The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

ASE 162M High-Speed Aerodynamics
Section 14275

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Supersonic Flow over a Sphere

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1 Introduction

This experiment investigates supersonic flow over a sphere, a fundamental problem in compressible aerodynamics with significant implications for high-speed vehicle design. When a supersonic flow encounters a blunt object such as a sphere, it forms a detached bow shock upstream of the body. The characteristics of this shock, particularly its standoff distance from the body, are crucial parameters in understanding the aerodynamic forces, heat transfer, and pressure distribution on the object.

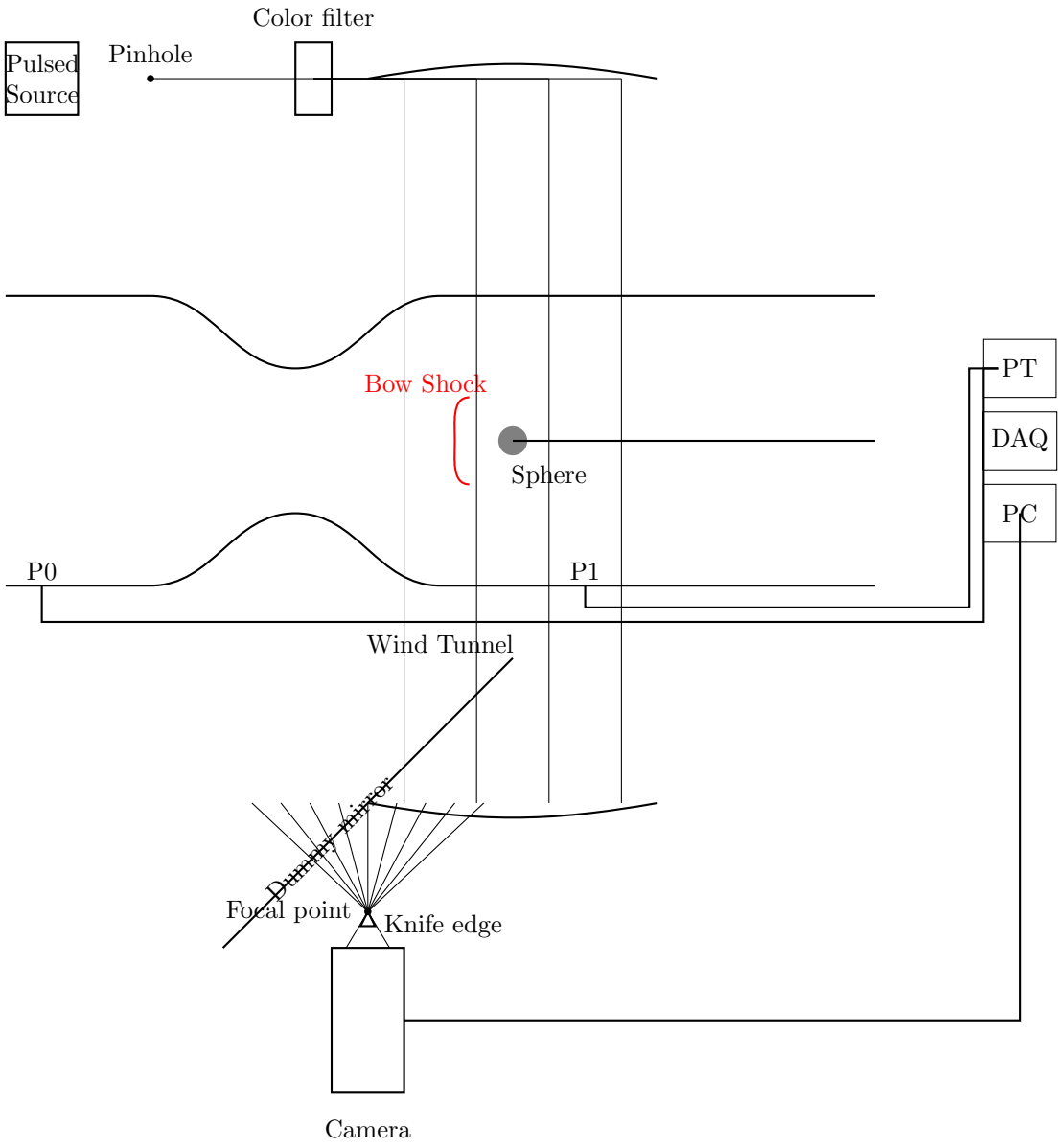
The primary objectives of this experiment are to:

1. Visualize the flow field and shock structure using Schlieren and shadowgraph imaging techniques.
2. Measure and analyze the non-dimensional shock standoff distance as a function of Mach number.
3. Compare experimental results with theoretical predictions through curve fitting of empirical relations.
4. Examine the effects of Mach number and Reynolds number on the flow characteristics.

This report presents the experimental methodology, data analysis, and discussion of results, contributing to our understanding of supersonic flow phenomena and providing valuable data for validating computational fluid dynamics (CFD) simulations and theoretical models.

2 Experimental Setup

The experiment was conducted using the Aerolab Variable Mach Number Wind Tunnel, capable of producing supersonic flows with Mach numbers ranging from approximately 1.63 to 3.25. The test section of the wind tunnel is nominally 3" * 3". A sphere model was mounted in the test section at a 0° angle of attack.



Schematic of the experimental setup. 1: Pulsed light source, 2: Pinhole, 3: Color filter, 4: Concave mirror, 5: Wind tunnel test section, 6: Sphere model, 7: Bow shock, 8: Knife edge, 9: Camera, 10: Data acquisition system. Flow direction is from left to right.

Note for Ryan, this was my first tikz diagram of this type, and I am still learning so please don't grade me too harshly :)

2.1 Calibration and Imaging

Prior to running the experiments, a calibration process was performed:

- A 30/30, 5mm grid points calibration sheet was used for spatial calibration.
- 10 grid spaces, equivalent to 50mm, were used to determine the pixel-to-length ratio and associated uncertainty.
- A blank image without the calibration sheet was taken, followed by an image with the calibration sheet in place.

Flow visualization was achieved using both shadowgraph and Schlieren imaging techniques. A folded Schlieren system utilizing a pulsed xenon arc lamp as the light source was employed. For each Mach number, three types of images were captured:

1. Shadowgraph
2. Vertical Schlieren
3. Horizontal Schlieren

For the Schlieren setup, a knife edge was placed at the focal point to enhance the visualization of density gradients. The vertical and horizontal indicate the orientation of the blade.

2.2 Data Acquisition

Pressure measurements were recorded using a LabVIEW-based data acquisition system with two pressure transducers:

- Channel 0: Stagnation pressure (P_0)
- Channel 1: Freestream static pressure

The pressure data was recorded in volts (gauge pressure) and later converted to appropriate units using calibration constants.

2.3 Experimental Procedure

The experiment followed these steps for each run:

1. The wind tunnel was started in the sequence: power, hydraulic, then run.
2. The initial Mach number was set to approximately 3, with subsequent runs decreasing by increments of 0.25.
3. Once the LabVIEW system showed stable pressure readings, images were captured using the Pulnix color CCD camera and XCAP for Windows acquisition software.
4. The wind tunnel was operated only while the run button was held, and shut down in the reverse order of startup once the airflow stopped.
5. Mach number was adjusted by changing the area ratio and chamber pressure according to the manufacturer's specifications, referencing the "Approximate Minimum Stagnation Pressure vs Mach Number" chart.

Care was taken to monitor the pressure of the compressed air tank to ensure consistent flow quality across all runs. The experiment was repeated for Mach numbers of 3, 2.75, 2.5, 2.25, 2, and 1.75.

3 Results and Discussion

3.1 Reynolds Number and Mach Number Calculations

To calculate the Reynolds numbers and Mach numbers, we used the following equations:

3.1.1 Mach Number Calculation

The Mach number was calculated using the isentropic flow equation:

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}} \tag{1}$$

where p_0 is the stagnation pressure, p is the static pressure, γ is the ratio of specific heats for air (1.4), and M is the Mach number.

3.1.2 Reynolds Number Calculation

The Reynolds numbers were calculated using:

$$Re_D = \frac{\rho_\infty U_\infty D}{\mu} \tag{2}$$

where ρ_∞ is the freestream density, U_∞ is the freestream velocity, D is the sphere diameter, and μ is the dynamic viscosity.

The viscosity was calculated using Sutherland's law:

$$\mu = \mu_0 \left(\frac{T}{T_0}\right)^{3/2} \frac{T_0 + C}{T + C} \tag{3}$$

where μ_0 is the reference viscosity, T_0 is the reference temperature, T is the flow temperature, and C is Sutherland's constant for air.

Using the data from the pressure measurements and the Python script, we calculated the unit Reynolds numbers and diametric Reynolds numbers (Re_D) for all runs, as well as the Mach numbers. Sutherland's law was used for viscosity calculations. The results are presented in the following plot:

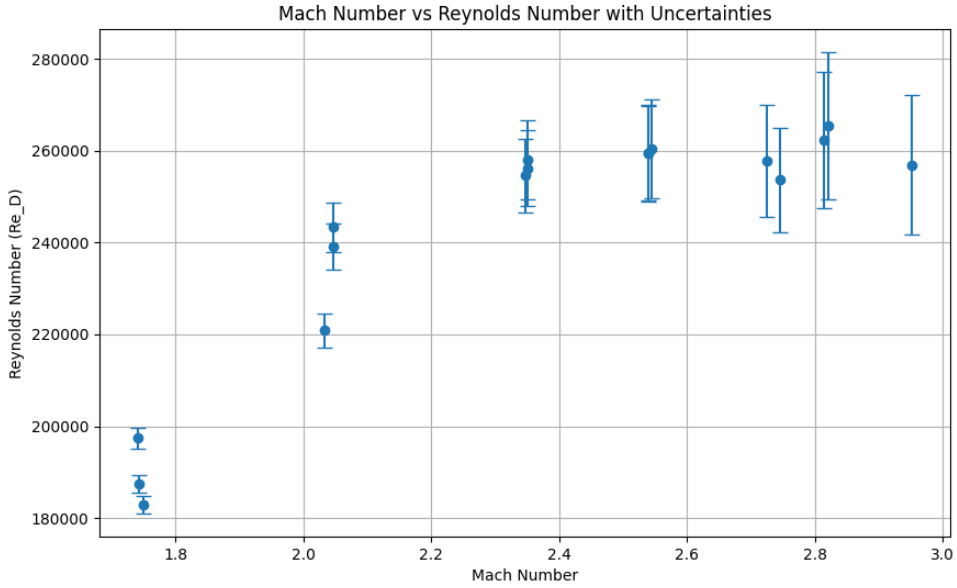


Figure 1: Mach number vs. Diametric Reynolds number

Figure **??** shows the relationship between the calculated Mach number and the diametric Reynolds number. We observe a positive correlation, with both Mach number and Reynolds number increasing together. This trend is expected due to higher flow velocities at increased Mach numbers.

The error bars represent propagated uncertainties from measurement errors. For Mach number, the uncertainty is primarily due to pressure measurement errors:

$$\delta M = \sqrt{\left(\frac{\partial M}{\partial p_0}\delta p_0\right)^2 + \left(\frac{\partial M}{\partial p}\delta p\right)^2} \tag{4}$$

where δp_0 and δp are uncertainties in stagnation and static pressure measurements, respectively.

For Reynolds number, uncertainties propagate from multiple sources:

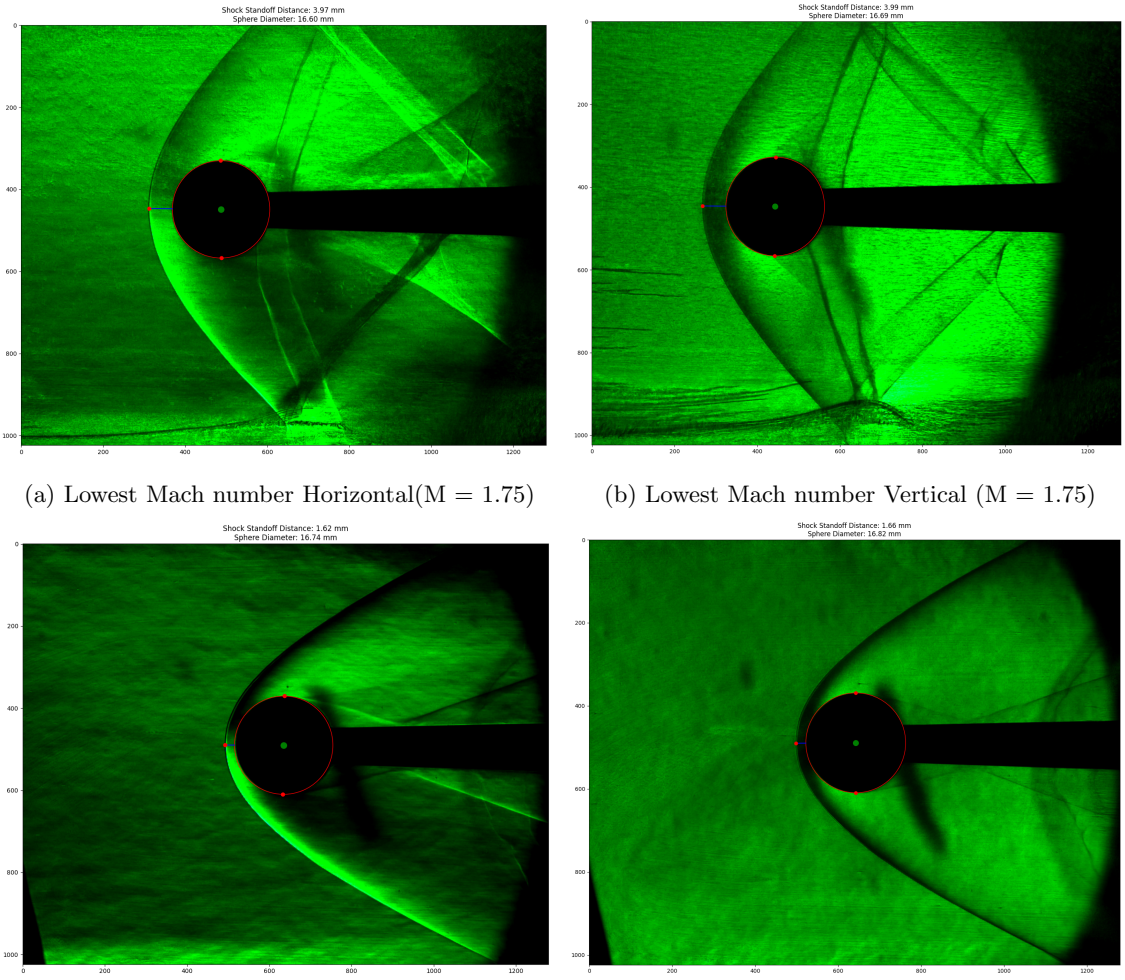
$$\delta Re_D = Re_D\sqrt{\left(\frac{\delta\rho}{\rho}\right)^2 + \left(\frac{\delta U}{U}\right)^2 + \left(\frac{\delta D}{D}\right)^2 + \left(\frac{\delta\mu}{\mu}\right)^2} \tag{5}$$

where $\delta\rho$, δU, δD, and $\delta\mu$ are uncertainties in density, velocity, diameter, and viscosity calculations, respectively.

The increasing size of error bars at higher Mach and Reynolds numbers indicates that measurement uncertainties have a more significant impact in these regimes, likely due to the nonlinear relationships in the governing equations.

3.2 Schlieren Images Analysis

We analyzed the Schlieren images for the highest and lowest Mach numbers to observe the flow features around the sphere.



(a) Lowest Mach number Horizontal(M = 1.75)

(b) Lowest Mach number Vertical (M = 1.75)

(c) Highest Mach number Horizontal (M = 3.0)

(d) Highest Mach number Vertical(M = 3.0)

Figure 2: Schlieren images at extreme Mach numbers

In Figure 2, we can observe the following key features:

1. Bow shock: A strong, detached shock wave is visible in front of the sphere for both Mach numbers. The shock is more oblique and closer to the sphere at the higher Mach number.
2. Shock standoff distance: The distance between the bow shock and the sphere's leading edge is noticeably larger for the lower Mach number (Figure 2a) compared to the higher Mach number (Figure 2c).
3. Expansion region: Behind the sphere, we can see an expansion fan where the flow accelerates and turns around the sphere.
4. Wake region: A turbulent wake is visible downstream of the sphere, characterized by density gradients in the Schlieren images.
5. Boundary layer: Although not clearly visible due to the image resolution, a thin boundary layer is expected to form on the sphere's surface.

The changes in the flow structure as the Mach number increases from 1.75 to 3.0 are consistent with theory:

1. The bow shock moves closer to the sphere at higher Mach numbers, reducing the shock standoff distance.

2. The shock becomes stronger and more oblique at higher Mach numbers, as evidenced by the sharper contrast in the Schlieren image.
3. The wake region appears to be more compressed and elongated at the higher Mach number, likely due to the increased dynamic pressure.
4. These changes correlate with the increase in Reynolds number observed in Figure 1.

3.3 Shock Standoff Distance Analysis

We analyzed the non-dimensional shock standoff distance δ/D as a function of Mach number using the formula:

$$\frac{\delta}{D} = f(M) \tag{6}$$

where δ is the shock standoff distance and D is the sphere diameter. The standoff distance δ and sphere diameter D were measured using a custom image analysis script developed for this experiment. This script processes the Schlieren images, allowing for precise measurement of the shock location and sphere dimensions. The visual can be shown below.

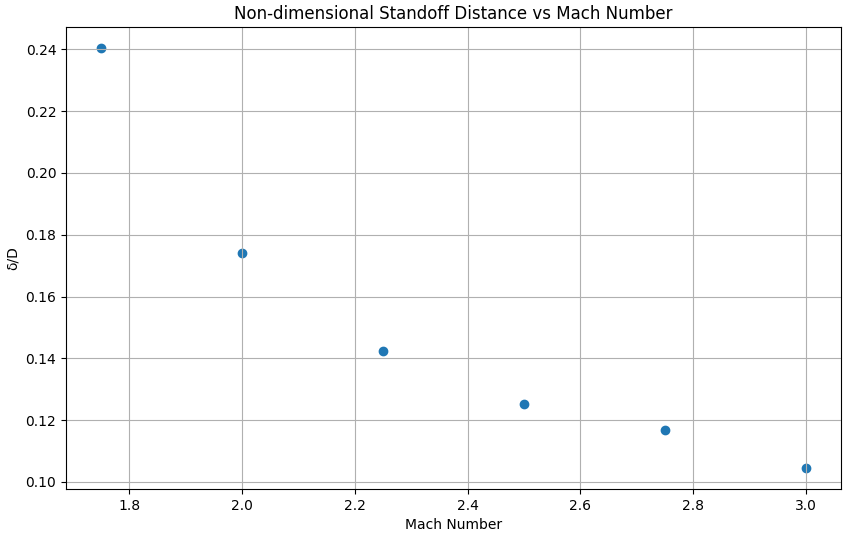


Figure 3: Non-dimensional standoff distance vs. Mach number

Figure 3 presents the relationship between the non-dimensional standoff distance and Mach number. The graph reveals several key observations:

- Inverse relationship: As the Mach number increases, the non-dimensional standoff distance decreases, consistent with our Schlieren image observations and theoretical expectations.
- Nonlinear trend: The relationship appears nonlinear, with a steeper decrease in standoff distance at lower Mach numbers (1.75 to 2.25) and a more gradual decrease at higher Mach numbers (2.5 to 3.0).
- Range: The non-dimensional standoff distance ranges from approximately 0.24 at Mach 1.75 to 0.10 at Mach 3.0, representing a significant change in shock structure over the tested Mach number range.
- Asymptotic behavior: The curve shape suggests an asymptotic approach to a minimum standoff distance as Mach number increases, which aligns with theoretical predictions for hypersonic flow regimes.

This trend is explained by the physics of shock formation in supersonic flows. As the Mach number increases, the bow shock forms closer to the sphere due to the increased shock strength and more efficient flow compression.

3.4 Curve Fitting

We performed curve fitting on the standoff distance data using three different relations:

3.4.1 Qualitative scaling dependence

$$\frac{\delta}{D} = c\sqrt{\frac{1+\frac{\gamma-1}{2}M_\infty^2}{M_\infty - 1}} \tag{7}$$

3.4.2 Basic fit

$$\frac{\delta}{D} = c\gamma^\alpha M^\beta \tag{8}$$

3.4.3 Offset fit

$$\frac{\delta}{D} = c\gamma^\alpha (M-1)^\beta \tag{9}$$

The results of these curve fits are presented in the following plot and table:

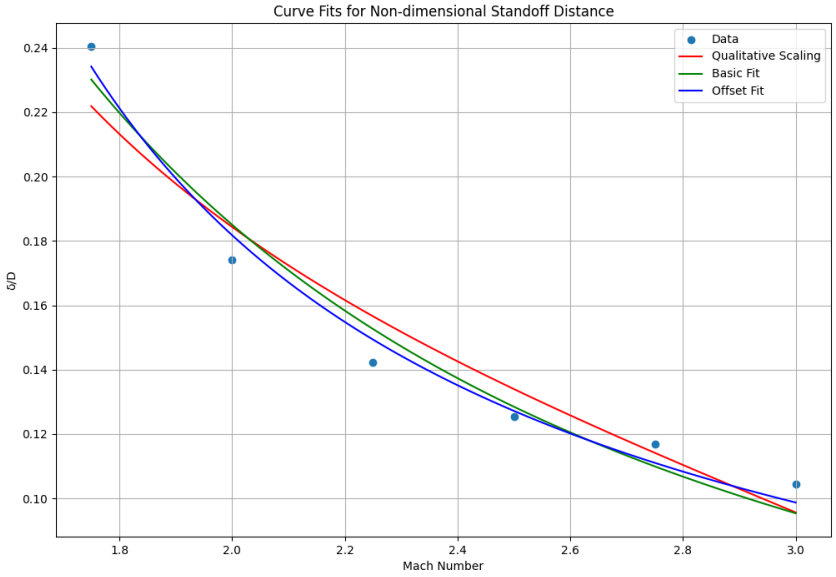


Figure 4: Curve fits for non-dimensional standoff distance

Table 1: Curve Fit Parameters

Fit Type	c	α	β
Qualitative	0.2157	N/A	N/A
Basic	0.2315	9.448	-1.635
Offset	0.1653	-1.774	-0.881

4 Discussion

The data exhibits good agreement with all three relations, which can be characterized as follows:

1. **Qualitative scaling**:
$$\frac{\delta}{D} = c\sqrt{\frac{1 + \frac{\gamma-1}{2}M_\infty^2}{M_\infty - 1}} \tag{10}$$
This fit captures the general trend of decreasing standoff distance with increasing Mach number. The single parameter c (0.2157) represents a scaling factor that accounts for the overall magnitude of the standoff distance. The qualitative scaling shows good agreement with the data, especially at higher Mach numbers, reflecting the physical expectation of asymptotic behavior as Mach number increases.

2. **Basic fit**:
$$\frac{\delta}{D} = c\gamma^\alpha M^\beta \tag{11}$$
The negative value of β (-1.635) confirms the inverse relationship between Mach number and standoff distance, aligning with physical expectations. The large positive value of α (9.448) suggests a strong dependence on the specific heat ratio γ. This is somewhat unexpected for a simple geometry like a sphere and may indicate that the fit is compensating for other factors not explicitly accounted for in the model.

3. **Offset fit**:
$$\frac{\delta}{D} = c\gamma^\alpha (M-1)^\beta \tag{12}$$
This fit attempts to account for the behavior near Mach 1 by using (M - 1) instead of M. The negative α (-1.774) and β (-0.881) indicate that the standoff distance decreases with increasing Mach number, but at a different rate compared to the basic fit. The offset fit appears to provide a good balance between accuracy and physical interpretation, especially for lower Mach numbers.

Regarding the dependence on composition (through γ) and Mach number:

1. For the basic fit, the strong dependence on γ (large α) is unexpected. We would typically expect a weaker dependence on composition for a simple geometry like a sphere. This suggests that the fit might be overcompensating for other factors or that there may be additional physics not captured by this simple model.

2. The offset fit shows a negative dependence on γ, which is also unexpected. However, the magnitude is smaller than in the basic fit, suggesting it might be a more realistic representation of the weak composition dependence we would expect.

3. Both fits show a strong dependence on Mach number (through β), which aligns with my physical understanding of shock formation in supersonic flows. The offset fit's use of (M - 1) provides a better representation of the behavior near Mach 1, which is consistent with theoretical expectations.

The consistency of our data with these fits is generally good, as evidenced by the close agreement between the fit curves and the experimental data points in Figure 4. However, there are some discrepancies, particularly at Mach numbers around 2.25 and 2.75, where the experimental data points deviate slightly from all three fit curves.

5 Conclusion

This experiment investigated supersonic flow over a sphere at Mach numbers from 1.75 to 3.0, utilizing Schlieren imaging to visualize the flow field and analyze non-dimensional shock standoff distances. The analysis revealed a positive correlation between Mach and Reynolds numbers, with increased uncertainties at higher Mach numbers. The offset fit provided the best balance of accuracy and interpretation, particularly near Mach 1, while both basic and offset fits indicated an unexpected dependence on gas composition. These findings underscore the complexity of supersonic flow and suggest further research into varying Mach numbers

and gas compositions. Overall, this work enhances our understanding of high-speed aerodynamics and its implications for supersonic vehicle design.

A References

1. Anderson, J. D. (2010). Fundamentals of aerodynamics. Tata McGraw-Hill Education.
2. Liepmann, H. W., & Roshko, A. (1957). Elements of gasdynamics. John Wiley & Sons.
3. Van Dyke, M. (1982). An album of fluid motion. Parabolic Press.

B Python Scripts

B.1 standoff2.py

```
import os
import cv2
import numpy as np
import matplotlib.pyplot as plt
import re
import argparse  # Add this import

def onclick(event):
    global points
    if event.button == 1 and len(points) < 3:  # Left mouse button
        points.append((event.xdata, event.ydata))
        plt.plot(event.xdata, event.ydata, 'ro')
        plt.draw()
        if len(points) == 3:
            plt.close()

def find_sphere_center_and_radius(top, bottom):
    center_x = (top[0] + bottom[0]) / 2
    center_y = (top[1] + bottom[1]) / 2
    radius = np.sqrt((top[0] - bottom[0])**2 + (top[1] - bottom[1])**2) / 2
    return (center_x, center_y), radius

def closest_point_on_circle(center, radius, point):
    dx = point[0] - center[0]
    dy = point[1] - center[1]
    distance = np.sqrt(dx**2 + dy**2)
    return (
        center[0] + radius * dx / distance,
        center[1] + radius * dy / distance
    )

def measure_shock_standoff(image_path, calibration_factor):
    global points
    points = []

    # Read the image
    img = cv2.imread(image_path)
    img_rgb = cv2.cvtColor(img, cv2.COLOR_BGR2RGB)

```

```
        # Display the image and wait for point selection
        fig, ax = plt.subplots(figsize=(12, 8))
        ax.imshow(img_rgb)
        ax.set_title("Click to select: 1) Sphere top, 2) Sphere bottom, 3) Shock wave point")
        fig.canvas.mpl_connect('button_press_event', onclick)
        plt.show()

        if len(points) != 3:
            print("Error: Three points were not selected.")
            return None

        # Find sphere center and radius
        top, bottom, shock = points
        center, radius = find_sphere_center_and_radius(top, bottom)

        # Find closest point on circle to shock point
        closest_point = closest_point_on_circle(center, radius, shock)

        # Calculate distances
        shock_distance = np.sqrt((shock[0] - center[0])**2 + (shock[1] - center[1])**2)
        standoff_distance_px = shock_distance - radius

        # Calculate measurements
        standoff_distance_mm = standoff_distance_px / calibration_factor
        sphere_diameter_mm = (2 * radius) / calibration_factor

        # Visualize the measurement
        fig, ax = plt.subplots(figsize=(12, 8))
        ax.imshow(img_rgb)
        circle = plt.Circle(center, radius, color='r', fill=False)
        ax.add_artist(circle)
        ax.plot([closest_point[0], shock[0]], [closest_point[1], shock[1]], 'b-')
        ax.plot([p[0] for p in points], [p[1] for p in points], 'ro')
        ax.plot(center[0], center[1], 'go', markersize=10)
        ax.set_title(f"Shock Standoff Distance: {standoff_distance_mm:.2f} mm\n"
                     f"Sphere Diameter: {sphere_diameter_mm:.2f} mm")
        plt.show()

        return standoff_distance_mm, sphere_diameter_mm

    def process_all_images(folder_path, calibration_factor):
        results = {}
        for filename in os.listdir(folder_path):
            if filename.endswith(('.bmp', '.jpg', '.png')):  # Add or remove file extensions as needed
                image_path = os.path.join(folder_path, filename)
                print(f"Processing {filename}...")

                standoff, diameter = measure_shock_standoff(image_path, calibration_factor)

                if standoff is not None:
```

```
                    # Extract Mach number from filename (assuming format like 'M1_75_horizontal.bmp')
                    match = re.search(r'M(\d+)_(\d+)', filename)
                    if match:
                        mach_number = float(f"{match.group(1)}.{match.group(2)}")
                        results[mach_number] = {'diameter': diameter, 'standoff': standoff}
                        print(f"Mach {mach_number}: Shock standoff distance: {standoff:.2f} mm, Sphere diameter: {diameter:.2f} mm")
                    else:
                        print(f"Warning: Couldn't extract Mach number from filename {filename}")
                else:
                    print(f"Warning: Couldn't process {filename}")

        return results

    # Add this function to parse command-line arguments
    def parse_arguments():
        parser = argparse.ArgumentParser(description="Process shock standoff images.")
        parser.add_argument("-image", type=str, help="Path to a specific image to process")
        return parser.parse_args()

    # Modify the main part of the script
    if __name__ == "__main__":
        args = parse_arguments()

        # Usage
        folder_path = r"Lab 1\Data\Images"
        calibration_factor = 14.3  # pixels/mm, as you provided

        if args.image:
            # Process a single image
            image_path = args.image
            print(f"Processing single image: {image_path}")
            standoff, diameter = measure_shock_standoff(image_path, calibration_factor)
            if standoff is not None:
                print(f"Shock standoff distance: {standoff:.2f} mm, Sphere diameter: {diameter:.2f} mm")
            else:
                print(f"Warning: Couldn't process {image_path}")
        else:
            # Process all images in the folder
            data_dict = process_all_images(folder_path, calibration_factor)

            # Now you can use data_dict for further analysis
            print("\nProcessed data:")
            for mach, data in data_dict.items():
                print(f"Mach {mach}: diameter = {data['diameter']:.2f} mm, standoff = {data['standoff']:.2f} mm")
```

B.2 mach-analysis.py

```python
import os
import re
import numpy as np
import matplotlib.pyplot as plt
from scipy.signal import find_peaks
from scipy.optimize import fsolve, curve_fit

def read_data_file(file_path):
    with open(file_path, 'r') as f:
        lines = f.readlines()

    data_start = next(i for i, line in enumerate(lines) if "X_Value" in line)
    data = np.genfromtxt(lines[data_start+1:], delimiter='\t', usecols=(1, 2))
    return data

def voltage_to_pressure(voltage, conversion_factor):
    return voltage * (conversion_factor / 0.1)

def find_steady_state(data, window_size=50):
    # Find the index of the maximum pressure
    peak_index = np.argmax(np.abs(data))

    # Define a region around the peak to search for the steady state
    search_start = max(0, peak_index - window_size*2)
    search_end = min(len(data), peak_index + window_size*2)

    # Calculate moving standard deviation
    std_dev = np.array([np.std(data[i:i+window_size]) for i in range(search_start,
    ↪  search_end-window_size)])

    # Find the region with the lowest standard deviation (most stable)
    stable_start = search_start + np.argmin(std_dev)

    return stable_start, stable_start + window_size

def calculate_mach_number(p0_gauge, p_gauge, p_atm=14.7, gamma=1.4):
    # Convert gauge pressures to absolute pressures
    p0 = p0_gauge + p_atm
    p = p_gauge + p_atm

    if p0 <= p or p <= 0:
        return np.nan

    # Use the correct isentropic flow equation for Mach number
    mach = np.sqrt((2 / (gamma - 1)) * ((p0 / p)**((gamma - 1) / gamma) - 1))

    return mach

def calculate_reynolds_numbers(mach_number, pressure, temperature, diameter):
    # Constants
    gamma = 1.4  # Ratio of specific heats for air
    R = 287.05  # Gas constant for air in J/(kg·K)
```

```
52
53          # Sutherland's law constants
54          C = 120  # Sutherland's constant for air in K
55          T0 = 291.15  # Reference temperature in K
56          mu0 = 1.827e-5  # Reference viscosity in Pa·s
57
58          # Calculate temperature ratio
59          T_ratio = 1 + (gamma - 1) / 2 * mach_number**2
60          T = temperature * T_ratio  # Static temperature
61
62          # Calculate density
63          rho = pressure / (R * T)
64
65          # Calculate velocity
66          V = mach_number * np.sqrt(gamma * R * T)
67
68          # Calculate viscosity using Sutherland's law
69          mu = mu0 * (T / T0)**(3/2) * (T0 + C) / (T + C)
70
71          # Calculate Reynolds numbers
72          Re_unit = rho * V / mu
73          Re_D = Re_unit * diameter
74
75          return Re_unit, Re_D
76
77      def mach_number_uncertainty(p0, p, dp0, dp, gamma=1.4):
78          # Function to solve for Mach number
79          def mach_equation(M):
80              return (p0/p) - (1 + (gamma-1)/2 * M**2)**(gamma/(gamma-1))
81
82          # Calculate Mach number
83          M = fsolve(mach_equation, 1.0)[0]
84
85          # Partial derivatives
86          dM_dp0 = M / (2*p0) * (1 + (gamma-1)/2 * M**2)
87          dM_dp = -M / (2*p) * (1 + (gamma-1)/2 * M**2)
88
89          # Uncertainty propagation
90          dM = np.sqrt((dM_dp0 * dp0)**2 + (dM_dp * dp)**2)
91
92          return M, dM
93
94      def process_file(file_path):
95          data = read_data_file(file_path)
96
97          stagnation_pressure = voltage_to_pressure(data[:, 0], 60)
98          static_pressure = voltage_to_pressure(data[:, 1], 15)
99
100         start, end = find_steady_state(stagnation_pressure)
101
102         avg_stagnation_pressure = np.mean(stagnation_pressure[start:end])
103         avg_static_pressure = np.mean(static_pressure[start:end])
104
```

```
105         mach_number, mach_uncertainty = mach_number_uncertainty(avg_stagnation_pressure +
            ↪  14.7,
106                                                                 avg_static_pressure +
                                                                    ↪  14.7,
107                                                                 0.01 *
                                                                    ↪  avg_stagnation_pressure,
108                                                                 0.01 *
                                                                    ↪  avg_static_pressure)
109
110         # Calculate Reynolds numbers
111         diameter = 0.0163  # Sphere diameter in meters
112         Re_unit, Re_D = calculate_reynolds_numbers(mach_number, avg_static_pressure *
            ↪  6894.75729,  # Convert psi to Pa
113                                                    297, diameter)  # Assuming 297 K (24°C)
                                                       ↪  ambient temperature
114
115         # Calculate Reynolds number uncertainty (simplified, assuming only Mach number
            ↪  contributes significantly)
116         dRe_D = Re_D * mach_uncertainty / mach_number
117
118         # Visualize the data
119         # plt.figure(figsize=(12, 6))
120         # plt.plot(stagnation_pressure, label='Stagnation Pressure')
121         # plt.plot(static_pressure, label='Static Pressure')
122         # plt.axvline(start, color='r', linestyle='--', label='Steady State Start')
123         # plt.axvline(end, color='r', linestyle='--', label='Steady State End')
124         # plt.axhline(avg_stagnation_pressure, color='g', linestyle=':', label='Avg
            ↪  Stagnation')
125         # plt.axhline(avg_static_pressure, color='m', linestyle=':', label='Avg Static')
126         # plt.legend()
127         # plt.title(f'Pressure Data for {os.path.basename(file_path)}')
128         # plt.xlabel('Sample')
129         # plt.ylabel('Pressure (psi)')
130         # plt.show()
131
132         print(f"File: {os.path.basename(file_path)}")
133         print(f"Stagnation Pressure (gauge): {avg_stagnation_pressure:.2f} psi")
134         print(f"Static Pressure (gauge): {avg_static_pressure:.2f} psi")
135         print(f"Stagnation Pressure (absolute): {avg_stagnation_pressure + 14.7:.2f}
            ↪  psi")
136         print(f"Static Pressure (absolute): {avg_static_pressure + 14.7:.2f} psi")
137         print(f"Pressure Ratio (p/p0): {(avg_static_pressure + 14.7) /
            ↪  (avg_stagnation_pressure + 14.7):.4f}")
138         print(f"Calculated Mach Number: {mach_number:.2f}")
139         print(f"Unit Reynolds Number: {Re_unit:.2e} 1/m")
140         print(f"Diametric Reynolds Number: {Re_D:.2e}")
141         print(f"Reynolds Number Uncertainty: {dRe_D:.2e}")
142         print("---")
143
144         return mach_number, mach_uncertainty, avg_stagnation_pressure,
            ↪  avg_static_pressure, Re_unit, Re_D, dRe_D
145
146     def extract_mach_number(filename):
147         match = re.search(r'M(\d+)_(\d+)', filename)
```

```
148         if match:
149             return float(f"{match.group(1)}.{match.group(2)}")
150         return None
151 
152     # Directory containing the data files
153     data_dir = r"Lab 1\Working-data"
154 
155     results = []
156 
157     for filename in os.listdir(data_dir):
158         if filename.endswith(".txt"):
159             file_path = os.path.join(data_dir, filename)
160             expected_mach = extract_mach_number(filename)
161             if expected_mach:
162                 calculated_mach, mach_uncertainty, stagnation_p, static_p, Re_unit, Re_D,
                    ↪  dRe_D = process_file(file_path)
163                 results.append((expected_mach, calculated_mach, mach_uncertainty,
                    ↪  stagnation_p, static_p, Re_unit, Re_D, dRe_D))
164                 print(f"Processed {filename}: Expected M={expected_mach:.2f}, Calculated
                    ↪  M={calculated_mach:.2f} ± {mach_uncertainty:.2f}")
165 
166     # Sort results by expected Mach number
167     results.sort(key=lambda x: x[0])
168 
169     # Plotting Mach number vs Re_D with error bars (vertical only)
170     plt.figure(figsize=(10, 6))
171     mach_numbers, re_d_numbers, re_d_uncertainties = zip(*[(calc_mach, re_d, dre_d)
172                                                            for _, calc_mach, _, _, _, _,
                                                               ↪  re_d, dre_d in results])
173 
174     # Use absolute values for uncertainties
175     re_d_uncertainties = np.abs(re_d_uncertainties)
176 
177     plt.errorbar(mach_numbers, np.abs(re_d_numbers), yerr=re_d_uncertainties, fmt='o',
        ↪  capsize=5)
178     plt.xlabel('Mach Number')
179     plt.ylabel('Reynolds Number (Re_D)')
180     plt.title('Mach Number vs Reynolds Number with Uncertainties')
181     plt.grid(True)
182     plt.show()
183 
184     # Print results
185     print("\nMach Number and Reynolds Number Comparison with Uncertainties:")
186     print("Expected M | Calculated M ± Uncertainty | Re_D ± Uncertainty")
187     print("-" * 70)
188     for expected, calculated, mach_unc, _, _, _, re_d, dre_d in results:
189         print(f"{expected:.2f}       | {calculated:.2f} ± {abs(mach_unc):.2f}
            ↪  | {re_d:.2e} ± {abs(dre_d):.2e}")
190 
191     # Calculate and print average error
192     valid_results = [(exp, calc) for exp, calc, _, _, _, _, _, _ in results if not
        ↪  np.isnan(calc)]
193     if valid_results:
194         errors = [abs(calc - exp) for exp, calc in valid_results]
```

```
        avg_error = np.mean(errors)
        print(f"\nAverage Mach number error: {avg_error:.4f}")
    else:
        print("\nNo valid Mach number calculations.")

    # Add these new functions after the existing functions

    def calculate_nondimensional_standoff(mach_numbers, diameters, standoff_distances):
        return np.array(standoff_distances) / np.array(diameters)

    def qualitative_scaling(M, c, gamma=1.4):
        return c * np.sqrt((1 + (gamma - 1) / 2 * M**2) / (M - 1))

    def basic_fit(M, c, alpha, beta, gamma=1.4):
        return c * gamma**alpha * M**beta

    def offset_fit(M, c, alpha, beta, gamma=1.4):
        return c * gamma**alpha * (M - 1)**beta

    def plot_nondimensional_standoff(mach_numbers, nondimensional_standoff):
        plt.figure(figsize=(10, 6))
        plt.scatter(mach_numbers, nondimensional_standoff)
        plt.xlabel('Mach Number')
        plt.ylabel('delta/D')
        plt.title('Non-dimensional Standoff Distance vs Mach Number')
        plt.grid(True)
        plt.show()

    def perform_curve_fits(mach_numbers, nondimensional_standoff):
        # Qualitative scaling fit
        popt_qual, _ = curve_fit(qualitative_scaling, mach_numbers,
        ↪  nondimensional_standoff)

        # Basic fit
        popt_basic, _ = curve_fit(basic_fit, mach_numbers, nondimensional_standoff)

        # Offset fit
        popt_offset, _ = curve_fit(offset_fit, mach_numbers, nondimensional_standoff)

        return popt_qual, popt_basic, popt_offset

    def plot_curve_fits(mach_numbers, nondimensional_standoff, popt_qual, popt_basic,
    ↪  popt_offset):
        plt.figure(figsize=(12, 8))
        plt.scatter(mach_numbers, nondimensional_standoff, label='Data')

        M_fit = np.linspace(min(mach_numbers), max(mach_numbers), 100)

        plt.plot(M_fit, qualitative_scaling(M_fit, *popt_qual), 'r-', label='Qualitative
        ↪  Scaling')
        plt.plot(M_fit, basic_fit(M_fit, *popt_basic), 'g-', label='Basic Fit')
        plt.plot(M_fit, offset_fit(M_fit, *popt_offset), 'b-', label='Offset Fit')

        plt.xlabel('Mach Number')
```

```
246         plt.ylabel('delta/D')
247         plt.title('Curve Fits for Non-dimensional Standoff Distance')
248         plt.legend()
249         plt.grid(True)
250         plt.show()
251 
252     # ... (keep all existing code up to the standoff_data dictionary)
253 
254     # Replace the first instance of standoff_data with this:
255     standoff_data = {
256         2.25: [16.65, 16.82, 16.65, 2.40, 2.29, 2.44],
257         1.75: [16.69, 16.65, 16.69, 3.95, 3.97, 4.11],
258         2.0: [16.87, 16.51, 16.74, 2.80, 2.95, 2.98],
259         2.5: [16.83, 16.69, 16.82, 2.02, 2.22, 2.07],
260         2.75: [16.78, 16.78, 16.78, 2.00, 1.95, 1.93],
261         3.0: [16.69, 16.78, 16.78, 1.71, 1.80, 1.74]
262     }
263 
264     # Add these new functions after the existing functions
265 
266     def calculate_nondimensional_standoff(mach_numbers, diameters, standoff_distances):
267         return np.array(standoff_distances) / np.array(diameters)
268 
269     def qualitative_scaling(M, c, gamma=1.4):
270         return c * np.sqrt((1 + (gamma - 1) / 2 * M**2) / (M - 1))
271 
272     def basic_fit(M, c, alpha, beta, gamma=1.4):
273         return c * gamma**alpha * M**beta
274 
275     def offset_fit(M, c, alpha, beta, gamma=1.4):
276         return c * gamma**alpha * (M - 1)**beta
277 
278     def plot_nondimensional_standoff(mach_numbers, nondimensional_standoff):
279         plt.figure(figsize=(10, 6))
280         plt.scatter(mach_numbers, nondimensional_standoff)
281         plt.xlabel('Mach Number')
282         plt.ylabel('delta/D')
283         plt.title('Non-dimensional Standoff Distance vs Mach Number')
284         plt.grid(True)
285         plt.show()
286 
287     def perform_curve_fits(mach_numbers, nondimensional_standoff):
288         # Qualitative scaling fit
289         popt_qual, _ = curve_fit(qualitative_scaling, mach_numbers,
            ↪ nondimensional_standoff)
290 
291         # Basic fit
292         popt_basic, _ = curve_fit(basic_fit, mach_numbers, nondimensional_standoff)
293 
294         # Offset fit
295         popt_offset, _ = curve_fit(offset_fit, mach_numbers, nondimensional_standoff)
296 
297         return popt_qual, popt_basic, popt_offset
298 
```

```
299     def plot_curve_fits(mach_numbers, nondimensional_standoff, popt_qual, popt_basic,
        ↪  popt_offset):
300         plt.figure(figsize=(12, 8))
301         plt.scatter(mach_numbers, nondimensional_standoff, label='Data')
302 
303         M_fit = np.linspace(min(mach_numbers), max(mach_numbers), 100)
304 
305         plt.plot(M_fit, qualitative_scaling(M_fit, *popt_qual), 'r-', label='Qualitative
            ↪  Scaling')
306         plt.plot(M_fit, basic_fit(M_fit, *popt_basic), 'g-', label='Basic Fit')
307         plt.plot(M_fit, offset_fit(M_fit, *popt_offset), 'b-', label='Offset Fit')
308 
309         plt.xlabel('Mach Number')
310         plt.ylabel('delta/D')
311         plt.title('Curve Fits for Non-dimensional Standoff Distance')
312         plt.legend()
313         plt.grid(True)
314         plt.show()
315 
316     # ... (keep the rest of your existing code)
317 
318     # After your existing code, add:
319 
320     # Process the standoff data
321     mach_numbers = list(standoff_data.keys())
322     diameters = [[d for d in data[:3]] for data in standoff_data.values()]
323     standoff_distances = [[s for s in data[3:]] for data in standoff_data.values()]
324 
325     # Calculate average values
326     avg_diameters = [np.mean(d) for d in diameters]
327     avg_standoffs = [np.mean(s) for s in standoff_distances]
328 
329     # Calculate non-dimensional standoff
330     nondimensional_standoff = calculate_nondimensional_standoff(mach_numbers,
        ↪  avg_diameters, avg_standoffs)
331 
332     # Plot non-dimensional standoff distance
333     plot_nondimensional_standoff(mach_numbers, nondimensional_standoff)
334 
335     # Perform curve fits
336     popt_qual, popt_basic, popt_offset = perform_curve_fits(mach_numbers,
        ↪  nondimensional_standoff)
337 
338     # Plot curve fits
339     plot_curve_fits(mach_numbers, nondimensional_standoff, popt_qual, popt_basic,
        ↪  popt_offset)
340 
341     # Print table of fit constants
342     print("\nFit Constants:")
343     print("Qualitative Scaling: c =", popt_qual[0])
344     print("Basic Fit: c =", popt_basic[0], ", alpha =", popt_basic[1], ", beta =",
        ↪  popt_basic[2])
345     print("Offset Fit: c =", popt_offset[0], ", alpha =", popt_offset[1], ", beta =",
        ↪  popt_offset[2])
```

B.3 calibration.py

```python
import cv2
import numpy as np
import matplotlib.pyplot as plt

def onclick(event):
    global points
    if event.button == 1 and len(points) < 2:  # Left mouse button
        points.append((event.xdata, event.ydata))
        plt.plot(event.xdata, event.ydata, 'ro')
        plt.draw()
        if len(points) == 2:
            plt.close()

def calculate_calibration_factor(image_path, grid_spacing_mm=5):
    global points
    points = []

    # Read the image
    img = cv2.imread(image_path)
    img_rgb = cv2.cvtColor(img, cv2.COLOR_BGR2RGB)

    # Display the image and wait for point selection
    fig, ax = plt.subplots()
    ax.imshow(img_rgb)
    ax.set_title("Click on two points 10 grid spaces apart")
    fig.canvas.mpl_connect('button_press_event', onclick)
    plt.show()

    if len(points) != 2:
        print("Error: Two points were not selected.")
        return None

    # Calculate distance between selected points
    distance_px = np.sqrt((points[1][0] - points[0][0])**2 + (points[1][1] -
    ↪  points[0][1])**2)

    # Calculate calibration factor (pixels per mm)
    calibration_factor = distance_px / (10 * grid_spacing_mm)

    # Visualize the selected points
    fig, ax = plt.subplots()
    ax.imshow(img_rgb)
    ax.plot([points[0][0], points[1][0]], [points[0][1], points[1][1]], 'r-')
    ax.plot([p[0] for p in points], [p[1] for p in points], 'ro')
    ax.set_title(f"Calibration: {calibration_factor:.2f} pixels/mm")
    plt.show()

    return calibration_factor

# Usage
calibration_image_path = "Lab 1\Data\Images\Wind_off_calibrate.bmp"
cal_factor = calculate_calibration_factor(calibration_image_path)
```

```
52      if cal_factor:
53          print(f"Calibration factor: {cal_factor:.2f} pixels/mm")
```