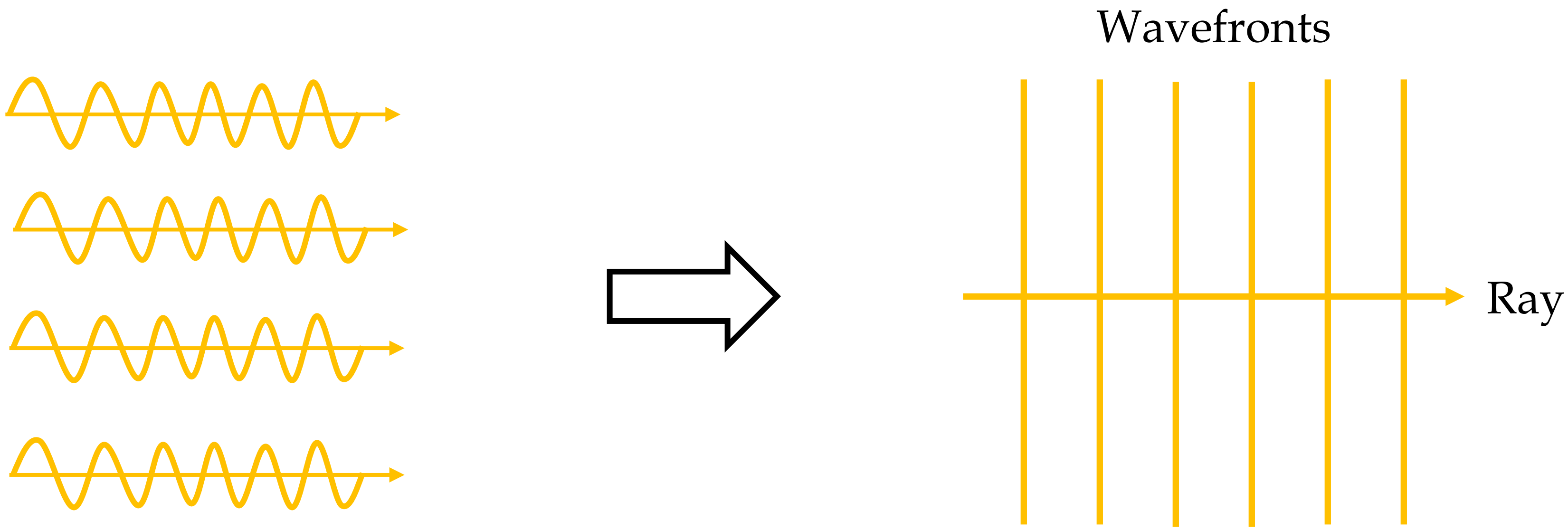
    - Speed = c
    - **Frequency = ν**



$$\nu = \frac{c}{\lambda}$$

Optics

- Can also think of light waves in terms of rays and wavefronts
  - Rays are vectors pointing along the direction the wave is travelling
  - Wavefronts symbolize parts of the wave that are in phase, are perpendicular to rays



Optics

- Can also think of light waves in terms of rays and wavefronts
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  - Consider a cylindrical lens

Cylindrical Lens

Optics

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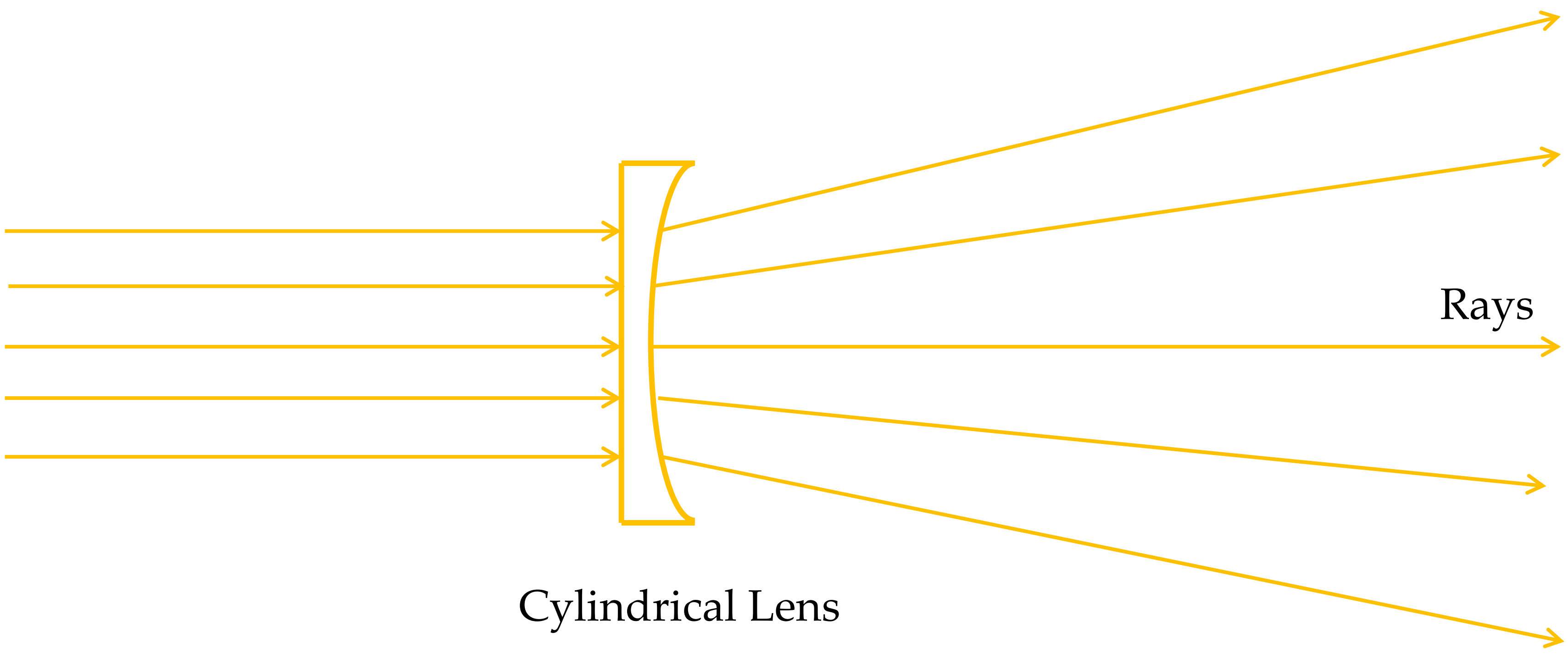

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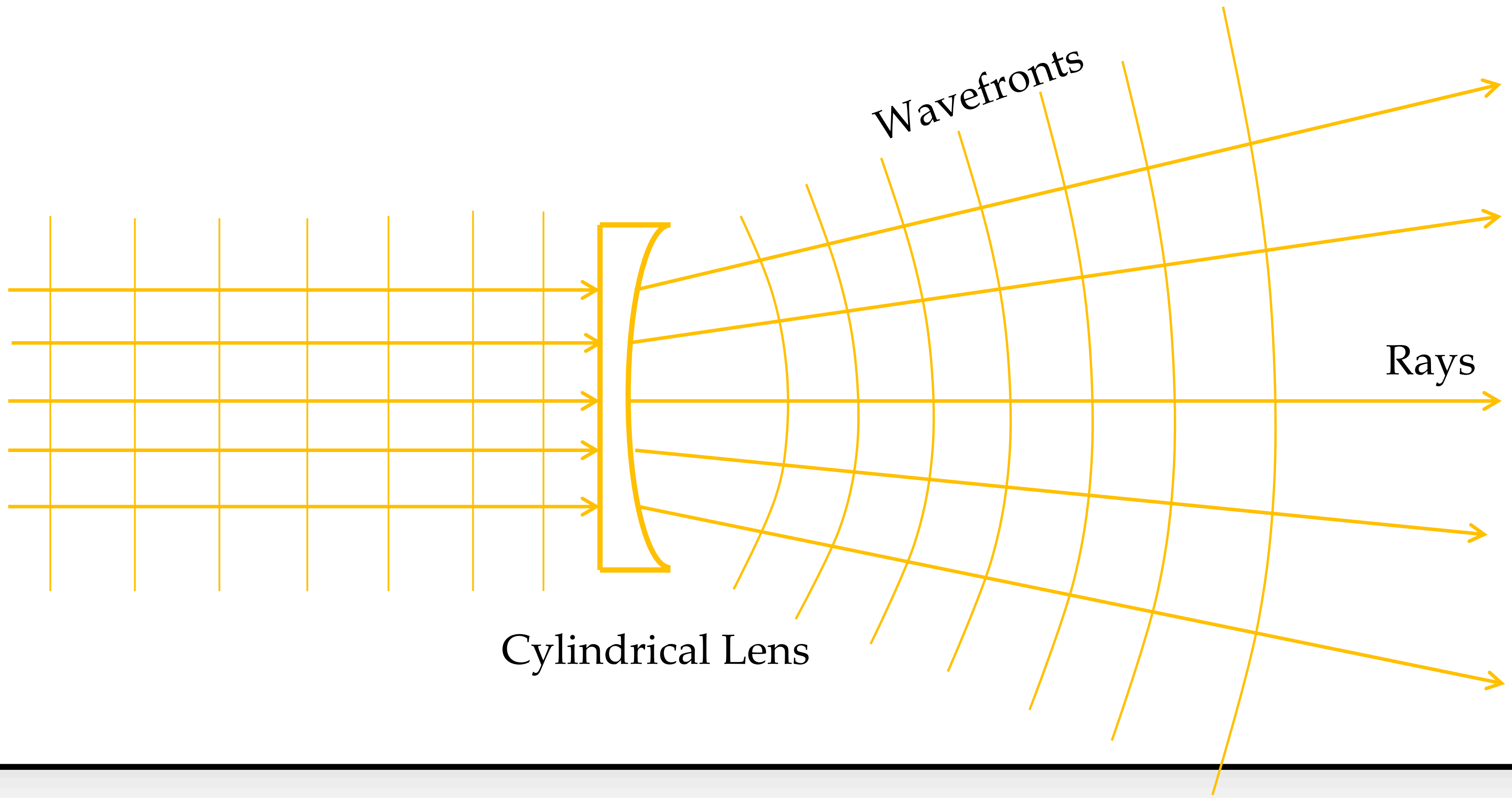
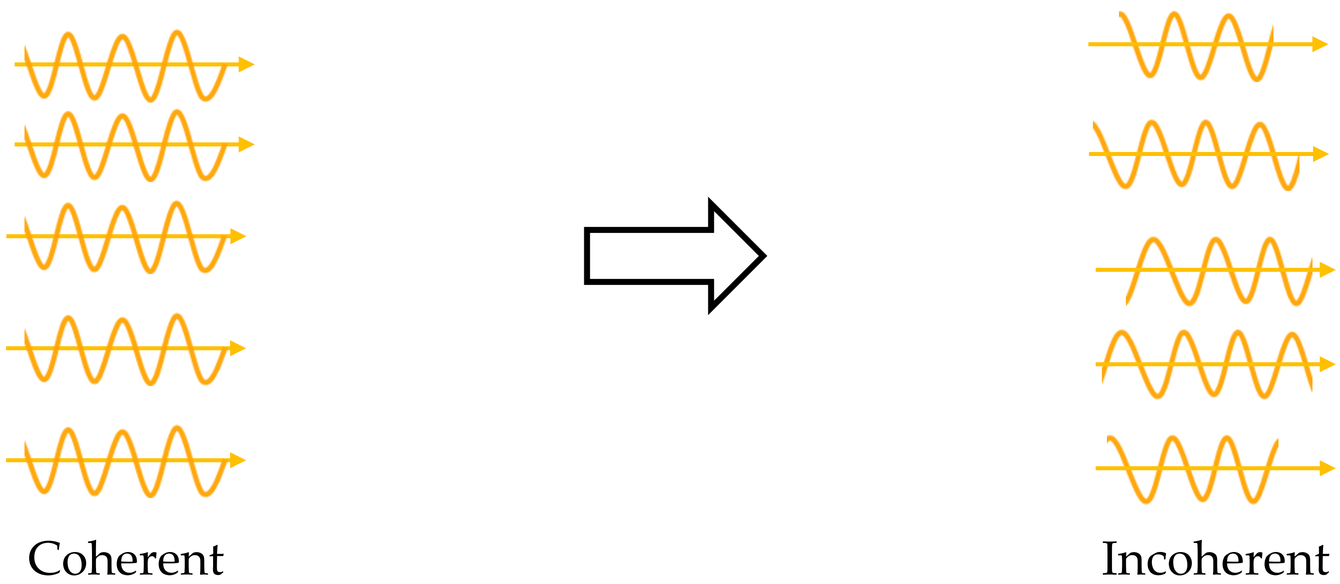


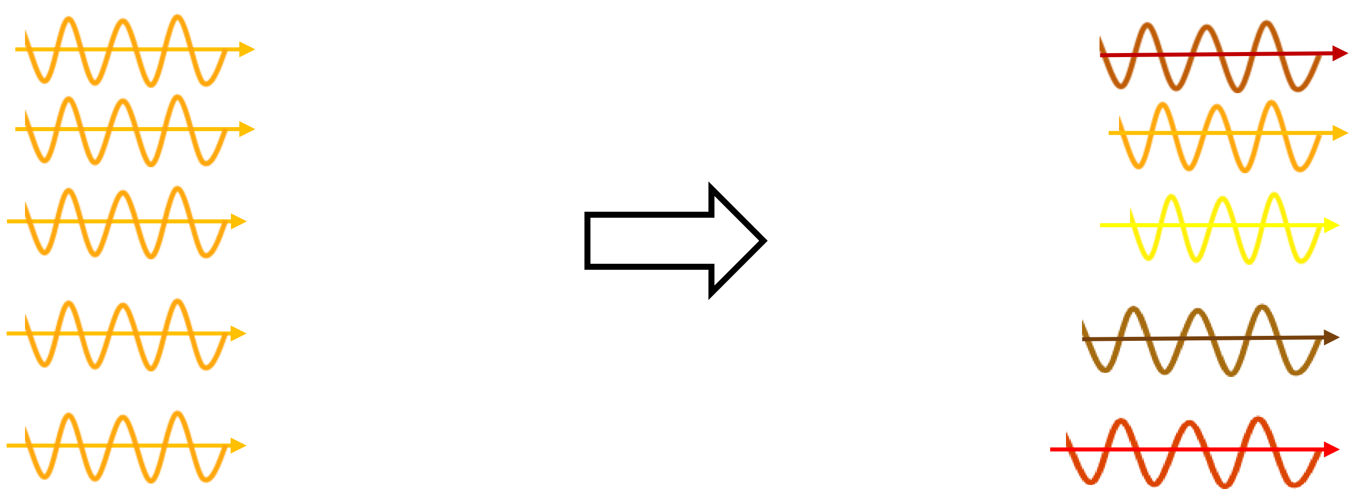
Optics

- Some other important properties of light:
  - Coherence
    - A measure of how close light is to being in phase
    - Most light is incoherent
    - Lasers produce coherent light



Optics

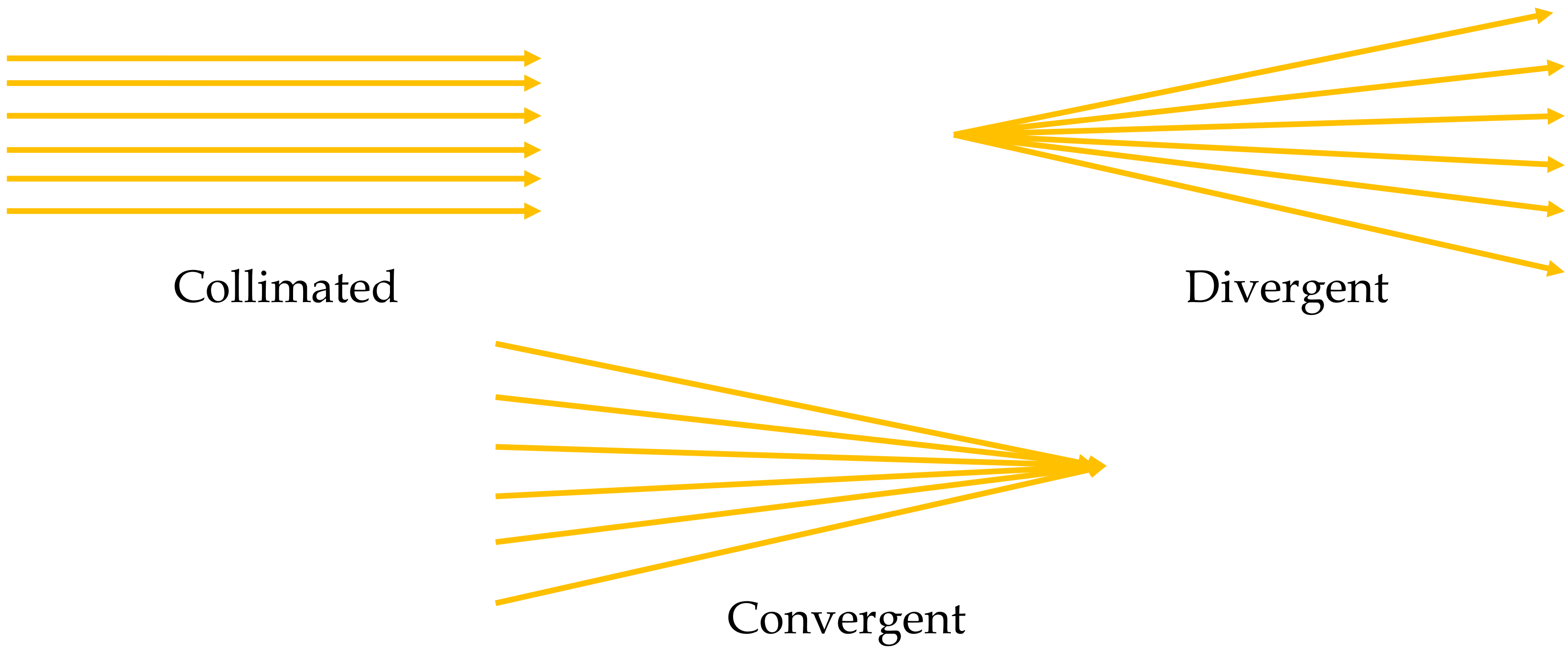
- Some other important properties of light:
  - Chromatism
    - Measure of how close the light is to being “one color”
    - Monochromatic – all light is of the same wavelength
      - Not physically possible due to quantum mechanics, but some lasers and LEDs are a close approximation
    - Sometimes called “bandwidth” or “linewidth”



Monochromatic   Polychromatic

Optics

- Some other important properties of light:
  - Collimation – how parallel the light rays are



Optics

- Some other important properties of light:
  - Collimation – how parallel the light rays are
  - Most light sources are “diffuse,” meaning the rays do not share a common origin

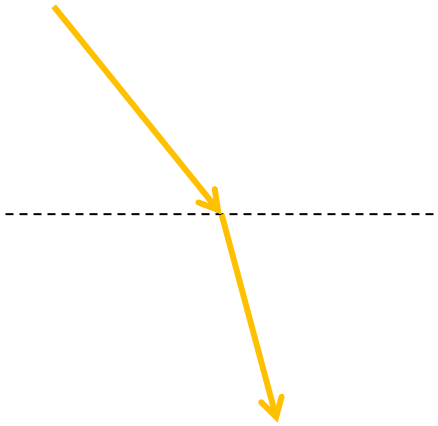
Collimated

Divergent Point Source

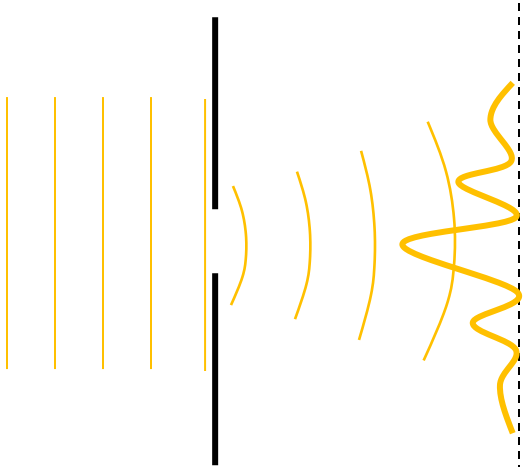
Divergent Diffuse Source

Optics

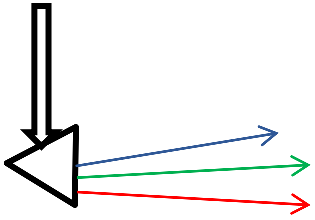
- Some other important properties of light:
  - Refraction – deflection of a wave
  - Diffraction – distortion of a wavefront due to a small aperture
  - Dispersion – separation of wavelengths



Refraction



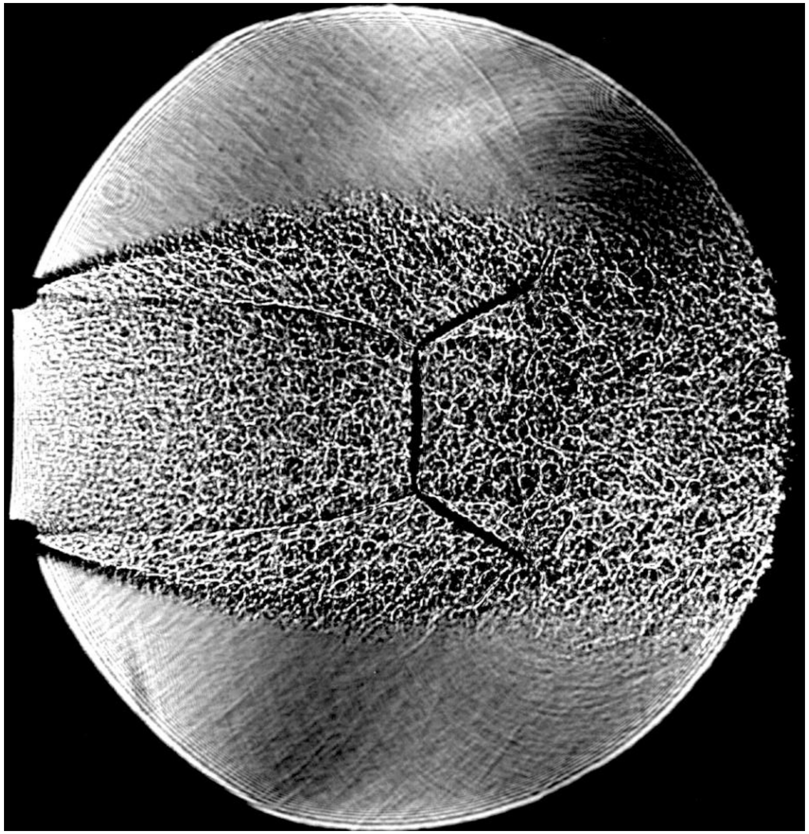
Diffraction



Dispersion

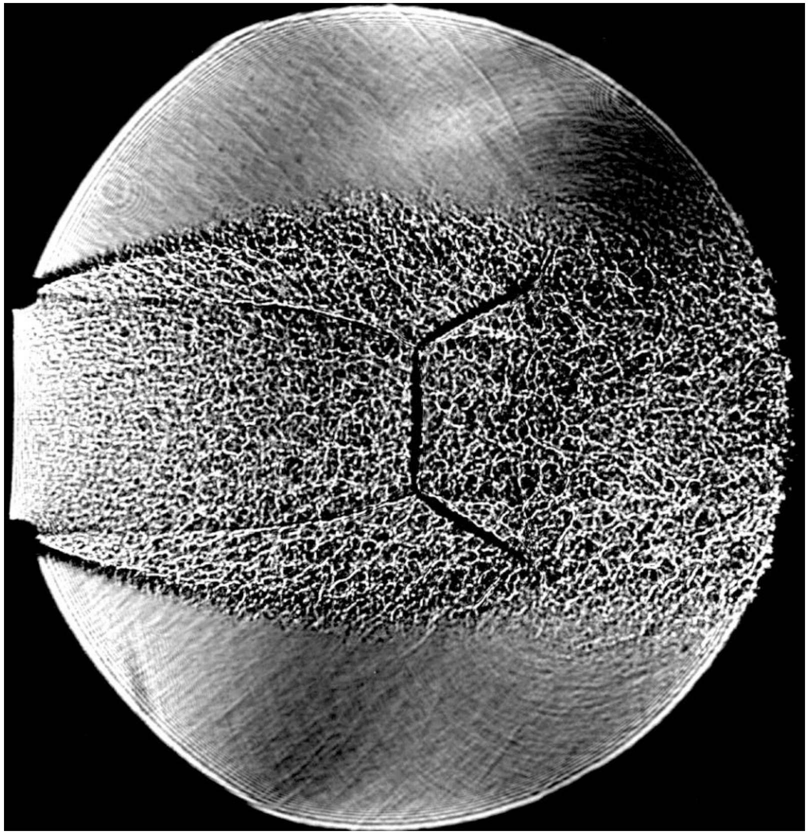
Shadowgraph

- Most basic interferometric technique
- Literally the imaging of shadows
- Commonly occurs in nature

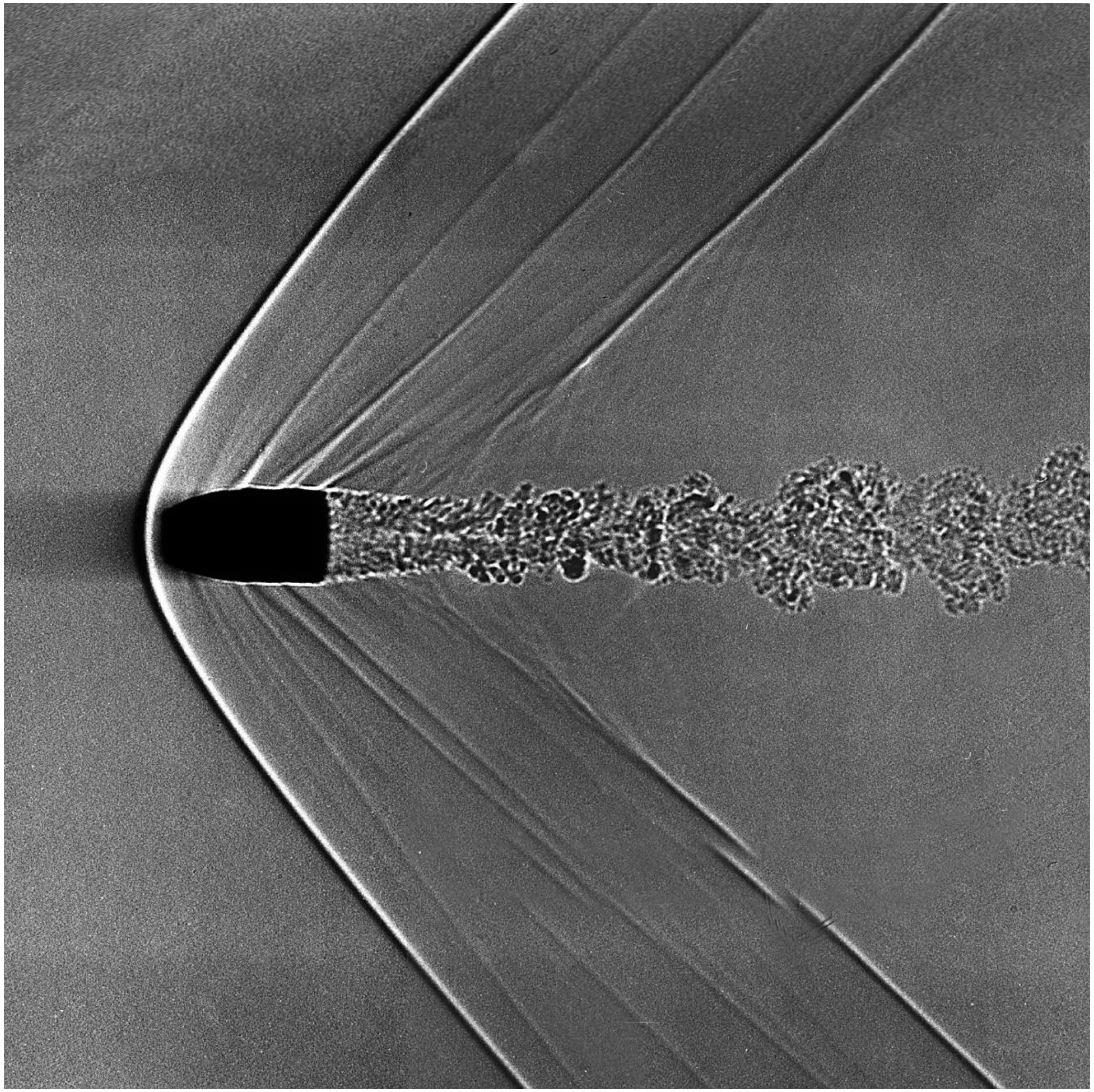


Shadowgraph

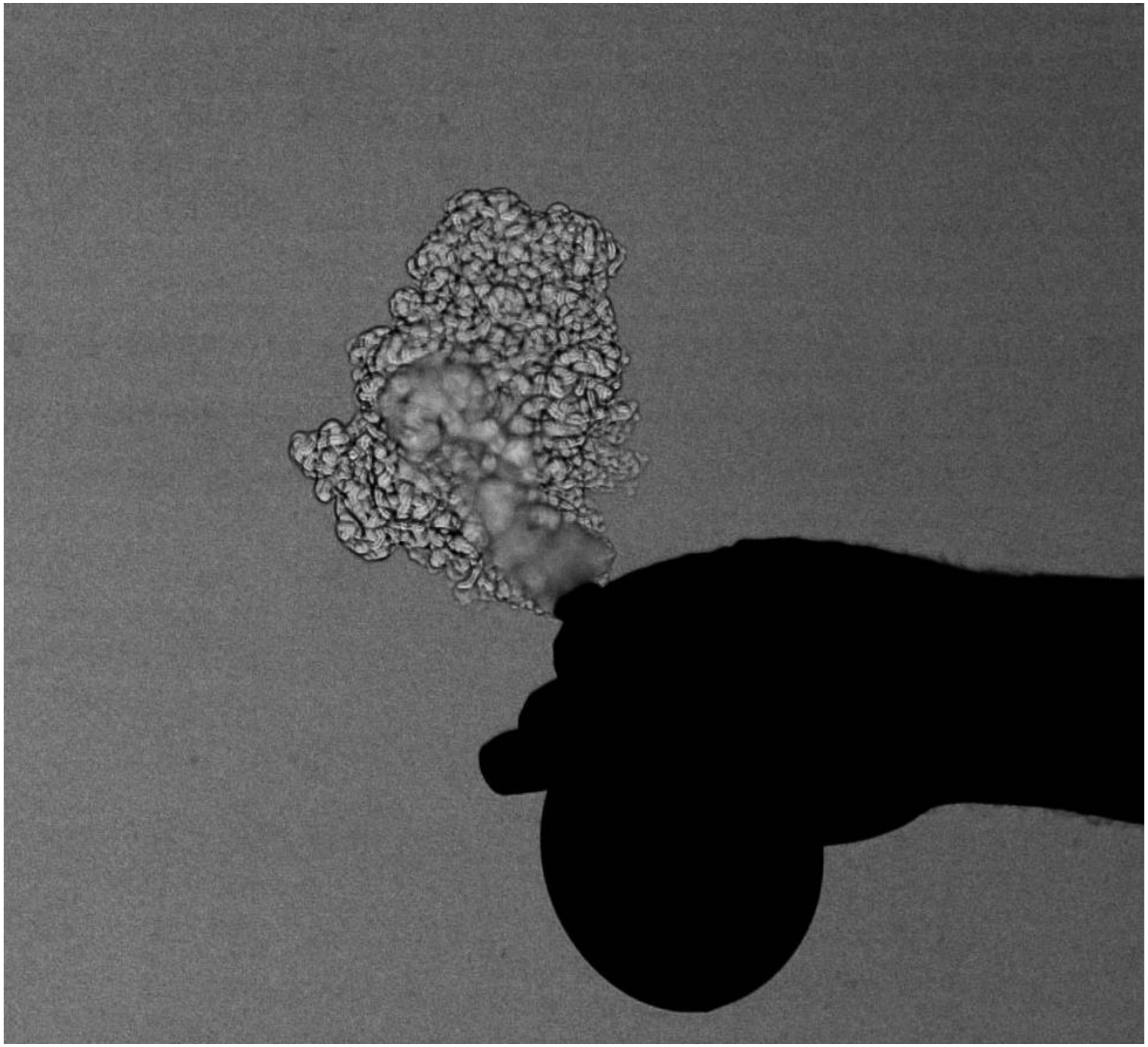
- Basic concept
  - Shine a light through the flow of interest
  - Gradients in the flow refract the light
  - Record the resulting image with a camera



Shadowgraph



Shadowgraph



Operating Principle

- The shadowgraph technique relies on changes in the index of refraction, *n*, of the medium in which light waves are propagating

$$n = \frac{c_o}{c} = \frac{speed\ of\ light\ in\ vacuum}{speed\ of\ light\ in\ medium}$$

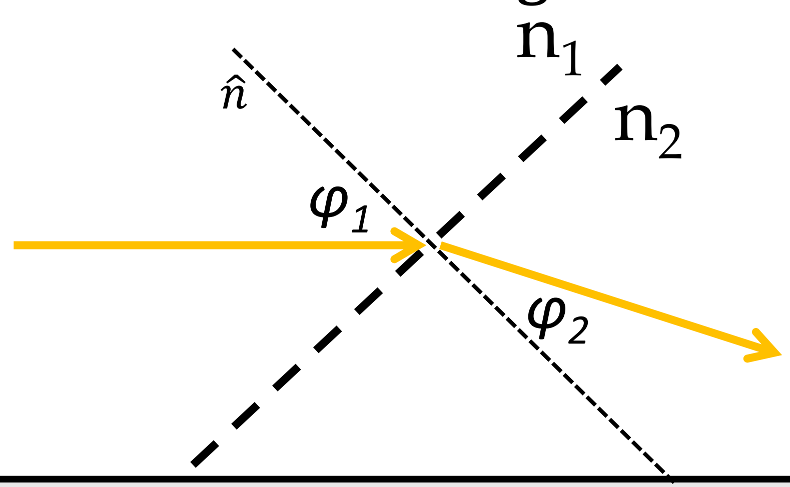
- Changes in *n* result in a refraction (bending) of incoming light rays
- Changes in *n* can be caused by changes in
  - Media (consider the surface of a body of water)
  - The density of the medium
  - The temperature of the medium
  - The incident wavelength of light

Snell's Law

- Refraction at a simple interface can be predicted using Snell's Law
- Consider light intersecting a boundary between two media (n_1 and n_2) at an angle φ_1
- Light will refract at angle φ_2 defined by:

$$n_1 \sin \varphi_1 = n_2 \sin \varphi_2$$

- If $n_2 > n_1$ it must be that $\varphi_1 > \varphi_2$
- **This means light bends towards regions of higher n**



Operating Principle

- In the flows of interest in this class, changes in index of refraction will be caused primarily by changes in gas density
- We can relate density to index of refraction using the Gladstone-Dale relation
  - For gases,

    $$n - 1 = \kappa\rho$$

  - Where κ is the Gladstone-Dale constant (but it depends on gas, temperature, and light wavelength)
    - For air at STP and visible light, this value is about 0.23 cm^3/g
  - Valid only for $n \approx 1$ (generally a good assumption for air)
- Light bends towards regions with larger n (Snell's Law)
- Therefore **light rays deflect towards regions of higher density!**

Operating Principle

- With some work, we can also develop a shadowgraph equation

$$\boxed{\frac{\Delta \mathrm{I}}{\mathrm{I}_0} = \Delta z \kappa \int_{\Delta z} \left(\frac{d^2 \rho(x,y,z)}{dx^2} + \frac{d^2 \rho(x,y,z)}{dy^2} \right) \mathrm{dz}}$$

- Where,
  - $\Delta \mathrm{I}$ = change in intensity
  - I_0 = intensity of light source
  - Δz = distance between image plane and camera
  - κ = Gladstone-Dale constant
  - ρ = density of gas

Operating Principle

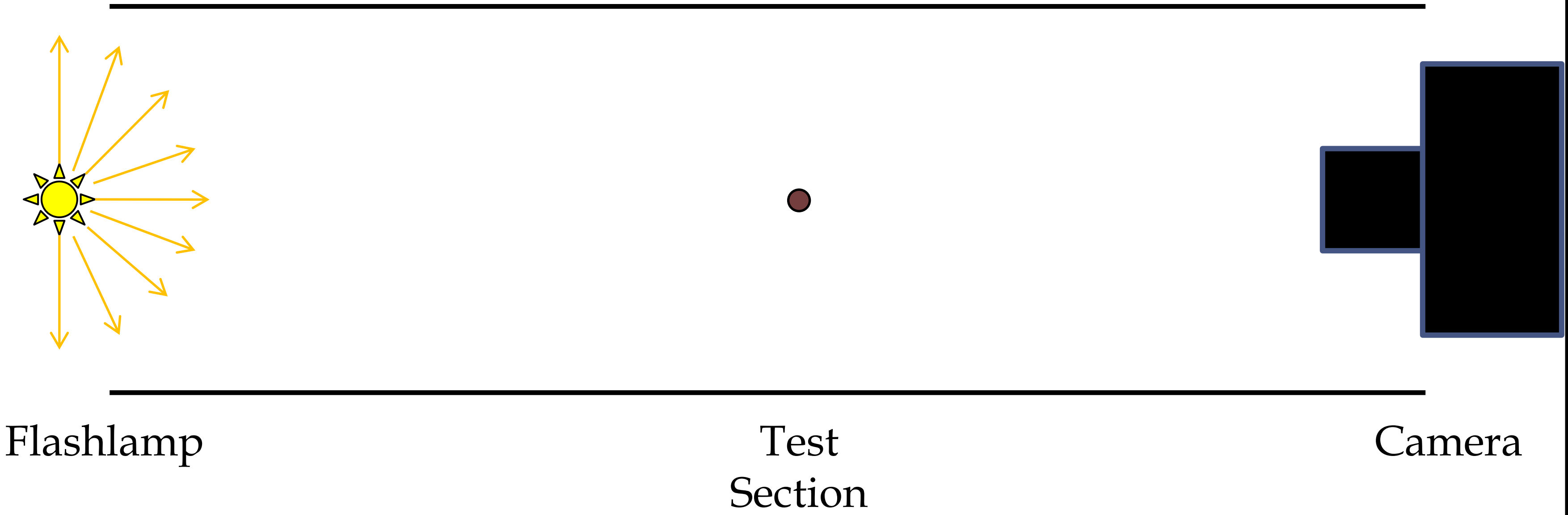
- With some work, we can also develop a shadowgraph equation

$$\boxed{\frac{\Delta \mathrm{I}}{\mathrm{I}_0} = \Delta z \kappa \int_{\Delta z} \left(\frac{d^2 \rho(x,y,z)}{dx^2} + \frac{d^2 \rho(x,y,z)}{dy^2} \right) \mathrm{dz}}$$

- So what do we learn?
  - Shadowgraph is sensitive to the **second derivative of density**
    - Less sensitive than if it were directly dependent on density or dependent on the first derivative
    - Uniform gradients (second derivative is zero) do not show up
  - This is a path-integrated technique
    - You see gradients along the entire path, not just on a plane
    - Sensitivity increases with increasing path length, but image becomes more blurred and distorted

Experimental Setup

- A relatively simple technique to set up
- For example:



Experimental Setup

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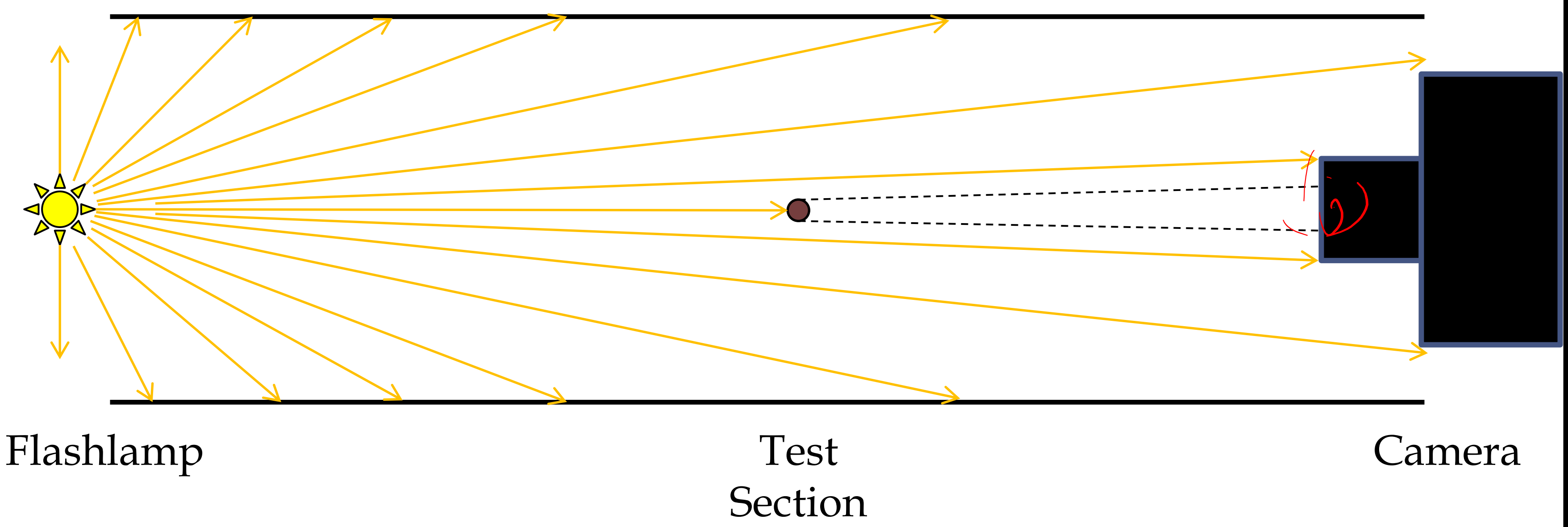
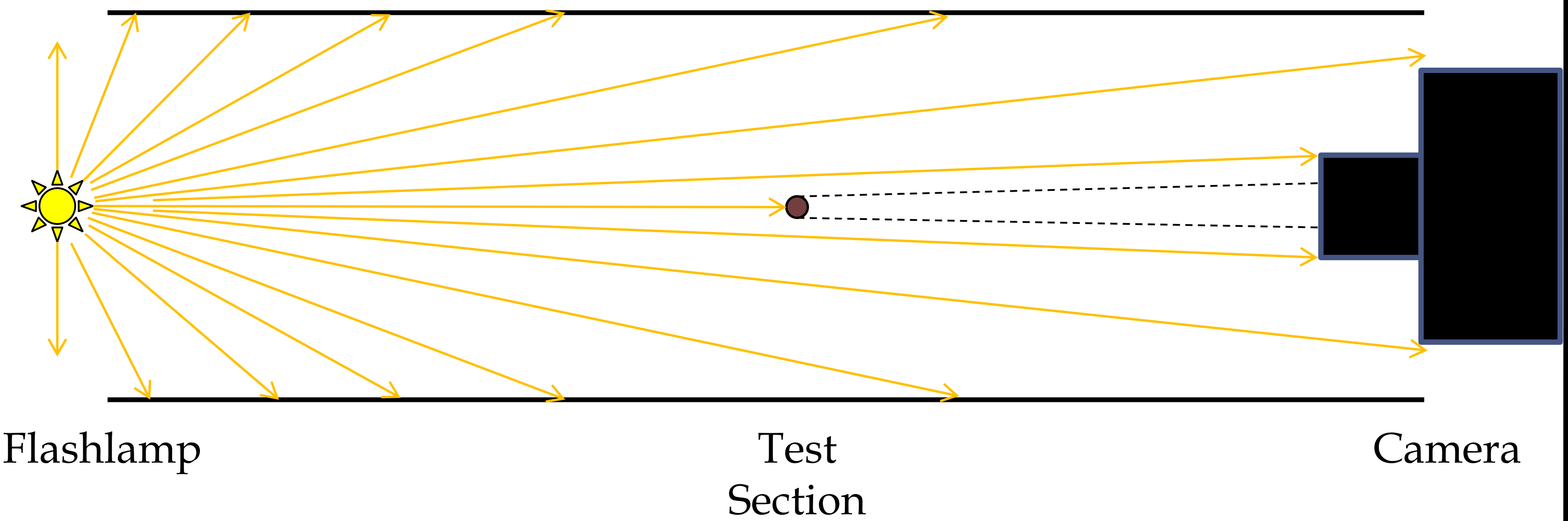


- This is known as the “direct shadow” method

Experimental Setup

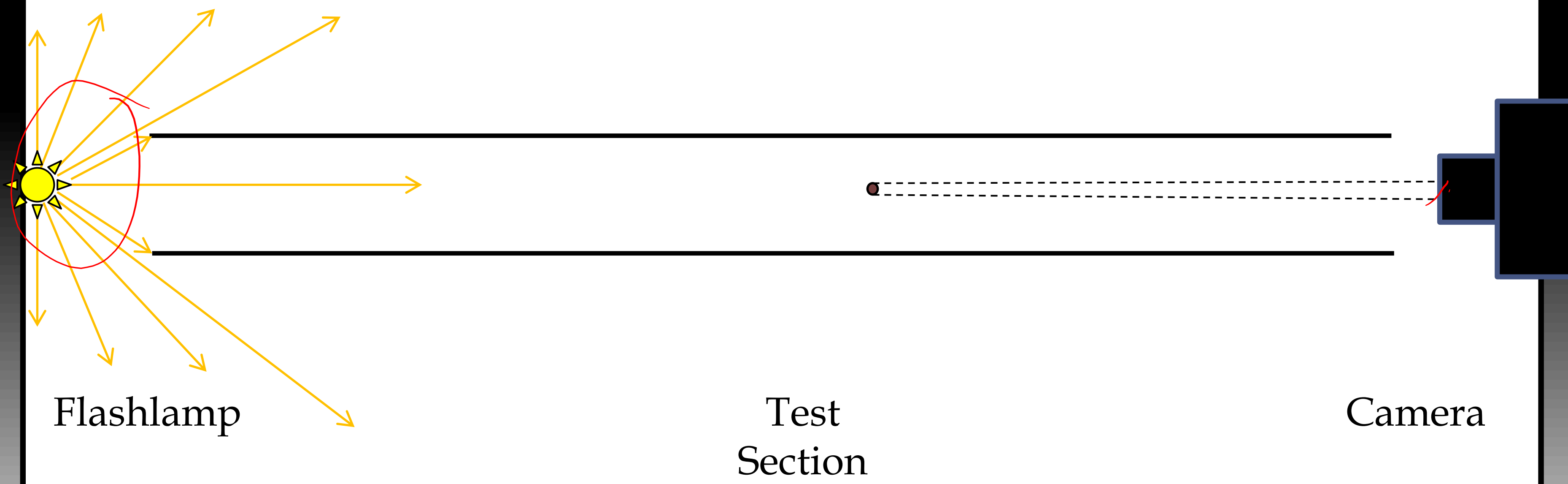
- The problem here is that the light rays are not collimated
  - An object closer to the light source will appear larger on the camera
  - Objects on one side of the test section appear larger than objects on the other side



- Less than ideal, but okay if area of interest is limited to a thin plane

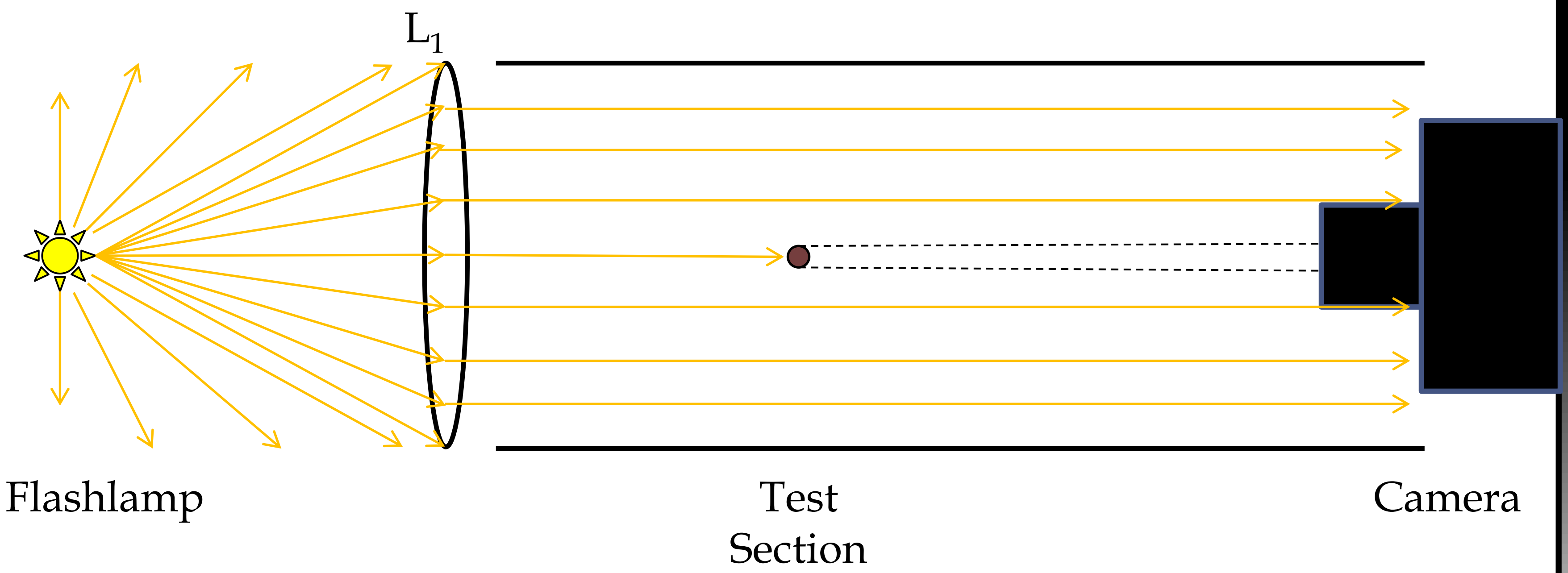
Experimental Setup

- One possible solution is to move the light source very far away
  - For example, light from the sun can be considered collimated
  - The drawback is that as you move your light source farther away, the required intensity for your source increases



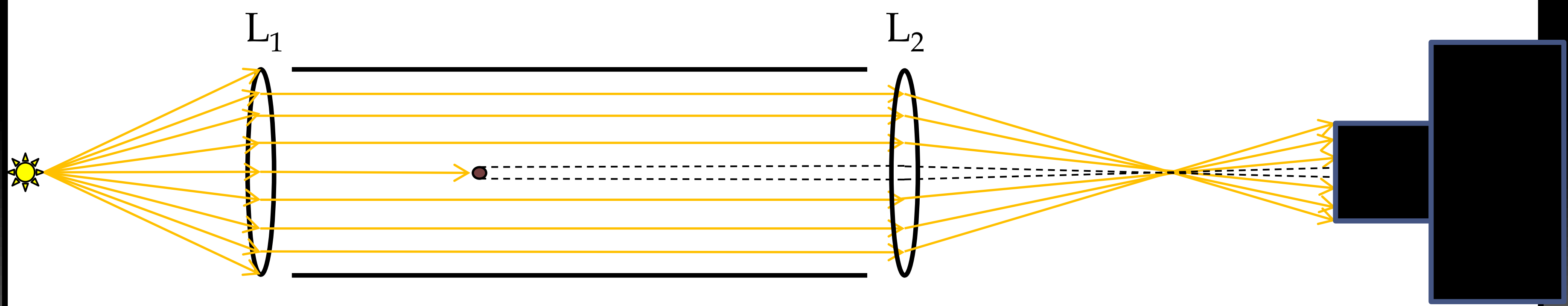
Experimental Setup

- You could also collimate the light source with optics
  - A spherical lens or mirror placed one focal length away from the light source will work
  - The drawback here is that you need an extremely large camera lens to capture the flowfield



Experimental Setup

- Perhaps the best solution is to first collimate the light and then refocus the light after the test section
  - Offers improved image quality and flexibility in camera location
  - Also makes it easier to set up Schlieren (next week)

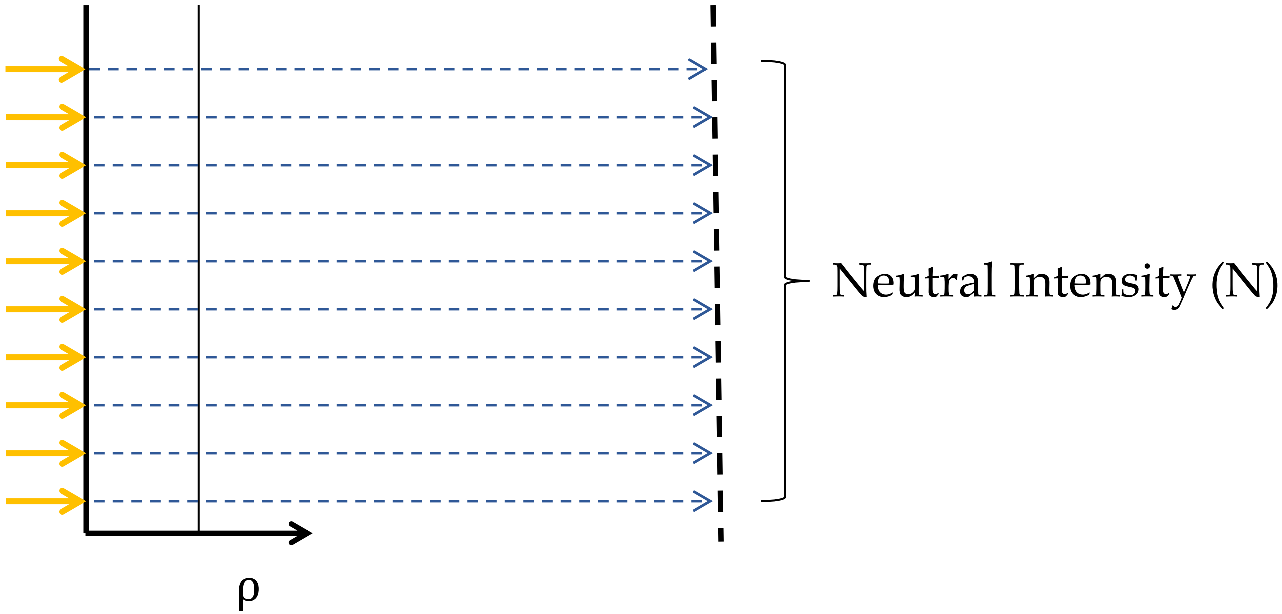


Shadowgraph Ray-Tracing

- When interpreting shadowgraph images, it can be helpful to think of density gradients as lenses that redirect incoming light rays
- Also remember, we are interested in relative intensity changes in the images, not absolute intensity
- When using ray-tracing to analyze density profiles, changes in intensity appear in the form of the relative spacing of incoming light rays

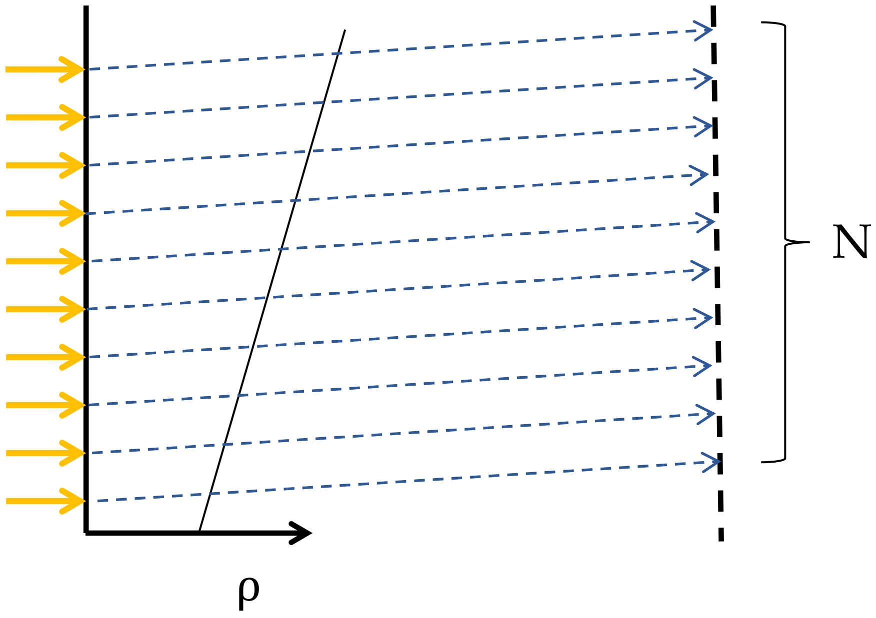
Shadowgraph Ray-Tracing

- Example 1
  - Uniform density field
  - Results in uniform illumination



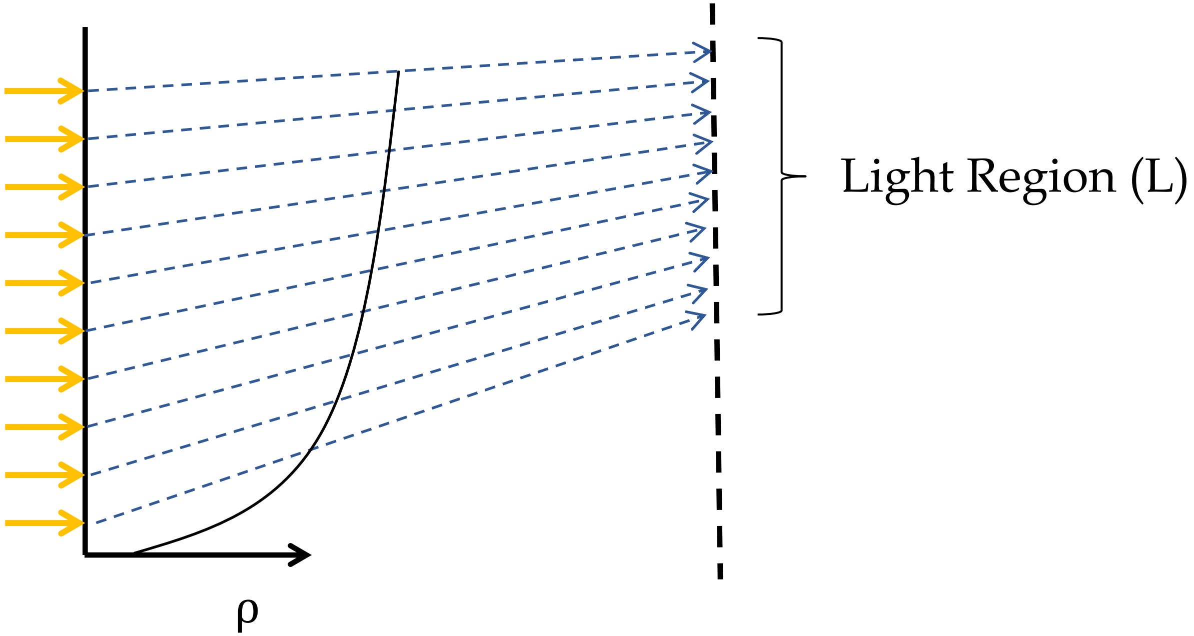
Shadowgraph Ray-Tracing

- Example 2
  - Constant density gradient ($\frac{\partial \rho}{\partial x}$ = constant)
  - Uniform deflection still results in a uniform illumination



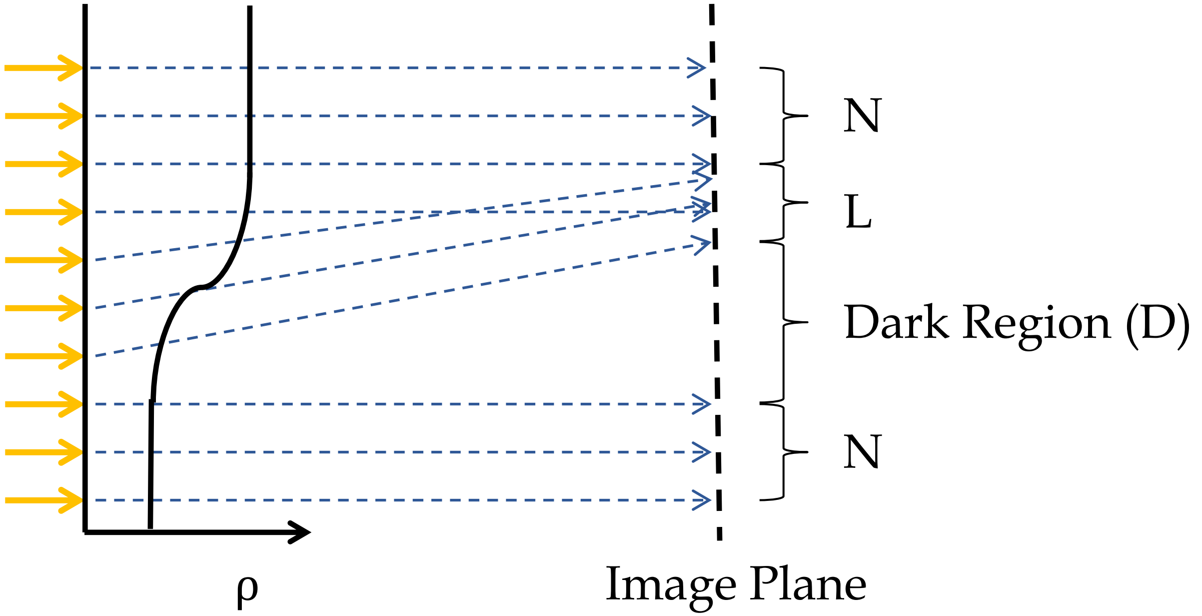
Shadowgraph Ray-Tracing

- Example 3
    - $\frac{\partial^2 \rho}{\partial x^2}$ = constant
    - Results in higher—but still uniform—intensity (lensing)



Shadowgraph Ray-Tracing

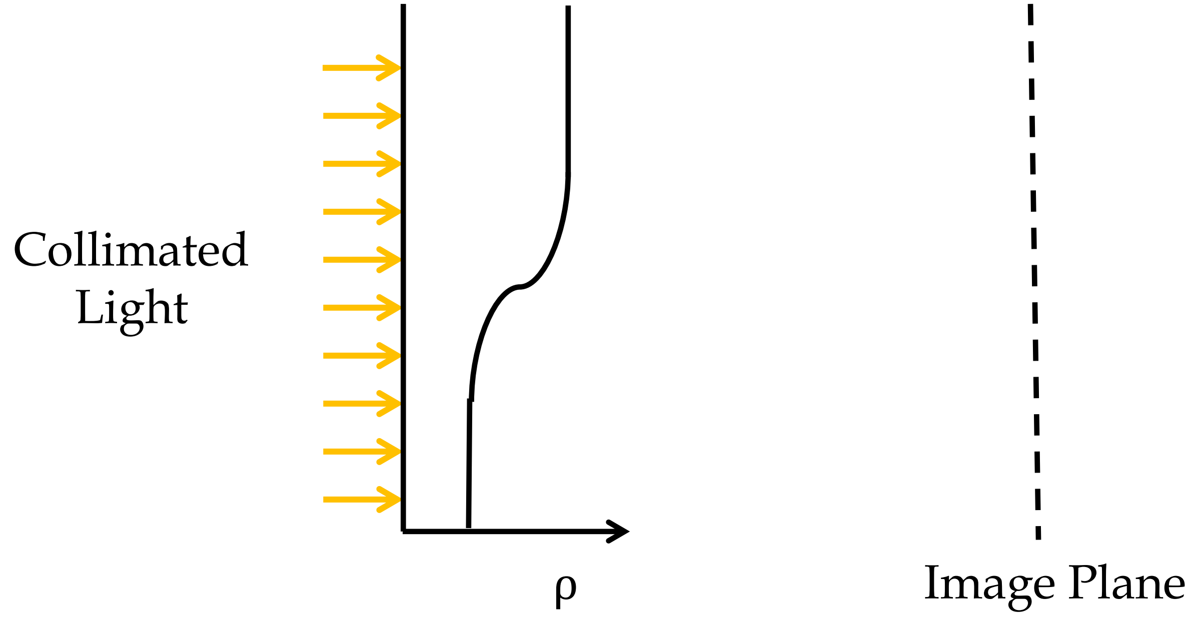
- Example 4
  - $\frac{\partial^2 \rho}{\partial x^2} \neq$ constant (like a shock wave)
  - Finally, we see something!



Shadowgraph Ray-Tracing

- So how do you do ray-tracing yourself?
- Consider a normal shock

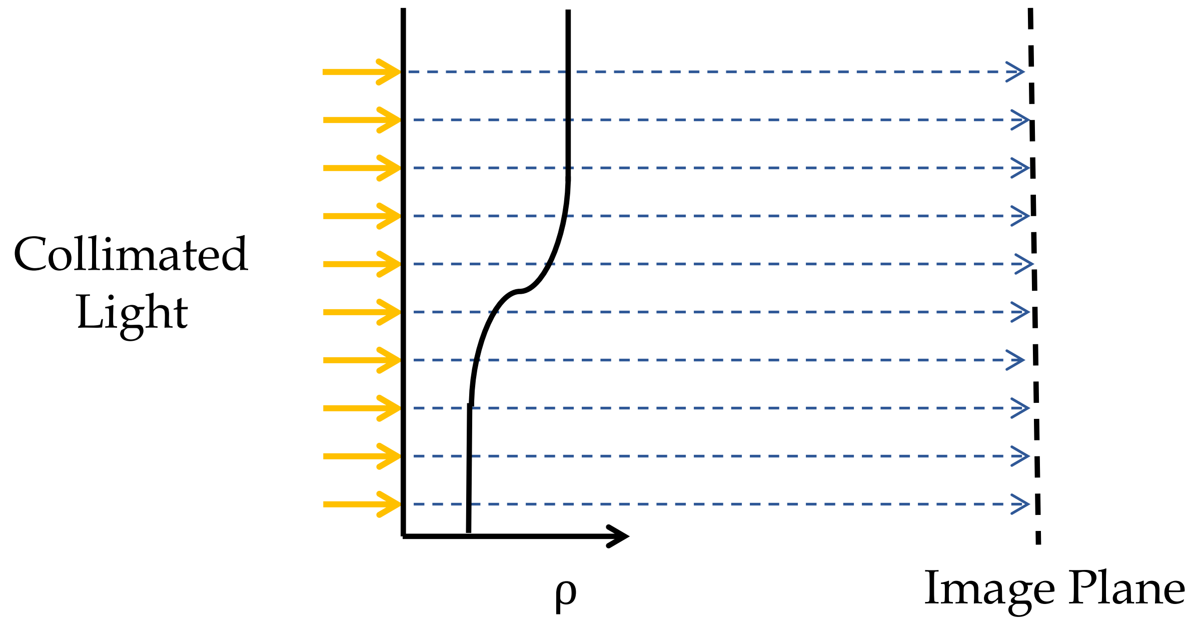
1) Figure out the density profile



Shadowgraph Ray-Tracing

- So how do you do ray-tracing yourself?
- Consider a normal shock

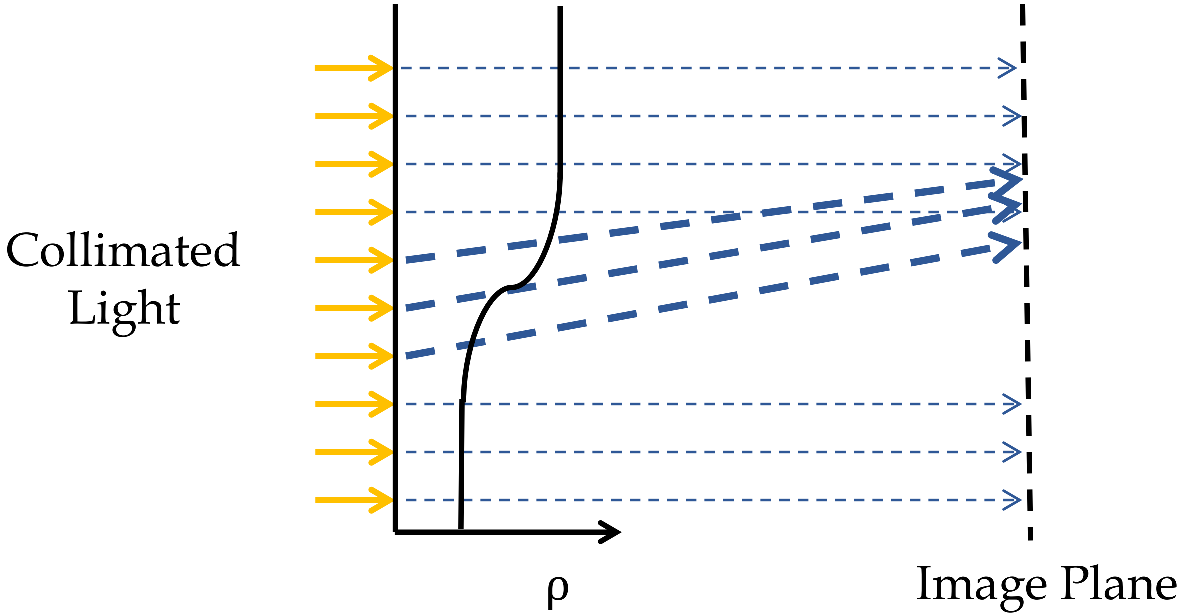
1) Figure out the density profile
2) **Trace rays through test section without deflection**



Shadowgraph Ray-Tracing

- So how do you do ray-tracing yourself?
- Consider a normal shock

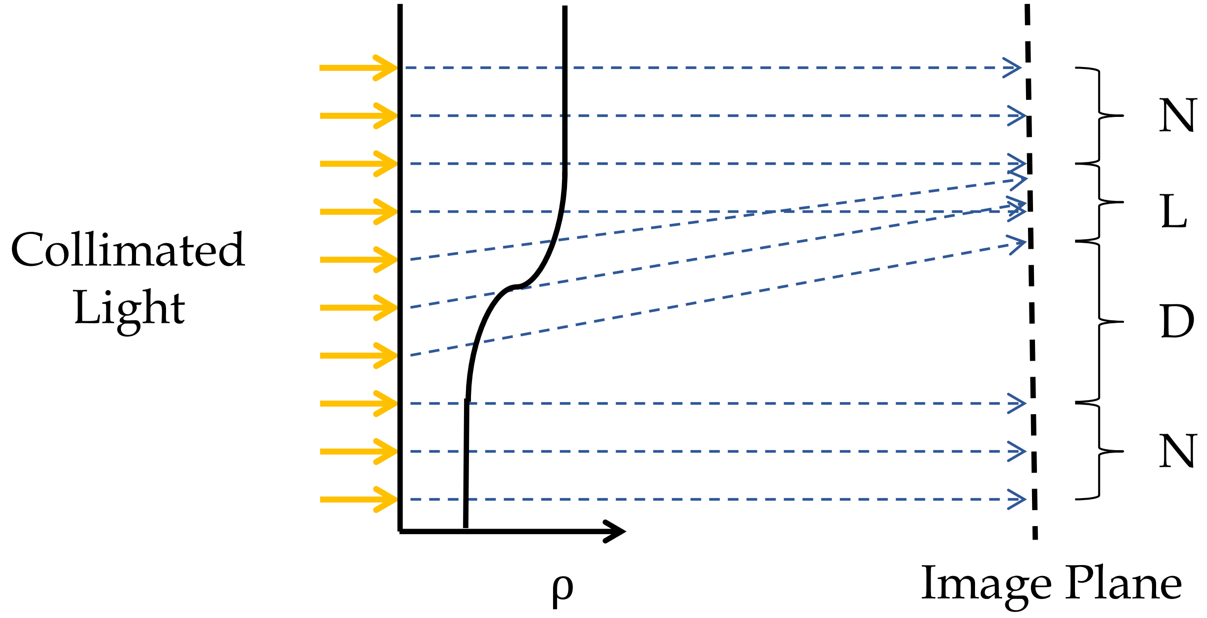
1) Figure out the density profile
2) Trace rays through test section without deflection
3) **Deflect rays passing through a density gradient towards <u>regions of higher density</u>**



Shadowgraph Ray-Tracing

- So how do you do ray-tracing yourself?
- Consider a normal shock

1) Figure out the density profile
2) Trace rays through test section without deflection
3) Deflect rays passing through a density gradient towards regions of higher density
4) **Identify regions of dark, light, and neutral intensity based on ray spacing**

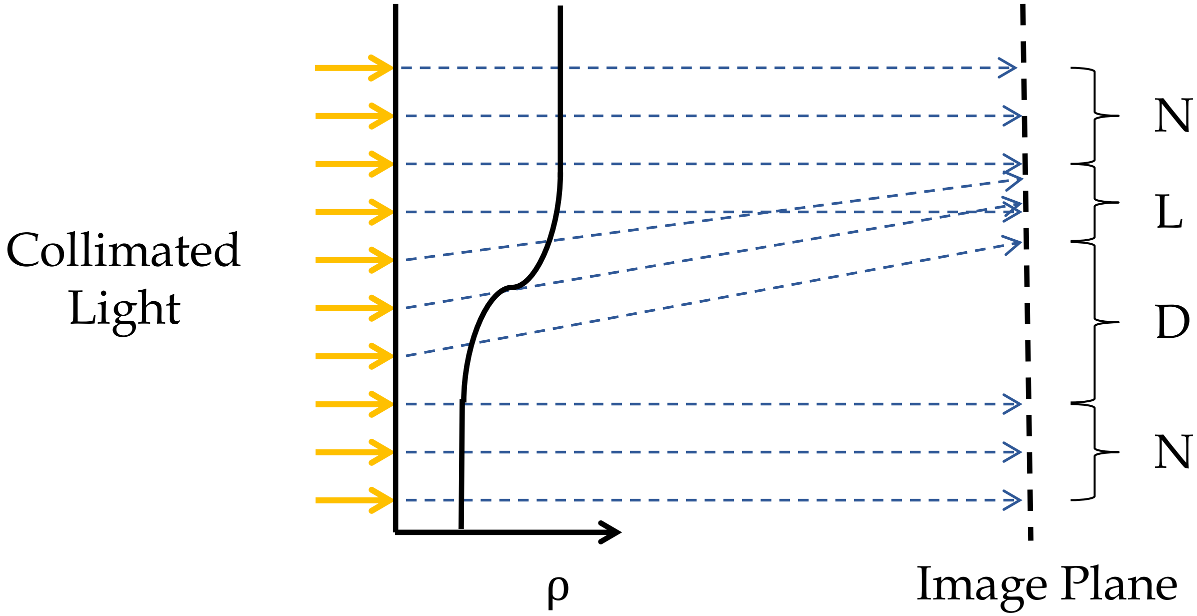


Shadowgraph Ray-Tracing

- So how do you do ray-tracing yourself?
- Consider a normal shock

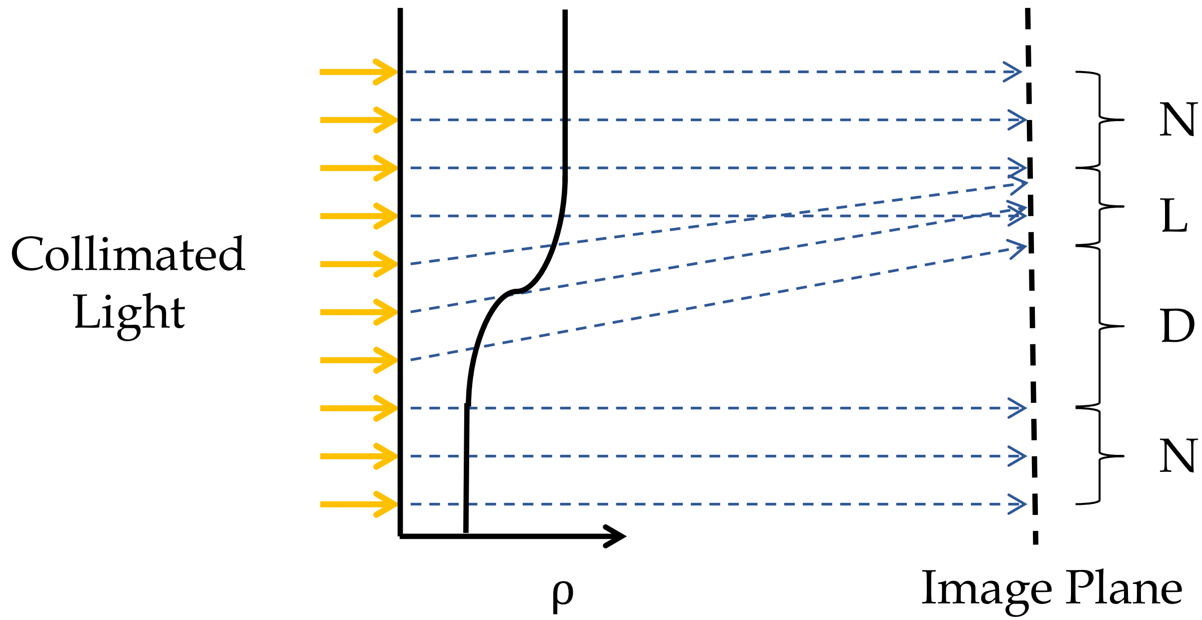
4) Identify regions of dark, light, and neutral intensity based on ray spacing

- Regions with the same spacing as the incident light are considered to have "neutral" or "background" intensity
- Regions with higher or lower spacing are considered light and dark regions, respectively



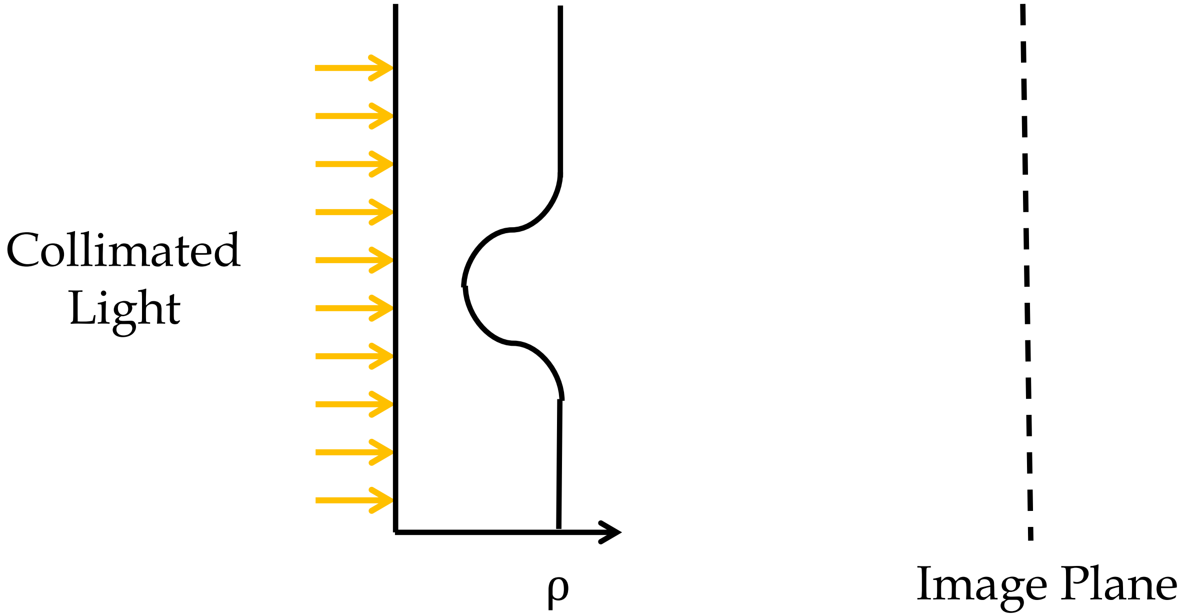
Shadowgraph Ray-Tracing

- What does this result tell us?
  - **Shadowgraph is only sensitive to changes in the second derivative of density**
  - **Rays bend towards region of higher density**
  - Shocks produce a dark region followed by a light region extending beyond the boundaries of the shock
  - The actual location of the shock is the start of the dark region
  - Cannot determine shock thickness from a shadowgraph image



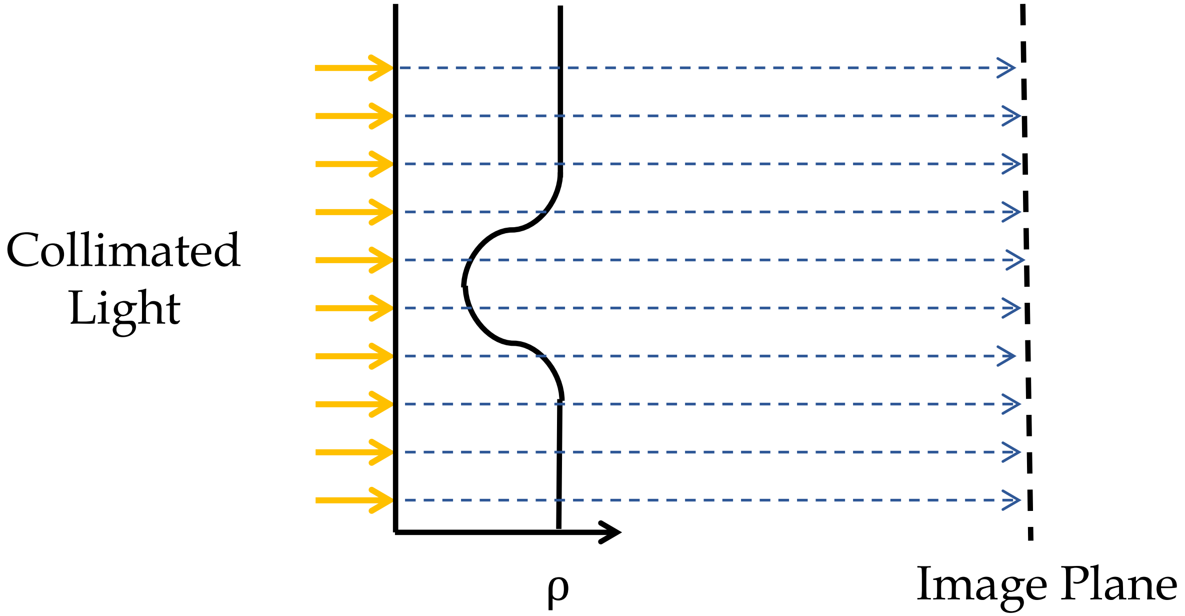
Shadowgraph Ray-Tracing

- Now try it yourself!
  - On a sheet of paper, use ray-tracing to predict the shadowgraph image resulting from the density field below



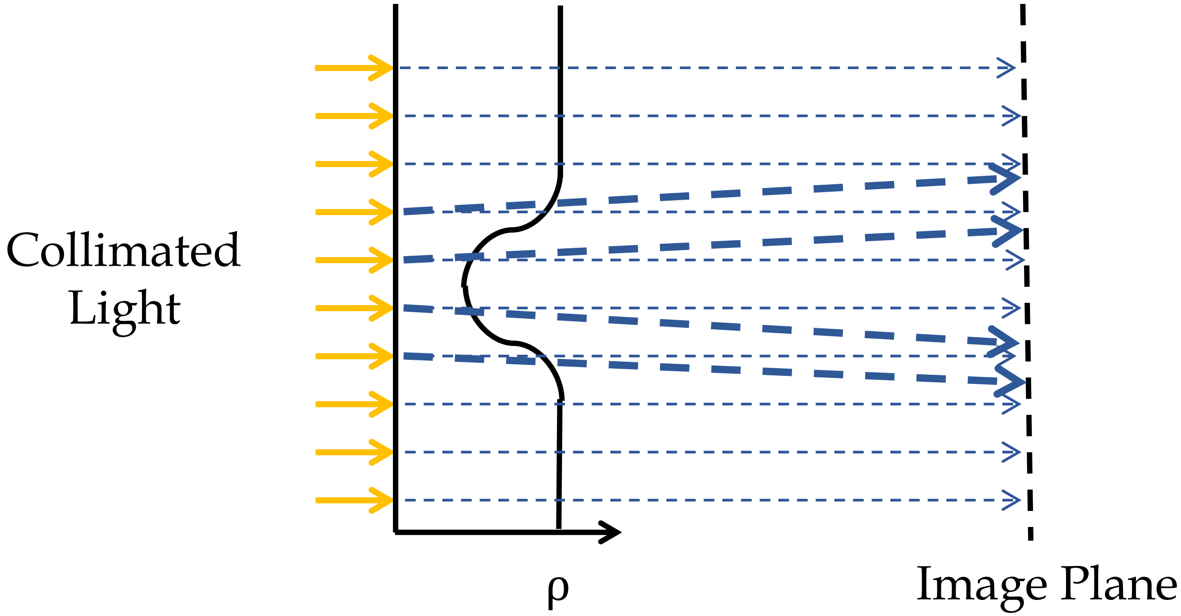
Shadowgraph Ray-Tracing

- Now try it yourself!
  - **Step 1 – draw unperturbed rays**



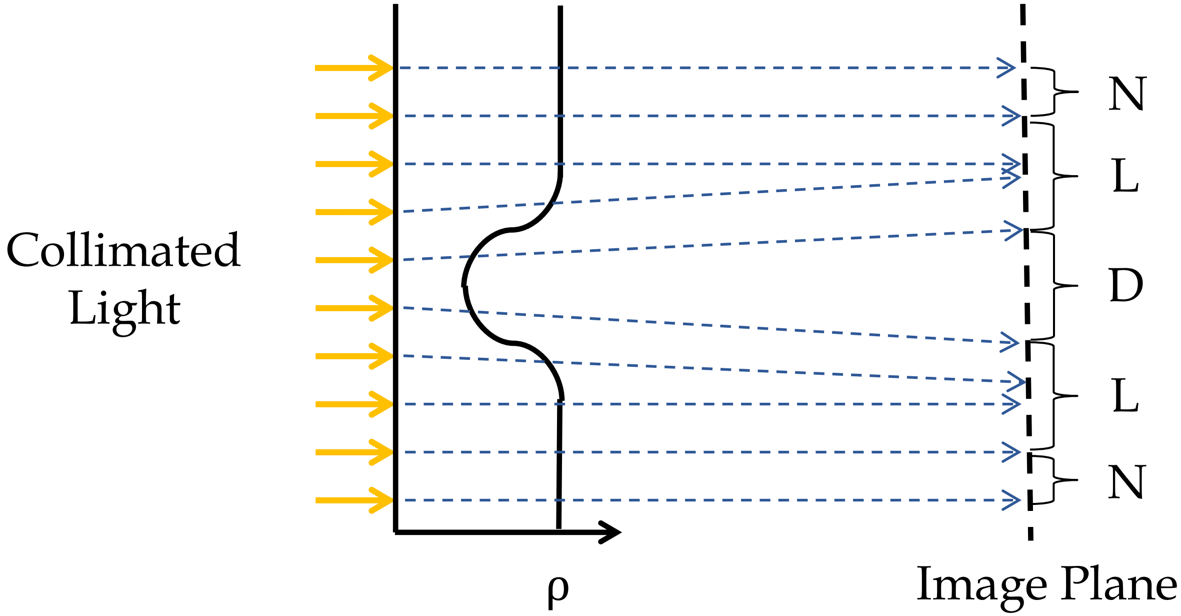
Shadowgraph Ray-Tracing

- Now try it yourself!
  - Step 1 – draw unperturbed rays
  - **Step 2 – for rays passing through a density gradient, deflect them towards the region of higher density**



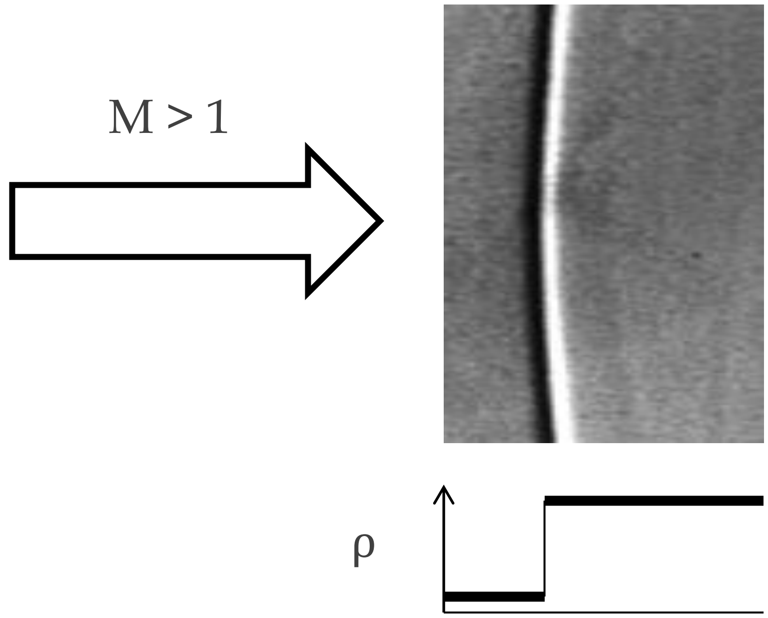
Shadowgraph Ray-Tracing

- Now try it yourself!
  - Step 1 – draw unperturbed rays
  - Step 2 – for rays passing through a density gradient, deflect them towards the region of higher density
  - **Step 3 – identify regions of light, dark, and neutral intensity**



Shadowgraph Ray-Tracing

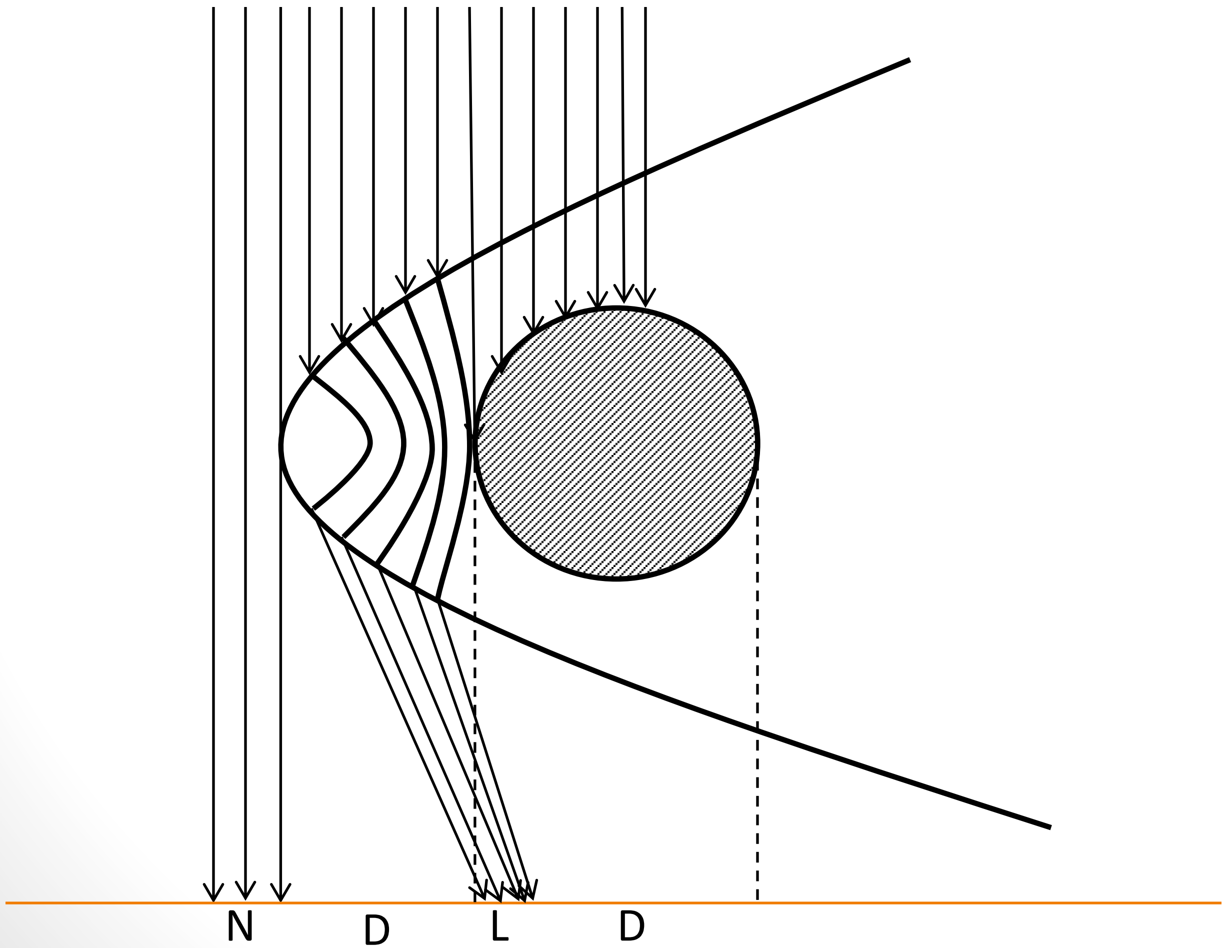
- Shadowgraph of a normal shock



- The shock appears much thicker in the image than it actually is
    - Shocks are thinner than the wavelength of visible light (which is on the order of hundreds of nanometers)
    - The shock acts as an aperture to the incoming light and the thickness in the image is limited by diffraction

General Features of Flowfield

- For spheres, shadowgraph appears to truncate entire sphere, not just a spot



Limitations of Shadowgraph

- A “line-of-sight” technique
  - Information is collected across the entire ray path or line-of-sight
  - Density gradients from the camera to the light source are all imaged, not just from the test section
  - This can sometimes result in odd three-dimensional effects
- Extremely difficult to make the results quantitative, so purely a visualization technique
- Only sensitive to strong variations in the flow
  - For example, cannot visualize expansion fans using shadowgraph
- Size of flow structures visualized in images is diffraction-limited (i.e., shock thickness)