MnS - Computer Practical - 4

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1 (a) $\dot{x} = -x$; $x_0 = 1$

$$\frac{dx}{dt} = -x$$

or $$\int_{x(0)}^{x(t)} \frac{dx}{x} = \int_0^t - dt$$

or $$\left[\ln x\right]_{x(0)}^{x(t)} = -\left[t\right]_0^t$$

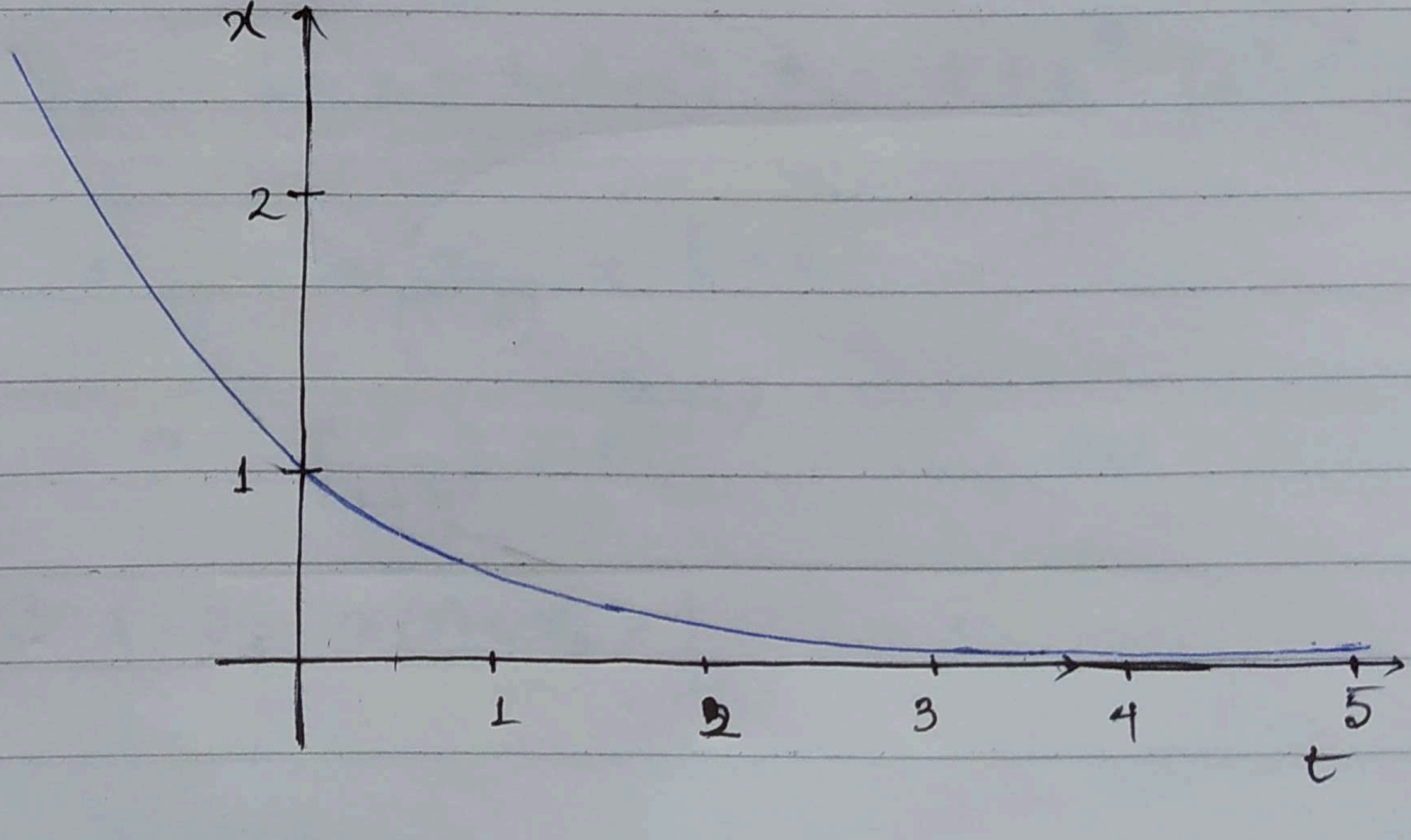
or $$\ln(x(t)) - \ln(x(0)) = -[t - 0]$$

or $$\ln(x(t)) - \ln(1) = -t$$

or $$\ln x - 0 = -t \qquad [x(t) = x]$$

or $$\ln x = -t$$

or $$x = e^{-t}.$$



1(b) $\dot{x} = x^{-1}, \quad x_0 = 1$

or $\dfrac{dx}{dt} = \cancel{x^{-1}}\ \dfrac{1}{x}$.

or $\displaystyle\int_{x(0)}^{x(t)} x\,dx = \int_0^t dt$

or $\left[\dfrac{x^2}{2}\right]_{x(0)}^{x(t)} = [t]_0^t$

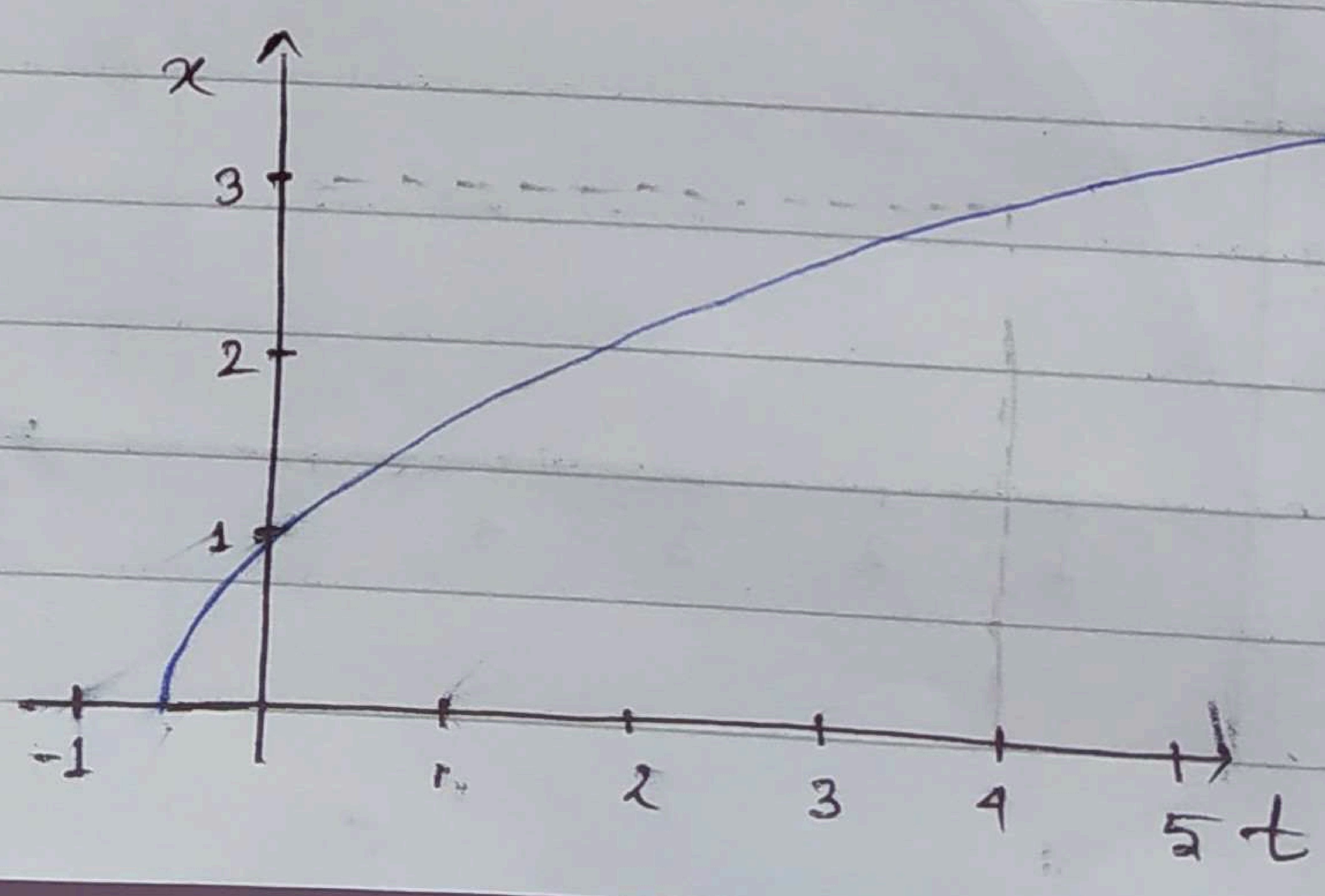
or $\dfrac{x(t)^2}{2} - \dfrac{1^2}{2} = t - 0$

or $\dfrac{[x(t)]^2}{2} = t + \dfrac{1}{2}$

or $x^2 = 2\left(t + \dfrac{1}{2}\right) \qquad [x(t) = x]$.

$= (2t + 1)$

or $x = \sqrt{2t+1}$



1(c) $\dot{x} = 1 - x \; ; \; x_0 = 0$

or $\dfrac{dx}{dt} = 1 - x$

or $\displaystyle\int_{x(0)}^{x(t)} \frac{dx}{1-x} = \int_0^t dt$

or $\left[-\ln\left(1 - x(t)\right)\right]_{x(0)}^{x(t)} = [t]_0^t$

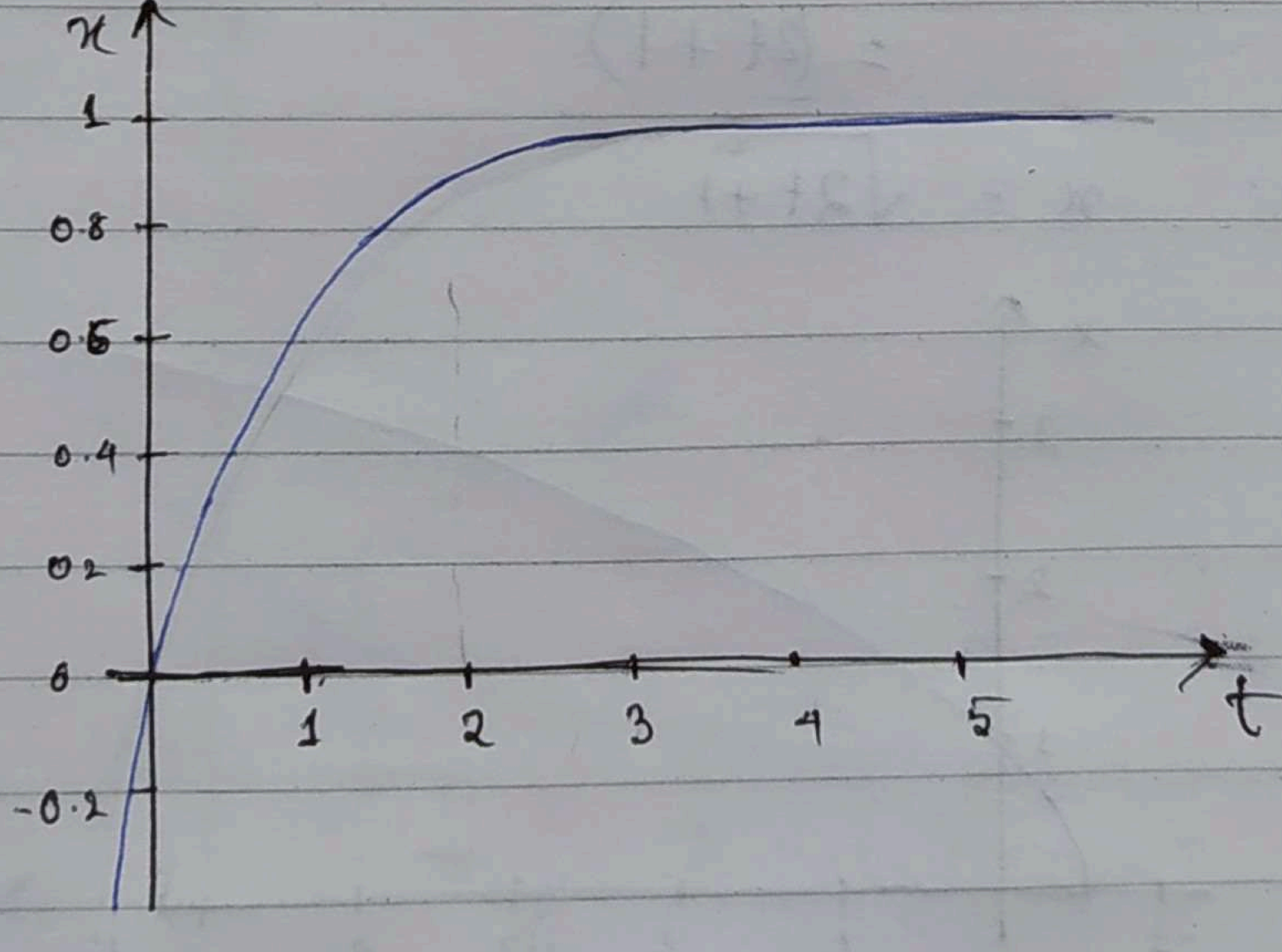
or $-\ln(1-x) + \ln(1-0) = t - 0 \quad [\because x(t) = x]$

or $-\ln(1-x) + 0 = t - 0$

or $\ln(1-x) = -t$

or $1 - x = e^{-t}$

or $x = 1 - e^{-t}$



1(d) $\dot{x} = x(1-x)$; $x_0 = \frac{1}{2}$

or $\frac{dx}{dt} = x(1-x)$

or $\int_{x(0)}^{x(t)} \frac{dx}{x(1-x)} = \int_0^t dt$

or $\int_{x(0)}^{x(t)} \frac{x + (1-x)}{x(1-x)} dx = \int_0^t dt$

or $\int_{x(0)}^{x(t)} \left[\frac{\cancel{x}}{\cancel{x}(1-x)} + \frac{\cancel{(1-x)}}{x\cancel{(1-x)}} \right] dx = [t]_0^t$

or $\int_{x(0)}^{x(t)} \frac{dx}{(1-x)} + \int_{x(0)}^{x(t)} \frac{dx}{x} = [t]_0^t$

or $[-\ln(1-x)]_{x(0)}^{x(t)} + [\ln x]_{x(0)}^{x(t)} = [t]_0^t$

or $-\ln(1-x(t)) + \ln(1-\frac{1}{2}) + \ln(x(t)) - \ln(\frac{1}{2})$

$= t - 0$

$[\because x_0 = \frac{1}{2}]$

or $-\ln(1-x) + \cancel{\ln(\frac{1}{2})} + \ln(x) - \cancel{\ln(\frac{1}{2})} = t$

$[\because x(t) = x]$

or $\ln x - \ln(1-x) = t$

or $\ln\left(\frac{x}{1-x}\right) = t$

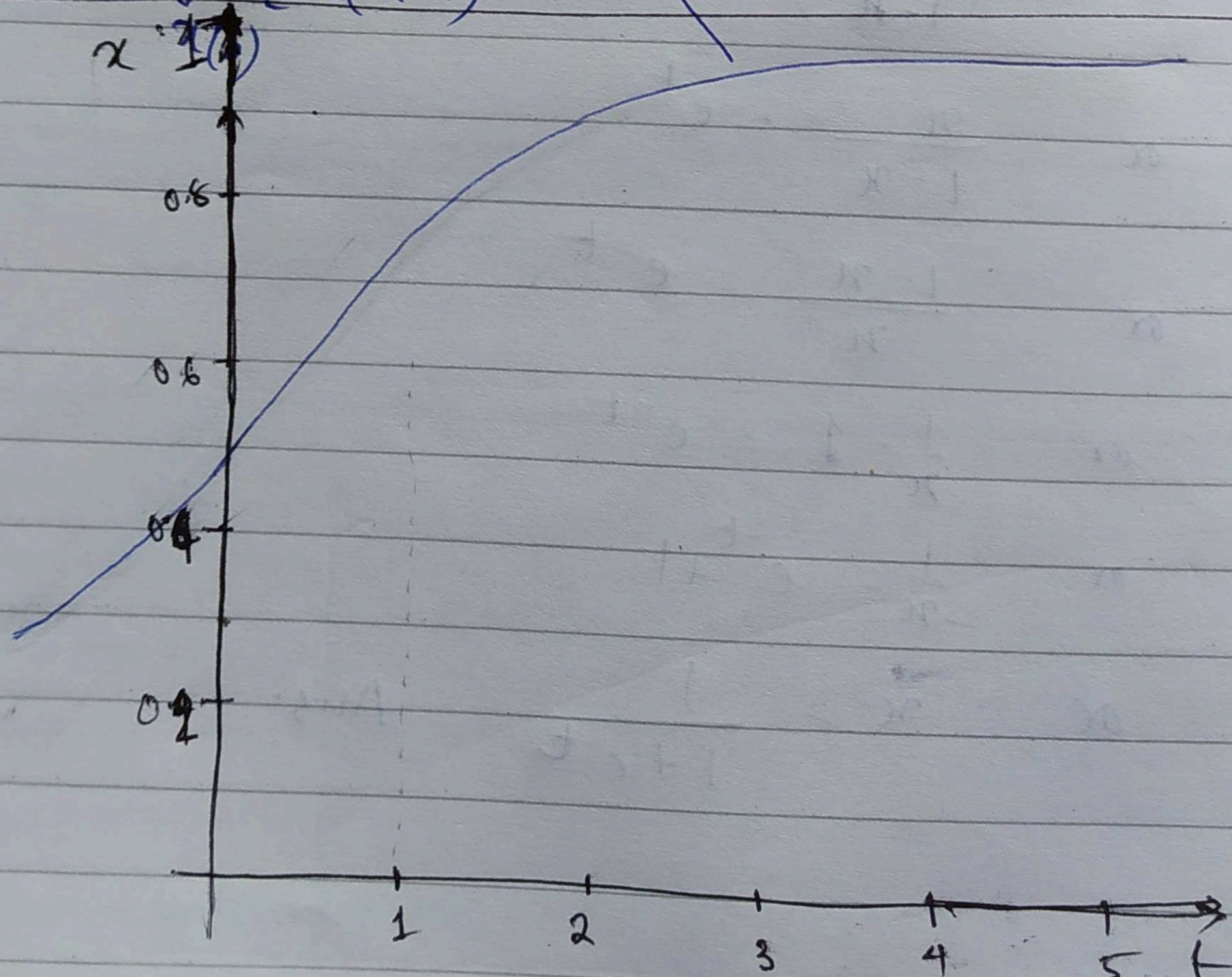
or $\frac{x}{1-x} = e^{t}$

or $\frac{1-x}{x} = e^{-t}$

or $\frac{1}{x} - 1 = e^{-t}$

or $\frac{1}{x} = e^{-t} + 1$

or $x = \frac{1}{1+e^{-t}}$ Ans.



x
1
0.8
0.6
0.4
0.2
1
2
3
4
5
t