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Models of Neural Systems, WS 2020/21
Computer Practical 3
Assignment due: December, 7th, 2020, 10 am

Visual Receptive Fields

1. **Visual stimulus.** A commonly used stimulus to study the (early) visual system is the sinusoidal grating:
$$s(x, y) = A\cos(Kx\cos\Theta + Ky\sin\Theta - \Phi), \tag{1}$$
where x, y are the spatial coordinates, K is the spatial frequency, Θ is the orientation, Φ is the spatial phase and A is the contrast amplitude.

   (a) Approximate the visual field (x, y) with a 2-dimensional grid of uniformly-distributed photoreceptors (retina) at $(x_i, y_j) = (-x_0 + i\Delta x, -y_0 + j\Delta y)$ for $i = 0, 1, \ldots, \frac{2x_0}{\Delta x}$ and $j = 0, 1, \ldots, \frac{2y_0}{\Delta y}$ where x_0 and y_0 determines the size of the visual field and Δx, Δy determines its sampling. Since visual field sizes are specified in degrees, we take this to be the appropriate unit for x and y. Take $x_0 = 5°$, $y_0 = 5°$ (hence a visual field of size $(10°)^2$).
   Hint: You can use `numpy.meshgrid` to generate such a grid.

   (b) Compute a sinusoidal grating on the grid specified in (a) with the following grating parameters: $\Theta = 0$, $\Phi = 0$, $K = \pi\,\frac{1}{\text{degree}}$, $A = 1$. Use the `matplotlib.pyplot.imshow` function to plot the 2-dimensional map.

   (c) Vary the orientation Θ and the spatial frequency K and plot the resulting gratings.

2. **A model of receptive fields**. A receptive field (RF) describes the "sensitivity" of a neuron to its inputs, depending on their position in the visual field. A convenient mathematical approximation of a receptive field is provided by the Gabor function:
$$D_s(x, y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right)\cos(kx - \phi), \tag{2}$$
where k is the preferred spatial frequency, ϕ is the preferred spatial phase and σ_x, σ_y define the receptive field sizes.

   (a) Implement a Gabor function as a model of a receptive field with preferred spatial frequency $k = \pi\,\frac{1}{\text{degree}}$, preferred spatial phase $\phi = 0$ and receptive field size $\sigma_x = 2°$, $\sigma_y = 1°$. Use the same dimension (x_0, y_0) as in Problem 1. Plot this Gabor receptive field.

   (b) What are the optimal stimuli for a neuron with such a RF? Where can you find a neuron with such a RF? Does this RF characterize a simple cell or a complex cell and why?

   (c) The linear response r_s of a neuron with receptive field D_s to a visual stimulus s, is given by
$$r_s = \int s(x, y) D_s(x, y) dx dy. \tag{3}$$
   Numerically this corresponds to
$$r_s = \Big(s \cdot D_s\Big)\Delta x\Delta y = \sum_{i=0}^{\frac{2x_0}{\Delta x}} \sum_{j=0}^{\frac{2y_0}{\Delta y}} s(x_i, y_j) D_s(x_i, y_j)\Delta x\Delta y. \tag{4}$$

Numerically calculate the response of a cell with a Gabor receptive field (parameters as in exercise 2a) to the grating from exercise 1b, and state the resulting number in your report. Compare it with a response to an uniformly illuminated visual field ($s(x, y) = A$). Explain the difference.

3. **Tuning Curve**. A tuning curve describes the dependence of the neuronal response on a parameter of the stimulus such as spatial frequency, orientation or phase. Plot the neuronal response r to the grating from the exercise 1b as a function of:

   (a) stimulus orientation Θ,

   (b) ratio of the stimulus' spatial frequency to the cell's preferred value K/k,

   (c) stimulus' spatial phase Φ.

   Characterize the receptive field in terms of its selectivity (spatial, orientation and spatial frequency) and preferred stimulus (the stimulus giving rise to maximal response).

4. **Bonus Exercise: Image Filtering**. Equation 4 assumes the visual stimulus and the neuron's receptive field to be aligned. However, in primary visual cortex (V1), the centers of the receptive fields of neurons cover the whole visual space. Assuming different cells to have just shifted versions of their receptive fields, the response of a population of V1 cells can be written as:

$$r_s(x_k, y_l) = \sum_{i=0}^{\frac{2x_0}{\Delta x}} \sum_{j=0}^{\frac{2y_0}{\Delta y}} s(x_i, y_j) D_s(x_i - x_k, y_j - y_l) \Delta x \Delta y, \tag{5}$$

   where $r_s(x_k, y_l)$ describes the response of a cell with its receptive field centered at (x_k, y_l).

   (a) Load an arbitrary image as a visual stimulus (see Moodle for examples or pick any picture of your choice). For further processing convert the image to grayscale.
   Hint: You can use `matplotlib.pyplot.imread`.

   (b) Implement equation 5 for cells with receptive fields centered on the same grid as the input stimulus. As before, assume the stimulus to range from $-5°$ to $5°$ in both x and y coordinates. Use $k = \frac{8\pi}{\text{degree}}$, $\phi = \frac{\pi}{2}$, $\sigma_x = \sigma_y = \frac{1}{8}^{\circ}$. Plot the original and the resulting image. Describe what this Gabor filter is doing with the image. What could it be good for?
   Hint: Instead of implementing the correlation (5) yourself, you can use `scipy.signal.fftconvolve(s,` D_s`, 'same')` which calculates a convolution. Be careful to transform the kernel accordingly!

Upload your solution to Moodle (see also submission checklist in the general information section). Please, comment your code and provide short answers to the questions in each task.

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