The University of Texas at Austin
Aerospace Engineering and Engineering Mechanics
Cockrell School of Engineering

ASE 375 Electromechanical Systems
Section 14115

Monday: 3:00 - 6:00 pm

Report 3: Measuring Displacement

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Contents

1 Introduction

In this experiment we take a look at (1) Using a linear potentiometer to measure displacement of a small scale wing model with weights attached at different locations, (2) Determining the twist angle from these displacement measurements, and (3) Calculating the shear center, bending stiffness, and torsional stiffness of the wing model.

This experiment attempts to accurately model measurements from the linear potentiometer in comparison to the ideal linear relationship from theoretical predictions. From these measurements we can then calculate the wing's shear center location and stiffness.

2 Equipment

Devices used in this lab include:

- Digital Calipers: Used for measuring outer and inner dimensions of objects. In our case, digital calipers were used to measure the chord length of the small scale wing platofrm, the distance steps taken to calibrate the linear potentiometer, and the markings indicating chordwise locations for weight placement on the wing model.

- Brass Slotted Weights with hanger: Used to induce bending on the wing model to acquire displacement measurements. Total 250 grams.

- LP804 Series Miniature Linear Potentiometer: Device used to measure linear position and displacement from wing bending and twisting.

- 'Rare Earth' Magnets: High strength magnets used for secure placement of linear potentiometer to the wing platform to acquire measurements without slippage.

- DAQ, NI-9215 Voltage Input Module, and LabVIEW: Data Acquisition System used to process data collected by digitizing the analog information into "bins" for a computer. NI-9215 used to measure the input voltage signals for the DAQ system. LabVIEW used to model voltages through the DAQ read from the linear potentiometer displacement.

- Solderless Breadboard and Jumper Wires: Used to make connections from the linear potentiometer to power, ground, and signals for data collection.

3 Procedure

This section covers our procedure for this experiment broken into three parts. NOTE: For gathering measurements, ensure the standard deviation has reached a very small approximate steady-state and then mark down the LabVIEW output (upto the thousandth decimal digit).

3.1 Calibration of Linear Potentiometer

This first part contains our procedure for calibrating the linear potentiometer. Before beginning calibration, this part required connection setup of the linear potentiometer to the DAQ, NI-9215, and LabVIEW to collect and process data to model the input voltage measurements.

1. Ensure connection between the DAQ system (NI-9215) and the Linear Potentiometer is correct.

2. Take the potentiometer and place it on its side for better control of its displacement.

3. Using the digital calipers, measure from the base of the moving shaft on the linear potentiometer to the first displacement step x_1. We let the displacement step $\Delta x \approx 10\,mm$ for our calibration process.

4. From the LabVIEW plot, gather the average voltage with the standard deviation for x_1. This will be a repeated process for each x_i.

5. Repeat the displacement and measurement process for $i >= 5$ times, i.e. the next step is $x_2 \approx 20\,mm$. We let $i = 6$ with an additional final calibration measurement $\approx 20\,mm$ from the x_6 step to cover the full range of the $5V$ input voltage.

6. We are now able to take the calibration data into Data Processing for plotting displacement mm versus voltage V to observe the linearity of the linear potentiomenter.

3.2 Chordwise Displacement

This is Part 1 of measuring displacements from weights attached to our wing model. Enusre proper setup of the linear potentiometer under the wing platform to begin displacement measurement at different chordwise locations.

1. With the digital caliper, measure the chord length of the wing model. With the caliper, make $n >= 10$ markings for chordwise locations between 0 and c (chord). For our setup, we marked chordwise locations $y_c \approx [0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100]\ mm$, from trailing edge to leading edge.

2. Place a 'rare earth' magnet to either the leading edge or trailing edge joint and secure the tip of the potentiometer to that joint.

3. Hang the weight (250 grams in our setup) to the first chordwise position and gather the average voltage with the standard deviation from the LabVIEW model. This will be used to process displacement measurement.

4. Repeat step (3) through each y_c element and mark down the measurements for each position.

5. Once finished with measurements at the first edge, switch to the other edge and follow steps (3) and (4) again. For example, if leading edge was chosen first, position the magnet and potentiomenter to the trailing edge joint next.

6. After finishing these procedure, take measurements into Data Processing for determining the Shear Center location of the wing model.

3.3 Spanwise Displacement

Part 2 of displacement measurements involve measuring spanwise displacement due to weight hanging at three different trailing edge joints throughout the span of the wing platform. Ensure proper setup of linear potentiomenter under the span of the wing platform.

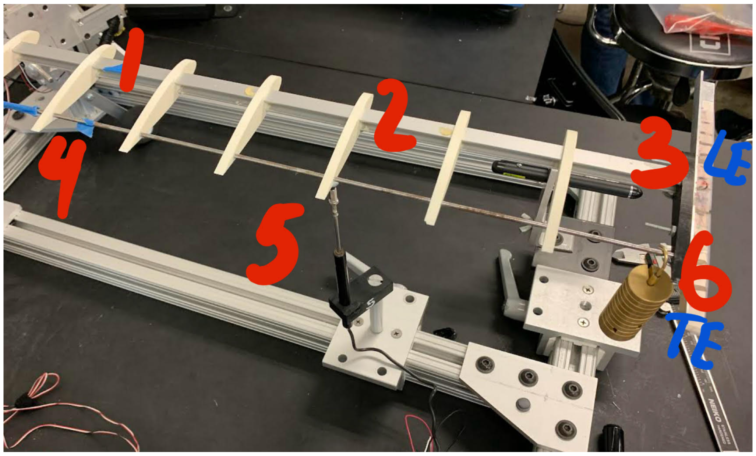

Figure 1: Spanwise position indexing with Leading and Trailing Edge labels

1. Start by hanging the weight at the 6^{th} position as shown in **Figure (1)**.
2. Place the magnet and potentiomenter at a spanwise location of choice and gather the input voltage measurement along with the standard deviation.
3. Repeat (2) measurements for all other spanwise locations.
4. Next, move the weight to the 5^{th} position.
5. Follow steps (2) and (3) for spanwise positions $x_S = [1, 2, 4, 5]$
6. Lastly, move the weight to position 4
7. Follow steps (2) and (3) for positions $x_S = [1, 4]$
8. Now the spanwise data can be taken to Data Processing for computing torque and stiffness on the wing model

4 Data Processing

Variables

i. (EI): Bending Stiffness

ii. (GJ): Torsional Stiffness

iii. (e): Location of Shear Center

iv. (S_j): Shear Force at a j coordinate

v. (x_j): Spanwise coordinate

vi. (y_f): Chordwise Coordinate

vii. (M_j): Moment acting at j coordinate

viii. (τ_j): Torque acting at j coordinate

ix. x_i: Spanwise locations

x. w_i: Bending deflection of the shear center

xi. θ_i: Sectional twist

Equations

$$M_j = S_j x_j \tag{1}$$

$$\tau = S(y_f - e) \tag{2}$$

$$w_i = \frac{S_j}{EI}\left(\frac{x_j x_i^2}{2} - \frac{x_i^3}{6}\right) \tag{3}$$

$$\theta = \frac{\tau}{GJ} x_i \tag{4}$$

$$\theta = \arctan\left(\frac{x_B - x_A}{c}\right) \tag{5}$$

5 Results and Analysis

Calibration (mm)	Voltage (V)	STD
10	0.5706	0.0007
20.09	1.013	0.00071
29.99	1.82	0.00088
40.04	2.485	0.0009
50.01	3.023	0.0008
60.09	3.699	0.00078
80.06	4.996	0.0004

Table 1: Calibration Data

X-pos.	LE Volt V	LE STD	LE Def. mm	TE Volt V	TE STD	TE Def. mm	Theta rad
0	2.014	0.00096	29.2859375	1.948	0.00093	28.2546875	-0.010230298
10	2.014	0.00096	29.2859375	1.966	0.00099	28.5359375	-0.007440339
20	2.005	0.00096	29.1453125	1.986	0.00095	28.8484375	-0.00294518
30	2.004	0.00095	29.1296875	2.002	0.001	29.0984375	-0.00031002
40	1.99	0.00096	28.9109375	2.029	0.00093	29.5203125	0.006045313
50	1.997	0.001	29.0203125	2.058	0.001	29.9734375	0.009455323
60	1.993	0.00099	28.9578125	2.081	0.00099	30.3328125	0.013640027
70	1.985	0.00097	28.8328125	2.108	0.00098	30.7546875	0.01906391
80	1.9898	0.00099	28.9078125	2.136	0.00097	31.1921875	0.022658572
90	1.988	0.001	28.8796875	2.165	0.001	31.6453125	0.027429874
100	1.979	0.00098	28.7390625	2.186	0.00096	31.9734375	0.032076048

Table 2: Leading and Trailing Edge Data

The calibration table in Table (1) represents how we gained the voltage to displacement relationship for the linear potentiometer. The voltage data was then plotted against the displacement data and a linear relationship was found, as shown in figure (2).

(2) contains the voltage data at various positions along the chord. It is then converted into distance using the calibration equation. The deflection at the Leading Edge and Trailing Edge is used to calculate the angle of twist with (5) and shear center of the wing. Plotting the angle of twist against the X position gives a

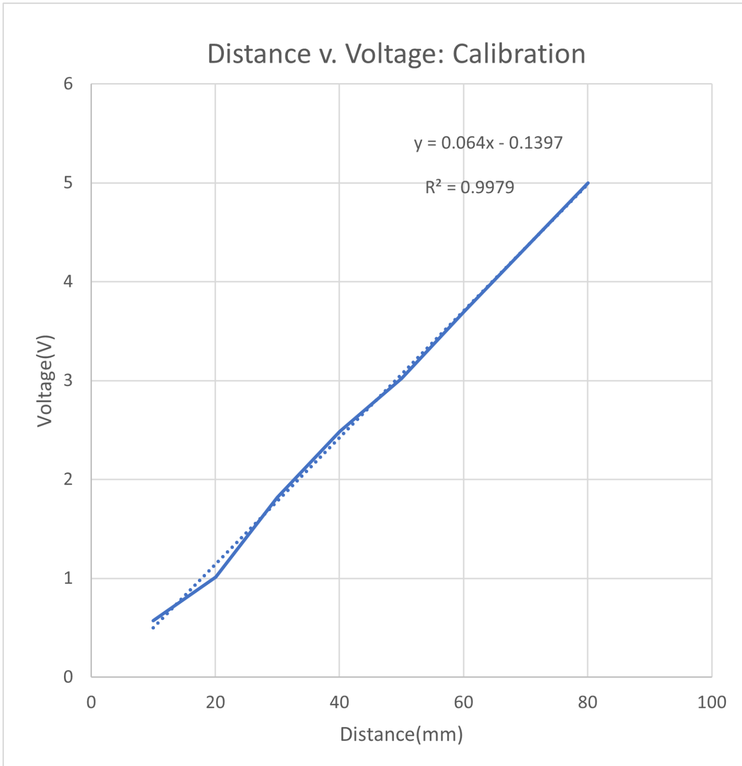

Figure 2: Calibrating our Linear Potentiometer

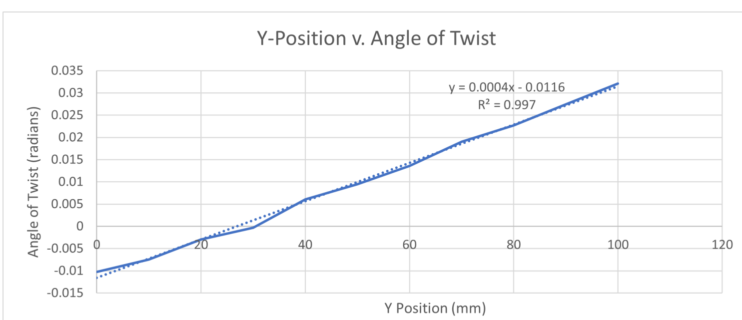

Figure 3: Y distance vs Twist Angle

linear representation where the intersect at 0 radians will be the shear center. This is shown in figure (3) and the shear center is determined to be at 31% of the chord length, or at 31.25 (mm).

Weight kg	0.25
Shear Force N	2.4525
Shear Center	0.31% Chord
Moment $N * cm$	67.44375
Torque $N * mm^2$	170.57628
Deflection $X_i = 11$	4004.091667
Deflection $X_i = 19.25$	11753.00729
Deflection $X_i = 27.5$	22945.88542
EI $N * m^2$	490.5
GJ $N * m/rad$	213.22035

Table 3: Deflection and Stiffness Data

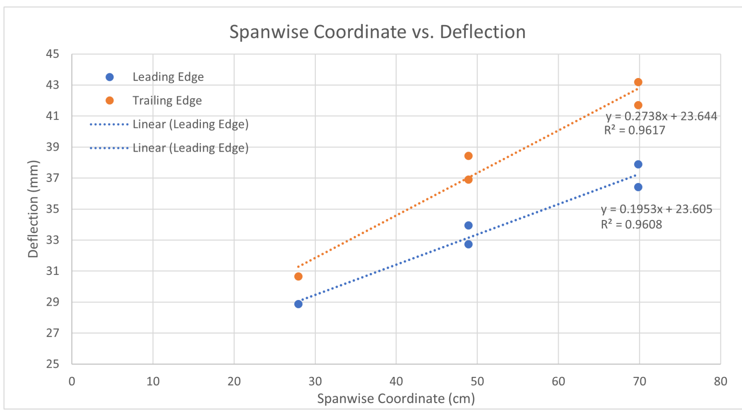

Figure 4: Spanwise Coordinates vs Deflection with varying weight placement

For this section, X_j was taken as 27.5 in, or 69.85 cm, the following values were calculated using equations (1), (2), (3), and (4). The measured deflection at the 3 X_i values were plotted against the theoretical deflection and the slope of the graph was set equal to $\frac{S_j}{EI}$. Solving for EI yielded a bending stiffness value of 490.5 $N * m^2$.

Leading Edge	Deflection	Trailing Edge	Deflection	Theta
48.895	32.7234375	48.895	36.8953125	0.041364042
69.85	36.4265625	69.85	41.7078125	0.052345491
27.94	28.8796875	27.94	30.6609375	0.017669292
48.895	33.9421875	48.895	38.4265625	0.044458532
69.85	37.8796875	69.85	43.1921875	0.052654657

Table 4: Actual Deflection

As for our uncertainty, we can use the least count of our different measuring devices as a basis for the overall uncertainty. That gives us the following equation:

$$\delta_{\text{total}} = \sqrt{\delta^2_{\text{caliper}} + \delta^2_{\text{STD}} + \delta^2_{\text{potentiometer}} + \delta^2_{\text{DAQ}}}$$

In this case, the caliper or ruler measurements are accurate to the nearest millimeter, the potentiomenter is accurate to the nearest hundredth of a millimeter, and the DAQ is accurate to the nearest thousandth of a volt. This gives us the following uncertainties:

- Caliper: $\delta_{\text{caliper}} = 0.5\,\text{mm}$
- Standard Deviation: $\delta_{\text{std}} = 0.001\,\text{mm}$
- Potentiometer: $\delta_{\text{potentiometer}} = 0.013\,\text{mm}$
- DAQ: $\delta_{\text{DAQ}} = 0.001\,\text{V}$

For our overall propagation of uncertainty, we have:

$$\delta_{\text{total}} = \sqrt{0.5^2 + +0.001^2 + 0.013^2 + 0.001^2} = 0.501\,\text{mm}$$

6 Conclusion

The first goal of this lab was to learn how to use a linear potentiometer to measure displacement. We successfully built a physical circuit containing the potentiometer and integrated it into the LabVIEW software, as well as calibrated it within 1% FS. The next goal was to determine the twist angle from displacement measurements. The potentiometer was used at various points along the chord with a weight at one end to measure deflection due to the weight at one end. Using geometric relations, the angle of twist was determined and the shear center location was calculated. The final goal was to obtain the flexural and torsional stiffness of the wing section. With several more potentiometer measurements along the span of the wing, these values were calculated using equations provided in the lab manual.

Appendices

Appendix: t-Distribution Tables

Table A11. ***t*-Distribution**

Values of z for given values of the distribution function $F(z)$ (cf. p. 754).
Example: For 9 degrees of freedom, $z = 1.83$ when $F(z) = 0.95$.

$F(z)$	Number of Degrees of Freedom									
	1	2	3	4	5	6	7	8	9	10
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.33	0.29	0.28	0.27	0.27	0.27	0.26	0.26	0.26	0.26
0.7	0.73	0.62	0.58	0.57	0.56	0.55	0.55	0.55	0.54	0.54
0.8	1.38	1.06	0.98	0.94	0.92	0.91	0.90	0.89	0.88	0.88
0.9	3.08	1.89	1.64	1.53	1.48	1.44	1.42	1.40	1.38	1.37
0.95	6.31	2.92	2.35	2.13	2.02	1.94	1.90	1.86	1.83	1.81
0.975	12.7	4.30	3.18	2.78	2.57	2.45	2.37	2.31	2.26	2.23
0.99	31.8	6.97	4.54	3.75	3.37	3.14	3.00	2.90	2.82	2.76
0.995	63.7	9.93	5.84	4.60	4.03	3.71	3.50	3.36	3.25	3.17
0.999	318.3	22.3	10.2	7.17	5.89	5.21	4.79	4.50	4.30	4.14

$F(z)$	Number of Degrees of Freedom									
	11	12	13	14	15	16	17	18	19	20
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.26
0.7	0.54	0.54	0.54	0.54	0.54	0.54	0.53	0.53	0.53	0.53
0.8	0.88	0.87	0.87	0.87	0.87	0.87	0.86	0.86	0.86	0.86
0.9	1.36	1.36	1.35	1.35	1.34	1.34	1.33	1.33	1.33	1.33
0.95	1.80	1.78	1.77	1.76	1.75	1.75	1.74	1.73	1.73	1.73
0.975	2.20	2.18	2.16	2.15	2.13	2.12	2.11	2.10	2.09	2.09
0.99	2.72	2.68	2.65	2.62	2.60	2.58	2.57	2.55	2.54	2.53
0.995	3.11	3.06	3.01	2.98	2.95	2.92	2.90	2.88	2.86	2.85
0.999	4.03	3.93	3.85	3.79	3.73	3.69	3.65	3.61	3.58	3.55

$F(z)$	Number of Degrees of Freedom									
	22	24	26	28	30	40	50	100	200	∞
0.5	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.6	0.26	0.26	0.26	0.26	0.26	0.26	0.26	0.25	0.25	0.25
0.7	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.53	0.52
0.8	0.86	0.86	0.86	0.86	0.85	0.85	0.85	0.85	0.84	0.84
0.9	1.32	1.32	1.32	1.31	1.31	1.30	1.30	1.29	1.29	1.28
0.95	1.72	1.71	1.71	1.70	1.70	1.68	1.68	1.66	1.65	1.65
0.975	2.07	2.06	2.06	2.05	2.04	2.02	2.01	1.98	1.97	1.96
0.99	2.51	2.49	2.48	2.47	2.46	2.42	2.40	2.37	2.35	2.33
0.995	2.82	2.80	2.78	2.76	2.75	2.70	2.68	2.63	2.60	2.58
0.999	3.51	3.47	3.44	3.41	3.39	3.31	3.26	3.17	3.13	3.09

Appendix: NI-9215 Datasheet

NI-9215 Datasheet