

Analysis of a Financial Product

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- Repayment cases
- Risks
- Estimation of parameters
- Geometric Brownian Motion

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- Probability of loss
- Histogram of the return
- Sensitivity analysis
- Comparison with Product A

Daiwa Securities Group Inc. is a Japanese investment bank which sells through its website a financial product hold by the Swiss bank UBS.

Is it valuable to invest in that financial product?

- ▶ Barrier option down and in, up and out : Knock-in 60% & Knock-out 105%
- ▶ 3% of interest per year semi-annually compounded paid for 3 years : 1.5% every 6 months
- ▶ 2 stock indexes at stake : Standard & Poor's 500 and Nikkei 225

► Knock-in :

- observed every end of business days (so we always generate closing prices)
- active if at least one of the indexes drop under 60%
- activation means the repayment will be the min of both index price percentages and 100%
- no more protection for the client if the knock-in activates

► Knock-out :

- observed every valuation day (for repayments)
- active if only one index raise over 105%
- activation means the repayment is done before the maturity at 100% rate (contract stopped)
- protection for the seller because there is high probability they will repay 100% at maturity, so they save some interest payments (they can raise the interest payments to lure clients)

Repayment rate		Knock-in	
		Not activated	Activated
Knock-out	Not activated	1	$\min(\frac{S_{j,T}}{S_{j,0}}, 1)$
	Activated	1	1

Table: The different rates of repayment cases, depending on realization of knock-in and knock-out

- ▶ The product has a credit rating of A (S&P), so it has an average cumulative default rate of 0.34

- ▶ We estimate over 3 or 6 months the mean μ , the volatility σ of each market price, plus the correlation ρ between them both.
- ▶ The formulas are, with $u_{j,i} = \log_e \left(\frac{S_{j,i-1}}{S_{j,i}} \right)$

$$\begin{aligned}\hat{\mu}_j &= \frac{1}{m} \sum_{i=1}^m u_{j,i} \\ \hat{\sigma}_j &= \sqrt{\frac{1}{m-1} \sum_{i=1}^m (u_{j,i} - \hat{\mu}_j)^2} \\ \hat{\rho} &= \frac{\frac{1}{m} \sum_{i=1}^m u_{1,i} u_{2,i}}{\hat{\sigma}_1 \hat{\sigma}_2}\end{aligned}$$

► Notations :

- S is the closing price of the index price
- Δt is the constant time step
- μ is the estimated mean of the historical data of the index price
- σ is the estimated volatility of the same historical data
- ξ is the independent standard normal distribution

► GBM in its exact discretization form:

$$S_j(t + \Delta t) = S_j(t) e^{\left(\mu_j - \frac{\sigma_j^2}{2}\right)\Delta t + \sigma_j \sqrt{\Delta t} \mathcal{E}_j}$$

$$\text{with } \begin{cases} \mathcal{E}_1 &= \xi_1 \\ \mathcal{E}_2 &= \rho \xi_1 + \sqrt{1 - \rho^2} \xi_2 \end{cases}$$

Result/ Probability of loss

Result/ Histogram of the return

Result/ Numerical efficiency

Result/ Comparison with Product A

- ▶ 2 indexes : EuroSTOXX50 & Nikkei225, $T = 5$ years, Knock-out 49%

$$r = \begin{cases} 1\% & \text{if one of the index is under 80\% of its initial value} \\ 5.7\% & \text{else, or it is the first interest payment} \end{cases}$$



Hull, J.C. (2012)

Options, Futures, and Other Derivatives

8th global edition, chap. 22-23