Vg101: Introduction to Computer and Programming

Spring 2021

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MATLAB Task 2: Fourier Series

1. Background

The theorem of Fourier Series has been widely applied in many engineering fields. One of the examples can be how we can use analog signals (sine and cosine) to represent digital data (square waves).

Fourier series:

Any reasonably behaved periodic function, g(t), with period T can be constructed by summing a (possibly infinite) number of sines and cosines:

$$g(t) = \frac{1}{2}c + \sum_{n=1}^{\infty} a_n \sin(2\pi nft) + \sum_{n=1}^{\infty} b_n \cos(2\pi nft)$$

where f=1/T is the fundamental frequency and a_n and b_n are the sine and cosine amplitudes of the nth harmonics.

Mathematically, we can have:

$$\int_0^T \sin(2\pi kft)\,\sin(2\pi nft)\,dt = \begin{cases} 0 \text{ for } k \neq n \\ T/2 \text{ for } k = n \end{cases}$$

$$a_n = \frac{2}{T}\int_0^T g(t)\sin(2\pi nft)\,dt \qquad b_n = \frac{2}{T}\int_0^T g(t)\cos(2\pi nft)\,dt \qquad c = \frac{2}{T}\int_0^T g(t)\,dt$$

2. Problem to be solved

Problem:

For a 8-bit randomized string (e.g. 01100010), please using MATLAB to plot the simulation of using different numbers of harmonics to represent the bit string.

a) Input: a 8-bit randomized string
b) Algorithm: Fourier Series. Using the sum of different numbers of harmonics to simulate the square wave for the bit string
c) Output: plots

Hints:

a) You can think the 8 bits are sent within period T, so that the signal is a periodic function, therefore Fourier Series can be applied
b) Think about how random number can be produced, but now we only want randomized 1s and 0s
c) In reality, we can not achieve infinity. Let's see how many harmonics can generate acceptable simulation of the square wave.
d) Try to use 1 harmonic, 2 harmonics, 4 harmonics, 8 harmonics and 16 harmonics (or even more harmonics, if you think it is necessary) to simulate the square wave respectively.
e) Plot them with the square wave in a single figure so that we can compare them easily
f) Only plots are required as the output of your program, but legends and titles should be displayed on the plots.

3. Assessment

a) On-site demo and explanation will be required; (20)
b) Code should be well commented and correctly named;(20)
c) Functionalities implemented;(60)

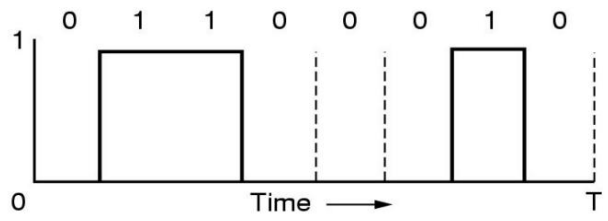
4. Submission

Demo: on the first week of our lab session. (Friday afternoon, Week 10)
Code: should be submitted before the demo. (deadline)
Naming conventions:
[studentID]_[name].m or [studentID]_[name].zip (if you want to hand in multiple files)
e.g. 202012345_张三丰.m or 202012345_张三丰.zip

Expecting to receive an All-In-One zip file for each class.

An example to help you understand the algorithm

Considering bit string 01100010, 8 bit for ASCII character 'b', its square wave looks like:



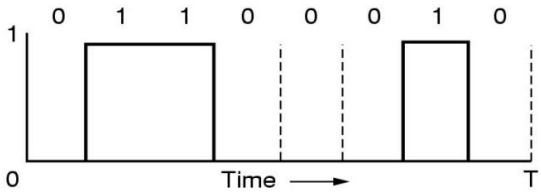
By applying Fourier Series, we can have: (why?)

$$a_n = 1/\pi n[\cos(\pi n/4) - \cos(3\pi n/4) + \cos(6\pi n/4) - \cos(7\pi n/4)]$$

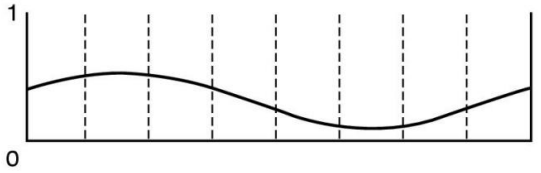
$$b_n = 1/\pi n[\sin(3\pi n/4) - \sin(\pi n/4) + \sin(7\pi n/4) - \sin(6\pi n/4)]$$

$$c_n = \frac{3}{4}$$

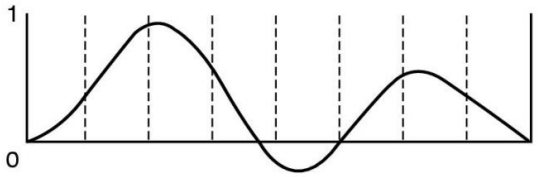
Compute its Fourier Series and plot them (your plots are not necessary to be the same as these)



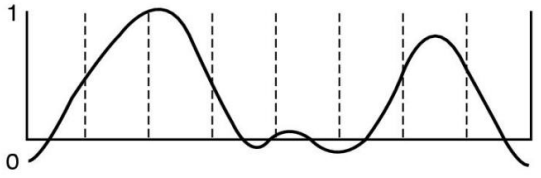
Square wave



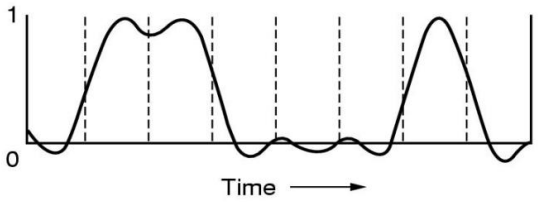
1 harmonic



2 harmonics



4 harmonics



8 harmonics