

Analysis of a Financial Product

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1 Introduction

2 Theory

- Barrier option
- Repayment cases
- Risks
- Geometric Brownian Motion
- Estimation of parameters

3 Result

- Simulation of indexes
- Expected return
- Probability of loss
- Numerical efficiency
- Sensitivity analysis

4 Conclusion

- ▶ Daiwa Securities Group Inc. is a Japanese investment bank that sells a barrier option
 - 2 stock indexes at stake: S&P500 and Nikkei225
 - down and in, up and out: Knock-in 60% & Knock-out 105%
 - 3% of interest per year (let's say semi-annually compounded) paid for 3 years : 1.5% every 6 months

- ▶ Is it valuable to invest in that financial product?

► Knock-in :

- observed every end of business days (so we always generate closing prices)
- active if at least one of the indexes drops under 60%
- activation means the repayment will be the min of both index price percentages and 100%
- no more protection for the client if the knock-in activates

► Knock-out :

- observed every valuation day of interest payments
- active if both indexes raise over 105%
- activation means the repayment is done before the maturity at 100%-rate (contract stopped)
- protection for the seller because there is high probability they will repay 100% at maturity, so they save some interest payments (they can raise the interest payments to lure clients)

Repayment rate		Knock-in	
		Not activated	Activated
Knock-out	Not activated	1	$\min(\frac{S_{j,T}}{S_{j,0}}, 1)$
	Activated	1	1

Table: The different rates of repayment cases, depending on realization of knock-in and knock-out

- ▶ The product has a credit rating of A (S&P), so it has an average cumulative default rate of 0.34 over 3 years
- ▶ The knock-out can stop prior to maturity (need to reinvest)
- ▶ Worst case scenario: at least one of both indexes is very low in 3 years

► Notations:

- S is the closing price of an index
- Δt is the constant time step
- μ is the estimated mean of the historical data of an index price
- σ is the estimated volatility of the same historical data
- ξ is the independent standard normal distribution
- ρ is the correlation between both indexes

► GBM in its exact discretization form:

$$S_j(t + \Delta t) = S_j(t) e^{\left(\mu_j - \frac{\sigma_j^2}{2}\right)\Delta t + \sigma_j \sqrt{\Delta t} \mathcal{E}_j}$$

$$\text{with } \begin{cases} \mathcal{E}_1 &= \xi_1 \\ \mathcal{E}_2 &= \rho \xi_1 + \sqrt{1 - \rho^2} \xi_2 \end{cases}$$

- ▶ We estimate over 3 months the mean μ , the volatility σ of each market price, plus the correlation ρ between them both.
- ▶ The formulas are, with the log-returns over one day $u_{j,i} = \log_e \left(\frac{S_{j,i-1}}{S_{j,i}} \right)$

$$\begin{aligned}\hat{\mu}_j &= \frac{1}{m} \sum_{i=1}^m u_{j,i} \\ \hat{\sigma}_j &= \sqrt{\frac{1}{m-1} \sum_{i=1}^m (u_{j,i} - \hat{\mu}_j)^2} \\ \hat{\rho} &= \frac{\frac{1}{m} \sum_{i=1}^m u_{1,i} u_{2,i}}{\hat{\sigma}_1 \hat{\sigma}_2}\end{aligned}$$

Simulation of S&P500 and Nikkei225 using GBM process:

- ▶ S&P500 $\sigma = 23\%$



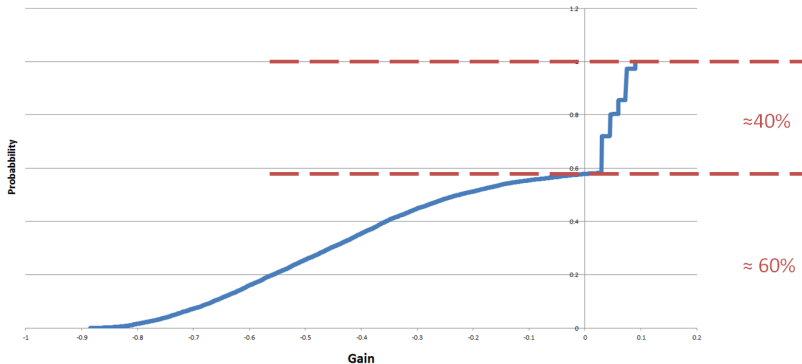
- ▶ Nikkei225 $\sigma = 43\%$



Estimation of the financial product return

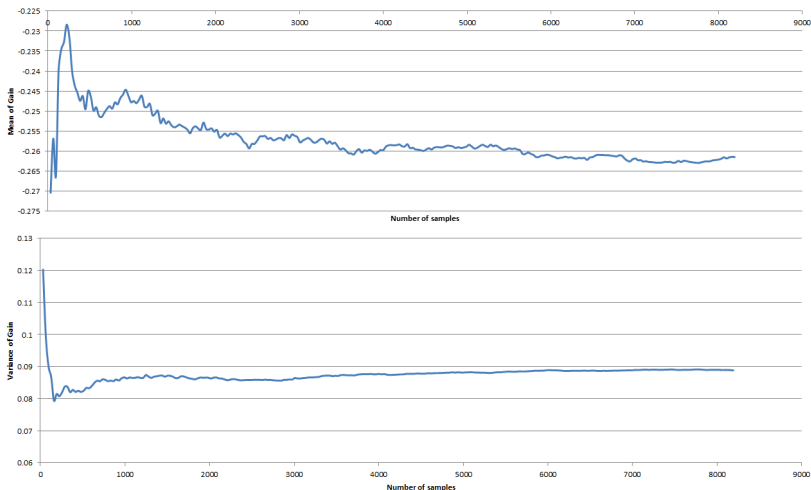
- ▶ Mean return : -26.1 %
 - ▶ Variance : 8,9 %
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- ▶ For an investment of 100 in this financial product, the expected amount returned is 73.9.

Cumulative density function of the return:



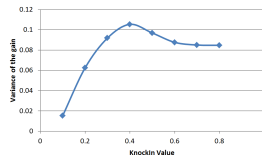
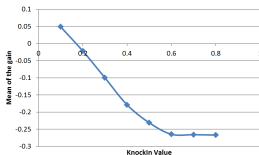
- ▶ The investor has less than half a chance to get a positive return with this product

Mean and variance depending on the number of samples

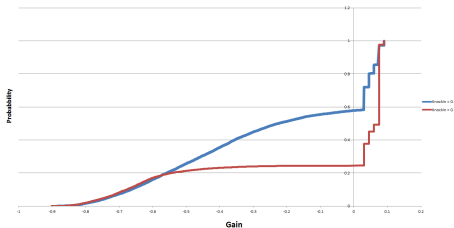


Return for different Knock-in value:

► Mean and Variance



► CDF comparison



- ▶ A possible way to make this product fair would be to set:
 - the knock-in value around 20 % (non realistic)
 - the interest rate to 14 %
- ▶ Removing the knock-out makes the product less valuable for the investor (-32 %)
- ▶ For a knock-in value of 30 % we can make the following comments:
 - Daiwa still makes a profit which is around 10 % of the investment
 - The probability of gain for the investor is around 80 % which could be an interesting commercial argument

Conclusion

- ▶ Risky product that benefits more for Daiwa
- ▶ Plus, S&P is very likely to fall in the next three years

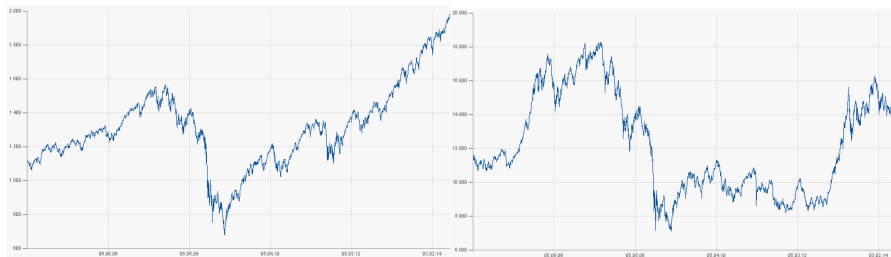


Figure: Evolution of S&P (left) and Nikkei (right) over 10 years



Hull, J.C. (2012)

Options, Futures, and Other Derivatives

8th global edition, chap. 22-23