In [2]:

```
import matplotlib.pyplot as plt
import numpy as np
from sklearn.model_selection import train_test_split
from data_gen import *
from drawer import *
%matplotlib inline
```

In [3]:

```
seed=0
```

In [20]:

```
def PLA(data,w_init, max_iter,if_draw=False):
    w = w_init
    X=data[0]
    y=data[1]
    X=np.hstack((np.ones((X.shape[0],1)),X))

    for i in range(max_iter):
        h = np.sign(X @ w)
        mistake_indices = np.where(h != y)[0]
        if len(mistake_indices):
            j = np.random.choice(mistake_indices)
            w = w + y[j]*X[j,:]
            if if_draw:
                draw(data,w)
                plt.title('after '+str(i)+' iterations')
                print(w)
        else:
            break


    print('PLA Training Accuracy:',str((X.shape[0]-len(mistake_indices))
/X.shape[0]))
    return w

```

In [21]:

```
def Pocket(data,w_init,max_iter,if_draw=False):
    w = w_init
    X = data[0]
    y = data[1]
    X = np.hstack((np.ones((X.shape[0],1)),X))

    for  i in range(max_iter):
        h = np.sign(X@w)
        mistake_indices = np.where(h!=y)[0]
        mistake_num = len(mistake_indices)
        if mistake_num>0:
            np.random.shuffle(mistake_indices)
            j = np.random.choice(mistake_indices)
            w_ = w + y[j]*X[j,:]
            h_ = np.sign(X@w_)
            mistake_indices_=np.where(h_!=y)[0]
            mistake_num_ = len(mistake_indices_)
            # print('after '+str(i)+' iterations',end='')
            # print(w.round(3))
            if mistake_num_ <= mistake_num:
                w = w_
                if if_draw:
                    draw(data,w)
                    plt.title('after '+str(i)+' iterations')
        else:
            break

    print('Pocket Training Accuracy:',str((X.shape[0]-len(mistake_indice
s))/X.shape[0]))
    return w
```

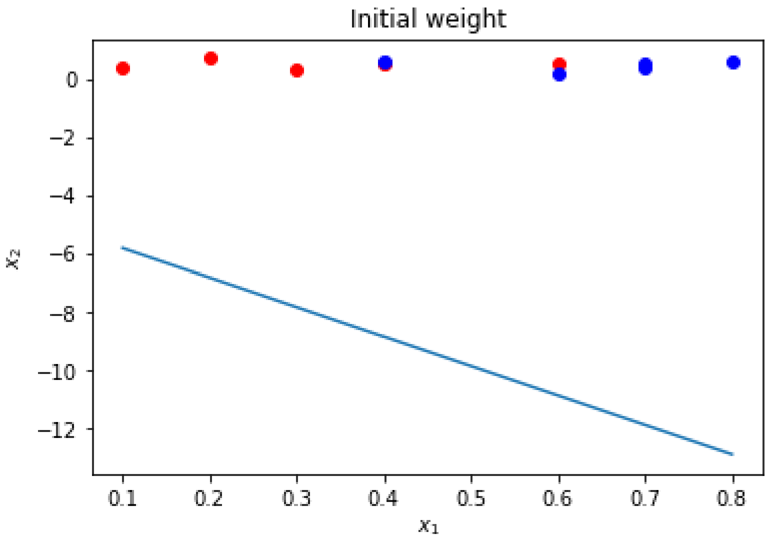
In [29]:

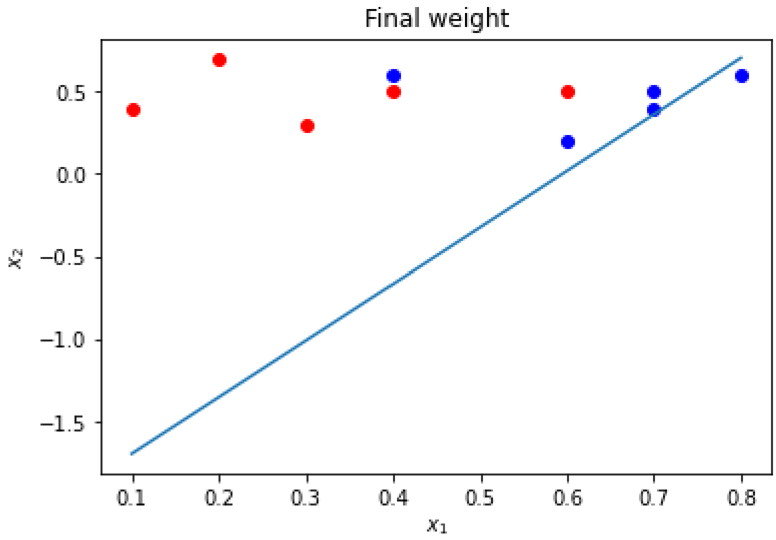
```
def validate(data,w):
    X=data[0]
    y=data[1]
    X = np.hstack((np.ones((X.shape[0],1)),X))#IMPORTANT!!!

    h=np.sign(X@w)
    mistake_indices = np.where(h!=y)[0]
    return (X.shape[0]-len(mistake_indices))/X.shape[0]
```

In [16]:

```
data = []
data.append(np.array([[0.2,0.7],[0.3,0.3],[0.4,0.5],[0.6,0.5],[0.1,0.4],
[0.4,0.6],[0.6,0.2],[0.7,0.4],[0.8,0.6],[0.7,0.5]]))
data.append(np.array([1,1,1,1,1,-1,-1,-1,-1,-1]))
w_init = np.random.randn(3)
print(w_init.round(3))
draw(data,w_init)
plt.title('Initial weight')

w = Pocket(data,w_init,20)
draw(data,w)
plt.title('Final weight')
print(w.round(3))
```

```
[-0.995 -2.091 -0.207]
Pocket Accuracy: 0.6
[ 1.005 -1.691  0.493]
```

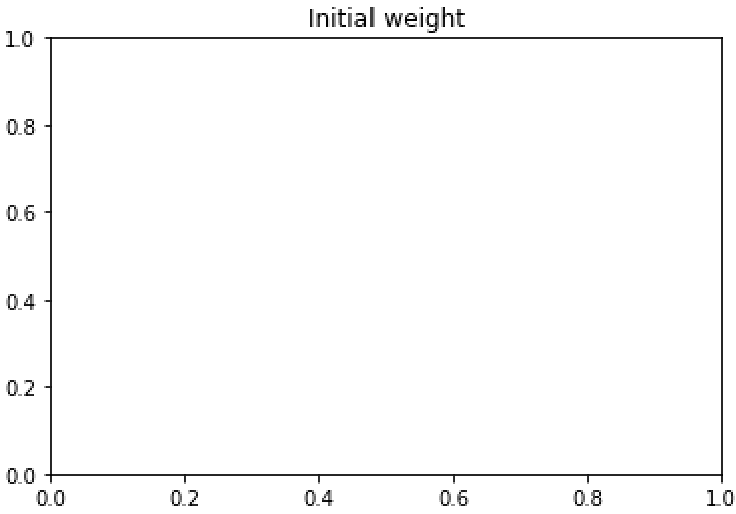


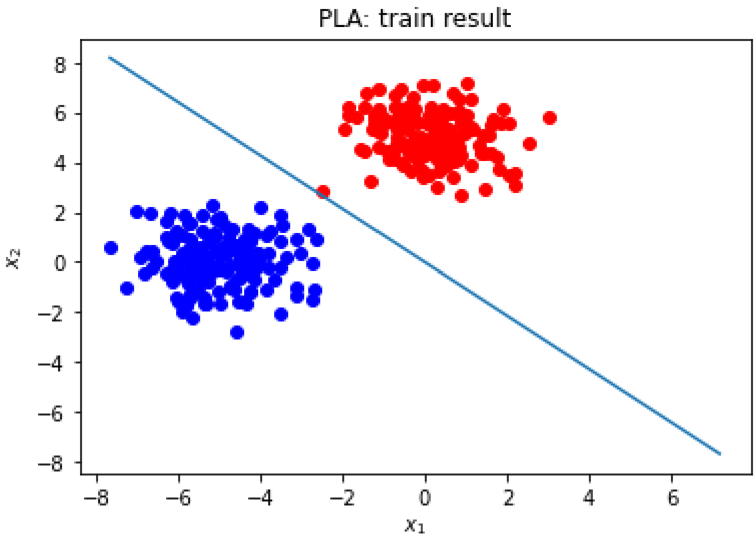


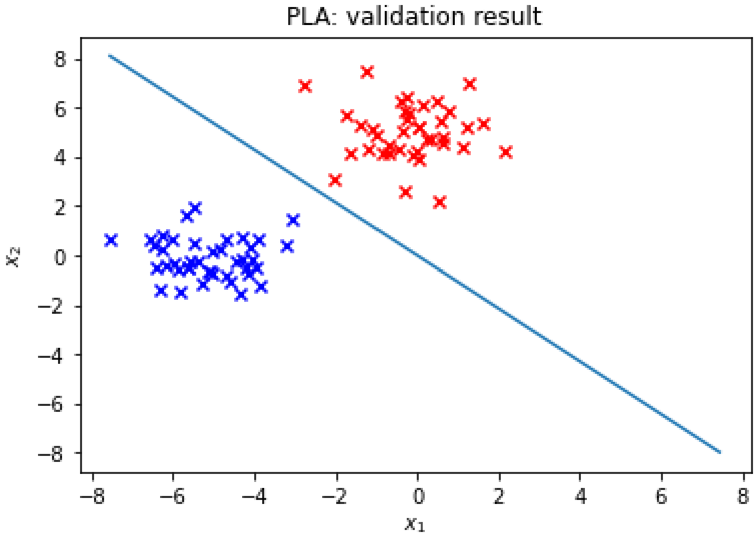
In [35]:

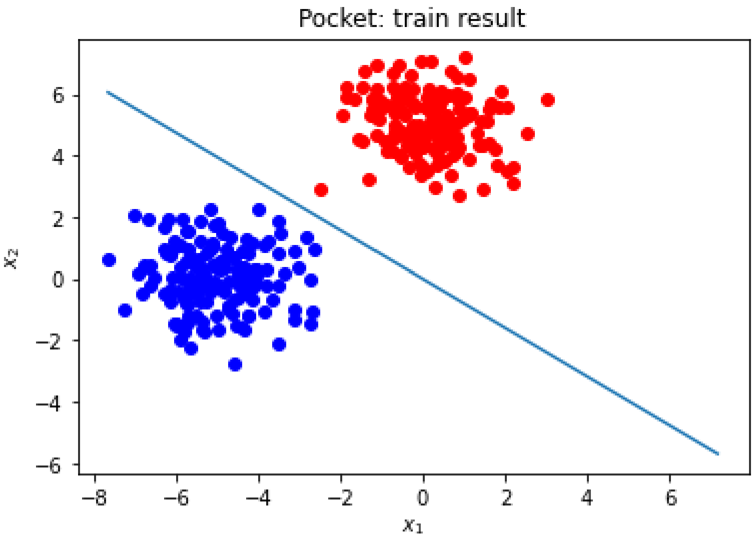
```
data = data_generator([-5,0],np.identity(2),[0,5],np.identity(2),400,seed=seed)
X_train,X_test,y_train,y_test=train_test_split(data[0],data[1],train_size=0.8,test_size=0.2,random_state=seed)
train_data=(X_train,y_train)
test_data=(X_test,y_test)
w_init = np.zeros(3)
draw(data,w_init)
plt.title('Initial weight')

%time w_pla = PLA(train_data,w_init,100)
draw(train_data,w_pla,'o')
plt.title('PLA: train result')

draw(test_data,w_pla,'x')
plt.title('PLA: validation result')
print('PLA Validation Accuracy '+str(validate(test_data,w_pla)))

%time w_pocket = Pocket(train_data,w_init,100)
draw(train_data,w_pocket,'o')
plt.title('Pocket: train result')

draw(test_data,w_pocket,'x')
plt.title('Pocket: validation result')
print('Pocket Validation Accuracy '+str(validate(test_data,w_pocket)))
```

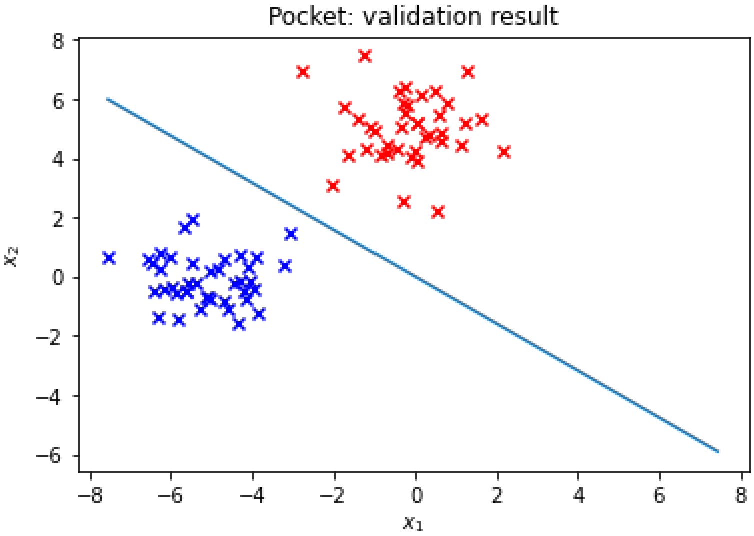
```
PLA Training Accuracy: 1.0
Wall time: 4 ms
PLA Validation Accuracy 1.0
Pocket Training Accuracy: 1.0
Wall time: 950 µs
Pocket Validation Accuracy 1.0
```


Initial weight
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


PLA: train result
8
6
4
2
0
−2
−4
−6
−8
x_2
−8
−6
−4
−2
0
2
4
6
x_1


PLA: validation result
8
6
4
2
0
−2
−4
−6
−8
x_2
−8
−6
−4
−2
0
2
4
6
8
x_1





从上面的结果我们可以看出:

- PLA
    - 在training set上的正确率为100%
    - 在validation set上的正确率为100%
    - 运行时间为4ms
- Pocket
    - 在training set上的正确率为100%
    - 在validation set上的正确率为100%
    - 运行时间为950us

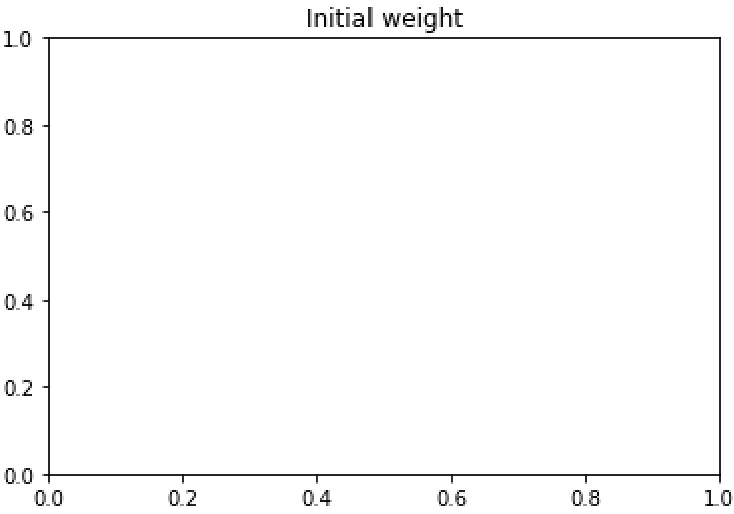
可见Pocket算法的速度远快于PLA算法

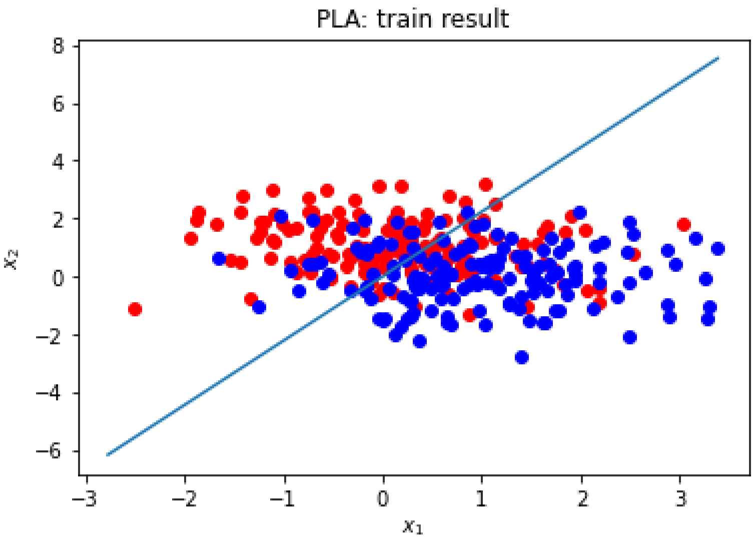
In [36]:

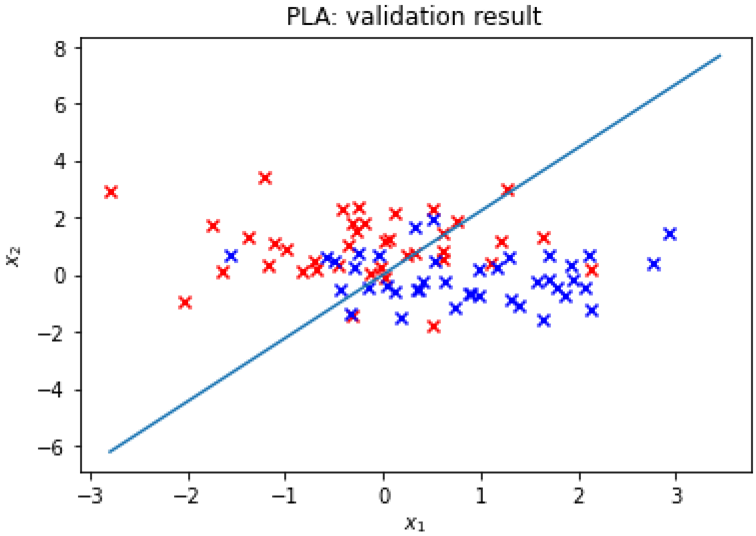
```
data = data_generator([1,0],np.identity(2),[0,1],np.identity(2),400,seed=seed)
X_train,X_test,y_train,y_test=train_test_split(data[0],data[1],train_size=0.8,test_size=0.2,random_state=seed)
train_data=(X_train,y_train)
test_data=(X_test,y_test)
w_init = np.zeros(3)
draw(data,w_init)
plt.title('Initial weight')

%time w_pla = PLA(train_data,w_init,100)
```

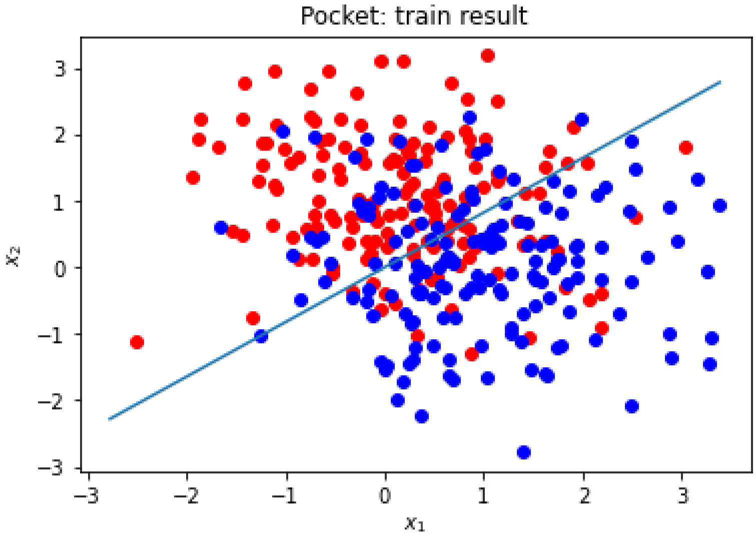
```
draw(train_data,w_pla,'o')
plt.title('PLA: train result')

draw(test_data,w_pla,'x')
plt.title('PLA: validation result')
print('PLA Validation Accuracy '+str(validate(test_data,w_pla)))

%time w_pocket = Pocket(train_data,w_init,100)
draw(train_data,w_pocket,'o')
plt.title('Pocket: train result')

draw(test_data,w_pocket,'x')
plt.title('Pocket: validation result')
print('Pocket Validation Accuracy '+str(validate(test_data,w_pocket)))
```

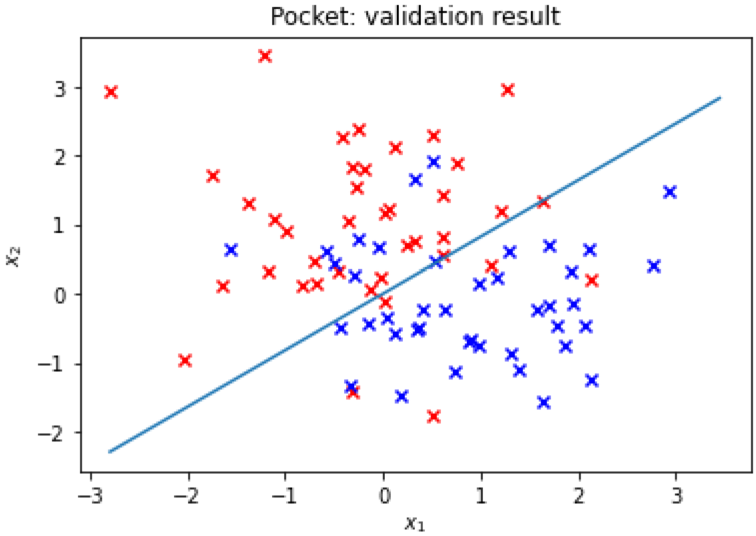
```
PLA Training Accuracy: 0.60625
Wall time: 8.97 ms
PLA Validation Accuracy 0.775
Pocket Training Accuracy: 0.75
Wall time: 12 ms
Pocket Validation Accuracy 0.8125
```











从上面的结果我们可以看出:

- PLA
  - 在training set上的正确率为60.625%
  - 在validation set上的正确率为77.5%
  - 运行时间为8.97ms
- Pocket
  - 在training set上的正确率为75%

- 在validation set上的正确率为81.25%
- 运行时间为12ms

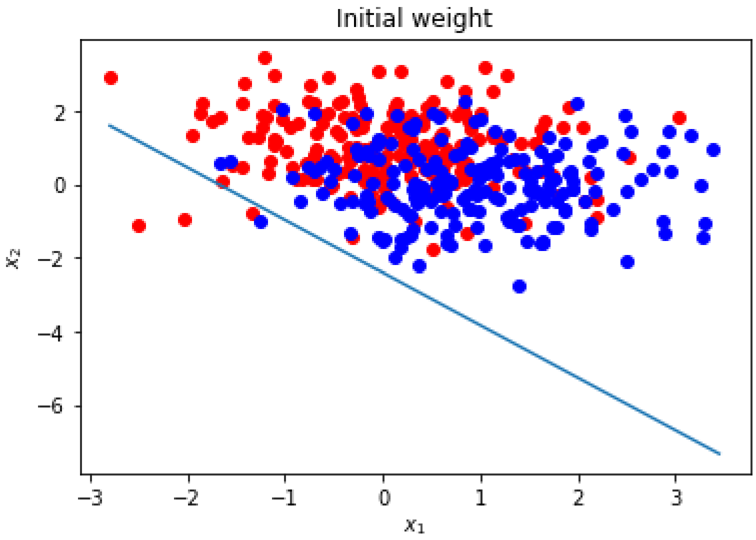
推测是因为样本线性不可分, 所以两种算法都一直迭代到所设置的最大迭代次数才停止, 而Pocket在每次迭代中要计算两次, PLA只用计算一次, 所以Pocket算法所花时间更短一点

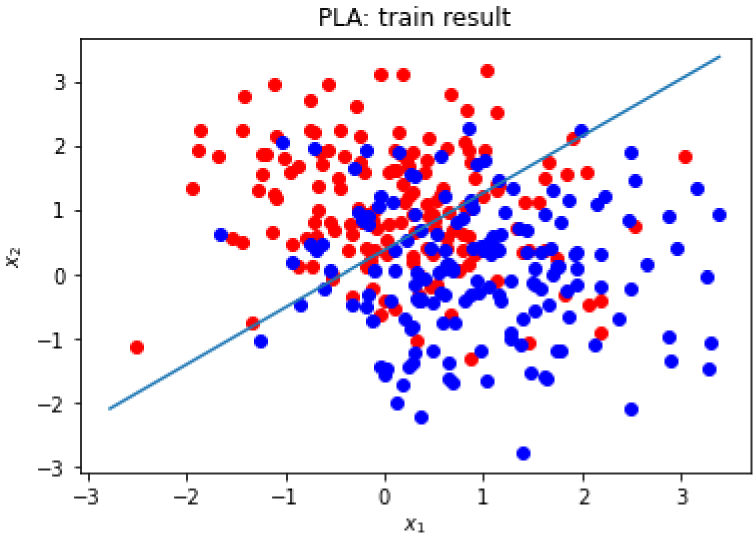
下面为测试实验

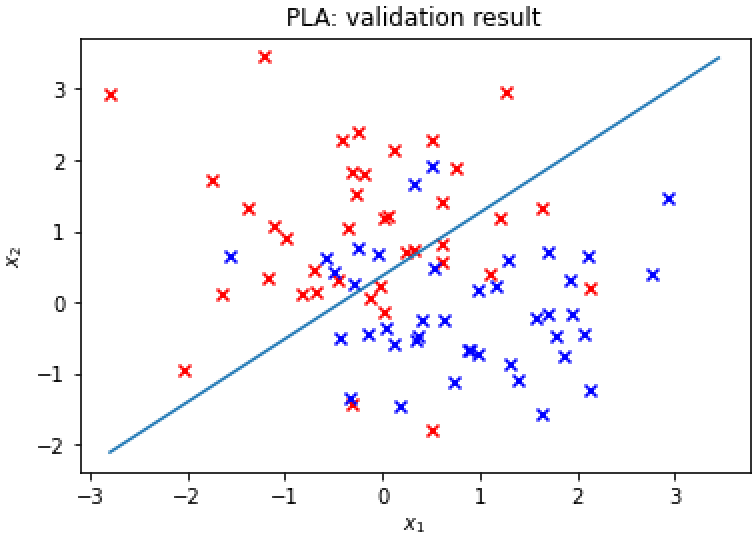
实验一: 改变初始化方式, 把零初始化变为随机初始化

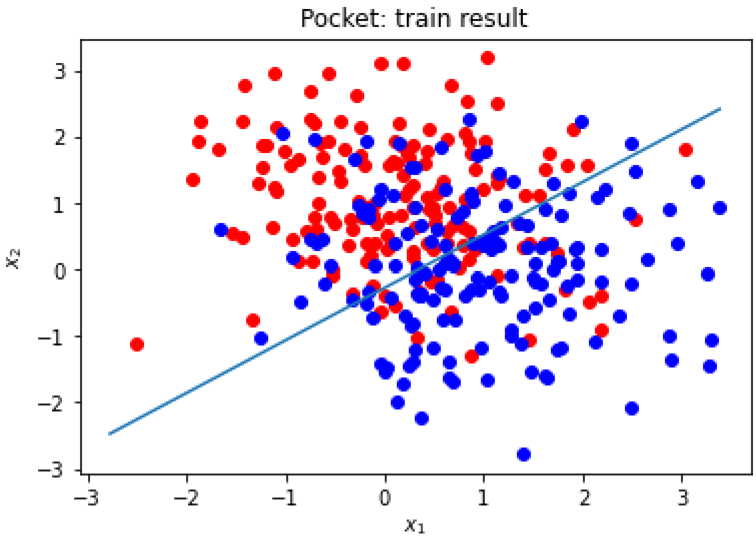
In [38]:
```
data = data_generator([1,0],np.identity(2),[0,1],np.identity(2),400,seed
=seed)
X_train,X_test,y_train,y_test=train_test_split(data[0],data[1],train_siz
e=0.8,test_size=0.2,random_state=seed)
train_data=(X_train,y_train)
test_data=(X_test,y_test)
w_init = np.random.randn(3)
draw(data,w_init)
plt.title('Initial weight')

%time w_pla = PLA(train_data,w_init,100)
draw(train_data,w_pla,'o')
plt.title('PLA: train result')

draw(test_data,w_pla,'x')
plt.title('PLA: validation result')
print('PLA Validation Accuracy '+str(validate(test_data,w_pla)))

%time w_pocket = Pocket(train_data,w_init,100)
draw(train_data,w_pocket,'o')
plt.title('Pocket: train result')

draw(test_data,w_pocket,'x')
plt.title('Pocket: validation result')
print('Pocket Validation Accuracy '+str(validate(test_data,w_pocket)))
```

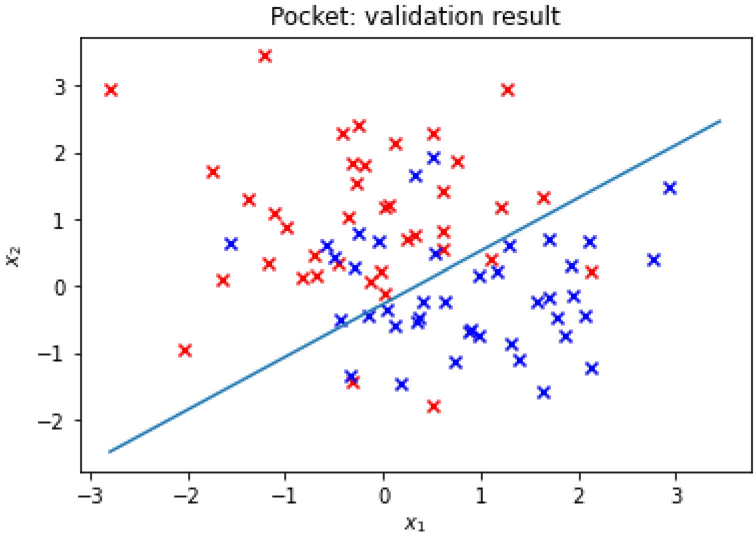
```
PLA Training Accuracy: 0.621875
Wall time: 11.9 ms
PLA Validation Accuracy 0.7625
Pocket Training Accuracy: 0.759375
Wall time: 11.9 ms
Pocket Validation Accuracy 0.825
```


Initial weight
2
0
−2
−4
−6
x_2
−3
−2
−1
0
1
2
3
x_1


PLA: train result
3
2
1
0
−1
−2
−3
x_2
−3
−2
−1
0
1
2
3
x_1


PLA: validation result
3
2
1
0
−1
−2
x_2
−3
−2
−1
0
1
2
3
x_1





可见初始化方式对结果并无显著性影响

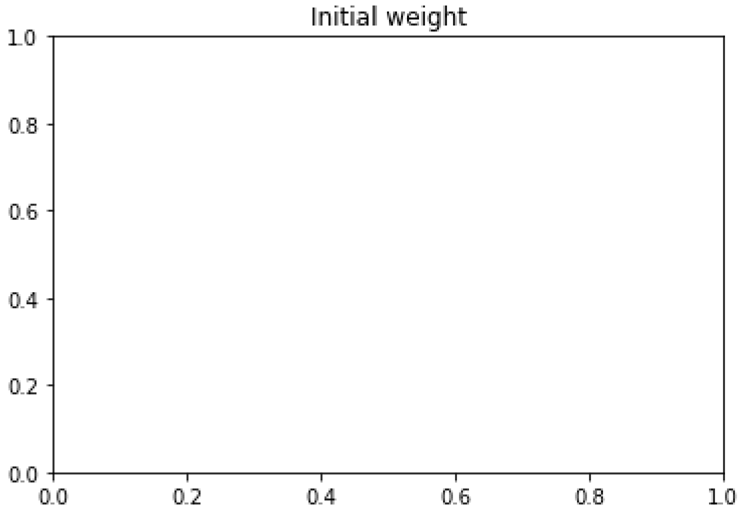
实验二: 尝试增加迭代次数

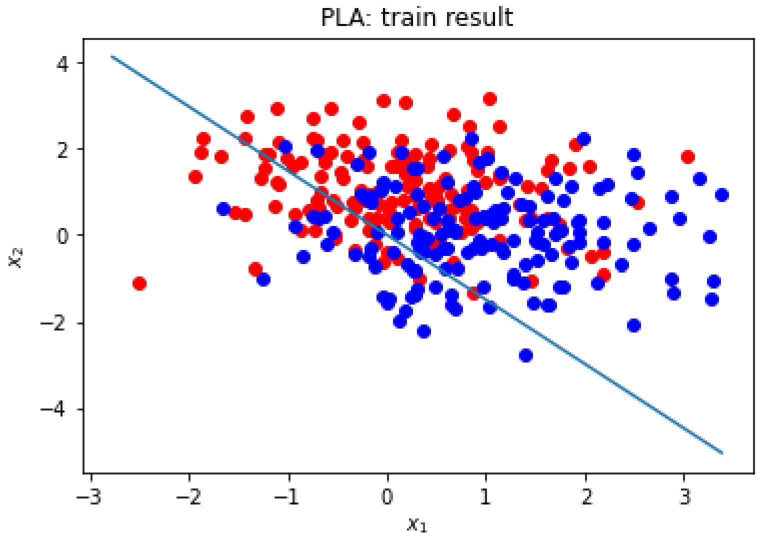
In [39]:

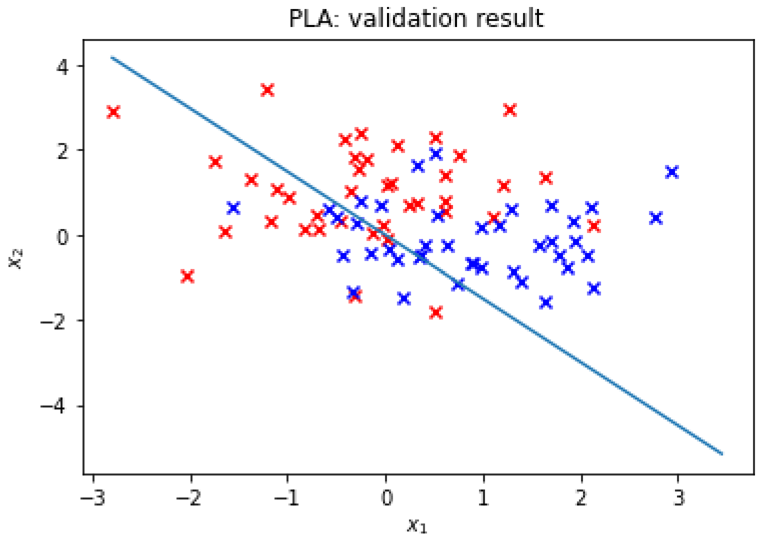
```
data = data_generator([1,0],np.identity(2),[0,1],np.identity(2),400,seed=seed)
X_train,X_test,y_train,y_test=train_test_split(data[0],data[1],train_size=0.8,test_size=0.2,random_state=seed)
train_data=(X_train,y_train)
test_data=(X_test,y_test)
w_init = np.zeros(3)
draw(data,w_init)
plt.title('Initial weight')

%time w_pla = PLA(train_data,w_init,1000)
draw(train_data,w_pla,'o')
plt.title('PLA: train result')

draw(test_data,w_pla,'x')
plt.title('PLA: validation result')
print('PLA Validation Accuracy '+str(validate(test_data,w_pla)))

%time w_pocket = Pocket(train_data,w_init,1000)
draw(train_data,w_pocket,'o')
plt.title('Pocket: train result')
```

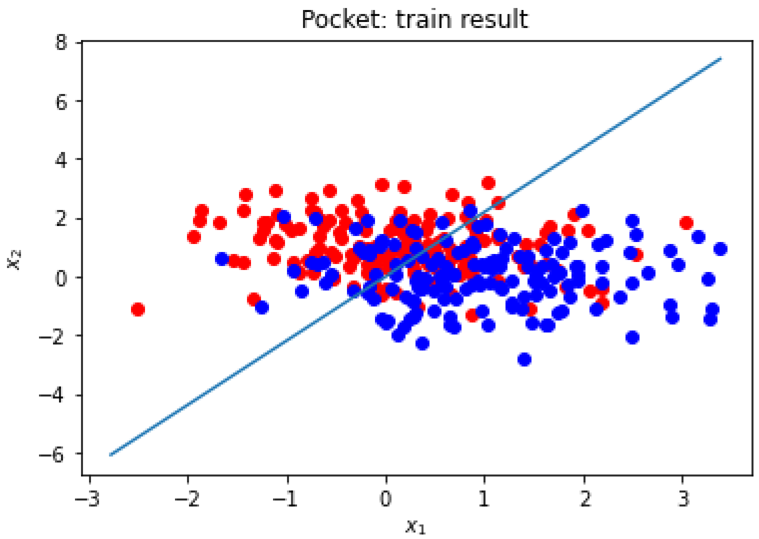
```
draw(test_data,w_pocket,'x')
plt.title('Pocket: validation result')
print('Pocket Validation Accuracy '+str(validate(test_data,w_pocket)))
```

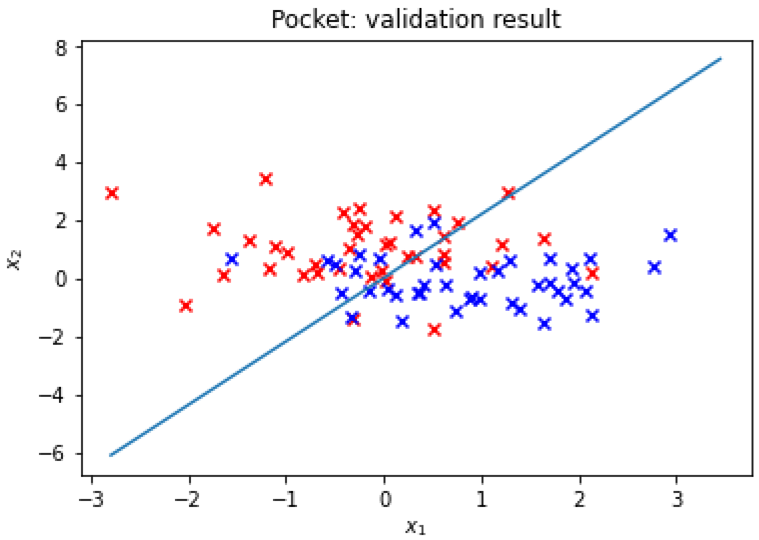
```
PLA Training Accuracy: 0.60625
Wall time: 141 ms
PLA Validation Accuracy 0.4375
Pocket Training Accuracy: 0.71875
Wall time: 158 ms
Pocket Validation Accuracy 0.775
```











可见对于线性不可分的情形, 增加迭代次数可能反而会造成模型发生较为严重的过拟合现象

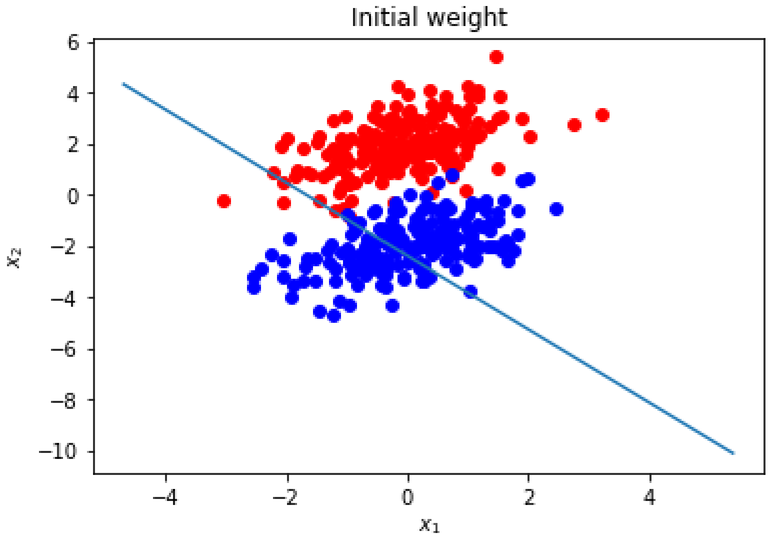
实验三: 尝试改变数据分布

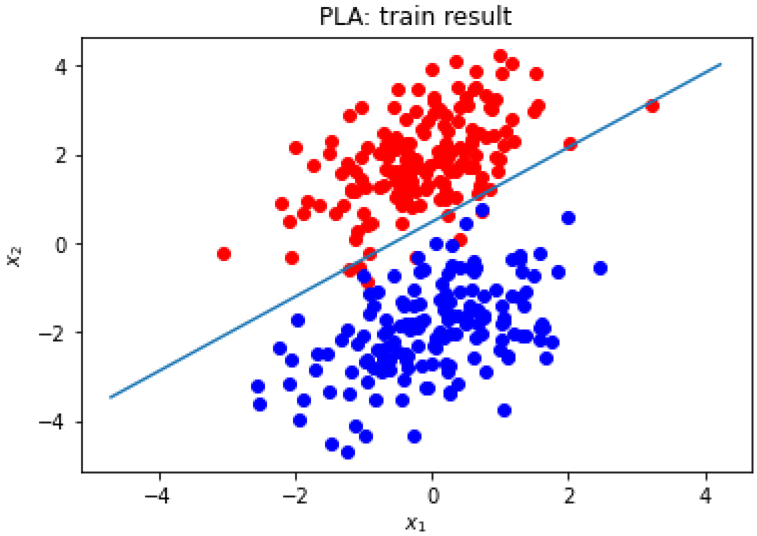
In [43]:

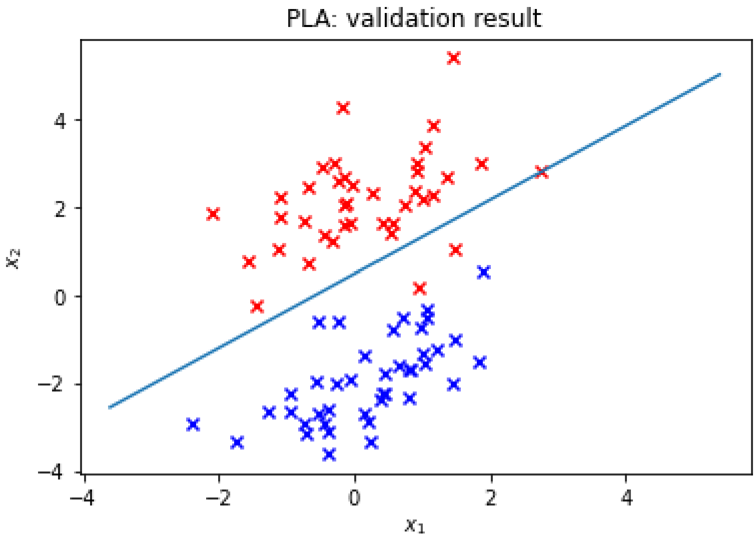
```
data = data_generator([0,-2],[[1,0.5],[0.5,1]],[0,2],[[1,0.5],[0.5,1]],400,seed=seed)
X_train,X_test,y_train,y_test=train_test_split(data[0],data[1],train_size=0.8,test_size=0.2,random_state=seed)
train_data=(X_train,y_train)
test_data=(X_test,y_test)
w_init = np.random.randn(3)
draw(data,w_init)
plt.title('Initial weight')

%time w_pla = PLA(train_data,w_init,100)
draw(train_data,w_pla,'o')
plt.title('PLA: train result')

draw(test_data,w_pla,'x')
plt.title('PLA: validation result')
print('PLA Validation Accuracy '+str(validate(test_data,w_pla)))

%time w_pocket = Pocket(train_data,w_init,100)
draw(train_data,w_pocket,'o')
```

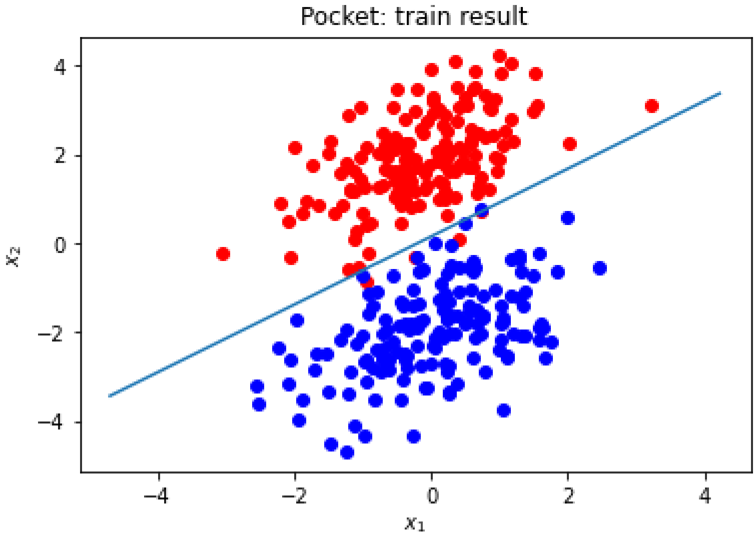
```
plt.title('Pocket: train result')

draw(test_data,w_pocket,'x')
plt.title('Pocket: validation result')
print('Pocket Validation Accuracy '+str(validate(test_data,w_pocket)))
```

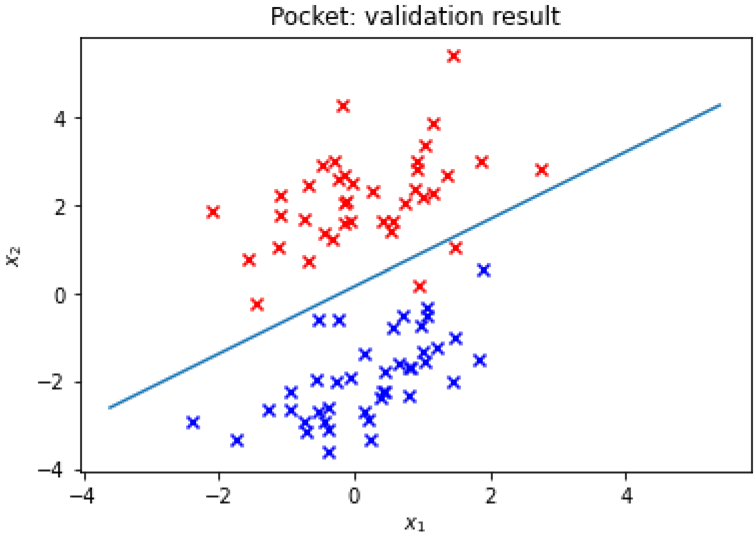
```
PLA Training Accuracy: 0.98125
Wall time: 6.94 ms
PLA Validation Accuracy 0.975
Pocket Training Accuracy: 0.9875
Wall time: 9.92 ms
Pocket Validation Accuracy 0.975
```











可见当模型具有较好的线性可分性时, 两种模型都能够具有较好的效果