T1 Prove that a Markov chain with a finite number of states cannot have any null recurrent state.

T2 Prove that for a renewal process $E[S_{N(t)+1}] = E[X](M(t)+1)$.

T3 Prove that for a Markov chain the n-step transition probabilities, $P_{ij}^{(n)}$, satisfy the relationship

$$P_{ij}^{(n)} = \sum_m P_{im}^{(k)} P_{mj}^{(n-k)}, k = 0, 1, \ldots, n$$

T1 State and prove the elementary renewal theorem.

T2 Prove that in a Markov chain the period is a class property.

T3 Prove that for a Poisson process $X(t)$ the statistics of $X(s)$ conditioned on $X(t), s < t$, is binomial, and provide the espression of $P[X(s) = k | X(t) = n]$.

T1 State and prove the elementary renewal theorem.

T2 Prove that if states i and j of a Markov chain communicate and i is recurrent, then j is also recurrent.

T3 Prove that a Markov chain with a finite number of states cannot have any null recurrent state.

T1 For a Poisson process of rate λ, prove that the interarrival times are iid exponential with mean $1/\lambda$.

T2 Prove that in a Markov chain the period is a class property.

T3 Prove that for a renewal process $E[S_{N(t)+1}] = E[X](M(t)+1)$.

T3 Consider a random walk over the non-negative integers with the following transition probabilities: $P_{01} = 1$, $P_{i,i+1} = p, P_{i,i-1} = q, i > 0$, with $p + q = 1$. Study its behavior, and in particular characterize its recurrence or transiency and derive the steady-state distribution.

T3 For a renewal process, give an expression for $E[S_{N(t)+1}]$, also providing a formal proof.

T2 For a Poisson process $X(t)$ of rate λ, state and derive the expression of $P[X(u) = k | X(t) = n]$ for the two cases (i) $0 < u < t, 0 \leq k \leq n$ and (ii) $0 < t < u, 0 \leq n \leq k$.

T3 For a renewal process, state precisely (also providing a formal proof) what is the value of

$$\lim_{t \to \infty} \frac{N(t)}{t}$$