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MA 323 - Monte Carlo Simulation
Assignment - 2

Ans 1:
Used a Linear Congruence Generator to generate the first 17 terms, with the following initial values:

a = 1597
b = 1
m = 244944
seed = 2931

The elements obtained were:

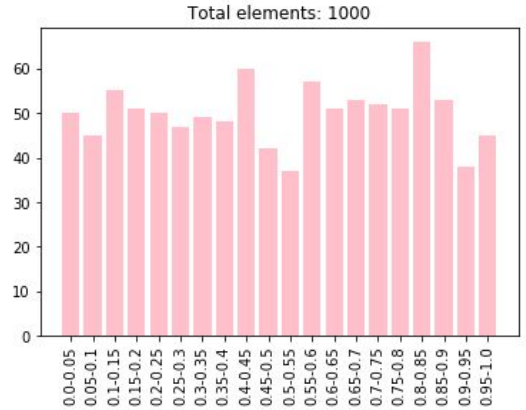
[0.1097, 0.2016, 0.9832, 0.1252, 0.8662, 0.3834, 0.2733, 0.4784, 0.9904, 0.6653, 0.4825, 0.582, 0.4122, 0.3231, 0.9359, 0.6252, 0.4445]

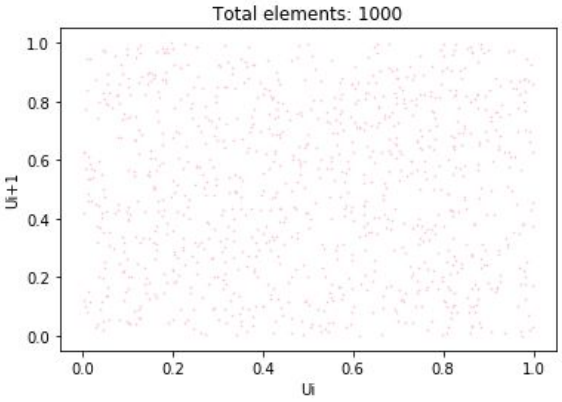
Following this, used a **Lagged Fibonacci Generator** to generate the remaining elements. The relation used was:

$U_{i+1} = (U_{i-17} - U_{i-5})$; If $U_i < 0$, set $U_i = U_i + 1$

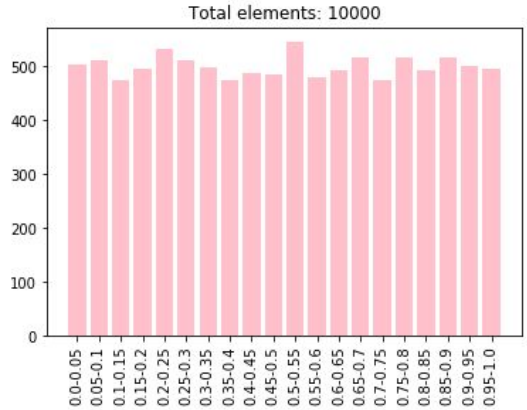
Three cases were considered, with the total number of elements present in the sequence being varied each time.
A **Bar Graph** and a **(U_i, U_{i+1})** plot was drawn for each case.

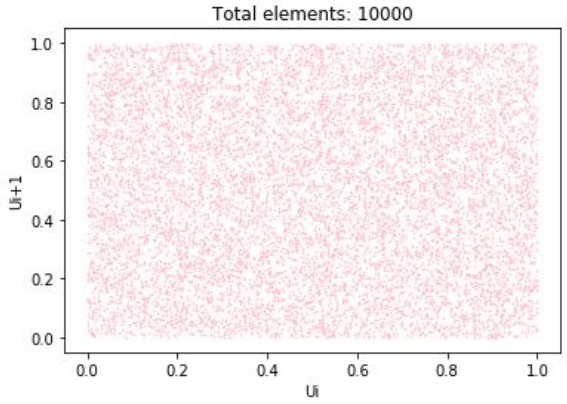
Case 1: Total number of elements = 1000



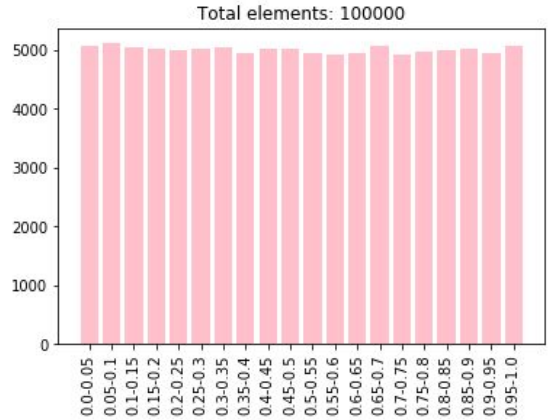


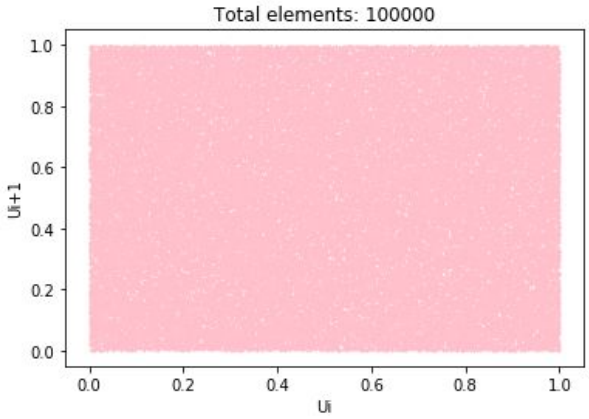
Case 2: Total number of elements = 10000





Case 3: Total number of elements = 100000





Observations:

- The uniformity of the Bar Graphs gets better as we increase the total number of elements, reaching nearly completely uniform for the third case.
- No matter which case, there seems to be no relation between consecutive elements of the sequences

Conclusions:

- The sequences generated by the Lagged Fibonacci Generator $U_{i+1} = (U_{i-17} - U_{i-5})$ are mutually independent elements
- As the total of elements increases, the distribution tends to Uniform Distribution.

Ans 2:

Used a **Lagged Fibonacci Generator** to generate **Uniformly Distributed** sequences, which were then used to obtain samples of the **Exponential Distribution**.
(NOTE : The Generator used to obtain the sequences was same as the one used in the previous question)

The analysis was done for 4 cases.

In each of the cases the value of Theta was fixed to be 0.2. Through this we can determine the value of the Mean and the Variance for the Actual Distribution to be:

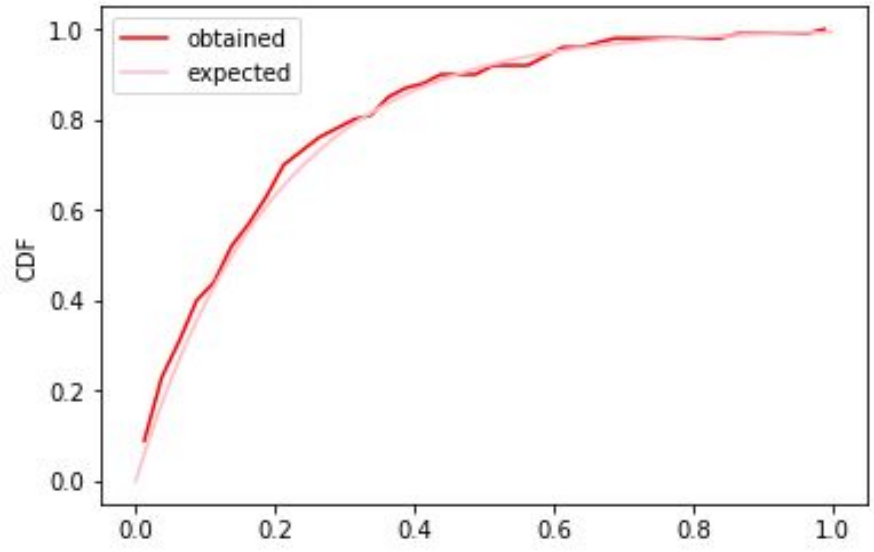
Mean = 0.2
Variance = 0.04 (theta squared)

Case 1: Elements used = 100

Mean (obtained) = 0.2037 Variance (obtained) = 0.0438

Absolute Difference between the obtained and actual:

Mean = 0.0037 Variance = 0.0038

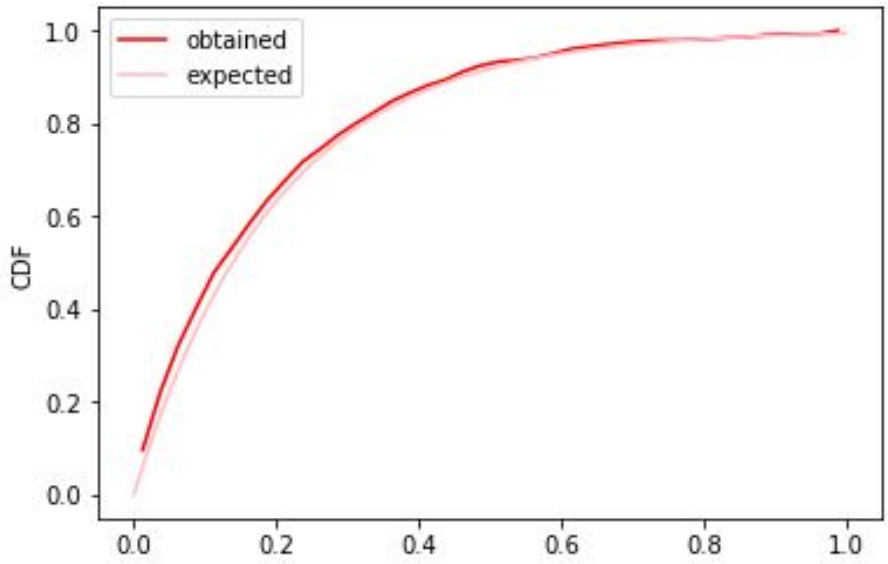


Case 2: Elements used = 1000

Mean (obtained) = 0.1979 Variance (obtained) = 0.038

Absolute Difference between the obtained and actual:

Mean =0.0021 Variance = 0.002

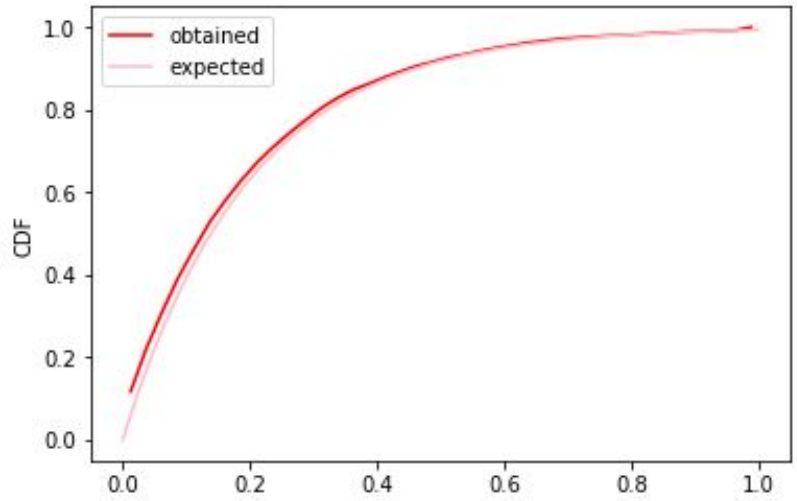


Case 3: Elements used = 10000

Mean (obtained) = 0.2004 Variance (obtained) = 0.0405

Absolute Difference between the obtained and actual:

Mean =0.0004 Variance = 0.0005

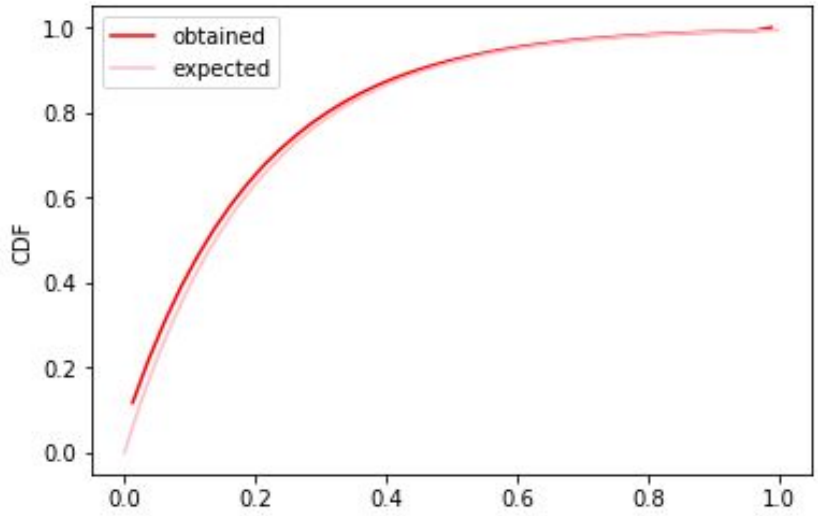


Case 4: Elements used = 1000000

Mean (obtained) = 0.1998 Variance (obtained) = 0.0396

Absolute Difference between the obtained and actual:

Mean =0.0002 Variance = 0.0004



Observations:

- As we increase the total number of elements under observations, the CDF of the sequence obtained from the transformation of a Uniform Distribution tends to that of the actual Exponential Distribution
- The Mean and Variance of the obtained exponential sequence also tends to that of the actual values, as we increase the count of elements

Ans 3:
Used a Lagged Fibonacci Generator to generate Uniformly Distributed sequences, which were then used to obtain samples of the Exponential Distribution.
(NOTE : The Generator used to obtain the sequences was same as the one used in the previous question)

The analysis was done for 4 cases.

Due to the nature of the ArcSine Distribution, we can determine the following values:

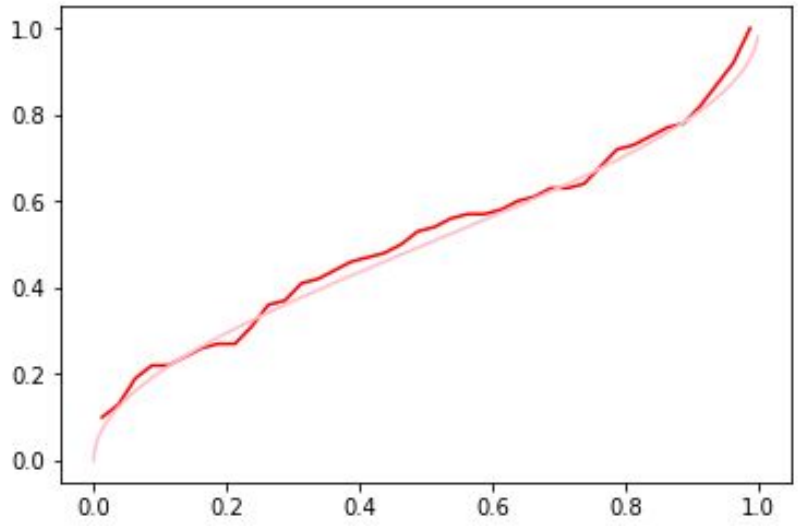
Mean = 0.5
Variance = 0.125

Case 1: Elements used = 100

Mean (obtained) = 0.4962 Variance (obtained) = 0.1240

Absolute Difference between the obtained and actual:

Mean = 0.0038 Variance = 0.0010

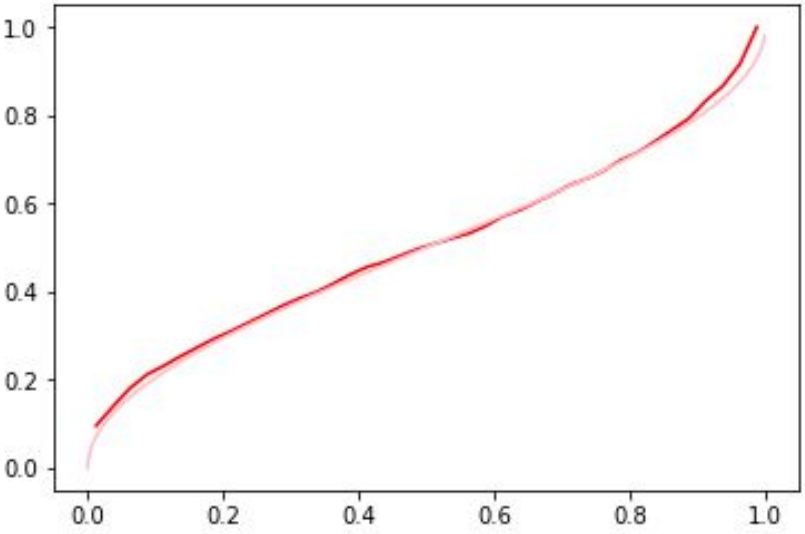


Case 2: Elements used = 1000

Mean (obtained) = 0.5031 Variance (obtained) = 0.1253

Absolute Difference between the obtained and actual:

Mean = 0.0031 Variance = 0.0003

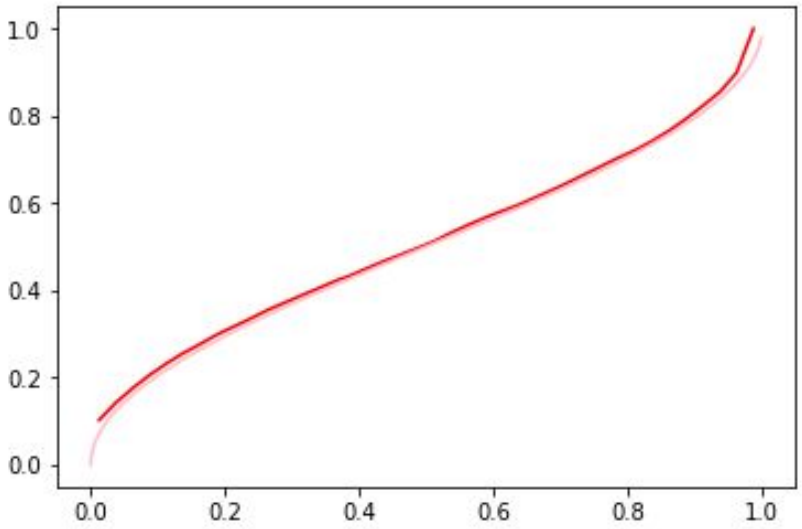


Case 3: Elements used = 10000

Mean (obtained) = 0.5001 Variance (obtained) = 0.1254

Absolute Difference between the obtained and actual:

Mean = 0.0001 Variance = 0.0004

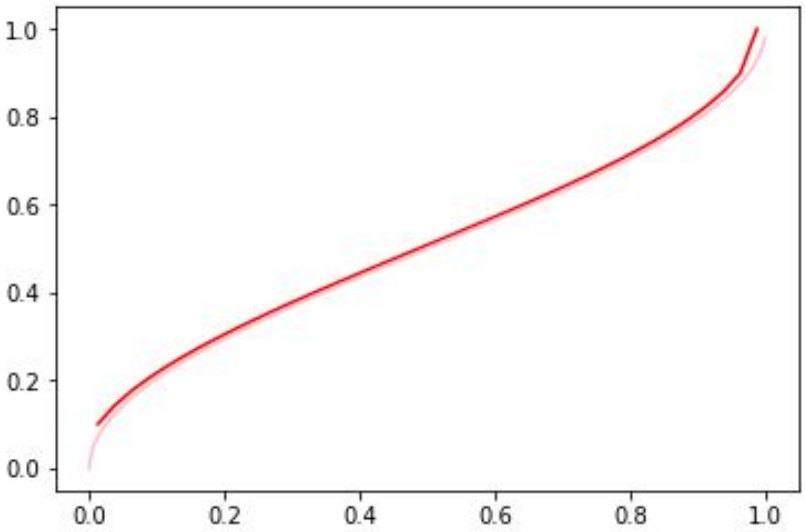


Case 4: Elements used = 1000000

Mean (obtained) = 0.4998 Variance (obtained) = 0.1249

Absolute Difference between the obtained and actual:

Mean = 0.0002 Variance = 0.0001



Observations:

- As we increase the total number of elements under observations, the CDF of the sequence obtained from the transformation of a Uniform Distribution tends to that of the actual ArcSine Distribution
- The Mean and Variance of the obtained exponential sequence also tends to that of the actual values, as we increase the count of elements