Preface

Zheng Xie

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Definition 0.1 (Vector Product)
Let $u, v \in \mathbb{R}^3$. The vector product (or cross product)u and v is the unique vector $u \wedge v \in \mathbb{R}^3$ characterized by

$$(u \wedge v) \cdot w = det(u, v, w)$$

for all $w \in \mathbb{R}^3$ Here $det(u, v, w)$ means that if we express u, v and w in the natural basis e_i, then

$$det(u, v, w) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

It follows that

$$u \wedge v = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} e_1 - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} e_2 + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} e_3$$

We also write $u \times v$.

Proposition 0.1
(1) $u \wedge v = -v \wedge u$ (anticommutativity)
(2) $(au + bv) \wedge w = au \wedge w + bv \wedge w$
(3) $u \wedge v = 0$ if and only if u and v are linearly dependent
(4) $(u \wedge v) \cdot u = 0$, $(u \wedge v) \cdot v = 0$

Remark
(1) $u, v, u \wedge v$ is a positive basis since

$$det(u, v, u \wedge v) = (u \wedge v) \cdot (u \wedge v) = |u \wedge v|^2 > 0$$

(2) $|u \wedge v| = |u||v|sin\theta$
(3) The vector product is **not** associative.