# Analysis of a Financial Product

BUTEAU Paul-Émile & WISSOCQ Julien

July 5, 2014

# Contents


1. Introduction
2. Theory
   - Barrier option
   - Repayment cases
   - Risks
   - Estimation of parameters
   - Geometric Brownian Motion
3. Result
   - Numerical efficiency
   - Probability of loss
   - Histogram of the return
   - Sensitivity analysis
   - Comparison with Product A

# Introduction

**Daiwa Securities Group Inc. is a Japanese investment bank that sells through its website a financial product hold by the Swiss bank UBS.**
**Is it valuable for an investor to invest in that financial product?**

- Barrier option down and in, up and out : Knock-in 60% & Knock-out 105%
- 3% of interest per year semi-annually compounded paid for 3 years : 1.5% every 6 months
- 2 stock indexes at stake : S&P500 and Nikkei225

- Knock-in :
  - observed every end of business days (so we always generate closing prices)
  - active if at least one of the indexes drop under 60%
  - activation means the repayment will be the min of both index price percentages and 100%
  - no more protection for the client if the knock-in activates
- Knock-out :
  - observed every valuation day (for repayments)
  - active if only one index raise over 105%
  - activation means the repayment is done before the maturity at 100% rate (contract stopped)
  - protection for the seller because there is high probability they will repay 100% at maturity, so they save some interest payments (they can raise the interest payments to lure clients)

| Repayment rate | | Knock-in: Not activated | Knock-in: Activated |
|---|---|---|---|
| Knock-out | Not activated | 1 | $min(\frac{S_{j,T}}{S_{j,0}}, 1)$ |
| | Activated | 1 | 1 |

Table: The different rates of repayment cases, depending on realization of knock-in and knock-out

- The product is rated

- We estimate over 3 or 6 months the mean $\mu$, the volatility $\sigma$ of each market price, plus the correlation $\rho$ between them both.
- The formulas are, with $u_{j,i} = \log_e\left(\frac{S_{j,i-1}}{S_{j,i}}\right)$

$$
\begin{aligned}
\hat{\mu}_j &= \frac{1}{m}\sum_{i=1}^{m} u_{j,i} \\
\hat{\sigma}_j &= \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(u_{j,i}-\hat{\mu}_j)^2} \\
\hat{\rho} &= \frac{\frac{1}{m}\sum_{i=1}^{m} u_{1,i}u_{2,i}}{\hat{\sigma}_1\hat{\sigma}_2}
\end{aligned}
$$

- Notations :
  - S is the closing price of the index price
  - $\Delta t$ is the constant time step
  - $\mu$ is the estimated mean of the historical data of the index price
  - $\sigma$ is the estimated volatility of the same historical data
  - $\xi$ is the independent standard normal distribution
- GBM in its exact discretization form:

$$S_j(t+\Delta t) = S_j(t)\, e^{\left(\mu_j - \frac{{\sigma_j}^2}{2}\right)\Delta t + \sigma_j \sqrt{\Delta t}\mathcal{E}_j}$$

$$\text{with} \begin{cases} \mathcal{E}_1 & = \xi_1 \\ \mathcal{E}_2 & = \rho\xi_1 + \sqrt{1-\rho^2}\xi_2 \end{cases}$$

# Result/ Numerical efficiency

## Result/ Probability of loss

# Result/ Histogram of the return

- 2 indexes : EuroSTOXX50 & Nikkei225, T = 5 years, Knock-out 49%

$$r = \begin{cases} 1\% & \text{if one of the index is under 80\% of its initial value} \\ 5.7\% & \text{else, or it is the first interest payment} \end{cases}$$

# References


Hull, J.C. (2012)
Options, Futures, and Other Derivatives
$8^{th}$ global edition, chap. 22-23