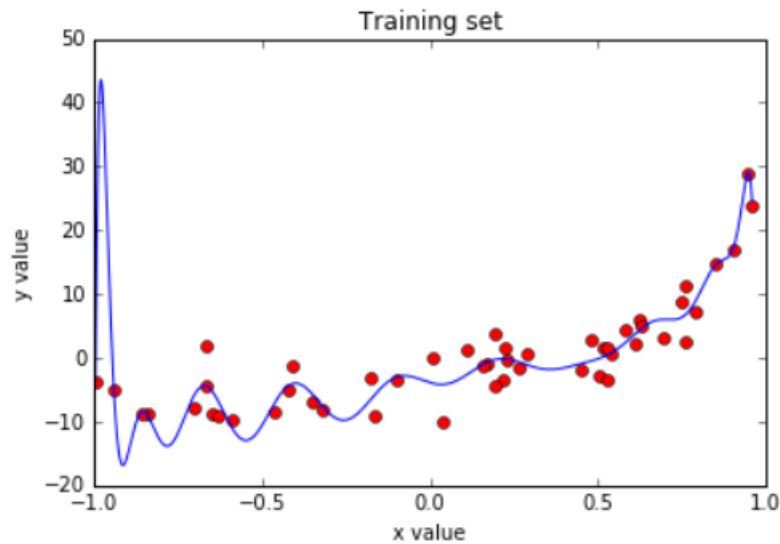


Comp 551 Assignment 1

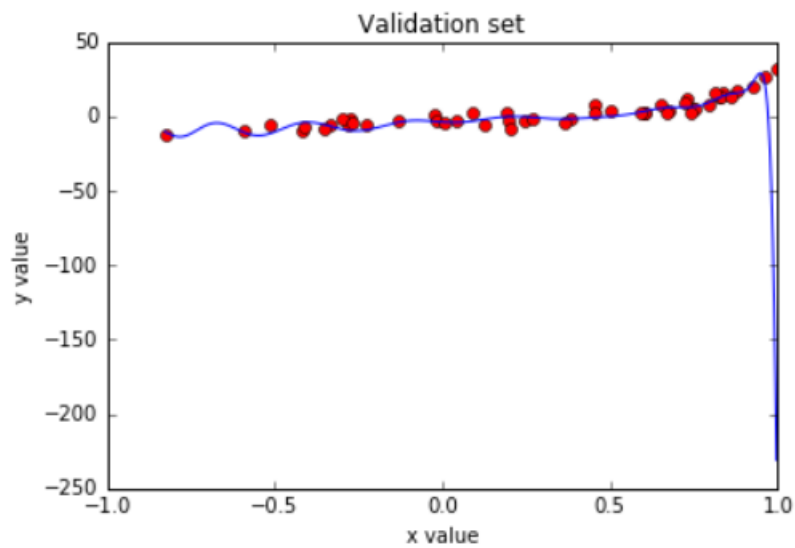
Qianyu Huang 260669624

1.

a) The curve seems to overfit the data. For example, the first spike is quite off.

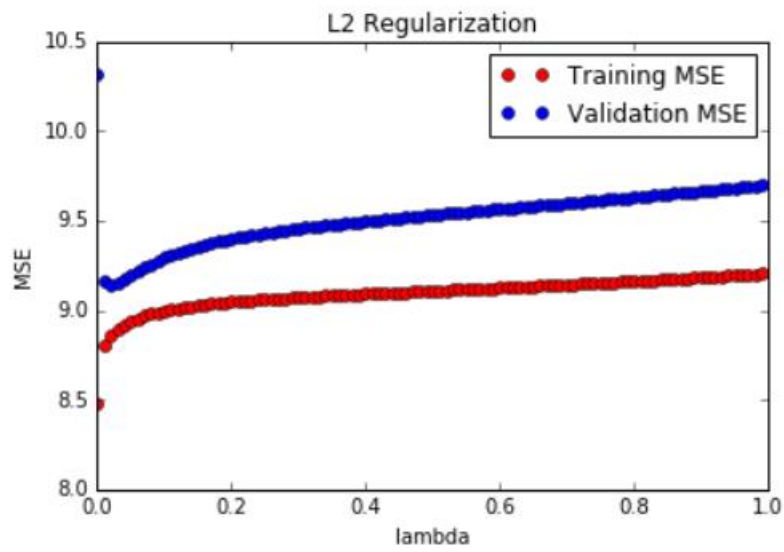


Training MSE: 6.474818801517948



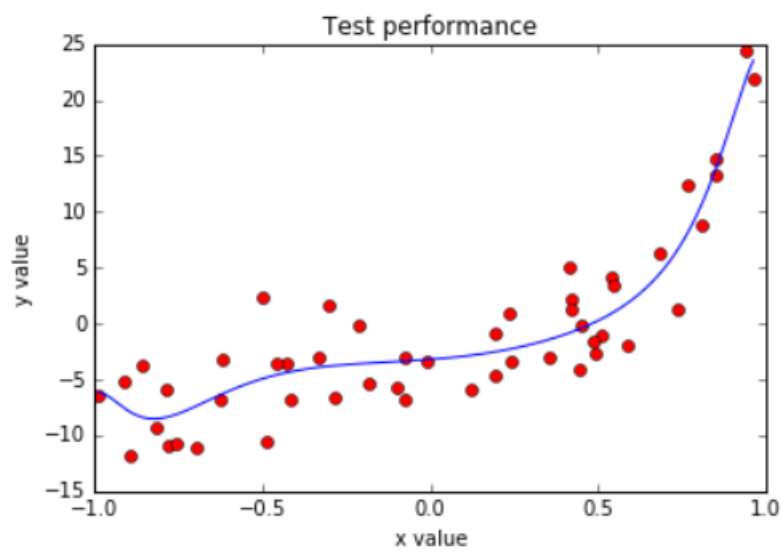
Validation MSE: 1415.1353642452727

b) The best lambda is 0.0201. Fitting curve is a lot smoother.



Best λ : 0.0201

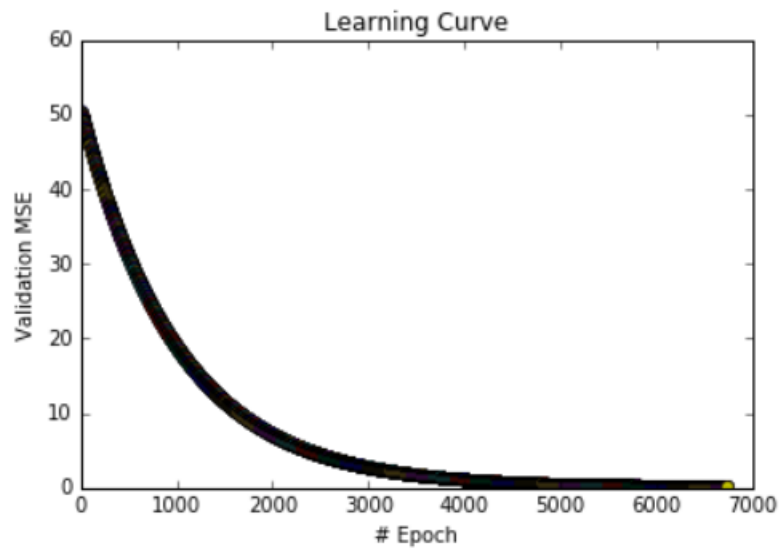
c) The degree of source polynomial is for sure smaller than 20, inferred from a positive λ .



Test MSE: 8.999412502533515

2.

a)



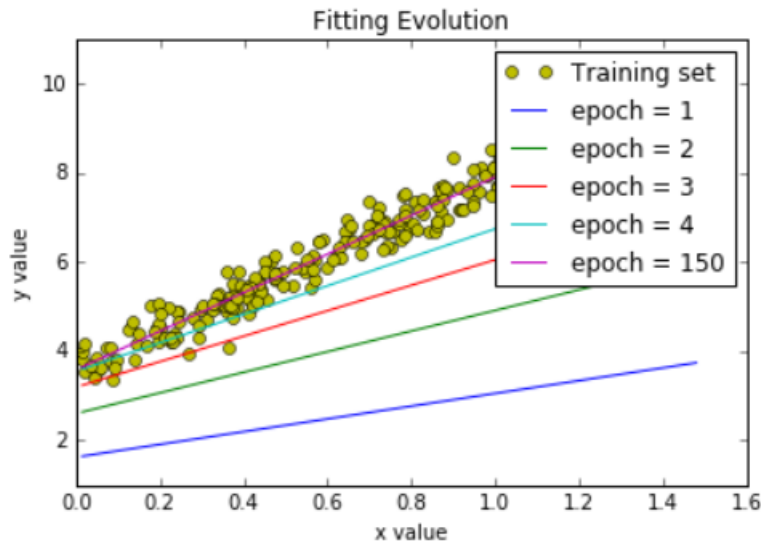
Final Validation MSE: 0.24186536821481688
 # Epoch: 6733
 Alpha: 1e-06
 Threshold 0.0001

b)

Alpha	Threshold	Epoch	Validation MSE
1e ⁻⁶	1e ⁻⁴	6733	0.242
1e ⁻⁵	1e ⁻⁴	3506	0.095
1e ⁻⁴	1e ⁻⁴	1083	0.075
1e ⁻³	1e ⁻⁴	182	0.074
1e ⁻²	1e ⁻⁴	19108	0.075

Best model step size: 1e⁻³
 Test MSE: 0.069

c)



Final Validation MSE: 0.07415999476751081
 # Epoch: 177
 Alpha: 0.001
 Threshold 0.0001

3.

a) Complete data is uploaded as complete_data.csv

b) 5-fold cross fit result:

Best MSE: 0.016665152266601146

Average MSE: 0.17994783839352194

Parameters learnt:

```
[[ -1.42529281e-01  2.46495039e-02  1.63954219e-01 -6.05964603e-02 -3.85973372e-04
  5.89445404e-02  6.25842661e-02 -1.94422924e-01 -6.37140301e-02  4.24766405e-02
  9.40245313e-02  3.41168332e-02 -1.98059709e-01 -2.07306008e-01  5.54211265e-02 -
  1.08713068e-01  1.13191181e-01 -3.37781227e-02 -9.83017854e-02  3.19365586e-01
  5.93298887e-02 -2.65502603e-01 -4.05647206e-02 -3.67582989e-02  2.76087890e-02
  3.63310301e-02  3.74557822e-02  1.98506987e-02 -1.45771690e-01 -1.31990369e-01
  1.44617033e-01  1.98900207e-02  7.59186200e-03  2.53047380e-01 -6.40812475e-02 -
  2.30380048e-03  8.15341952e-03  8.83438455e-02  5.40582986e-01  1.37277697e-01
  3.66662249e-01 -8.06452507e-01  1.92842335e-03  2.73817178e-02 -4.28753293e-01 -
  4.44732351e-02  1.58265609e-02  4.93237765e-02 -1.85978197e-01 -5.13361160e-03
  9.52640117e-02 -2.01496652e-01 -2.15420193e-02  1.81155915e-02 -2.58996524e-02
  2.69995646e-02  2.95247502e-03  2.04127837e-02  3.05412994e-01 -3.20447406e-01 -
  5.05394257e-02 -1.74506802e-01 -3.22698387e-02 -1.46681952e-01  4.82101362e-01 -
  2.92835904e-02 -2.27879633e-01 -5.52794854e-01  2.06337369e-01  1.15223739e-01
  3.48713375e-02  1.25778640e-01 -3.68661374e-02  4.60897295e-01  1.86776582e-02 -
  6.68945864e-02 -4.60218906e-02  1.19788747e-02 -1.47626965e-02 -2.38317958e-01
  1.35894013e-01 -1.58978691e-02 -2.48487757e-01 -1.22090005e-01 -2.88939407e-02
  4.66627504e-01  3.21731770e-02 -1.82071295e-02 -9.95822363e-02  9.03053462e-02]
```

1.86787411e-01 1.10020158e-01 2.07354588e-02 -2.44356308e-02 1.25146514e-02 -
3.62494710e-03 -1.08356924e+00 -2.58367412e+01 -8.70828839e-01 8.81510793e-01 -
8.92206727e-02 -6.03853794e-02 1.72418749e-01 2.50561803e+01 -9.72711899e-03 -
2.74399713e-02 3.16443013e-02 7.07491465e-02 7.50154063e-02 -4.32448999e-02 -
1.56876600e-01 -2.44080547e-02 -1.98770081e-02 6.60775885e-02 -1.10501387e-02 -
2.94298449e-02 4.94573451e-02 3.68469208e-01 -1.30809594e-01 3.09422333e-02
2.60335742e-02 1.45124957e+00]

[5.47687694e-01 -3.95427910e-02 1.56952894e-01 -8.63992128e-02 -4.35793445e-02 -
2.33165935e-02 9.75352576e-02 -2.16031194e-01 -1.80553568e-01 8.63104210e-02 -
8.29994538e-01 5.85420469e-02 -1.46761724e-01 -1.40556219e-01 4.33251177e-02 -
1.55618071e-01 1.24752159e-01 -4.37594944e-02 -9.68377247e-02 2.48871339e-01
2.76339916e-01 -6.01109629e-01 -4.01405972e-02 -2.17542054e-02 1.37891532e-02
4.45290940e-02 2.58936389e-02 2.08466041e-01 -1.26566732e-01 -8.35723073e-02 -
1.85612269e-02 3.46589406e-02 2.62074904e-02 2.78194152e-01 -6.29053751e-02 -
8.50218988e-03 1.03740747e-01 1.24383243e-01 2.25691936e-01 2.48490385e-01 -
7.66790748e-02 -5.96583438e-02 -1.70997743e-01 1.88020443e-01 -4.27211032e-01 -
7.47421409e-02 -1.28512012e-02 2.98977312e-02 -1.70405908e-01 -3.35548472e-01
1.75959593e-01 3.31329080e-02 1.59878191e-02 -1.08411236e-02 -6.63759541e-02
8.62998349e-02 -7.03369068e-02 -4.40137141e-03 2.89054651e-01 -2.08994389e-01 -
8.96077910e-02 -1.20875734e-01 -9.59823163e-02 -4.34388846e-02 7.74019283e-01 -
1.78034630e-01 -2.63048069e-01 -4.74156953e-01 1.98824607e-01 7.70248706e-02
3.26434284e-02 2.38750880e-01 -4.22822563e-02 3.53450056e-01 5.86354041e-02 -
8.89847867e-02 -9.69664090e-03 8.15204934e-03 -2.62029195e-03 -1.86381025e-01 -
1.35173735e-02 1.89151057e-01 -2.37299211e-01 -2.08274428e-02 -4.46604645e-02
3.37523351e-01 5.83033722e-02 -5.93536863e-02 -7.12366909e-02 2.26363121e-01
6.25803433e-02 9.61208034e-02 4.70980926e-02 -4.30006121e-02 1.63810459e-02
1.48994137e-03 -4.80240336e-01 -2.71800372e+01 -8.00751789e-01 2.13009733e-01 -
3.71890411e-01 1.11354857e-01 6.87969329e-02 2.69353476e+01 -1.00045982e-01 -
1.13052983e-01 1.03561181e-01 2.58447866e-02 -1.06971196e-02 -1.52108239e-01 -
4.67026754e-01 -4.63834904e-02 -2.56956416e-02 -1.44650903e-02 -2.62337077e-03 -
3.73179931e-02 1.02579997e-01 4.47699812e-01 1.46860138e-02 3.90557895e-02
4.95460229e-02 1.47229933e+00]

[-2.47982966e-01 2.94159479e-03 1.45890370e-01 -4.47436807e-02 -3.45813776e-02
3.99238653e-02 8.90644435e-02 -2.51226950e-01 -7.63735147e-02 1.25592707e-01
9.44461925e-02 4.45030001e-02 -3.11247229e-01 -1.80588913e-01 5.16664350e-02 -
2.74611226e-01 -2.80357606e-02 9.76292424e-02 -8.06035437e-02 3.86003928e-01 -
2.60227665e-04 -2.20644968e-01 -2.48310429e-02 -3.98981086e-02 4.82564708e-02
4.04442334e-02 3.34057105e-02 1.30787200e-01 -2.43296264e-01 -1.21827061e-01
1.06643639e-01 1.46984713e-01 -3.17780231e-02 1.95708918e-01 -6.49817330e-02 -
5.95874041e-02 4.64594594e-02 7.23277160e-02 5.06993453e-01 2.25595035e-01
1.56197891e-01 -6.36250157e-01 -1.55232546e-01 -2.21409941e-02 -2.23590877e-01 -
3.71183615e-02 -2.51522283e-02 8.68657857e-02 -1.94329675e-01 -1.67458516e-01

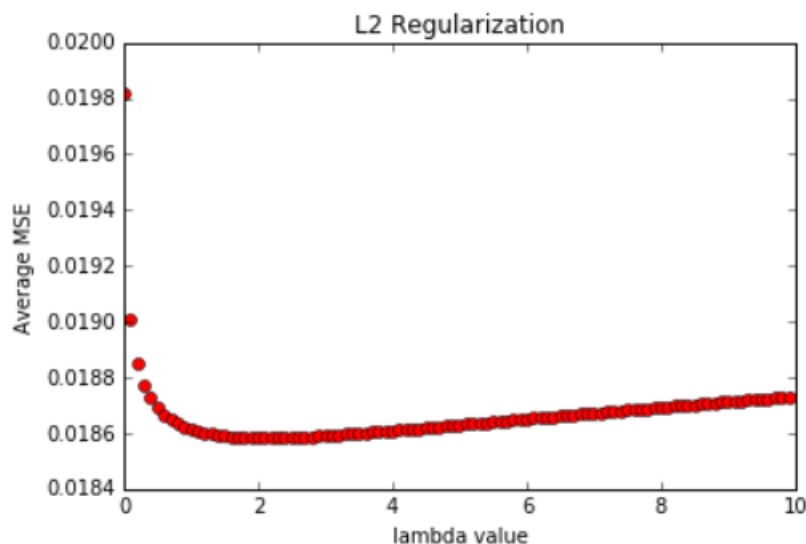
1.44495994e-01 -2.75346087e-01 7.96279886e-02 3.22347594e-02 -1.61155498e-01
7.79745501e-02 -1.65225335e-01 -1.86439204e-01 6.36382897e-01 -3.47829622e-01 -
3.40565109e-02 -2.28225711e-01 2.27180600e-01 -4.25554741e-01 6.00994499e-01 -
8.38533641e-03 -1.88370115e-01 -6.63113473e-01 1.94728335e-01 1.22757900e-01
3.35626071e-02 1.41711343e-01 -6.31826433e-02 5.41127724e-01 6.87140038e-02 -
9.72713421e-02 -2.54494301e-02 2.33759899e-02 -3.16733018e-02 -3.15982410e-01
2.18481666e-01 5.17753440e-03 -1.73759273e-01 -1.29009323e-01 3.24133485e-04
3.32401695e-01 5.87982798e-02 -4.33602844e-02 -9.51730753e-02 1.63801393e-02
2.72823176e-01 2.49032740e-01 1.47044825e-02 1.88876594e-02 4.42638632e-02 -
1.20774542e-03 -4.06992063e-01 1.76883914e+03 -5.11766328e-01 -1.27337538e-02 -
2.53247839e-01 -8.28296348e-02 1.86751379e-01 -1.76870346e+03 -1.29361247e-01 -
1.91008597e-02 -7.53837581e-02 4.71552680e-03 4.71244625e-02 6.85690844e-02 -
5.12449594e-02 -1.71193159e-02 -1.97452025e-02 7.45299872e-02 -1.02323523e-02 -
5.53396204e-02 9.98531835e-02 2.43712222e-01 -4.21675643e-02 2.09755270e-02 -
3.18313709e-02 1.17925283e+00]

[4.24963665e-02 -5.69861950e-02 2.56097625e-01 -6.29272370e-02 -1.28327115e-02
8.10567391e-02 1.20837050e-01 -1.95916171e-01 -1.64596514e-01 -4.94804528e-02 -
2.74879162e-01 5.88208080e-02 -9.61892025e-02 -2.20494280e-01 3.78283469e-02 -
1.97105583e-01 2.20767834e-01 5.89409863e-03 -1.04730374e-01 2.30175683e-01
2.09186249e-01 -4.99921639e-01 -4.40562753e-02 -2.89871461e-02 3.33223234e-02
5.42938634e-02 3.24991057e-02 1.97897849e-01 -2.07763765e-01 -1.49828060e-01
1.16269877e-01 1.22680100e-01 3.26872908e-02 2.91172414e-01 -7.51767716e-02 -
1.09341646e-02 9.82827083e-02 1.02296993e-01 5.63812257e-01 2.45706129e-01
1.55803696e-01 -7.01985042e-01 -6.90796959e-02 -1.17163918e-02 -2.92448398e-01 -
2.75932530e-02 -3.38638576e-04 9.62961936e-02 -2.36580242e-01 -2.55746037e-01
6.12539515e-02 -6.14957257e-02 1.93001923e-02 5.59364779e-02 -5.05426813e-02
9.19822057e-04 -2.46624709e-02 -1.48333449e-01 2.95874910e-01 -1.08978666e-01 -
5.33361318e-02 -1.47142122e-01 -1.37087349e-01 -3.95129566e-02 5.63600026e-01 -
1.28805766e-01 -2.32223653e-01 -6.88130409e-01 2.32964607e-01 1.02071939e-01
3.33028080e-02 2.11262882e-01 -2.04882865e-02 5.39355217e-01 3.59842690e-02 -
6.71714768e-02 -1.43458740e-02 -9.29168646e-03 -1.97570059e-03 -3.23462736e-01
3.23393206e-01 -5.91410698e-02 -2.36145674e-01 -5.14089752e-04 -5.49073952e-02
3.42464305e-01 1.65669907e-02 -6.17641198e-02 -6.95134814e-02 1.58393032e-01
1.54066895e-01 6.67521534e-02 -3.66573491e-02 -8.69716117e-02 6.61701945e-02
6.08096135e-02 9.64315666e-01 -1.91598459e+01 8.97559402e-01 5.87323551e-01 -
2.95429831e-01 -7.23195995e-02 2.09916220e-01 1.86384539e+01 -1.04637648e-01
1.00137451e-01 1.02336445e-01 1.15322046e-01 5.20321729e-02 -2.51317768e-02 -
3.62136409e-01 -1.25570809e-02 -8.97194143e-02 3.65954244e-02 -3.36750146e-02 -
4.37685044e-02 3.64422036e-02 4.77152934e-01 -7.56420189e-02 4.11535544e-02
5.32796875e-02 -3.46316278e-01]

[4.66640357e-01 -3.08065024e-02 1.91174097e-01 -4.14388824e-02 -2.36139106e-02
1.53129480e-01 1.61928673e-01 -2.48480938e-01 -2.04581603e-01 4.35632016e-02 -

5.02731371e-01 4.97466628e-02 -1.66021365e-01 -2.14633830e-01 4.49640826e-02 -
 1.62117415e-01 7.42807108e-02 -1.06527210e-02 -1.17649599e-01 2.74042617e-01
 1.14430826e-01 -3.48279829e-01 -2.74185664e-02 -4.61067522e-02 -1.46655281e-02
 3.23412489e-02 3.40101870e-02 4.80469635e-02 -1.28007152e-01 -8.16228596e-02
 2.70699299e-02 1.12689951e-02 1.30304785e-02 2.34994489e-01 -6.07512588e-02 -
 3.87843927e-02 6.36085974e-02 1.42611074e-01 4.06693732e-01 2.43223607e-01
 1.41157607e-01 -5.41212341e-01 -1.63158763e-01 1.57431223e-02 -3.78233990e-01 -
 1.90598386e-03 -4.45120722e-03 2.27580979e-02 -1.19198203e-01 -4.47468033e-02
 1.15251779e-01 -2.37132911e-01 2.50462384e-02 3.16494139e-02 -8.03577333e-02
 5.10127142e-02 1.83545360e-02 -3.77711137e-01 5.29701327e-01 -2.06675004e-01
 6.99457167e-02 -1.85423929e-01 5.06431362e-02 -1.43785776e-01 8.44046546e-01 -
 2.62801286e-01 -2.88705291e-01 -4.94035424e-01 1.25315641e-01 9.76318280e-02
 1.49699078e-02 1.13897551e-01 -5.05863276e-02 3.34661091e-01 5.70016450e-02 -
 6.65432006e-02 -5.89010384e-03 2.07735051e-02 -1.91096105e-02 -5.60791770e-01
 6.00309480e-01 -1.28777088e-01 -2.40243392e-01 7.74812604e-02 -1.16894825e-01
 3.55090331e-01 3.18505395e-02 -5.96701739e-02 -8.55897803e-02 1.46210813e-01
 1.60884668e-01 2.62007642e-01 2.16116366e-02 3.70066250e-02 -7.10278587e-03 -
 9.57171499e-03 -4.52757181e-01 -1.92179776e+01 -5.02541557e-01 2.56619233e-01 -
 2.03249000e-01 1.53519218e-02 1.20965484e-01 1.89900158e+01 -4.52805595e-02 -
 8.84953330e-02 1.96667462e-02 4.63045870e-02 8.60901391e-02 -9.56035252e-02 -
 1.43506006e-01 -7.41184090e-03 -1.00934085e-02 4.63873780e-02 2.07536634e-02 -
 4.75583703e-02 5.63822541e-02 1.82432532e-01 -4.65829441e-02 4.03009202e-02
 3.66957102e-02 1.11442757e+00]]

c) L2 Regularization:



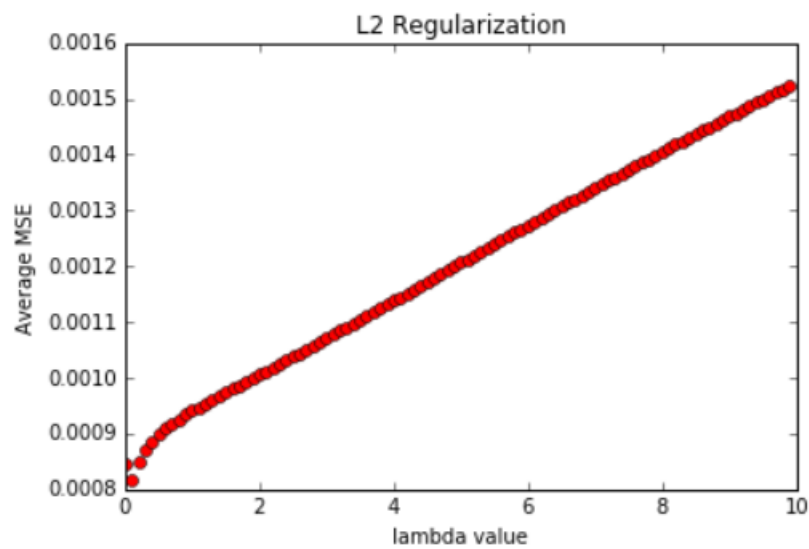
Lowest Average MSE: 0.01858304507066562

Best Lambda: 2.10001

Fitted parameters are uploaded as L2_params.csv

L2 Regularization with feature selection:

Using the best lambda value found in previous L2 regularization to find coefficients with averaged value over 0.01 across 5 runs. The corresponding features are kept and the rest are ignored during the following training.



Lowest Average MSE: 0.0008178528287246065

Best Lambda: 0.10001

Feature number kept: 108

Fitted parameters are upladed as L2_params_feature.csv