HUMBOLDT-UNIVERSITÄT ZU BERLIN
BERNSTEIN CENTER FOR COMPUTATIONAL NEUROSCIENCE

HUMBOLDT-UNIVERSITÄT ZU BERLIN
PHILIPPSTR. 13 HOUSE 4
PROF. RICHARD KEMPTER
PROF. BENJAMIN LINDNER

PHONE: 030/2093-9110
FAX: 030/2093-6771
WEBPAGE: HTTP://WWW.BCCN-BERLIN.DE/

Models of Neural Systems, WS 2020/21
Computer Practical 5
Assignment due: January, 4th, 2021, 10 am

1. Synaptic current
Simulate a linear membrane that receives an external synaptic input:

$$\tau_\mathrm{m}\frac{\mathrm{d}V(t)}{\mathrm{d}t} = -V(t) + E_\mathrm{m} - R_\mathrm{m}I_\mathrm{syn}(t) + R_\mathrm{m}I_\mathrm{e}, \tag{1}$$

where $I_\mathrm{syn}(t) = g_\mathrm{syn}(t)(V(t) - E_\mathrm{syn})$ is the post-synaptic current. The synaptic conductance $g_\mathrm{syn}(t)$ is modeled by two differential equations:

$$\tau_\mathrm{syn}\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -x(t) + \tau_\mathrm{syn}g^*\delta(t - t_\mathrm{spike}) \tag{2}$$

$$\tau_\mathrm{syn}\frac{\mathrm{d}g_\mathrm{syn}(t)}{\mathrm{d}t} = x(t) - g_\mathrm{syn}(t). \tag{3}$$

By virtue of the delta function in the second term of eq. (2) the variable x is increased by a value g^* whenever a presynaptic spike arrives (i.e. at $t = t_\mathrm{spike}$). In the absence of input spikes, the initial condition for both x and g_syn is $x(0) = g_\mathrm{syn}(0) = 0$. In what follows, take τ_syn=10 ms, τ_m=10 ms, $R_\mathrm{m} = 10^7\,\Omega$, $g^* = 30\,\mathrm{nS}$, $E_\mathrm{m} = -80\,\mathrm{mV}$, $R_\mathrm{m}I_\mathrm{e} = 0\,\mathrm{mV}$, $V(0) = E_\mathrm{m}$. Remember to be consistent with the units of measurement you use!

a) Characterize the response to *a single pre-synaptic spike* by plotting the following curves in one figure: synaptic conductance $g_\mathrm{syn}(t)$, synaptic current $I_\mathrm{syn}(t)$, membrane current $I_\mathrm{m}(t) = (V(t) - E_\mathrm{m})/R_\mathrm{m}$, membrane potential $V(t)$. If you show multiple curves in a single plot, use different axes for different physical quantities. Consider inhibitory ($E_\mathrm{syn} = E_\mathrm{inh} = -100\,\mathrm{mV}$) and excitatory ($E_\mathrm{syn} = E_\mathrm{exc} = 0\,\mathrm{mV}$) synapses separately.
(*Hint: The peak of the synaptic conductance in response to a single pre-synaptic spike should be approximately 11nS.*)

b) *Shunting inhibition. In vivo*, neurons are constantly bombarded by balanced excitatory and inhibitory inputs. To model the background activity, assume that, in addition to the time-dependent synaptic conductance, the membrane receives tonic inhibitory and excitatory inputs, which can be described by constant conductances. The total synaptic current equals then:

$$I_\mathrm{syn}(t) = g_\mathrm{syn}(t)(V(t) - E_\mathrm{syn}) + g_\mathrm{exc}(V(t) - E_\mathrm{exc}) + g_\mathrm{inh}(V(t) - E_\mathrm{inh})$$

with $g_\mathrm{exc} = 50\,\mathrm{nS}$ and $g_\mathrm{inh} = 200\,\mathrm{nS}$. Show that the resting potential of the membrane does not change due to the tonic synaptic inputs (note that this is a special case due to the choice of parameters). Simulate the membrane response to a single excitatory presynaptic spike and plot the results together with the membrane voltage from Exercise 1(a). How would you explain the differences?

2. Integrate-and-fire neuron.

Linear models are unable to produce realistic action potentials, but spike generation can be mimicked with a simple fire-and-reset mechanism.

a) Modify the Euler method so that every time at which the membrane potential reaches the threshold $V_{\mathrm{th}} = -54\,\mathrm{mV}$ the potential is set back to the value $V_{\mathrm{reset}} = E_{\mathrm{m}}$. The reset times correspond to the spike times of the neuron model.

b) Use the integration with reset and simulate the neuron from Eq. (1) with a constant input current $I_{\mathrm{e}} = 3\,\mathrm{nA}$ (remaining parameters: $R_{\mathrm{m}} = 10^7\,\Omega$, $I_{\mathrm{syn}}(t) = 0$). Start the neuron with random initial conditions and plot the voltage traces for a simulation time of 100 ms. Estimate the firing rate by dividing the spike count by the respective time window. Does this result coincide with the inverse of the inter-spike interval (ISI)? Repeat the estimation of the firing rate for a simulation time of 500 ms. How does the result change?

c) Calculate numerically the mean firing rate as a function of input current in the range 0 nA to 5 nA – using either spike counts or ISIs. Compare both estimates to the theoretical expression

$$r_0(I_{\mathrm{e}}) = \left[\tau_{\mathrm{m}} \ln\left(\frac{R_{\mathrm{m}}I_{\mathrm{e}} + E_{\mathrm{m}} - V_{\mathrm{reset}}}{R_{\mathrm{m}}I_{\mathrm{e}} + E_{\mathrm{m}} - V_{\mathrm{th}}}\right)\right]^{-1}. \tag{4}$$

How do you interpret the results? What would you expect this curve to look like for a real neuron?

3. Synaptically coupled IF neurons.

The smallest possible recurrent network consists of two interconnected neurons, (Fig. 1). For simplicity, we assume both synapses to be of the same type (excitatory or inhibitory) and of equal strength, g^*.

a) Implement this model with integrate-and-fire neurons: the temporal evolution of membrane voltage for each neuron is described by eq. (1), with the additional property of being reset to $V_{\mathrm{reset}} = -80\,\mathrm{mV}$, when it exceeds the threshold potential $V_{\mathrm{th}} = -54\,\mathrm{mV}$ (see exercise 2). The synaptic conductance of each neuron is changed according to the (presynaptic) spikes of the other neuron. Use for both model neurons $E_{\mathrm{m}} = -70\,\mathrm{mV}$, $R_{\mathrm{m}} = 10^7\,\Omega$, $g^* = 30\,\mathrm{nS}$, $I_{\mathrm{e}} = 3.5\,\mathrm{nA}$, $\tau_{\mathrm{syn}} = 10\,\mathrm{ms}$, $\tau_{\mathrm{m}} = 10\,\mathrm{ms}$. Assume that one of the neurons fires at time $t = 0$, while the other one is at V_{reset}.

b) Plot and compare the time course of the membrane potential for the two neurons in case of excitatory and inhibitory synaptic interactions. Explain the observed interaction between the neurons. Can you notice some change in the firing synchrony?

c) Here you will show how different parameters can influence the interaction between the coupled neurons. Plot the voltages in the case of $I_{\mathrm{e}} = 3\,\mathrm{nA}$. What happens if you double the synaptic conductance g^* or the synaptic time constant τ_{syn}? Plot the corresponding cases.

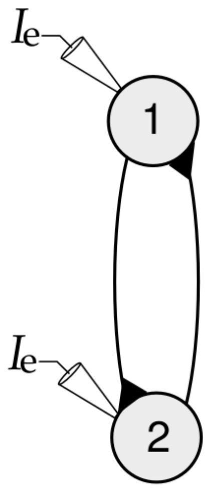

Figure 1: Mutually coupled Integrate-and-fire neurons.

Upload your solution to Moodle (see also submission checklist in the general information section). Please, comment your code and provide short answers to the questions in each task.

Contact	Location	Phone	Email
Gregory Knoll	Haus 2	2093 6247	gregory@bccn-berlin.de
Paula Kuokkanen	ITB, Haus 4, Room 107	2093 98407	paula.kuokkanen@hu-berlin.de
Eric Reifenstein	ITB, Haus 4, Room 013	2093 98413	eric.reifenstein@bccn-berlin.de
Natalie Schieferstein	ITB, Haus 4, Room 106	2093 98406	natalie.schieferstein@bccn-berlin.de