

# NETWORK MODELING

## SOLUTIONS FOR 26/06/2017

### Problem 1

$$\Pr[X_1(1) = 1 | X(2) = 3] = \binom{3}{1} \left( \frac{\lambda_1}{2(\lambda_1 + \lambda_2)} \right)^1 \left( 1 - \frac{\lambda_1}{2(\lambda_1 + \lambda_2)} \right)^{3-1}$$

$$\Pr[X(2) = 3 | X_1(1) = 1] = \Pr[X_1(2) + X_2(2) - X_1(1) = 3 - 1]$$

$$\Pr[X(2) = 3 | X_1(3) = 0] = \Pr[X_2(2) = 3] = \frac{e^{-2\lambda_2} (2\lambda_2)^3}{3!}$$

$$\begin{aligned} \Pr[X(2) = 3 | X_1(3) = 1] &\stackrel{(\text{TPL})}{=} \Pr[X(2) = 3 | X_1(2) = 0] \Pr[X_1(2) = 0 | X_1(3) = 1] + \\ &\quad + \Pr[X(2) = 3 | X_1(2) = 1] \Pr[X_1(2) = 1 | X_1(3) = 1] \\ &= \Pr[X_1(2) + X_2(2) - X_1(2) = 3 - 0] \binom{1}{0} \left( \frac{2}{3} \right)^0 \left( 1 - \frac{2}{3} \right)^1 + \\ &\quad + \Pr[X_1(2) + X_2(2) - X_1(2) = 3 - 1] \binom{1}{1} \left( \frac{2}{3} \right)^1 \left( 1 - \frac{2}{3} \right)^0 \end{aligned}$$

### Problem 2

Average number of consecutive good slots is 100, so  $p_{01} = \frac{1}{100} = 0.01$  and so  $p_{00} = 1 - p_{01} = 0.99$ .

Then we solve  $\begin{cases} \pi_G = p_{00}\pi_G + p_{10}\pi_B \\ \pi_G = 1 - \pi_B \end{cases} \Rightarrow p_{10} = \frac{\pi_G(1 - p_{00})}{\pi_B} = 0.49$ . So  $p_{11} = 1 - p_{10} = 0.51$ .

Embedded MC is  $C = \begin{bmatrix} 0.99 & 0.01 \\ 0.49 & 0.51 \end{bmatrix}$ .

For the first question we have  $\mathcal{T}_1 = \frac{p_{10}}{p_{10} + p_{01}} \simeq 0.98$ .

Protocol matrix is  $C = \begin{bmatrix} - & 0.015 \\ 0.735 & - \end{bmatrix}$ .

For the second question we have  $\mathcal{T}_2 = \frac{p_{10}^{(2)}}{p_{10}^{(2)} + 2p_{01}} \simeq 0.9735$ .

For the last question we first compute  $\mathcal{T}_3 = \frac{(1 - \delta)p_{10}^{(2)}}{(1 - \delta)p_{10}^{(2)} + 2 \left( (1 - \delta)p_{01} + \delta p_{01}^{(2)} + \delta p_{10}^{(2)} \right)} \simeq 0.7974$ .

Then average throughput is  $\mathcal{T}_\Theta = \frac{E[\text{perfect}]T_2 + E[\text{iid}]T_3}{E[\text{cycle}]} \simeq 0.88545$

### Problem 3

$A = \{0, 2\}$  is positive recurrent periodic ( $d = 2$ ) class.

$\{3\}$  is transient class.

$B = \{1, 4\}$  is positive recurrent aperiodic class.

Using first step analysis we have:

$$\Pr[\text{absorption in } \{0, 2\} | X_0 = 3] = \pi_3(\{0, 2\}) = \frac{5}{8}.$$

$$\Pr[\text{absorption in } \{2, 4\} | X_0 = 3] = \pi_3(\{2, 4\}) = \frac{3}{8}.$$

$$\lim_{n \rightarrow \infty} P^n = \begin{bmatrix} X & 0 & X & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ X & 0 & X & 0 & 0 \\ X & \frac{1}{3} & X & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix} \quad \lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} P^k = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{2} & 0 & \frac{2}{3} \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix}$$

Then  $[\bar{\theta}_{04}, \bar{\theta}_{14}, \bar{\theta}_{24}, \bar{\theta}_{34}, \bar{\theta}_{44}] = [\infty, \frac{5}{3}, \infty, \infty, \frac{3}{2}]$  and  $\mathbf{m} = [m_0, m_1, m_2, m_3, m_4] = [2, 3, 2, \infty, \frac{3}{2}]$ .

## Problem 4

$$\Pr[\text{both OFF}] = \left( \frac{\frac{1}{\beta}}{\frac{1}{\alpha} + \frac{1}{\beta}} \right)^2 = \left( \frac{1}{20} \right)^2 = 0.0025.$$

$$\Pr[\text{both ON}] = \left( \frac{\frac{1}{\alpha}}{\frac{1}{\alpha} + \frac{1}{\beta}} \right)^2 = \left( \frac{19}{20} \right)^2 = 0.9025.$$

$$E[\text{both OFF}] = \min\{\xi(\beta), \xi(\beta)\} = \frac{1}{2\beta} = 500 \text{ seconds.}$$

Then we can compute average cycle duration:  $E[\text{cycle}] = \frac{E[\text{both OFF}]}{\Pr[\text{both OFF}]} = 200000 \text{ seconds.}$

The probability that at least one processor is working is  $\Pr[\geq 1 \text{ ON}] = 1 - \Pr[\text{both OFF}] = \frac{399}{400} = 0.9975$ .

So  $E[\geq 1 \text{ ON}] = \Pr[\geq 1 \text{ ON}] E[\text{cycle}] = 199500 \text{ seconds.}$

Finally  $T = \Pr[\text{both ON}] \cdot 1 \text{ Gbps} + (1 - \Pr[\text{both ON}] - \Pr[\text{both OFF}]) \cdot 0.3 \text{ Gbps} = 0.931 \text{ Gbps.}$