The University of Texas at Austin
Aerospace Engineering
and Engineering Mechanics
Cockrell School of Engineering

ASE 375 Electromechanical Systems
Section 14115

Monday: 3:00 - 6:00 pm

Report 5: Strain-gage Measurements and Fourier Analysis

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Contents

1 Introduction

This experiment presents results of strain-gage measurements performed at the root of a cantilever beam undergoing (1) Different weights placed at its tip and (2) An applied force which leaves the beam freely vibrating. The purpose of this experiment is to learn about the Wheatstone bridge configurations in gathering our strain-gage data on the beam and how different sampling frequencies affect these strain measurements.

In (1), results of strain measurements at the root of the beam are gathered using two different half-bridge configurations. Using the Euler-Bernoulli Beam theory, we make comparisons between the ideal and real strain-gage results. In (2), we model the strain as a function of time at different sampling frequencies. For (2), we utilize one of the half-bridge configurations and perform Fourier analysis to estimate the natural frequency of the cantilever beam.

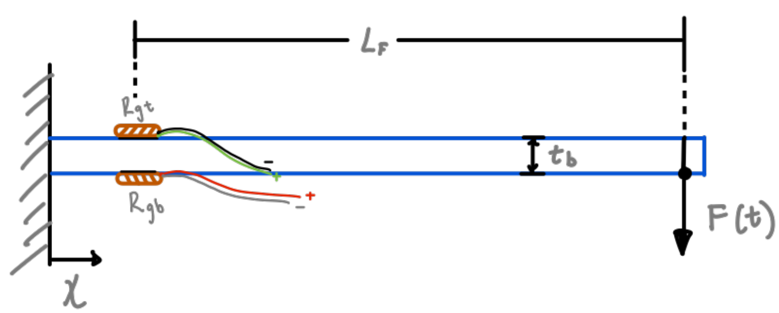
2 Equipment

Measurement devices and hardware used in this lab include:

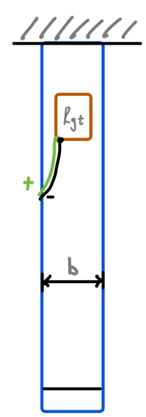
- Cantilever Beam:

  A uniform aluminum beam that is cantilevered at one end (root) with a string attached at the other end (tip) for holding weights.

Side View



Top View



- Digital Calipers, Bias Error = 0.005 mm:

  Used for measuring outer and inner dimensions of objects. In our case we use it to the measure the wall thickness, t_b, and width, b, of the cantilever beam.

- Ruler/Tape Measure, Bias Error = 0.5 mm:

  Used to measure the length of the cantilever beam, L_F.

- Brass Slotted Weights with hanger:

  Used to induce bending on the tip of the cantilever beam to acquire strain measurements. Hanger is 50 g along with ten 20 g weights. Total 250 grams.

- Strain Gauge:

  Sensor used to measure surface strains which then converts it into a change in electrical resistance. An output voltage is produced when a surface strain occurs since there is a resistance change within the bridge configuration.

  In this lab, we have 2 strain gauges placed at the root of the cantilever beam. One of them is on the top surface of the beam, R_{gt}, and the other is placed on the bottom surface, R_{gb}.

- DAQ, NI-9215 Voltage Input Module, and LabVIEW:

  Data Acquisition System used to process sample measurements into digital data. NI-9215 is an analog input module used to measure the output voltage signals of sensors and send it through the DAQ system. LabVIEW used to model these output voltages read from the DAQ of the strain-gage measurements. We connect to the 5V port of the DAQ for our experiment.

- Solderless Breadboard, Jumper Wires, and 350Ω Resistors:

  Used to make connections to the input analog modules and to construct circuits. In this lab we build two configurations of the Wheatstone half-bridge circuit with two 350Ω dummy resistors, labeled R_D, and the two strain gauges, R_{gt} and R_{gb}. Configurations are shown below, with an example of the configuration B circuit on the breadboard:

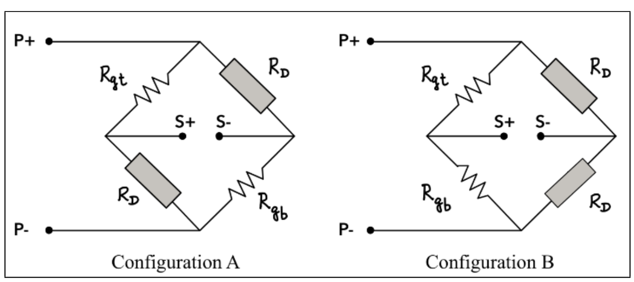


Figure 1: Half-Bridge Configurations, V_{OUT} between S^+ and S^-

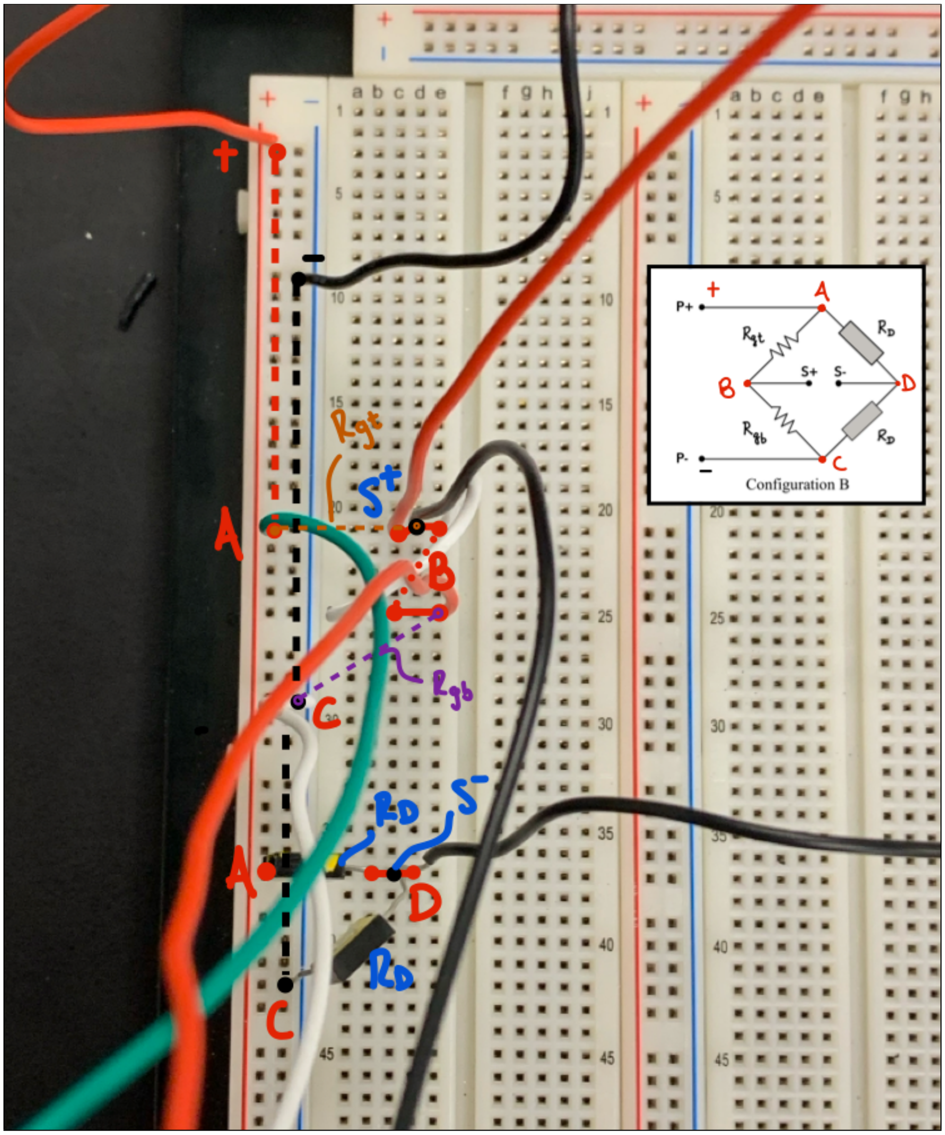


Figure 2: Configuration B Half-Bridge Circuit on the Breadboard

To switch from Configuration B to Configuration A, rearrange connections at B and D such that the dummy resistor from $\overrightarrow{DC}$ is now $\overrightarrow{BC}$ and bring the positive wire of the bottom strain-gage, R_{gb}, from B to D.

3 Procedure

This experiment is broken up into two parts. In Part 1, we will be gathering the average output voltage read from the two half-bridge configurations as shown in Figure 1 as weights hanging off the tip of the cantilever beam. In Part 2, we will be testing different sampling frequencies and collecting the output voltage read from the half-bridge circuit following Configuration B as the beam undergoes free vibration as a result of an applied force at the tip.

Before experimenting, we must set up the LabVIEW model following the form:

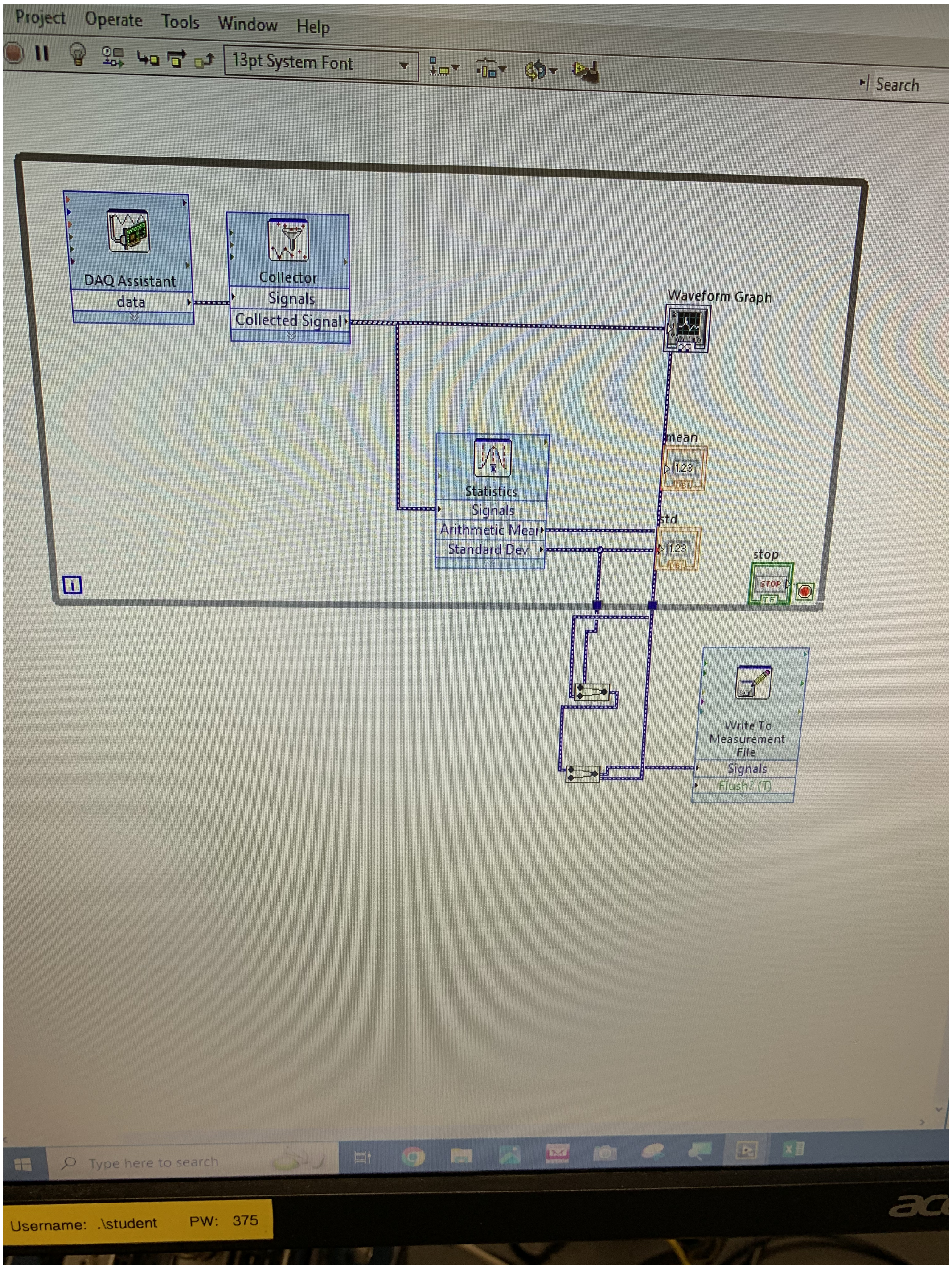


Figure 3: LabVIEW Block Diagram for modeling $V_{OUT} = f(W)$

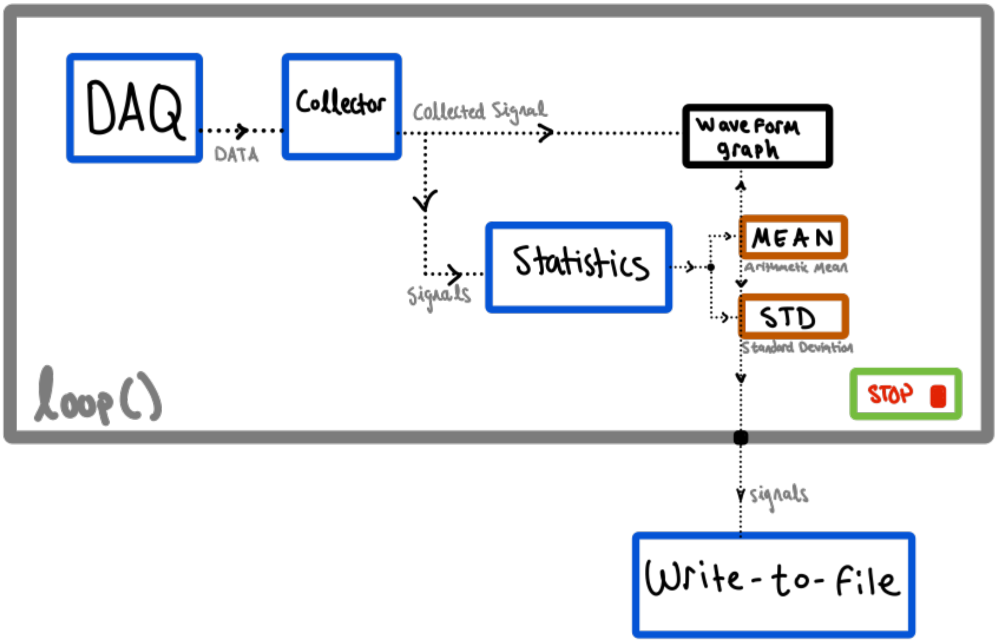


Figure 4: LabVIEW Block Diagram for modeling $V_{OUT} = f(t)$

The Write-to-File block will be used for Part 2 as we model the strain and output voltage as a function of time.

3.1 Part 1

1. Create the half-bridge circuit following Configuration A as shown in Figure 1.
2. Verify that the output is modeled correctly through LabVIEW by applying force to the beam. Once the setup is working well continue to the next step.
3. Begin with no weight hanging at the tip of the beam. This is the "zero" reading. Once the output stabilizes, note down the mean and standard deviation of V_{OUT}.
4. Continue collecting the output readings at each weight (in grams) as follows:

   $\mathbf{W}_{\text{up}} = [0, 50, 90, 130, 170, 210, 250]$.

   - $\mathbf{W}_{\text{up}}$ is the sequence we follow as we add weight to the tip of the beam. Take another reading for every 40 g added until a total of 250 g.
5. Now repeat step 3 as go down in weight, which follows the sequence:

   $\mathbf{W}_{\text{down}} = [230, 210, 190, 170, 150, 130, 110, 90, 70, 50, 0]$.

   - We record the output reading as we remove one 20 g at a time until there is no more weight.
6. Switch to the half-bridge circuit following Configuration B as shown in Figure 1 and repeat steps 2 through 5.
7. We note down our observations and begin **Data Processing** to model the strain as we loaded and unloaded the beam with weights.

3.2 Part 2

In this part, we will be utilizing the write-to-file block to collect our data through time. Instead of adding weight to the tip of the beam, we will flick the tip of the beam so it will freely oscillate up and down.

1. Use the half-bridge circuit following Configuration B as shown in Figure 1.
2. In LabVIEW, we can change the sampling frequency, f_S, and number of samples collected, N, by going into the DAQ and Collector block. We will collect output voltage readings through this sequence of sampling frequencies:

   $\mathbf{f}_S = [10,\ 20,\ 50,\ 100,\ 1000]$ Hz
3. For each f_{S_i}, run the sampling model on LabVIEW and apply a force at the tip of the beam so that it will freely oscillate until the sampling reaches its end. Stop and Save the sample data.
4. Once finished, note down observations of the effects of different sampling frequencies and continue to **Data Processing** to do time-frequency analysis and find the natural frequency, f_N, of the cantilevered aluminum beam.

4 Data Processing

4.1 Variables and Equations

Euler-Bernoulli Beam Theory:

$$\varepsilon_b = \frac{F L_f t_b}{2EI} \tag{1}$$

Variables:

I. ϵ_b: Bending Strain

II. F: Force

III. L_f: Length of the Beam

IV. b: Width of Beam

V. t_b: Thickness of the Beam

VI. E: Young's Modulus

VII. I: Moment of Inertia

VIII. V: Voltage

IX. W: Weight

Table 1: Constants:

Property	**Value**
L_f (mm)	335 ± 0.5
b (mm)	25.8 ± 0.005
t_b (mm)	3.32 ± 0.005
E_{aluminum} (GPa)	70
I (Moment of Inertia) mm^4	78.677 ± 0.00158
Gravity m/s^2	9.81

5 Results and Analysis

Table 2: Loading

Weight(g)	Force (N)	Strain	Voltage (A)	Strain (A)	Voltage (B)	Strain (B)
0	0	0	2.49077	0	0.00458	0
50	0.4905	4.95269E-05	2.49093	6.72E-05	0.00435	4.79167E-05
90	0.8829	8.91484E-05	2.49106	0.0001218	0.00419	8.125E-05
130	1.2753	0.00012877	2.49116	0.0001638	0.00404	0.0001125
170	1.6677	0.000168391	2.49125	0.0002016	0.00385	0.000152083
210	2.0601	0.000208013	2.49136	0.0002478	0.00366	0.000191667
250	2.4525	0.000247635	2.49147	0.000294	0.00348	0.000229167

As shown in table 2, the measured values for both bridge A and B closely resemble the theoretical values. They all increa saercni l

Table 3: Unloading

Weight (g)	Force (N)	Strain	Voltage (A)	Strain (A)	Voltage (B)	Strain (B)
230	2.2563	0.000227824	2.49145	0.0002856	0.00358	0.000208333
210	2.0601	0.000208013	2.49144	0.0002814	0.00366	0.000191667
190	1.8639	0.000188202	2.49143	0.0002772	0.00378	0.000166667
170	1.6677	0.000168391	2.49141	0.0002688	0.00386	0.00015
150	1.4715	0.000148581	2.49139	0.0002604	0.00396	0.000129167
130	1.2753	0.00012877	2.49137	0.000252	0.00405	0.000110417
110	1.0791	0.000108959	2.49133	0.0002352	0.00411	9.79167E-05
90	0.8829	8.91484E-05	2.49129	0.0002184	0.00421	7.70833E-05
70	0.6867	6.93377E-05	2.49127	0.00021	0.00428	0.0000625
50	0.4905	4.95269E-05	2.49125	0.0002016	0.00436	4.58333E-05
0	0	0	2.49113	0.0001512	0.00456	4.16667E-06

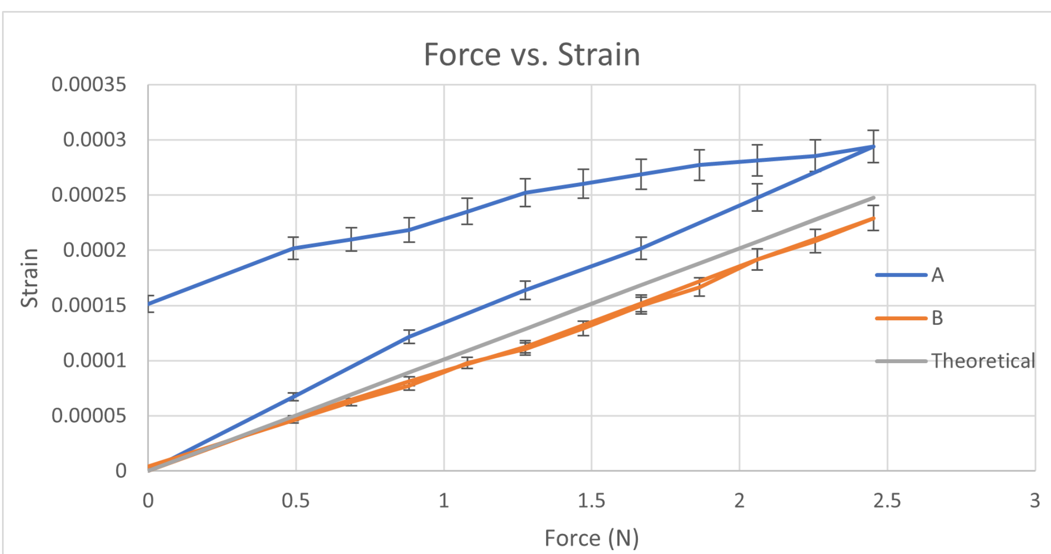


Figure 5: Force vs Strain

For all three cases in figure 5, the loading stage is linear and at a similar rate. For unloading, Configuration

B follows the same curve as the loading stage, but configuration A does not. This is because the bridge A has resistors in parallel, where bridge B is in series.

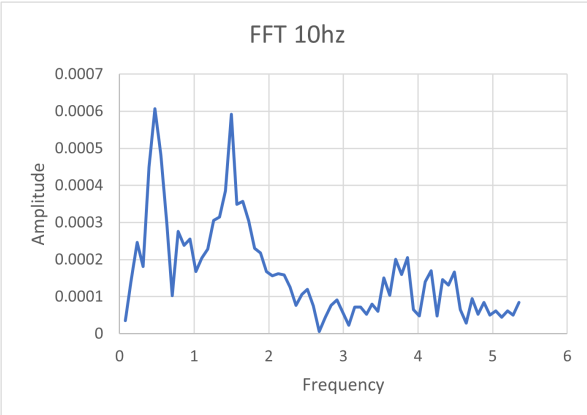


Figure 6: FFT at 10Hz

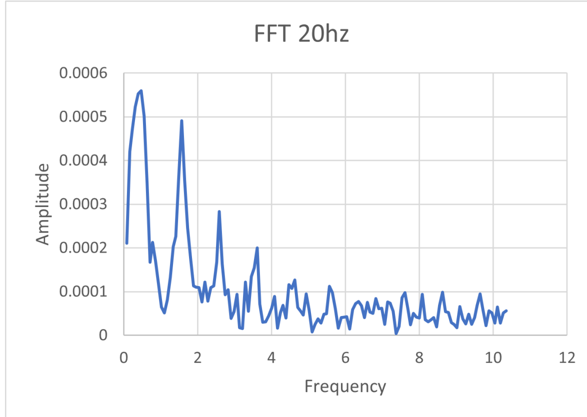


Figure 7: FFT at 20Hz

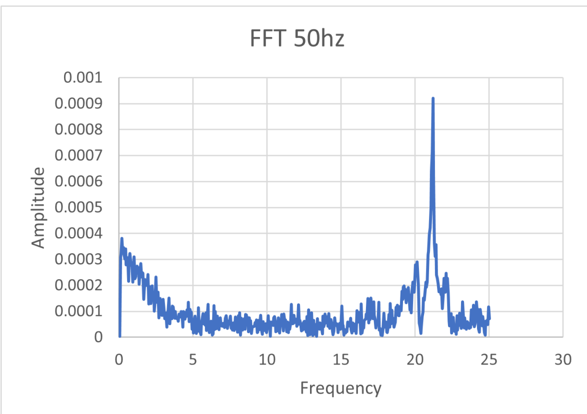


Figure 8: FFT at 50Hz

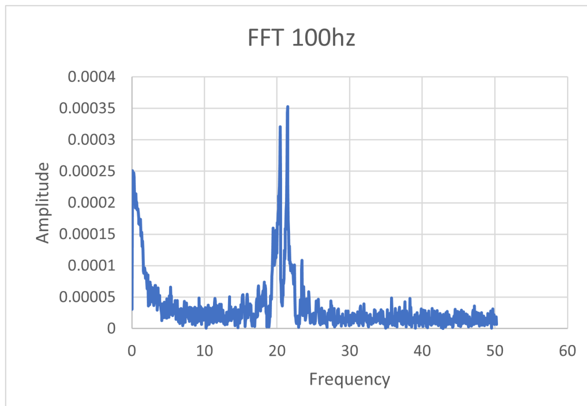


Figure 9: FFT at 100Hz

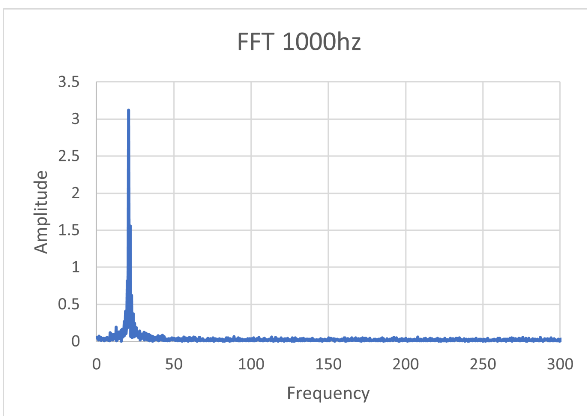


Figure 10: FFT for all Sampling Frequencies

The sampling frequencies $\mathbf{f}_S = [10,\ 20,\ 50,\ 100,\ 1000]$ Hz has the natural frequencies $\mathbf{f}_N = [1.5,\ 3.15,\ 21.21,\ 21.39,\ 20.75]$ Hz. The natural frequency increases as the sampling frequency increases until it levels off at around 20 Hz. This is because we need double or greater for the sampling frequency compared to the natural frequency in order to have the natural frequency show up in the FFT.

Nyquist Theorem states that

6 Conclusion

In this lab, we learned how to use two different Wheatstone Bridge configurations to measure strain. After subjecting a cantilever beam to a tip force, we measured the strain using strain gages and the wheatstone bridge. After acquiring the data, a fourier transform was performed and the natural frequency was found.

Appendices

t-Distribution Tables

Table A11. *t*-Distribution

Values of z for given values of the distribution function $F(z)$ (cf. p. 754).
Example: For 9 degrees of freedom, $z = 1.83$ when $F(z) = 0.95$.

$F(z)$	Number of Degrees of Freedom									
	1	2	3	4	5	6	7	8	9	10
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.33	0.29	0.28	0.27	0.27	0.27	0.26	0.26	0.26	0.26
0.7	0.73	0.62	0.58	0.57	0.56	0.55	0.55	0.55	0.54	0.54
0.8	1.38	1.06	0.98	0.94	0.92	0.91	0.90	0.89	0.88	0.88
0.9	3.08	1.89	1.64	1.53	1.48	1.44	1.42	1.40	1.38	1.37
0.95	6.31	2.92	2.35	2.13	2.02	1.94	1.90	1.86	1.83	1.81
0.975	12.7	4.30	3.18	2.78	2.57	2.45	2.37	2.31	2.26	2.23
0.99	31.8	6.97	4.54	3.75	3.37	3.14	3.00	2.90	2.82	2.76
0.995	63.7	9.93	5.84	4.60	4.03	3.71	3.50	3.36	3.25	3.17
0.999	318.3	22.3	10.2	7.17	5.89	5.21	4.79	4.50	4.30	4.14

$F(z)$	Number of Degrees of Freedom									
	11	12	13	14	15	16	17	18	19	20
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
0.7	0.54	0.54	0.54	0.54	0.54	0.54	0.53	0.53	0.53	0.53
0.8	0.88	0.87	0.87	0.87	0.87	0.87	0.86	0.86	0.86	0.86
0.9	1.36	1.36	1.35	1.35	1.34	1.34	1.33	1.33	1.33	1.33
0.95	1.80	1.78	1.77	1.76	1.75	1.75	1.74	1.73	1.73	1.73
0.975	2.20	2.18	2.16	2.15	2.13	2.12	2.11	2.10	2.09	2.09
0.99	2.72	2.68	2.65	2.62	2.60	2.58	2.57	2.55	2.54	2.53
0.995	3.11	3.06	3.01	2.98	2.95	2.92	2.90	2.88	2.86	2.85
0.999	4.03	3.93	3.85	3.79	3.73	3.69	3.65	3.61	3.58	3.55

$F(z)$	Number of Degrees of Freedom									
	22	24	26	28	30	40	50	100	200	∞
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.25
0.7	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.52
0.8	0.86	0.86	0.86	0.86	0.85	0.85	0.85	0.85	0.84	0.84
0.9	1.32	1.32	1.32	1.31	1.31	1.30	1.30	1.29	1.29	1.28
0.95	1.72	1.71	1.71	1.70	1.70	1.68	1.68	1.66	1.65	1.65
0.975	2.07	2.06	2.06	2.05	2.04	2.02	2.01	1.98	1.97	1.96
0.99	2.51	2.49	2.48	2.47	2.46	2.42	2.40	2.37	2.35	2.33
0.995	2.82	2.80	2.78	2.76	2.75	2.70	2.68	2.63	2.60	2.58
0.999	3.51	3.47	3.44	3.41	3.39	3.31	3.26	3.17	3.13	3.09

NI-9215 Datasheet

https://www.amc-systeme.de/files/pdf/ni-9215-amc.pdf