

# Rectification of GNSS-based Collaborative Positioning using 3D Building Models in Urban Areas

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## Abstract

GNSS collaborative positioning receives great attention because of the rapid development of vehicle-to-vehicle (V2V) communication. Its current bottleneck is in urban areas. During the relative positioning using GNSS double difference pseudorange measurements, the multipath effects and non-line-of-sight (NLOS) reception cannot be eliminated, or even worse, both might be aggregated. It has been widely demonstrated that 3D map aided (3DMA) GNSS can mitigate or even correct the multipath and NLOS effects. We, therefore, investigate the potential of aiding GNSS collaborative positioning using 3D city models. These models are used in two phases. First, the building models are used to exclude NLOS measurements at a single receiver using GNSS shadow matching (SDM) positioning. Second, the models are used together with broadcast ephemeris data to generate a predicted GNSS positioning error map. Based on this error map, each receiver will be identified as experiencing healthy or degraded conditions. The receiver experiencing degraded condition will be improved by the receiver experiencing the healthy condition, hence the aspect of collaborative positioning. Five low-cost GNSS receivers are used to conduct experiments. According to the result, the positioning accuracy of the receiver in a deep urban area improves from 46.2 to 14.4 meters.

24 INTRODUCTION

One of the bottlenecks of intelligent transportation system (ITS) is the positioning accuracy of vehicles. To improve the accuracy of positioning, an inertial navigation system (INS) is always integrated with GNSS (Groves 2013). Due to progress in computing capability, LiDAR is employed for simultaneous localization and mapping (SLAM) (Levinson et al. 2007). Unlike other

29 sensors measuring the relative position, the GNSS also provides absolute positions without  
30 accumulated error. Therefore, the GNSS solution is still a key technology to provide the  
31 positioning service for autonomous driving (Kamijo et al. 2015).

32 Due to the expected maturity of vehicle-to-vehicle (V2V) communications in the near  
33 future (Qiu et al. 2015), the positioning via V2V cooperation becomes possible. By making use of  
34 numerous measurements from surrounding vehicular, the positioning accuracy of the target vehicle  
35 can be much improved (de Ponte Müller 2017). The collaborative positioning can be mainly  
36 divided into transponder-based and GNSS-based relative positioning (Elazab et al. 2016; Liu et al.  
37 2017). By combining various types of transponder-based measurements (Xu et al. 2015), the  
38 positioning accuracy can be optimized through a weighted solution (Elazab et al. 2016), least  
39 squares estimation (Van Nguyen et al. 2015), or the application of a probability density filter  
40 (Zhang et al. 2014). However, the transponder-based approach suffers from signal reflection or  
41 blockage and unsynchronized clock, making practical implementation difficult (Blumenstein et al.  
42 2015). The GNSS-based approaches directly exchange the GNSS data between vehicles to  
43 improve the positioning performance (Lassoued et al. 2017), and most of them use the double-  
44 difference (DD) method (Alam et al. 2013). The idea behind the DD technique is to eliminate  
45 common pseudorange error between two GNSS receivers, including ionospheric, tropospheric,  
46 and satellite clock/orbit errors. As mentioned by Liu et al. (2014), the DD-based collaborative  
47 positioning is still difficult in urban areas due to multipath and NLOS errors.

48 In urban canyons, the GNSS signal can be reflected by a building surface, experiencing an  
49 extra traveling distance. The signal multipath and NLOS effects are introducing GNSS positioning  
50 errors that can in extreme cases exceed 100 meters in urban areas (Hsu 2018). One of the feasible  
51 solutions is to apply fault detection and exclusion (FDE) for the multipath or NLOS affected  
52 signals. A GNSS consistency check has been proposed to select consistent measurements for  
53 positioning based on pseudorange residuals (Groves and Jiang 2013). Similarly, a Forward-  
54 Backward receiver autonomous integrity monitoring (RAIM) technique has been developed to  
55 improve the performance of GNSS in the urban environment (Angrisano et al. 2012). The random  
56 sample consensus (RANSAC) method is further employed to improve the performance of RAIM  
57 in case of multiple outliers (Castaldo et al. 2014). Due to the arrival of multi-GNSS, the availability  
58 of GNSS is enhanced even in a dense urban area, which further improves its positioning

59 performance (Hsu et al. 2017). However, multi-GNSS could also increase the number of outliers  
60 (multipath or NLOS), rendering FDE unable to obtain satisfactory performance in dense urban  
61 area. Because multipath and NLOS effects are produced by buildings, a 3D building model can be  
62 employed to evaluate and mitigate such effects (Tiberius and Verbree 2004). The shadow matching  
63 (SDM) is a widely used 3DMA GNSS positioning method (Groves 2011). Instead of using  
64 pseudorange, it uses satellite visibility as measurement to estimate the receiver position. Satellite  
65 visibility is defined by the blockage of LOS signal transmission. If a satellite is not tracked by a  
66 receiver, it is very likely the signal is blocked by the buildings and vice versa. The SDM determines  
67 the receiver position by matching the satellite visibility computed from receiver measurements  
68 with the visibility for hypothesized positions using 3D models. If the computed visibility matched  
69 the visibility of a hypothesized position, then the receiver is very likely located at that hypothesized  
70 position. The performance assessment and of the 3DMA GNSS and the effect of mapping quality  
71 are summarized in Adjrad et al. (2018) and Groves and Adjrad (2018).

72 It is interesting to note the 3DMA GNSS and GNSS-based collaborative positioning are  
73 complementary; the former one can greatly mitigate multipath and NLOS effects while it is still  
74 suffering from various other factors to achieve highly accurate positioning. The latter one can  
75 eliminate the systematic errors by sharing raw GNSS data between vehicles, but it is limited to  
76 using multipath-free measurements. In addition, the receiver will be identified as experiencing  
77 healthy or degraded conditions based on 3DMA GNSS (Bradbury et al. 2007; Zhang and Hsu  
78 2018), which provides an appropriate receiver selection for collaborative positioning. Accordingly,  
79 we propose GNSS-based collaborative positioning using 3D building models. The 3DMA GNSS  
80 algorithm is employed for preliminary NLOS detection and exclusion, mitigating the uncorrelated  
81 errors during DD. The 3DMA GNSS is further used to select reliable receivers for collaborative  
82 positioning. Finally, the collaborative positioning solution is integrated with the 3DMA GNSS  
83 solution based on their complementary characteristics, improving the positioning accuracy in  
84 dense urban areas.

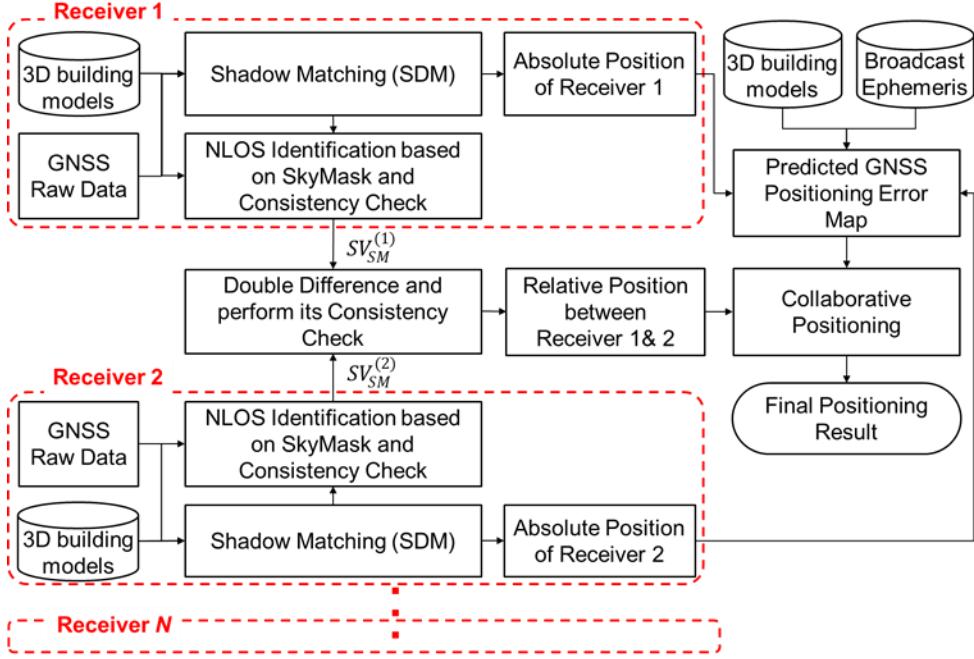
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## 86 **Overview of the Proposed 3DMA GNSS-Based Collaborative Positioning**

87 The flowchart of the proposed collaborative positioning algorithm is shown in Fig. 1. At the single  
88 receiver level, the received GNSS measurements will be used with the GNSS shadow matching

89 (SDM) based on the 3D building models (Wang et al. 2013), to obtain an improved initial  
90 positioning solution. Based on the SDM solution, satellite visibility can be identified using the  
91 *skymask* (skyplot with building boundaries). Therefore, the identification and exclusion of the  
92 NLOS measurements can be conducted. Then, the remaining GNSS measurements will be  
93 subjected to a consistency check. After the two exclusions, the surviving measurements are  
94 considered to be clean GNSS measurements. The surviving pseudorange measurements will be  
95 double differenced to obtain the relative positions between receivers. Meanwhile, the second-layer  
96 of consistency check will be employed during the double difference estimation, ensuring further  
97 the consistency of measurements (Zhang et al. 2018).

98 Among all measurements, an inaccurate measurement may lead to a large error during  
99 position computation. Therefore, it is important to classify whether the measurement is reliable.  
100 Due to the multipath and NLOS effects, it is difficult to evaluate the positioning performance  
101 mainly relying on measurements (Hsu 2017). Based on the 3D building model in the vicinity of  
102 receiver and the broadcast ephemeris, the multipath and NLOS delay of GNSS pseudorange  
103 measurement can be predicted using a ray-tracing algorithm (Hsu et al. 2016; Ziedan 2017).  
104 Simplifications have also been studied for 3DMA GNSS pseudorange simulation to lower the  
105 computation load for real-time implementation (Ng et al. 2019). Then, a positioning error map for  
106 predicting each location's GNSS error can be constructed (Zhang and Hsu 2018), and employed  
107 to predict each receiver's positioning performance based on its error estimate. Based on the  
108 predicted performance, the positioning solutions are obtained by applying the proposed  
109 collaborative positioning algorithm (which is a weighted average approach) to their absolute and  
110 relative positioning solutions.



111

112

**Fig. 1** Flowchart of the proposed 3DMA collaborative positioning algorithm.

113

114 **GNSS Shadow Matching Algorithm**

115 Conventional least squares estimation suffers from absorbing unmodeled multipath and NLOS  
 116 effect in the urban area. Hence, we use an advanced 3DMA GNSS positioning, also referred to as  
 117 shadow matching (SDM), to provide the absolute position of a single receiver. Here, a basic SDM  
 118 algorithm is employed (Wang et al. 2015) to determine the receiver location by searching for a  
 119 candidate position having a satellite visibility that is the most similar to the actual measured  
 120 satellite visibility. The satellite visibility is categorized into LOS and NLOS; the LOS signal  
 121 transmission is not blocked and the NLOS signal blocked by obstacles, respectively. The actual  
 122 measured satellite visibility is usually determined by C/N<sub>0</sub>. If it is weaker than a certain threshold,  
 123 the sight is NLOS and otherwise it is LOS. For the satellite visibility prediction at each candidate  
 124 position, the surrounding 3D building model from Google Earth (Fig. 2 left) can be plotted in a  
 125 polar coordinate overhead with azimuth and elevation, generating the skymask (right panel). Based  
 126 on the skymask, the satellite with an elevation below the building boundaries is considered as  
 127 NLOS. Otherwise it is LOS. For the measured satellite visibility, since the reflected NLOS signal  
 128 may be received in the urban area, only the measurement with C/N<sub>0</sub> over 40 dB-Hz will be regarded

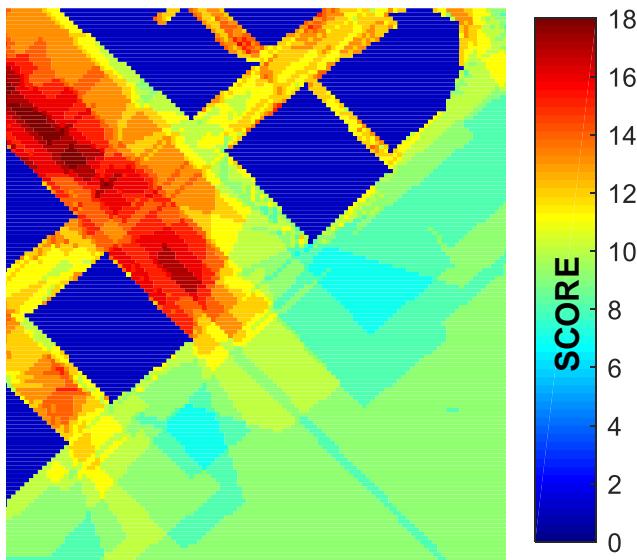
129 as LOS measurement, indicating a strong signal (Wang et al. 2013). After obtaining the predicted  
130 satellite visibility for different candidate locations and having the satellite visibility estimated from  
131 actual measurements, the receiver location is determined by finding a candidate position with a  
132 skymask-predicted satellite visibility that is the most similar to the measured satellite visibility.  
133 Fig. 3 demonstrates the match score with color for each candidate position; the higher score  
134 indicates the candidate position has a better match with the computed visibility from the  
135 measurements, which means the receiver has a higher possibility of being located at this candidate  
136 position. Finally, the SDM positioning solution is estimated by the weighted average of all  
137 predicted locations.

138



139  
140 **Fig. 2** Demonstration of the skymask based on the 3D building model corresponding to different  
141 locations. The skymask (right) indicates the sky-view with the building blockage (gray area)  
142 projected by the corresponding building models on Google Earth (left).  
143

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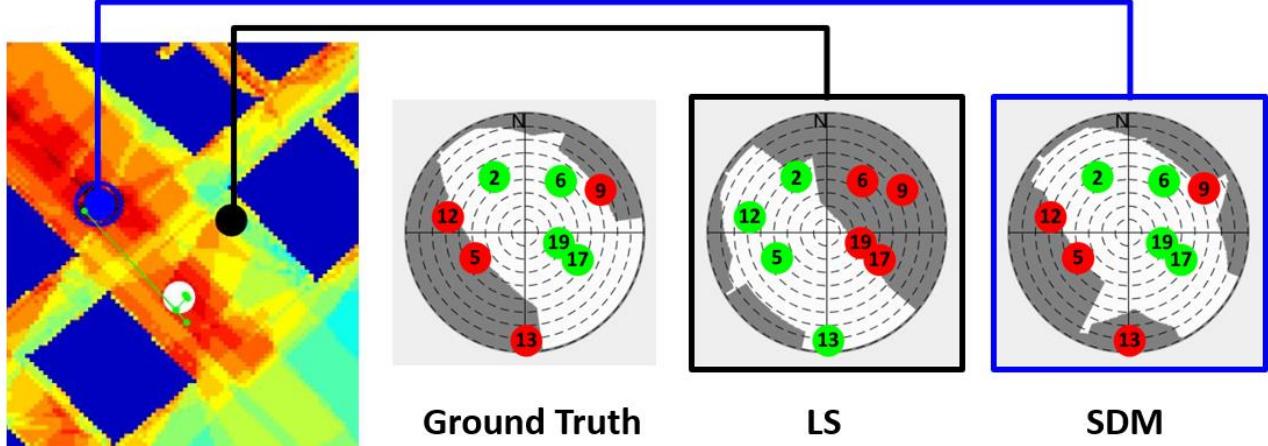


145 **Fig. 3** Distribution of match score between the measured satellite visibility and predicted satellite  
146 visibility of different candidate positions. The color indicates the similarity score for each  
147 candidate.

148

#### 149 Identification and Exclusion of NLOS Measurement

150 NLOS exclusion based only on  $C/N_0$  is usually not reliable, since the reflected signal could  
151 possibly have a  $C/N_0$  larger than the LOS measurement. A straightforward NLOS exclusion  
152 approach is to further use the 3D building model and the satellite positions to identify which  
153 satellite is blocked by buildings. Since the receiver location is unknown, a feasible approach is to  
154 generate the skymask based on a relatively accurate positioning solution. Interestingly, the GNSS  
155 SDM gives good positioning performance in the across-street direction (Wang et al. 2015), as  
156 shown by the blue dot in Fig.4. Theoretically, its error in along-street direction may only slightly  
157 affect the NLOS identification based on the skymask. The skymasks, the associated NLOS/LOS  
158 identification results for true location, and the LS and SDM solutions are shown in Fig.4. The true  
159 skymask of the receiver identifies that satellites 5, 9, 12, 13 are blocked by buildings. The incorrect  
160 LS solution lays on the wrong side with different skymask, resulting in erroneous NLOS  
161 identification. The SDM solution always falls on the correct side of the streets, which makes its  
162 estimated skymask similar to the truth even through having a large positioning error in along-street  
163 direction.



164

165      **Fig. 4** Illustration of NLOS/LOS identification result using the skymasks generated based on  
 166      ground-truth location, least squares solution (LS) and shadow matching solution (SDM). The  
 167      blue area on the map indicates buildings. The red and green markers on the skymask denote the  
 168      NLOS and LOS signals, respectively.

169

170      After obtaining the positioning solution from SDM, the corresponding skymask is  
 171      generated to classify NLOS from all GNSS measurements, using

$$172 \quad SV_{NLOS} = \{SV \in SV^i | ele^i < ele^{skymask}(azi^i)\} \quad (1)$$

173      For the  $i^{th}$  satellite  $SV$ ,  $azi$  and  $ele$  denote the azimuth and elevation angles of the satellite,  
 174      respectively. The satellites with an elevation angle below the skymask elevation angle on the same  
 175      satellite azimuth angle are identified as NLOS satellite. Rather than only based on the C/N<sub>0</sub> of the  
 176      measurements, the NLOS effect can be greatly mitigated by the proposed 3DMA NLOS exclusion.

177

## 178      **Relative Positioning Algorithm**

179      By using GNSS LOS measurements from different receivers, the relative position between  
 180      receivers can be estimated using double differencing. However, the multipath and NLOS error may  
 181      increase during DD, which requires it to be mitigated beforehand. Here, after applying the 3DMA  
 182      NLOS exclusion, a double-layer consistency check algorithm (Zhang et al. 2018) is further  
 183      employed with DD to mitigate the multipath and NLOS errors.

184

185 First-Layer of Consistency Check on Single Point Positioning

186 The surviving pseudorange measurements having passed the 3DMA exclusion will be applied to  
187 an equal weighted least squares estimation as follows:

188 
$$\hat{\mathbf{x}} = \mathbf{x}_0 + (\mathbf{H}^T \mathbf{H})^{-1} \mathbf{H}^T (\mathbf{p} - \mathbf{p}_0) \quad (2)$$

189 where  $\mathbf{p}$  and  $\mathbf{p}_0$  are the pseudorange measurements and predictions respectively.  $\mathbf{H}$  denotes the  
190 geometry matrix of satellites.  $\hat{\mathbf{x}}$  and  $\mathbf{x}_0$  indicates the estimated and predicted state vectors  
191 respectively, including position and receiver clock bias. The pseudorange residual  $\hat{\mathbf{\epsilon}}_{LS}$   
192 corresponding to each measurement can be calculated by:

193 
$$\hat{\mathbf{\epsilon}}_{LS} = \mathbf{p} - \mathbf{H} \cdot \hat{\mathbf{x}} \quad (3)$$

194 Then, the measurement consistency can be evaluated by the sum of square error  $SSE_{LS}$ , using

195 
$$SSE_{LS} = \hat{\mathbf{\epsilon}}_{LS}^T \cdot \hat{\mathbf{\epsilon}}_{LS} \quad (4)$$

196 A small value of  $SSE_{LS}$  indicates the measurements are consistent. A threshold is determined by  
197 chi-square test with  $10^{-5}$  probability of false alarm to guarantee the measurements are consistent  
198 enough (Blanch et al. 2015). A small probability of false alarm is used to ensure the healthy  
199 measurements are less unlikely to be mistakenly excluded. If the  $SSE_{LS}$  is over the threshold, the  
200 measurements will be excluded one by one and the corresponding  $SSE_{LS}$  recalculated. The subset  
201 of measurements with lowest  $SSE_{LS}$  is selected as the consistent measurements. By repeating the  
202 exclusion process, the inconsistent measurement will be excluded one by one until the  $SSE_{LS}$  is  
203 below the threshold. The survived measurements are considered to be consistent enough for  
204 positioning (Hsu et al. 2017).

205

206 Second Layer of Consistency Check on Relative Positioning

207 By sharing the survived measurements, the DD technique is used for relative positioning between  
208 receivers. For the  $i^{th}$  and  $j^{th}$  measurement both received by receivers  $n$  and  $m$ , the double difference  
209 of the shared measurement  $D_{n,m}^{i,j}$  is derived as following:

210                    $D_{n,m}^{i,j} = (\vec{e}^i - \vec{e}^j) \cdot \Delta\vec{x}_{n,m} + [(\varepsilon_n^i - \varepsilon_m^i) - (\varepsilon_n^j - \varepsilon_m^j)]$                    (5)

211 where  $\vec{e}$  denotes the unit LOS vector,  $\Delta\vec{x}_{n,m}$  denotes the relative position vector between receivers  
 212  $n$  and  $m$ ,  $\varepsilon_n^i$  indicates the uncommon error from the  $i^{th}$  GNSS measurement with regarding to the  
 213 receiver  $n$ . The DD (5) does not cancel the multipath and NLOS errors, or even worse, the error  
 214 may be aggregated. By conducting the double difference between a reference satellite and other  
 215 satellites for the receivers  $n$  and  $m$ , the relative positioning solution can be derived using:

216                    $\Delta\vec{x}_{n,m} = (\mathbf{E}^T \mathbf{E})^{-1} \mathbf{E}^T \mathbf{D}_{n,m}$                    (6)

217 where  $\mathbf{E}$  is the geometry matrix.  $\mathbf{D}_{n,m}$  is the DD measurements vector. Hence, the relative  
 218 positioning solution can be obtained.

219  
 220 The second layer of consistency check, which is similar to the first layer but pertains to the double  
 221 differences, is employed to further mitigate uncorrelated errors such as multipath and NLOS. After  
 222 estimating the relative position  $\Delta\vec{x}$  from DD, the measurement residual  $\hat{\varepsilon}_{DD}$  and the corresponding  
 223 sum of square error  $SSE_{DD}$  can be calculated by

224                    $\hat{\varepsilon}_{DD} = \mathbf{D} - \mathbf{E} \cdot \Delta\vec{x}$                    (7)

225                    $SSE_{DD} = \hat{\varepsilon}_{DD}^T \cdot \hat{\varepsilon}_{DD}$                    (8)

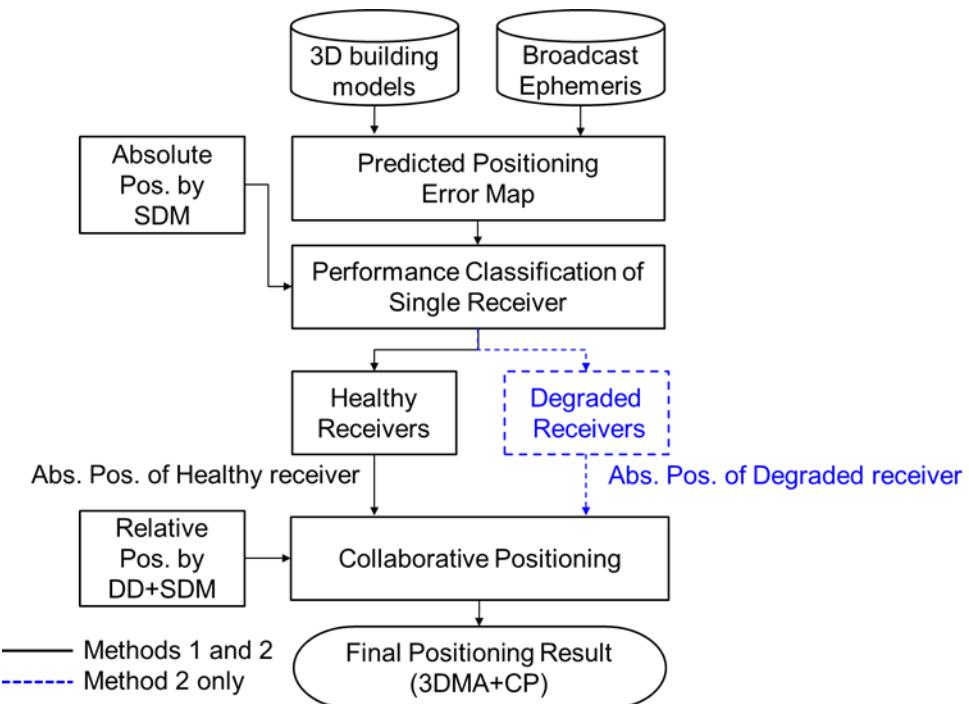
226 Again, if the  $SSE_{DD}$  is over the chi-square test threshold, the DD measurement will be excluded  
 227 one by one until finding a measurement subset with a  $SSE_{DD}$  below the threshold, which are  
 228 consistent enough for final double differencing. Finally, the improved relative positioning solution  
 229 between different receivers can be obtained by the proposed DD method.

230  
 231 **3DMA GNSS Collaborative Positioning**

232 In general, GNSS-based collaborative positioning, the absolute and relative positions from  
 233 available receivers are all combined to optimize the final positioning solution. However, the  
 234 multipath and NLOS reception will cause severe errors for the receiver operating in deep urban  
 235 canyons, degrading the overall collaborative positioning performance. Therefore, it is necessary to  
 236 identify the positioning performance of each receiver, selecting the receiver with healthy GNSS

237 signal reception to aid the one with degraded GNSS signal reception. Here, a GNSS positioning  
238 error map from ray-tracing simulation is used to predict the positioning performance of each  
239 receiver. The healthy receivers are selected to aid the degraded receivers with two different  
240 collaborative positioning methods: anchor-based method (Method 1) and complementary  
241 integration method (Method 2). The flowchart of the proposed collaborative positioning is shown  
242 in Fig.5.

243



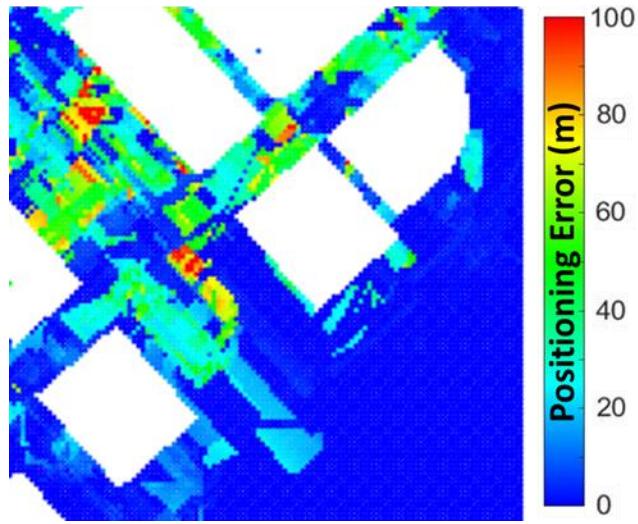
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245 **Fig. 5** Flowchart of the proposed 3DMA GNSS-based collaborative positioning algorithm.

246

247 First, the 3D building models and ephemeris are applied with the ray-tracing algorithm,  
248 simulating the GNSS range measurements including reflections. Then, the positioning error of a  
249 specific location can be predicted with the conventional least square solution from simulated  
250 measurements. The positioning error of each location can be constructed into a positioning error  
251 map (Zhang and Hsu 2018), as shown in Fig.6.

252



253

**Fig. 6** Demonstration of the predicted positioning error map using the ray-tracing algorithm and

3D building models. The color bar denotes the positioning error in the unit of meter.

255

256       Based on the SDM solution of each receiver, the corresponding GNSS positioning error  
 257   can be predicted by the positioning error map. The positioning error of neighboring locations  
 258   within a range of 10 m are selected to calculate the predicted positioning error of the receiver.  
 259   Considering the positioning accuracy of commercial GNSS receiver, the receiver with positioning  
 260   error less than 5 m is classified as a healthy receiver, otherwise, a degraded receiver.

261

### 262   *Method I*

263   The positioning solution estimated by LS or SDM of the degraded receiver still includes large  
 264   errors, which are difficult to be reduced by its own measurements. Since the healthy receivers  
 265   contain enough LOS measurements, both the absolute and relative positioning solutions achieve  
 266   better accuracy compared with that of the degraded receiver. It can use the positioning solutions  
 267   of the healthy receiver to estimate the position of the degraded receiver. Therefore, the position of  
 268   the degraded receiver can be derived as follows:

$$269 \quad \mathbf{x}_{M1,degraded} = \mathbf{x}_{SDM,healthy} + \Delta \vec{\mathbf{x}}_{DD,healthy-degraded} \quad (9)$$

270   where  $\mathbf{x}$  denotes the position of the receiver, the subscript  $M1$  denotes the estimated positioning

271 solution from Method 1.  $\mathbf{x}_{SDM,healthy}$  denotes the SDM solution of the healthy receiver.  
272  $\Delta\vec{\mathbf{x}}_{DD,healthy-degraded}$  denotes the relative positioning vector between healthy and degraded  
273 receiver obtained by the proposed DD method. Using the healthy receiver as an anchor, the position  
274 of the degraded receiver can be determined with better accuracy.

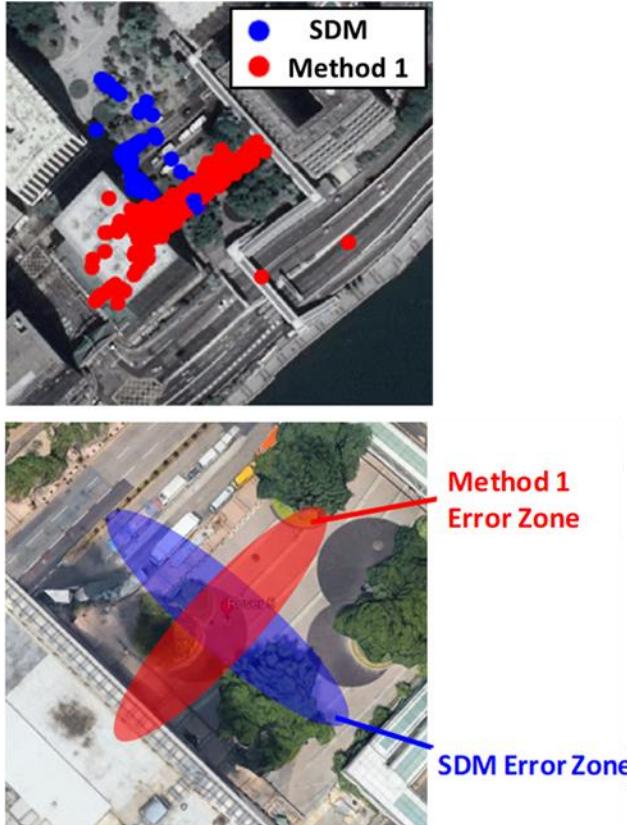
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276 *Method 2*

277 For Method 2, the positioning result of the degraded receiver from Method 1 is further integrated  
278 with the absolute positioning solution of degraded receiver estimated by SDM. The final position  
279 can be calculated as follows:

280 
$$\mathbf{x}_{M2,degraded} = \frac{1}{2}(\mathbf{x}_{M1,degraded} + \mathbf{x}_{SDM,degraded}) \quad (10)$$

281 where  $\mathbf{x}$  with the subscript of  $M2$  indicates the final solution estimated by Method 2 of the proposed  
282 algorithm. As shown in Fig.7, the positioning error distribution of Method 1 and SDM solutions  
283 are complementary. The SDM solution is known for its performance in the across-street direction.  
284 Method 1 is greatly based on the relative positioning using the common LOS measurements  
285 between two receivers. In the case of urban canyon, the common satellites are very likely visible  
286 in the along-street direction. Although an uncertainty-based weighted averaging could better  
287 integrate the two algorithms, the SDM determines the position by a candidate-searching method,  
288 which is hard to evaluate in terms of positioning uncertainty. Therefore, equal weight averaging is  
289 employed for simplicity. By integrating the solutions of Method 1 and SDM, the final positioning  
290 accuracy can be significantly enhanced.



291

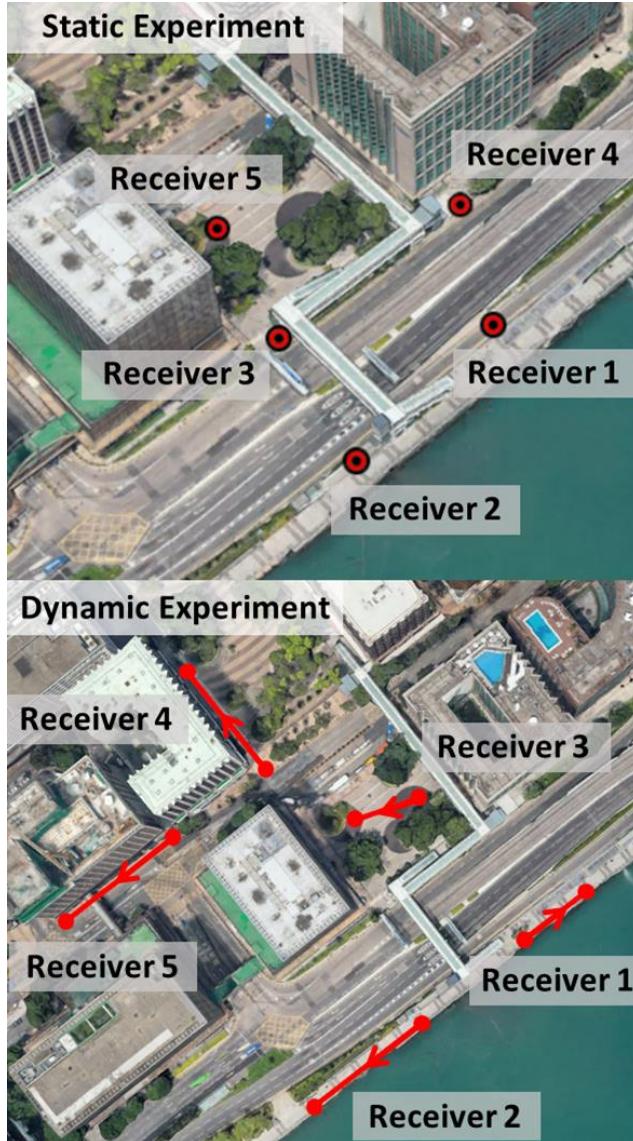
292 **Fig. 7** Demonstration of the complementary positioning error distributions of SDM and Method  
 293 1 of the proposed 3DMA GNSS-based collaborative positioning algorithm. The upper panel  
 294 shows the positioning distributions based on real data. The lower picture demonstrates the idea  
 295 of the complementary characteristics.

296

## 297 **Experiment Setup and Result**

298 To verify the proposed 3DMA GNSS-based collaborative positioning algorithm, a static  
 299 experiment is designed as shown in Fig.8 (top). Five locations are selected to represent 5 users in  
 300 different environments. For each location, the u-blox M8T is used to collect 10 minutes of GPS  
 301 and GLONASS measurements. Similarly, a dynamic experiment is designed as Fig.8 (bottom) to  
 302 verify the performance under a vehicle-like environment, where each receiver is carried by a  
 303 walking pedestrian. For the dynamic test, Receiver 1 and Receiver 2 are in the open-sky  
 304 environment, while Receiver 3 and Receiver 4 are in the urban area. Receiver 5 is located on a  
 305 narrow street with tall buildings on both sides, which is a harsh environment for positioning. The

306 recorded measurements are post-processed by the proposed algorithm.



307

308 **Fig. 8** Receiver locations of the static experiment (top) and dynamic experiment (bottom) in the  
309 urban area for the proposed 3DMA collaborative positioning algorithm.

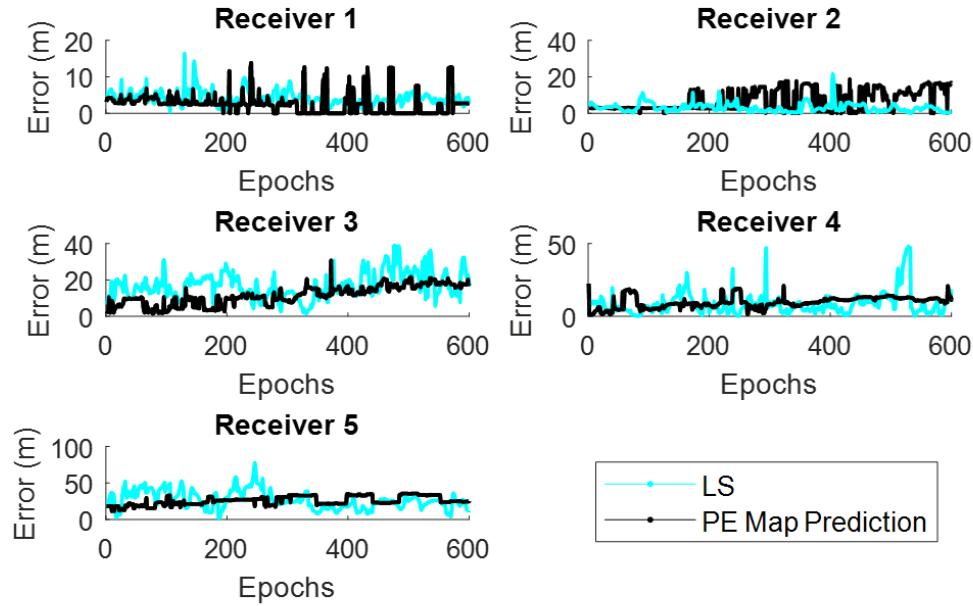
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311 Receiver performance classification during the static test

312 Based on the predicted GNSS positioning error map from ray-tracing simulation and SDM  
313 solutions, the positioning performance of each receiver can be predicted. The predicted positioning  
314 error distribution of each receiver is compared with its real-time least-squares estimation in Fig.9.

315 The corresponding mean errors and classification results are shown in Table 1.

316



317

318 **Fig. 9** Predicted positioning error obtained from the positioning error map and real positioning  
319 error based on least squares estimation for different receivers. LS stands for least square  
320 estimation and PE Map Prediction stands for predicted positioning error map.

321

322 **Table 1** Mean positioning error (m) and class of each receiver obtained from the least-squares  
323 estimation (LS) and predicted positioning error map (PEM).

Receiver	1	2	3	4	5
LS (m)	4.3	3.1	16.9	8.7	26.6
PEM (m)	2.6	7.0	11.5	9.7	25.8
Class	Healthy	Degraded	Degraded	Degraded	Degraded

324

325 Comparing the positioning error between the error map (black line) and LS (cyan line) in  
326 Fig.9, the predicted error of each receiver is similar to the real positioning error from LS, although  
327 the deviation of the true positioning error is larger. Therefore, the result verifies that the positioning

328 error map can predict the positioning error of each receiver. In the case of Receiver 1, the predicted  
329 error is less than 5 meters, which will be classified as a healthy receiver for collaborative  
330 positioning. For the other receivers, the predicted positioning errors are larger than 5 meters and  
331 classified as degraded receivers. The degraded receivers may suffer multipath or NLOS reception,  
332 requiring the aids of collaborative positioning.

333

334 Positioning performance of the static test

335 The performance of the proposed collaborative positioning algorithm will be compared with the  
336 following five approaches:

337 1) LS: Conventional least squares positioning algorithm

338 2) SDM: shadow matching, an innovative 3DMA GNSS positioning method.

339 3) CP-DD2CC: Collaborative positioning based on double layers consistency check.

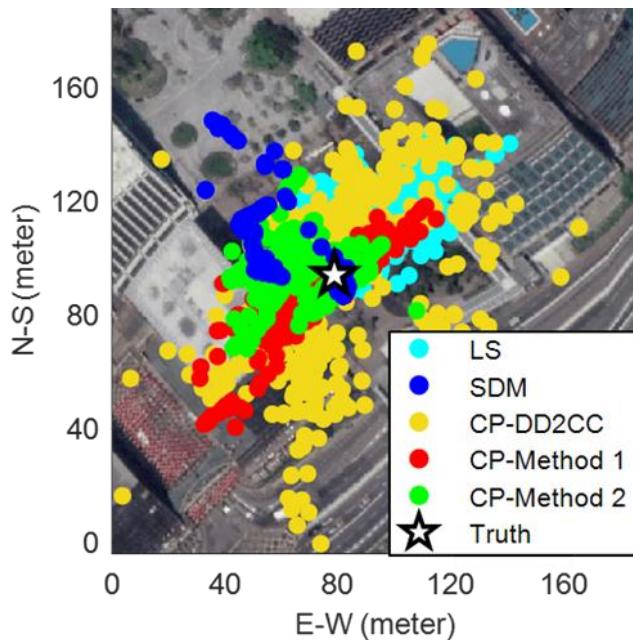
340 4) CP-Method 1: The proposed anchor based 3DMA GNSS collaborative positioning.

341 5) CP-Method 2: The proposed complementary integration based 3DMA GNSS  
342 collaborative positioning.

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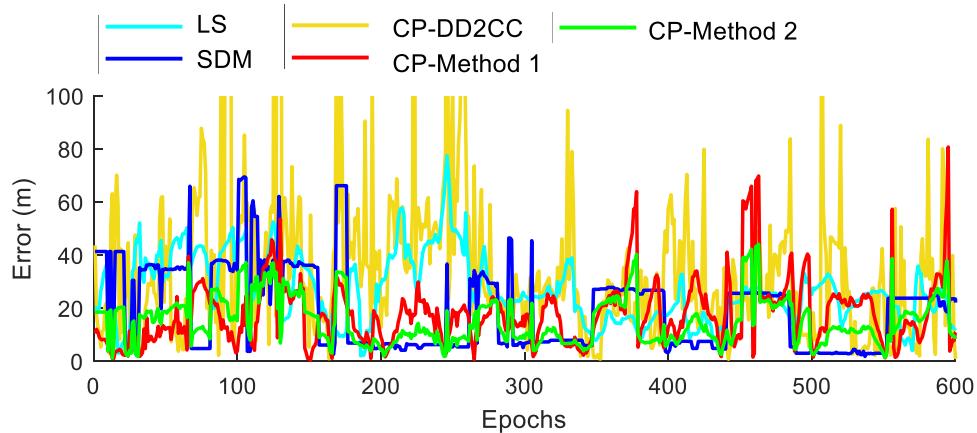
344 For Receiver 5, the positioning solutions of LS, SDM, CP-DD2CC, CP-Method 1 and CP-Method  
345 2 compared to its true location are shown on the Google Earth map in Fig.10. The positioning  
346 errors per epoch of the different approaches are shown in Fig.11. The mean and standard deviation  
347 of the positioning error for each degraded receiver (Receivers 2, 3, 4 and 5) are shown in Table 2.

348



349

350 **Fig. 10** Positioning solution of LS, SDM, CP-DD2CC, CP-Method 1 and CP-Method 2 for  
351 Receiver 5.



352

353 **Fig. 11** Positioning error distributions of LS, SDM, CP-DD2CC, CP-Method 1 and CP-Method 2  
354 for Receiver 5.

355

356

357 **Table 2** Mean positioning error and standard deviation of the classified degraded receivers by  
358 LS, SDM, CP-DD2CC, CP-Method 1 and CP-Method 2

Receiver	Method	LS	SDM	CP-DD2CC	CP-Method 1	CP-Method 2
2	Mean (m)	3.1	3.6	10.4	4.2	3.3
	STD (m)	2.4	2.8	43.3	2.5	1.9
3	Mean (m)	16.9	12.7	21.8	18.2	12.5
	STD (m)	7.0	7.1	65.7	14.4	7.7
4	Mean (m)	8.7	8.3	13.2	10.8	6.8
	STD (m)	7.7	4.0	23.8	8.8	4.7
5	Mean (m)	26.6	19.3	36.3	17.9	15.3
	STD (m)	12.4	15.7	41.2	12.1	8.9

359

360 Focusing on the case of Receiver 5, the estimated positions of the conventional LS have  
 361 significantly drifted from the true location, showing a 26.6 m mean error. Since the NLOS to LOS  
 362 measurements ratio is large, the consistency check algorithm may suffer from the fake consistency  
 363 issue. The healthy measurements may be mistakenly excluded and further increase the mean error  
 364 of collaborative positioning algorithm to 36.3 meters with 41.2 meters in STD. Aided by the 3D  
 365 building model, the SDM avoids using the multipath/NLOS affected pseudorange measurements  
 366 and improves the positioning error to 19.3 m in the mean. However, the positioning error is still  
 367 large because the NLOS cannot be all correctly classified based on the C/N<sub>0</sub>. The proposed  
 368 algorithm first excludes the NLOS measurements based on the satellite visibility from SDM. Then,  
 369 the classified healthy receiver further collaborates with degraded receivers by double differencing  
 370 their pseudorange measurements with double-layer consistency check. Hence, the multipath effect  
 371 and NLOS reception can be largely mitigated, contributing a more accurate result with 17.9 m in  
 372 mean and 12.1 m in STD (Method 1). Based on the complementary error distribution illustrated in  
 373 Fig 10, the CP-Method 1 solution can be further integrated with degraded receiver's SDM solution  
 374 as Method 2. The proposed CP-Method 2 can mitigate the enormous positioning error of shadow  
 375 matching or CP-Method 1 seen in Fig 11, thus contributing a more stable and accurate positioning  
 376 solution with 15.3 meters mean error and 8.9 m in STD.

377 For Receivers 3 and 4 located at an environment that half of the sky is blocked by buildings,  
 378 the shadow matching technique is effective and outperforms the CP-Method 1, since it mitigates

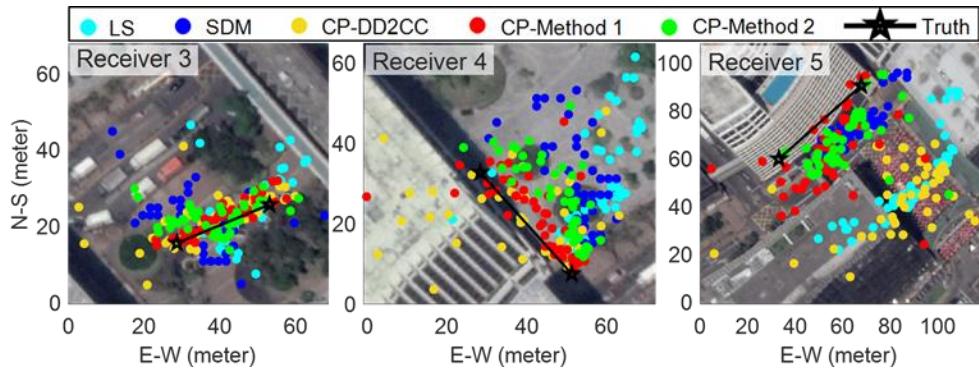
379 the positioning error from pseudorange measurements. The proposed CP-Method 2 further  
380 employs the solution of Method 1 to compensate for the positioning error in the direction in which  
381 shadow matching is ineffective, obtaining a better positioning result. Noticed that Receiver 3 is  
382 near a bridge that is not modeled in the 3D building model, causing the proposed algorithm to  
383 achieve limited improvements. Receiver 2 in the open-sky situation is inappropriately classified  
384 as a degraded receiver due to the prediction error. However, the proposed algorithm is still able to  
385 maintain its positioning performance of 3.3 meters in the mean with 1.9 meters in STD. After all,  
386 the proposed 3DMA GNSS collaborative positioning algorithm can improve the positioning  
387 performance of the receivers in an urban area as well as maintaining the performance of the ones  
388 in open-reception areas.

389

390 Positioning performance of the dynamic test

391 Based on the proposed receiver performance classification method, Receiver 1 and Receiver 2 are  
392 classified as healthy receivers with predicted positioning errors of about 0.1 m and 1.5 m.  
393 Receivers 3, 4 and 5 are classified as degraded receivers with 35.6 m, 33.6 m and 17.0 m predicted  
394 positioning error respectively. Therefore, we proposed to collaborate the measurements from  
395 Receiver 1 (healthy) with Receivers 3, 4 and 5 to improving the accuracy of each of these degraded  
396 receivers. The positioning solutions of the proposed and conventional SPP methods for each  
397 degraded receiver are shown in Fig. 12 and with mean and STD given in Table 3. Both Methods 1  
398 and 2 can achieve a mean positioning error of less than half the conventional LS method, and  
399 significantly improve the accuracy compared to SDM and CP-DD2CC solutions. For Receiver 5,  
400 Method 2 makes use of the complementary behavior of Method 1 and SDM to further reduce the  
401 positioning error to 14.4 meters, which is twice as good as the LS method. However, the proposed  
402 Method 2 does not achieve better performance for Receiver 3 and Receiver 4. This is because the  
403 SDM performance is not satisfactory, whereas the SDM-based NLOS classification is very  
404 accurate. Most of the NLOS measurements are correctly excluded, resulting in an accurate Method  
405 1 solution. Since the SDM is performing much worse with regard to Method 1, the positioning  
406 accuracy of Method 2 using equal averaging may be degraded by the SDM solution. As a result,  
407 an improvement from complementarily integrating SDM and Method 1 may not occur when the  
408 two methods perform at very different accuracy.

409



410

411 **Fig. 12** Positioning solutions of LS, SDM, CP-DD2CC, CP-Method 1, CP-Method 2 regarding  
 412 and true receiver location (Truth) for Receiver 3 in the middle between buildings (left), Receiver  
 413 4 closed to the building (middle) and Receiver 5 on a narrow street closed to buildings (right).

414

415 **Table 3** Mean positioning error and standard deviation of the classified degraded receivers by  
 416 LS, SDM, CP-DD2CC, CP-Method 1 and CP-Method 2 in a dynamic test

Receiver	Method	LS	SDM	CP-DD2CC	CP-Method 1	CP-Method 2
3	Mean (m)	11.4	10.3	8.1	3.0	5.4
	STD (m)	9.3	5.8	7.1	1.7	3.3
4	Mean (m)	21.7	17.8	15.0	5.6	10.6
	STD (m)	13.1	6.1	14.5	6.1	4.5
5	Mean (m)	46.2	16.7	49.8	19.0	14.4
	STD (m)	5.1	5.4	11.3	19.9	10.2

417

## 418 Conclusions

419 In this study, a new 3DMA GNSS collaborative positioning algorithm is developed. By estimating  
 420 the satellite visibility based on SDM, the NLOS measurements in dense urban area are correctly  
 421 distinguished and excluded. Based on the predicted GNSS positioning error map, the healthy  
 422 receiver can be identified and then used to collaborate with degraded receivers. The DD method  
 423 with double-layer consistency check is employed during the relative positioning, which further

424 mitigates the multipath effect and NLOS reception. The proposed collaborative positioning uses  
425 the measurements of the healthy receiver to aid positioning of degraded receivers and further  
426 integrates with the complementary SDM solution, achieving better positioning performance in  
427 dense urban areas.

428 The collaborative process of the proposed algorithm is simply based on equal weighted  
429 averaging. A more effective and suitable optimization approach such as factor-graph optimization  
430 is worth to be studied to improve the integration performance.

431

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435

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