Analysis of a Financial Product

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Contents

Introduction

- Daiwa Securities Group Inc. is a Japanese investment bank that sells a barrier option
  - 2 stock indexes at stake: S&P500 and Nikkei225
  - down and in, up and out: Knock-in 60% & Knock-out 105%
  - 3% of interest per year (let's say semi-annually compounded) paid for 3 years : 1.5% every 6 months

- Is it valuable to invest in that financial product?

- Knock-in :
  - observed every end of business days (so we always generate closing prices)
  - active if at least one of the indexes drops under 60%
  - activation means the repayment will be the min of both index price percentages and 100%
  - no more protection for the client if the knock-in activates

- Knock-out :
  - observed every valuation day of interest payments
  - active if both indexes raise over 105%
  - activation means the repayment is done before the maturity at 100%-rate (contract stopped)
  - protection for the seller because there is high probability they will repay 100% at maturity, so they save some interest payments (they can raise the interest payments to lure clients)

Repayment rate		Knock-in: Not activated	Knock-in: Activated
Knock-out	Not activated	1	$min(\frac{S_{j,T}}{S_{j,0}}, 1)$
	Activated	1	1

Table: The different rates of repayment cases, depending on realization of knock-in and knock-out

- The product has a credit rating of A (S&P), so it has an average cumulative default rate of 0.34 over 3 years
- The knock-out can stop prior to maturity (need to reinvest)
- Worst case scenario: at least one of both indexes is very low in 3 years

- Notations:
  - S is the closing price of an index
  - Δt is the constant time step
  - μ is the estimated mean of the historical data of an index price
  - σ is the estimated volatility of the same historical data
  - ξ is the independent standard normal distribution
  - ρ is the correlation between both indexes

- GBM in its exact discretization form:

$$S_j(t+\Delta t) = S_j(t)\, e^{\left(\mu_j - \frac{\sigma_j{}^2}{2}\right)\Delta t + \sigma_j \sqrt{\Delta t}\mathcal{E}_j}$$

$$\text{with} \begin{cases} \mathcal{E}_1 & = \xi_1 \\ \mathcal{E}_2 & = \rho\xi_1 + \sqrt{1-\rho^2}\xi_2 \end{cases}$$

- We estimate over 6 months the mean μ, the volatility σ of each market price, plus the correlation ρ between them both.
- The formulas are, with the log-returns over one day $u_{j,i} = \log_e\left(\frac{S_{j,i-1}}{S_{j,i}}\right)$

$$\hat{\mu}_j = \frac{1}{m}\sum_{i=1}^{m} u_{j,i}$$

$$\hat{\sigma}_j = \sqrt{\frac{1}{m-1}\sum_{i=1}^{m}(u_{j,i} - \hat{\mu}_j)^2}$$

$$\hat{\rho} = \frac{\frac{1}{m}\sum_{i=1}^{m} u_{1,i}u_{2,i}}{\hat{\sigma}_1\hat{\sigma}_2}$$

Result/ Histogram of the return

Result / Numerical efficiency

Result/ Sensitivity analysis

- 2 indexes : EuroSTOXX50 & Nikkei225, T = 5 years, Knock-out 49%

$$r = \begin{cases} 1\% & \text{if one of the index is under 80\% of its initial value} \\ 5.7\% & \text{else, or it is the first interest payment} \end{cases}$$

Conclusion

- Risky product that benefits more for Daiwa

- Plus, S&P is very likely to fall in the next three years

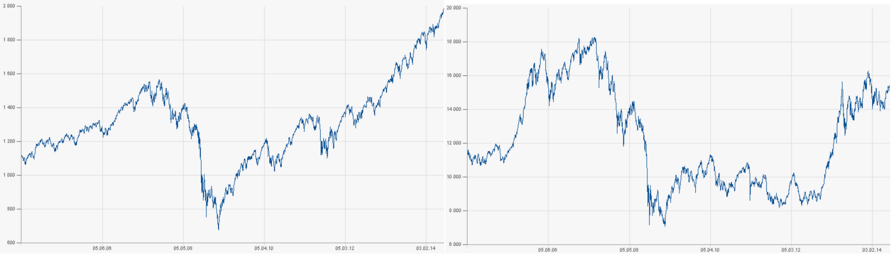

Figure: Evolution of S&P (left) and Nikkei (right) over 10 years

References

Hull, J.C. (2012)
Options, Futures, and Other Derivatives
8^{th} global edition, chap. 22-23