Art

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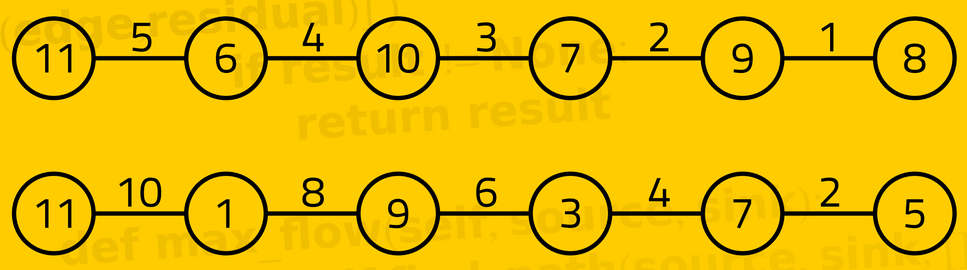
Easy: Graceful path labeling

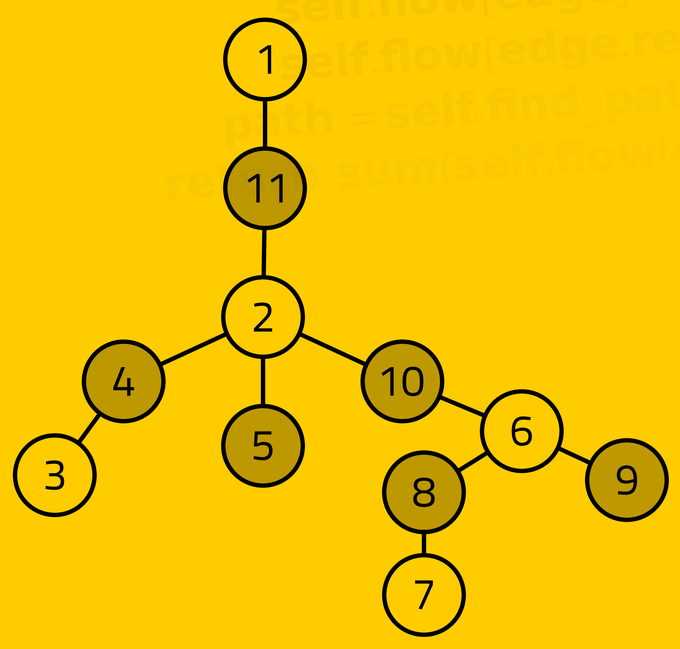
Many solutions exist, for example:



The graceful tree conjecture is an unsolved problem:

Do all trees have a graceful labeling?

Medium: A colorful tour



A solution exists even for trees, and it is constructive!

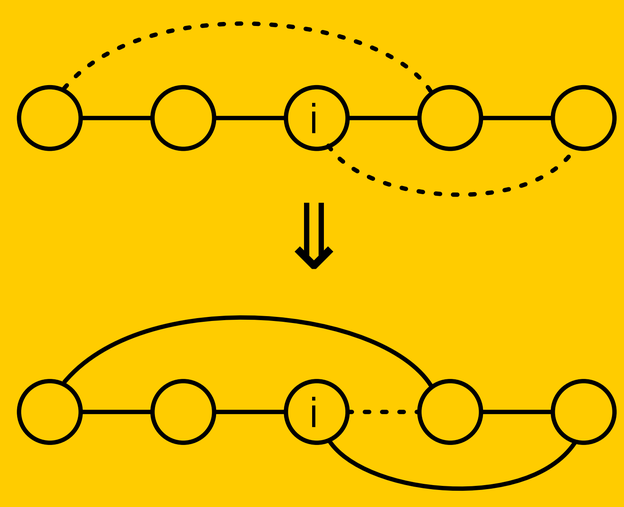
- traverse nodes using DFS
- Let nodes be odd (light) or even (dark)
- Color odd nodes when you enter them
- Color even nodes when you leave them

More info: Donald Knuth, TAoCP, Vol 4, Fasc 3

Hard: Hamiltonian Cycle in a dense graph

To find a Hamiltonian Cycle efficiently, exploit the fact that the graph has many edges.

- Start with a path that is as long as possible
  ⇒ already $\left\lceil \frac{N}{2} \right\rceil$ animals
- Now the path can be transformed into a cycle:
  There must be a node n_i such that n_i is connected to the last animal, and n_{i+1} is connected to Heidi.
- All other animals can be inserted into the cycle at some point (re-transforming it into a path if necessary)



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