Surfaces

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Definition 2.1 (Regular Surface)
A subset $S \subset \mathbb{R}^3$ is a regular surface if for each $p \in S$, there exists a neighborhood $V \in \mathbb{R}^3$, an open set $U \in \mathbb{R}^2$ and an onto map $x : U \to V \cap S$ such that
(1)x is differentiable, i.e. if $x(u,v) = (x_1(u,v), x_2(u,v), x_3(u,v))$, $(u,v) \in U$, then $x_i(u,v)$ have continuous partial derivatives of all orders in U.
(2)x is a homeomorphism, i.e. $x^{-1} : V \cap S \to U$ is continuous.
(3)(regularity condition)
For each $q \in U$,

Definition 2.2 (Principle Curvature)
Let $S \subset \mathbb{R}^3$ be a regular surface.