# Network Modeling
# Solutions for 21/06/2016

## Problem 1

Average number of consecutive good slots is 50, so $p_{01} = \frac{1}{50} = 0.02$ and so $p_{00} = 1 - p_{01} = 0.98$.

Then we solve $\begin{cases} \pi_G = p_{00}\pi_G + p_{10}\pi_B \\ \pi_G = 1 - \pi_B \end{cases} \Rightarrow p_{10} = \frac{\pi_G(1-p_{00})}{\pi_B} = 0.18$. So $p_{11} = 1 - p_{10} = 0.82$.

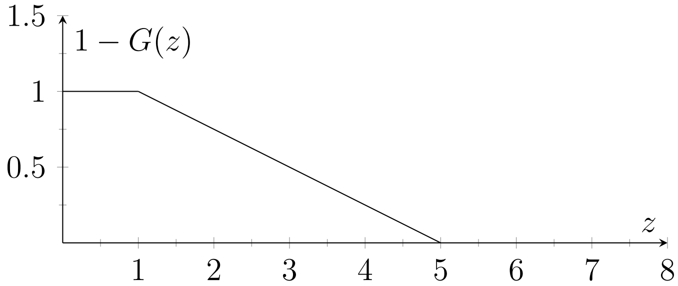
Embedded MC is $C = \begin{bmatrix} 0.98 & 0.02 \\ 0.18 & 0.82 \end{bmatrix}$. Protocol matrix is $C = \begin{bmatrix} - & 0.036 \\ 0324 & - \end{bmatrix}$.

For the first question we have $\mathcal{T}_1 = \frac{p_{10}^{(2)}}{p_{10}^{(2)} + 2p_{01}} \simeq 0.8901$.

For the second question we have $\mathcal{T}_2 = \frac{(1-\delta)p_{10}^{(2)}}{(1-\delta)p_{10}^{(2)} + 2\left((1-\delta)p_{01} + \delta p_{01}^{(2)} + \delta p_{10}^{(2)}\right)}$, where $\delta = 0.05$.

## Problem 2

First we consider $\lambda = \lambda_1 + \lambda_2$. Let $M(t)$ be the counter for packets in the system.




$M(t) \sim \mathcal{P}(\Lambda)$, where $\Lambda = \lambda \int_0^t [1 - G(z)]dz = \begin{cases} \lambda & t = 1 \\ 2.5\lambda & t = 3 \\ 3\lambda & t = 10 \end{cases}$.

Notice that for $t = 3$ one could see that it corresponds to the mean of the random variable, so $1 - G(3) = 0.5$.

Then $\Pr[M(t) = 0] = \frac{e^{-\Lambda}(\Lambda)^0}{0!} = \begin{cases} e^{-0.75} & t = 1 \\ e^{-1.875} & t = 3 \\ e^{-2.25} & t = 10 \end{cases}$.

For the second point we introduce $M_2(t)$ counting only type-2 arrivals.

Proceeding as before$M(t) \sim \mathcal{P}(\Lambda_2)$, where $\Lambda_2 = \lambda_2 \int_0^t [1 - G(z)]dz = \begin{cases} \lambda_2 & t = 1 \\ 2.5\lambda_2 & t = 3 \\ 3\lambda_2 & t = 10 \end{cases}$.

Then $\Pr[M_2(t) = 0] = \frac{e^{-\Lambda_2}(\Lambda_2)^0}{0!} = \begin{cases} e^{-0.5} & t = 1 \\ e^{-1.25} & t = 3 \\ e^{-1.5} & t = 10 \end{cases}$.

For the last point $\Pr[X_2(2) = 1 | X(3) = 3] = \binom{3}{1} \left(\frac{2\lambda_2}{3(\lambda_1 + \lambda_2)}\right)^1 \left(1 - \frac{2\lambda_2}{3(\lambda_1 + \lambda_2)}\right)^2 = \frac{100}{243}$.

# Problem 3

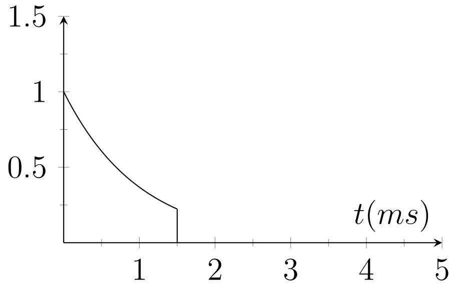
$A = \{0, 3\}$ is positive recurrent periodic ($d = 2$) class.
$\{2\}$ is transient class.
$B = \{1, 4\}$ is positive recurrent aperiodic class.
Using first step analysis we have:
$\Pr[\text{absorption in } \{0, 3\} | X_0 = 2] = \pi_2(A) = \frac{1}{2}$.
$\Pr[\text{absorption in } \{1, 4\} | X_0 = 2] = \pi_2(B) = \frac{1}{2}$.

$$\lim_{n\to\infty} P^n = \begin{bmatrix} X & 0 & 0 & X & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ X & \frac{1}{3}\frac{1}{2} & 0 & X & \frac{2}{3}\frac{1}{2} \\ X & 0 & 0 & X & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix} \qquad \lim_{n\to\infty} \sum_{k=0}^{n-1} P^k = \begin{bmatrix} \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \\ \frac{1}{2}\frac{1}{2} & \frac{1}{3}\frac{1}{2} & 0 & \frac{1}{2}\frac{1}{2} & \frac{2}{3}\frac{1}{2} \\ \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & \frac{1}{3} & 0 & 0 & \frac{2}{3} \end{bmatrix}$$

Then $\left[\overline{\theta}_{00}, \overline{\theta}_{10}, \overline{\theta}_{20}, \overline{\theta}_{30}, \overline{\theta}_{40}\right] = \left[2, \infty, \infty, 1, \infty\right]$ and $\mathbf{m} = \left[m_0, m_1, m_2, m_3, m_4,\right] = \left[2, 3, \infty, 2, \frac{3}{2}\right]$.

# Problem 4

The distribution of the first arrival is exponential, hence $E[\text{empty}] = \dfrac{1}{\lambda} = 1$ ms.
After the first arrival, we wait until another arrival or up to 1.5 ms, then we send. The distribution is a truncated exponential: $E[\text{busy}] = \int_0^{1.5} e^{-\lambda t} dt = \dfrac{1 - e^{-1.5\lambda}}{\lambda} \simeq 0.7768$ ms.




Fraction of time spent waiting is $P_{waiting} = \dfrac{E[\text{busy}]}{E[\text{empty}] + E[\text{busy}]} = 0.4371$.
If a packet finds the queue non empty, then transmission is immediate. Else it has to wait 0.77 ms on average. By law of total probability we have:
$E[\text{delay}] = E[\text{delay}|\text{empty}]P_{empty} + E[\text{delay}|\text{busy}]P_{waiting} = 0.7768(1 - 0.4371) = 0.4372$ ms.
The probability that one packet is sent is $P_1 = \Pr[exp(\lambda) > T] = e^{-\lambda T} = 0.2231$.
The probability that one packet is sent is $P_2 = 1 - P_1 = 0.7769$.
Efficiency is$\eta = \dfrac{1000P_1 + 2000P_2}{1320P_1 + 2320P_2} = \dfrac{1776}{2096} \simeq 0.8473$