

Social Structure Emergence:

A multi-agent Reinforcement Learning Framework for
Relationship Building

Yang Chen and Jiamou Liu

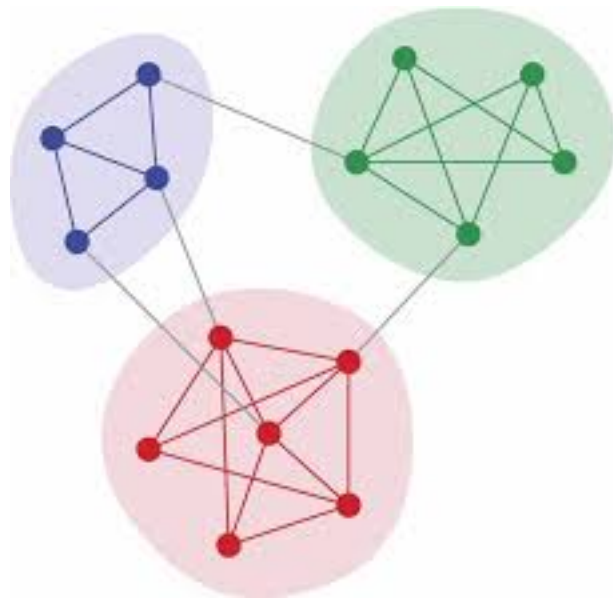


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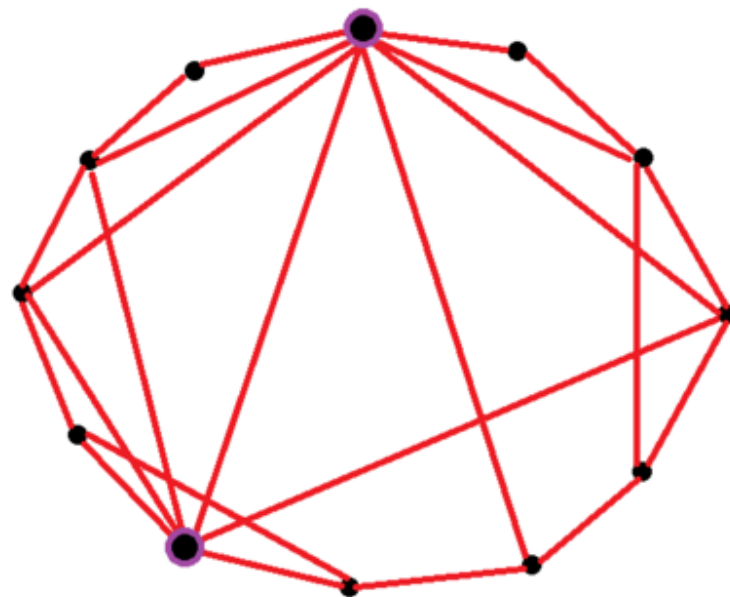
Motivation

Social Network Structures

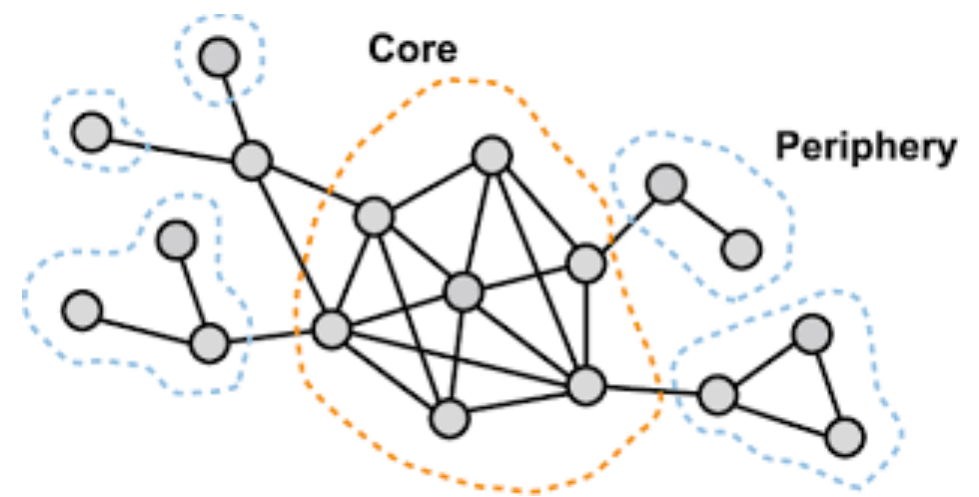
Community



Small-World



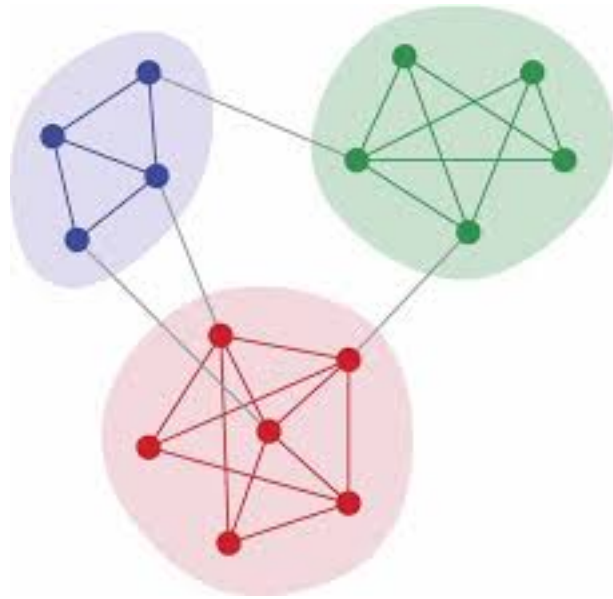
Core-Periphery



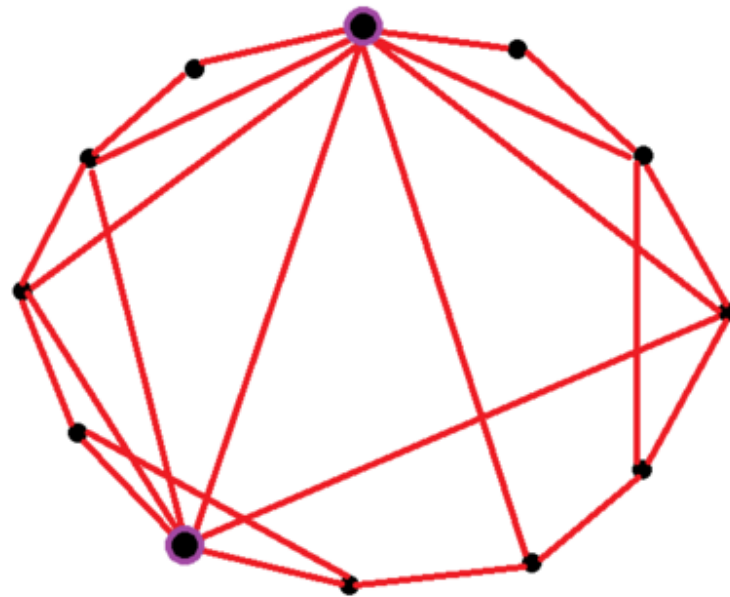
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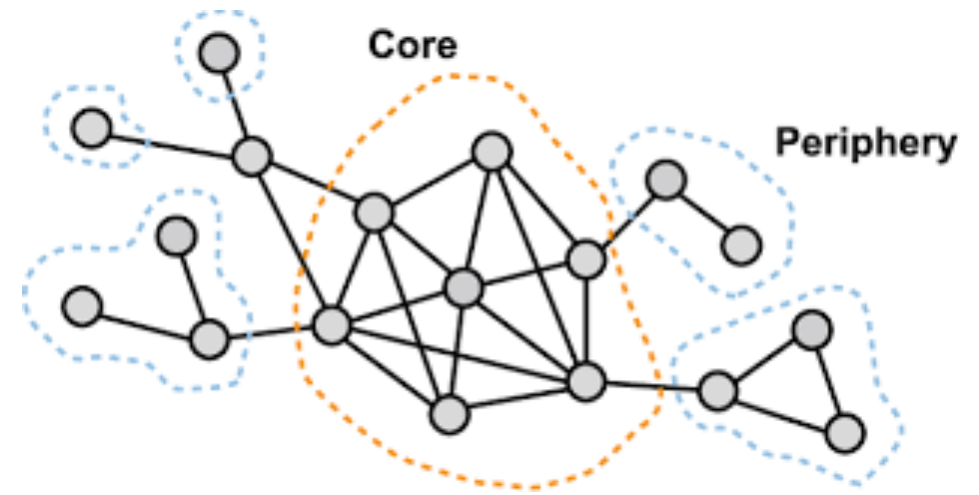
Community



Small-World



Core-Periphery



**How to explain the formation process
of different social structures?**

Motivation

Related Works

- **Random event based models**

[1] *Matthew O Jackson and Brian W Rogers. 2007. Meeting strangers and friends of friends: How random are social networks? American Economic Review 97, 3 (2007), 890–915.*

[2] *Jure Leskovec, Lars Backstrom, Ravi Kumar, and Andrew Tomkins. 2008. Microscopic evolution of social networks. In Proceedings of the 14th ACM SIGKDD international conference on Knowledge discovery and data mining. ACM, 462–470.*

- **Strategic decision based models**

[3] *Matthew O Jackson and Asher Wolinsky. 1996. A strategic model of social and economic networks. Journal of economic theory 71, 1 (1996), 44–74.*

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Pros: can generate networks with desired structural properties

Cons: pay limited attention to agents' behavioural acquisitions

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Pros: can explain the formation of social structures using game theory

Cons: they are one-shot models so that neglect dynamics of network formation

Motivation

What we did

- We view network formation as a process of **sequential** decision making of **all** agents in a social network.
- We propose a novel network formation model based on the (partially observable) **stochastic games**.
- We adopt the notion of **social capital** to define the utility in the game.
- We use **multi-agent reinforcement learning** method to train agents.
- Our model **unifies** the explanation for the natural emergence of various classical social structures.

Methods

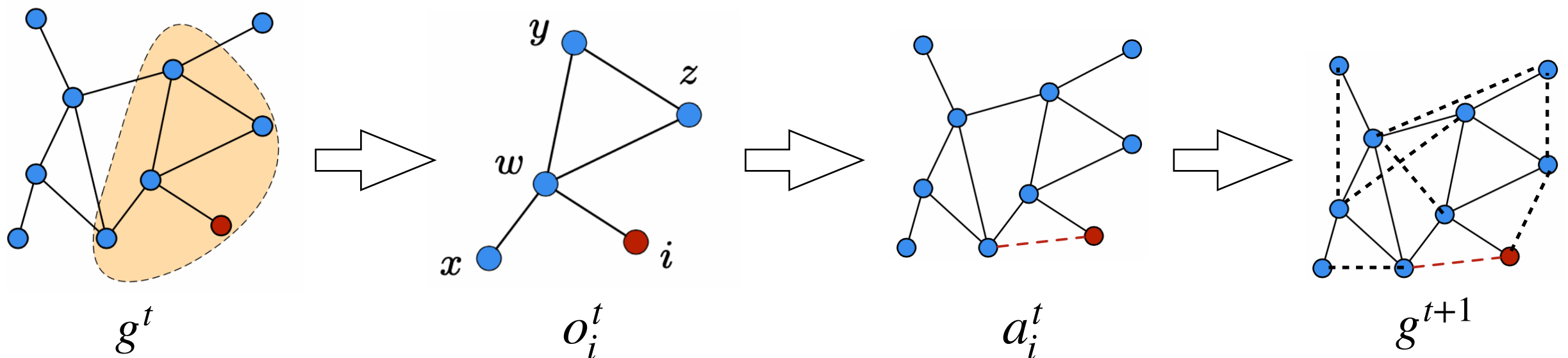
Partially Observable Stochastic Games

A POSG is defined by:

1. N is a finite set of agents.
2. S is a finite set of states.
3. s^0 is an initial state.
4. O_i is a finite set of observations of i .
5. A_i is a finite set of actions of i .
6. $P(s', \vec{o} | s, \vec{a})$ the probability that taking joint action \vec{a} in state s results in a transition to state s' and joint observation \vec{o} .
7. $R_i : S \times \vec{A} \rightarrow \mathbb{R}$ is the reward function of i .

Our model:

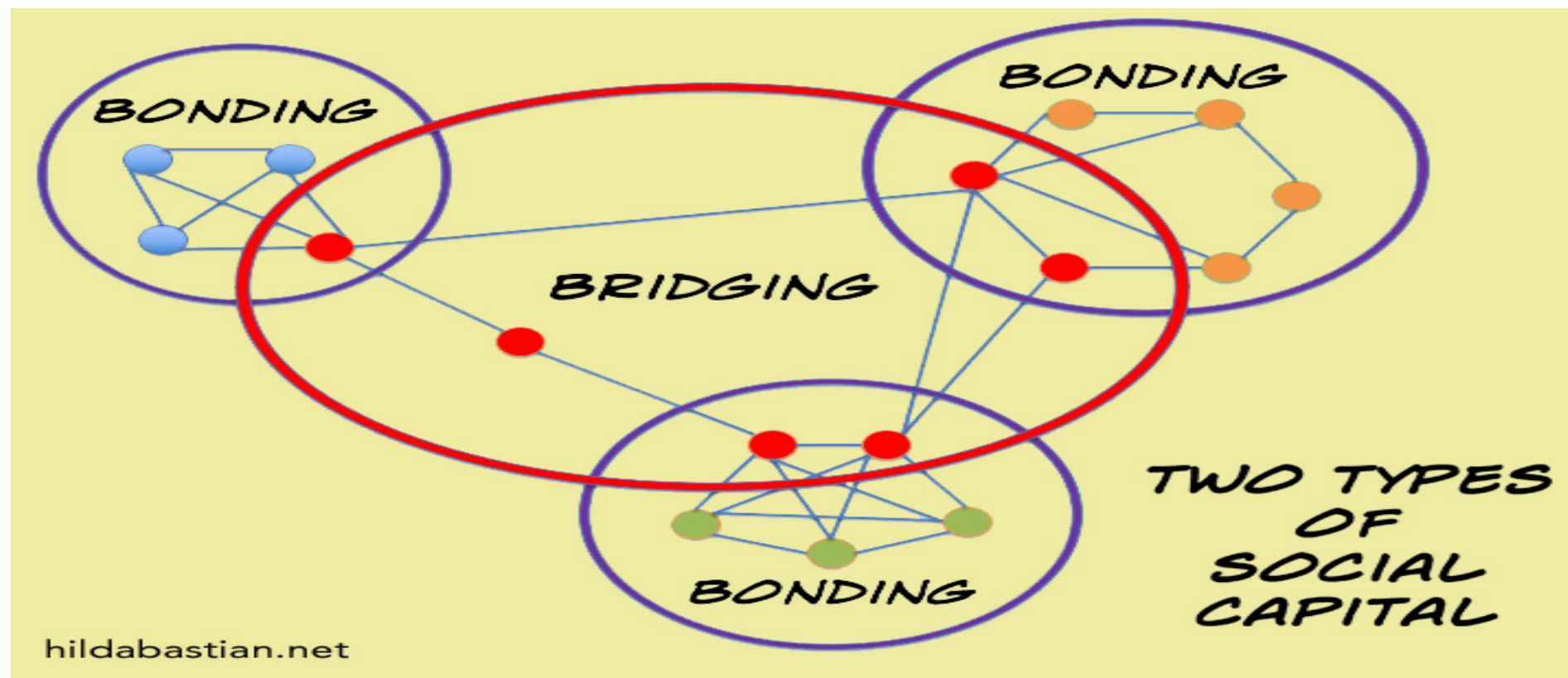
1. All N nodes.
2. A state is a set of edges g^t .
3. g^0 is a set of edges.
4. Agent i 's observation o_i is its neighbours **within** distance 2.
5. Agent i 's action a_i is building a link with an agent in o_i .
6. g^{t+1} is uniquely determined by g^t and \vec{a}^t .
7. The reward of i is the increase of **social capital**.



Methods

Reward Design – Social Capital (1)

- The concept of **social capital** captures the **benefits** attained by individuals via social interactions [5].
- Social capital consists of two types:
 - **bonding capital** refers to benefits an individual draws from its closed **neighbourhood**, in the form of, e.g., trust and support.
 - **bridging capital** is an embodiment of benefits of accessibility to **information** and **control** over information flow.



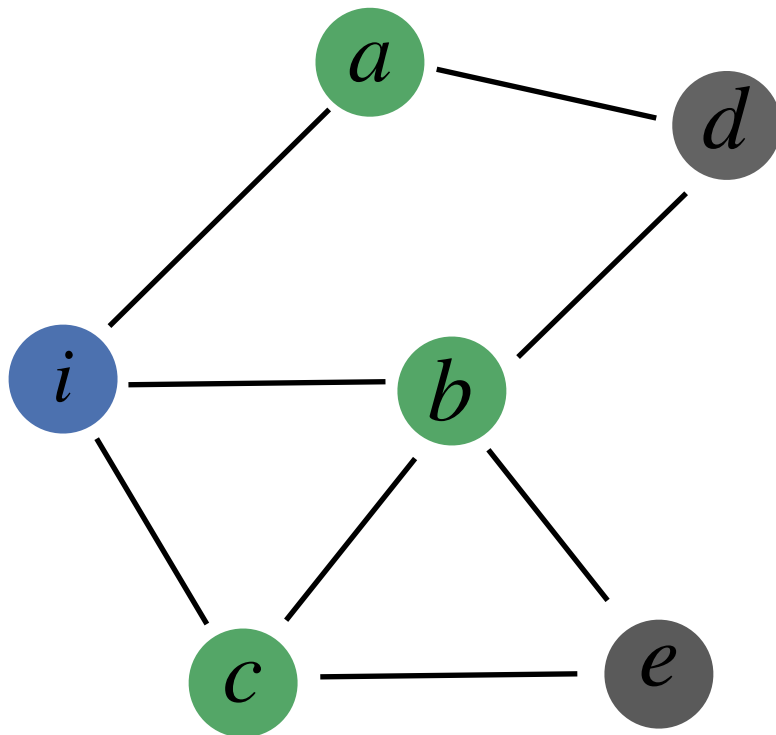
[5] James S Coleman. 1988. Social capital in the creation of human capital. *American journal of sociology* 94 (1988), S95–S120.

Methods

Reward Design — Social Capital (2)

Formalisation of Bonding Capital:

[6][7] use **personalised Page rank** to formalise bonding capital.



Page rank score:

$$PR_j = \beta r_j + (1 - \beta) \left(\sum_{k \in \mathcal{N}_j} P_k \right) / |\mathcal{N}_j|$$

$$r_j = 1 \text{ if } j = i \text{ and } 0 \text{ otherwise}$$

Bonding Capital:

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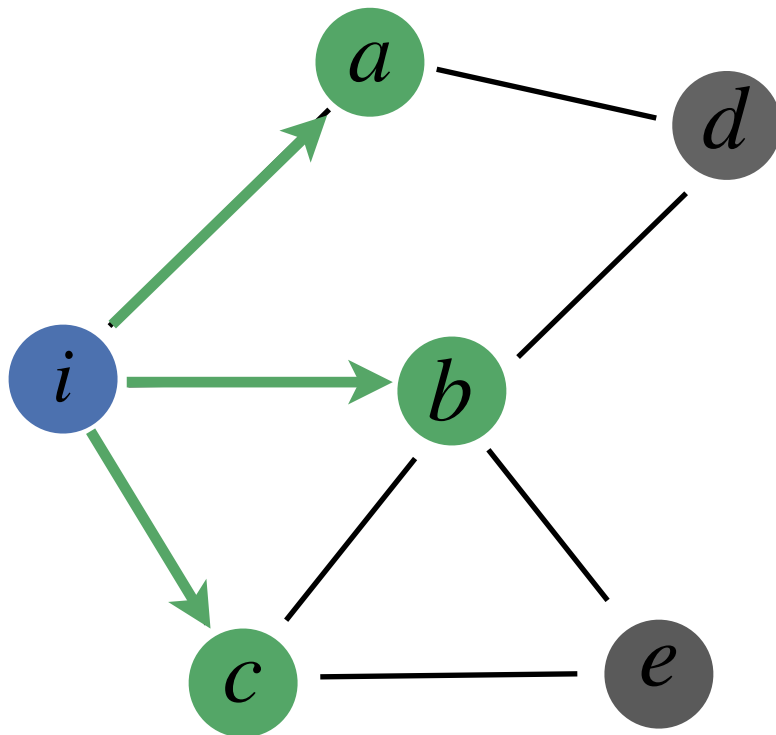
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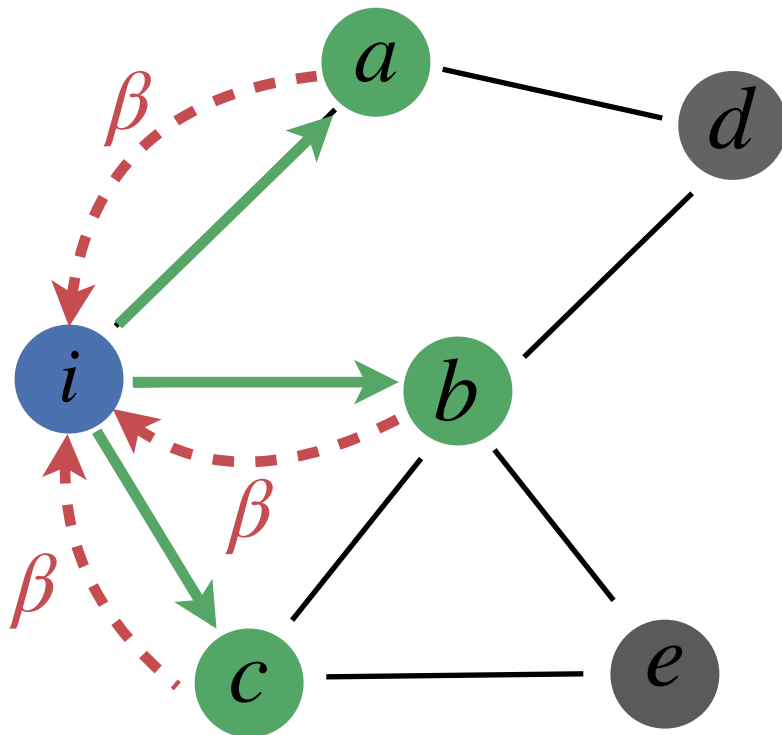
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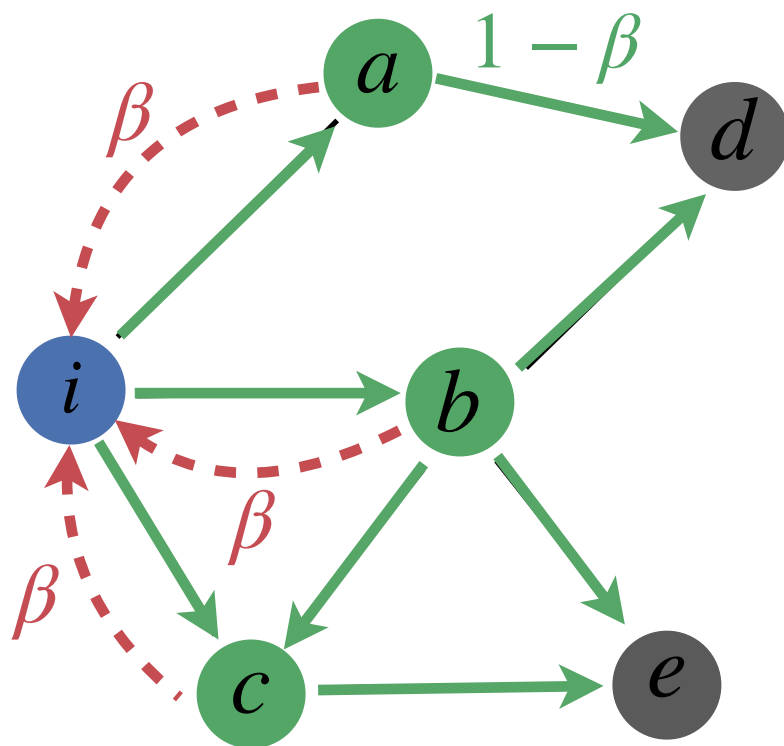
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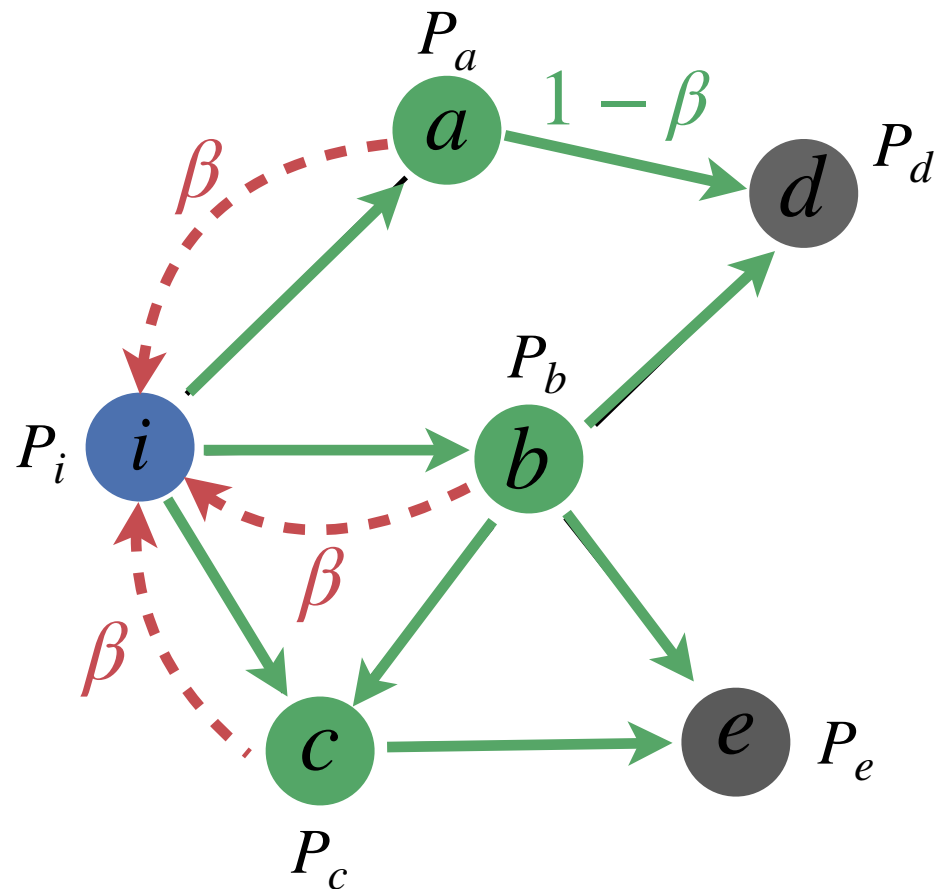
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Methods

Reward Design — Social Capital (3)

Formalisation of Bridging Capital:

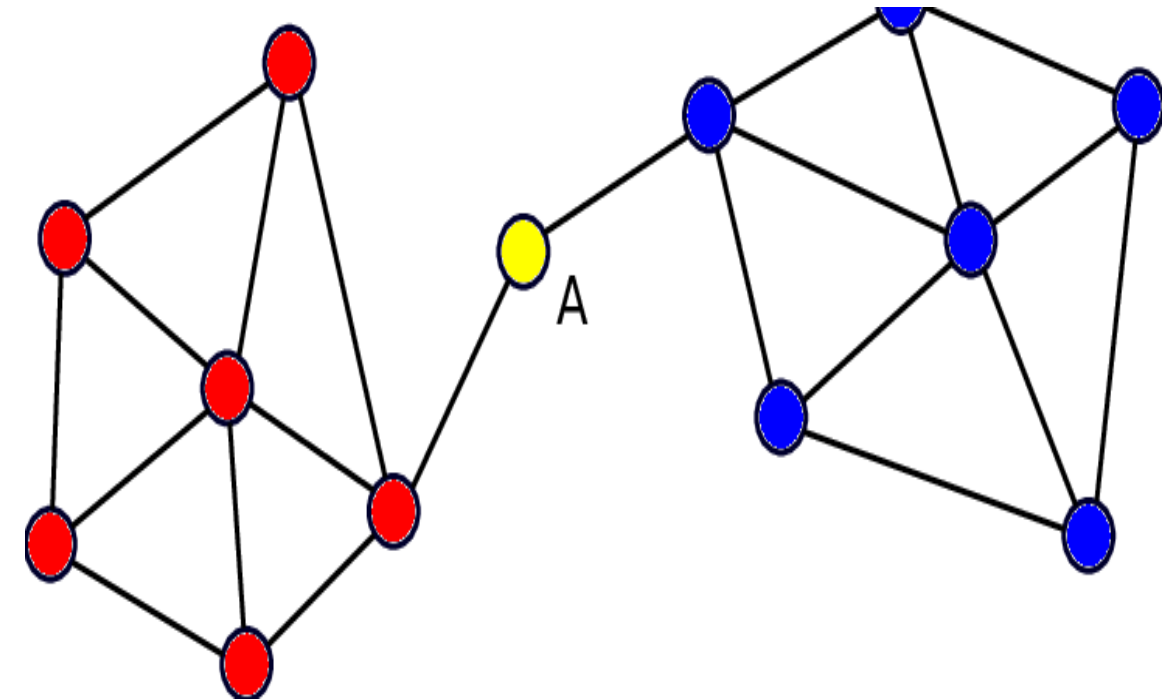
[8] use **betweenness centrality** to formalise bridging capital.

Bridging Capital:

$$br_i = \sum_{j \neq i \neq k \in N} \sigma_{jk}(i) / \sigma_{jk}$$

σ_{jk} is the number of shortest paths between j and k

$\sigma_{jk}(i)$ is the number of shortest paths between j and k passing i



[8] Ahmed M Alaa, Kartik Ahuja, and Mihaela van der Schaar. 2017. A micro foundation of social capital in evolving social networks. *IEEE Transactions on Network Science and Engineering* 5, 1 (2017), 14–31.

Methods

Reward Design — Social Capital (3)

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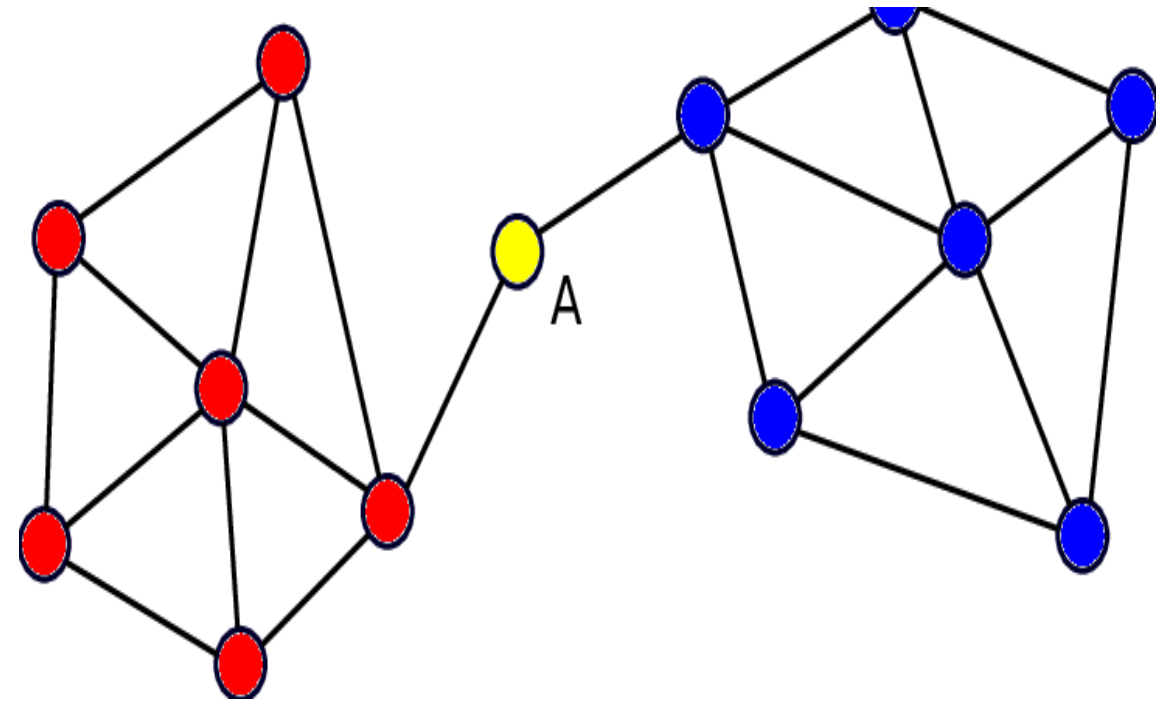
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Mixed Capital:

$mix_{i,w} = w \cdot bo_i + (1 - w)br_i$, **preference weight** $w \in [0,1]$ is a hyper-parameter.

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Methods

Novel Network Formation Model – Social Capital Games (SCG)

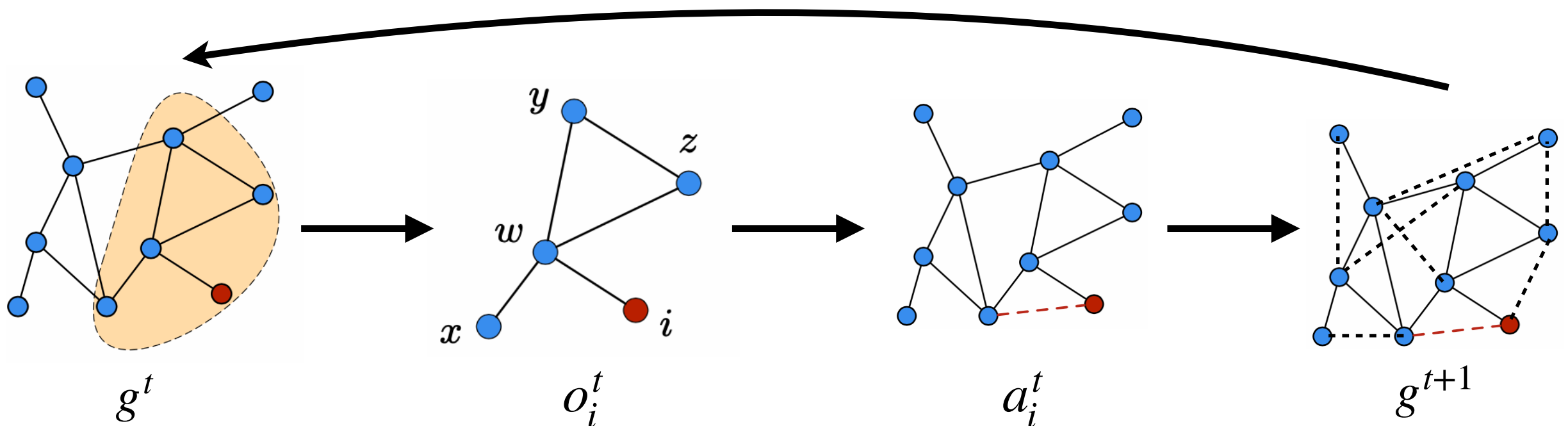
An **SCG** is a tuple (N, W, g^0, ℓ) , where

- $N = \{1, 2, \dots, n\}$ is a set of agents;
- $W = (w_1, w_2, \dots, w_n)$ is set of preference weights;
- g^0 is the initial network;
- $\ell \in \mathbb{N}^+$ is the termination time step.

Reward: $u_i(o_i^t, a_i^t) = \text{mix}_{i, w_i}^{t+1} - \text{mix}_{i, w_i}^t$

Policy: $\pi_i(o_i) = a_i$

Goal: maximise the cumulative reward $U_i^t = \sum_{t=0}^{\ell-1} u_i(o_i^t, a_i^t) = \text{mix}_{i, w_i}^{t+1} - \text{mix}_{i, w_i}^t$



Methods

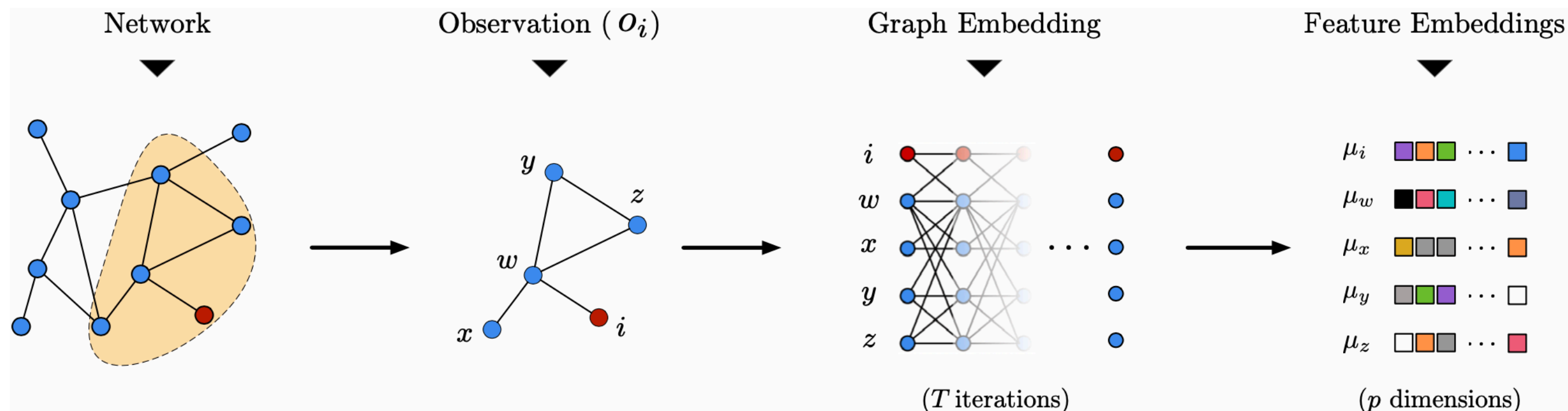
State representation — Graph embedding

The space of all possible observations of an agent can be very large due to the **Curse of Dimensionality**.

We use **graph embedding** to generate a representation of an observation.

Node representation: $\mu_j = GE(o_i; \Theta_i)$

State (observation) representation: $\phi(o_i) = \sum_{j \in o_i} \mu_j$

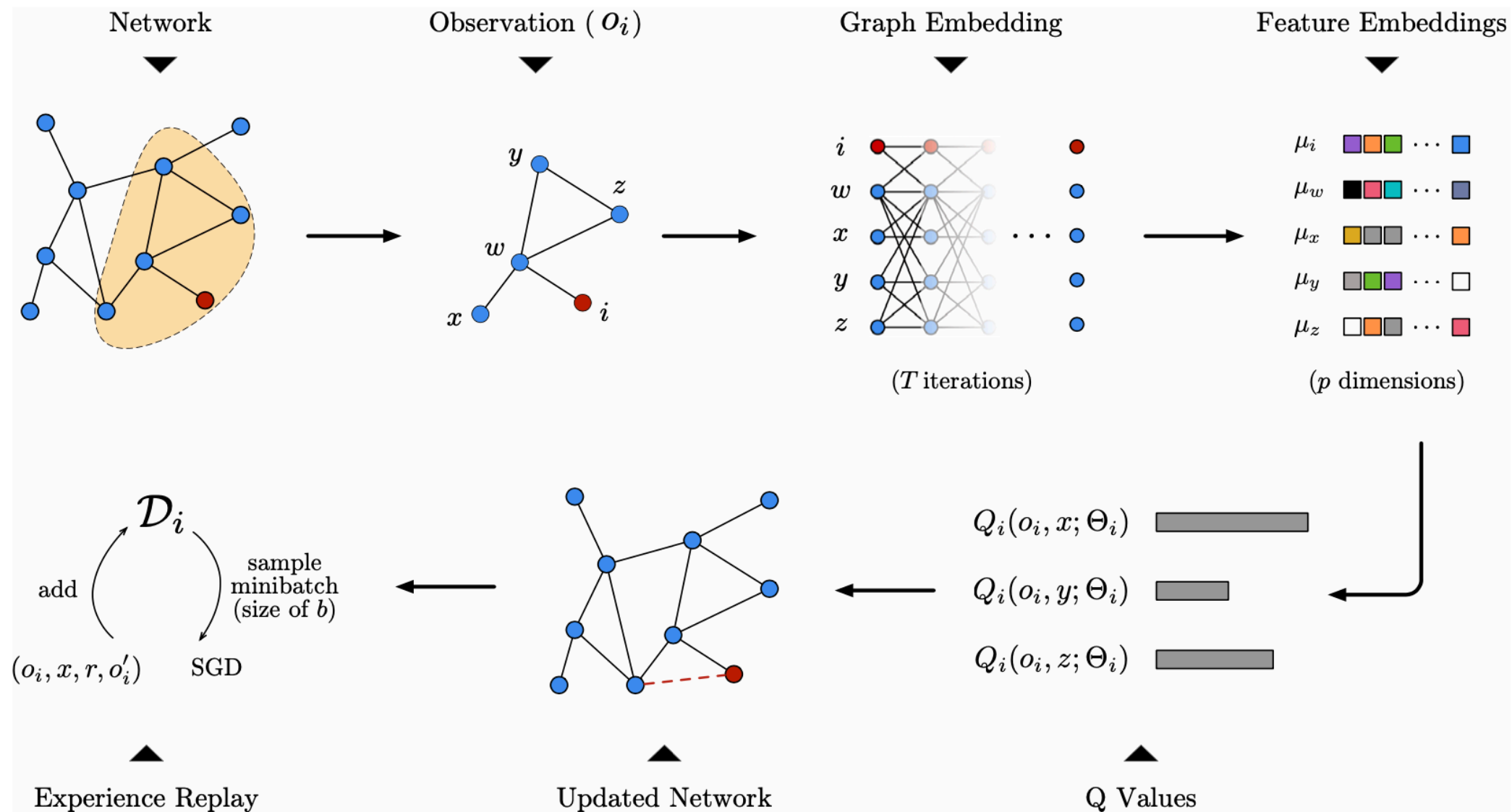


Methods

Policy Learning — Multi-agent RL

Policy $\pi(o_i) = \arg \max_a Q_i(o_i, a; \Theta_i)$

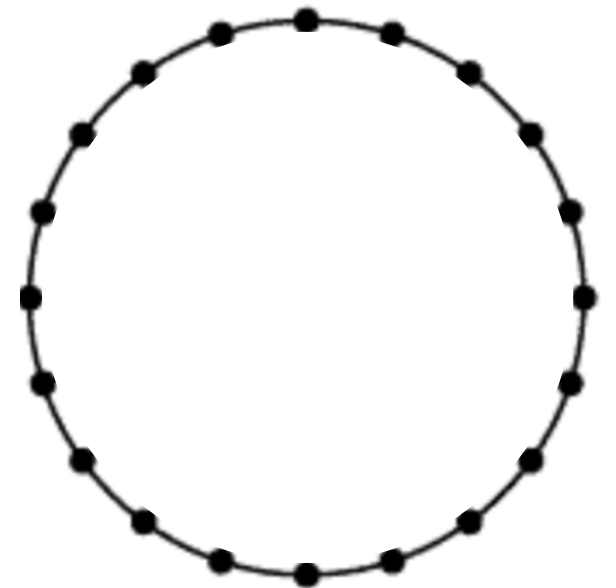
Loss Function $L(\Theta_i) = \mathbb{E}_{(o,a,r,o') \sim D_i} [(r + \max_{a'} Q_i(o', a'; \Theta_i) - Q_i(o, a; \Theta_i))^2]$



Experiments

General Settings

- Number of agents: $N = 100$
- Initial Network : A **regular lattice (ring)**
- Termination step $\ell \in \{2,5,8\}$
- Baselines:
 - Randomly created graphs
 - Graph generation models
- Observe the change of g^ℓ during the learning process.
- Goal: observe the emergence of community, Small-World and Core-Periphery structure under different settings of preference weights.



A ring network

Results

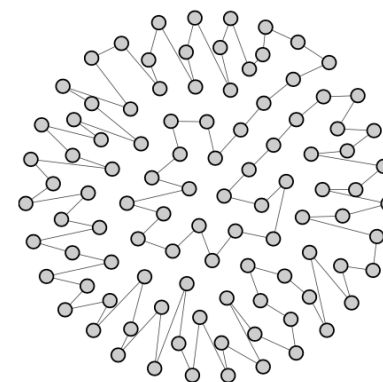
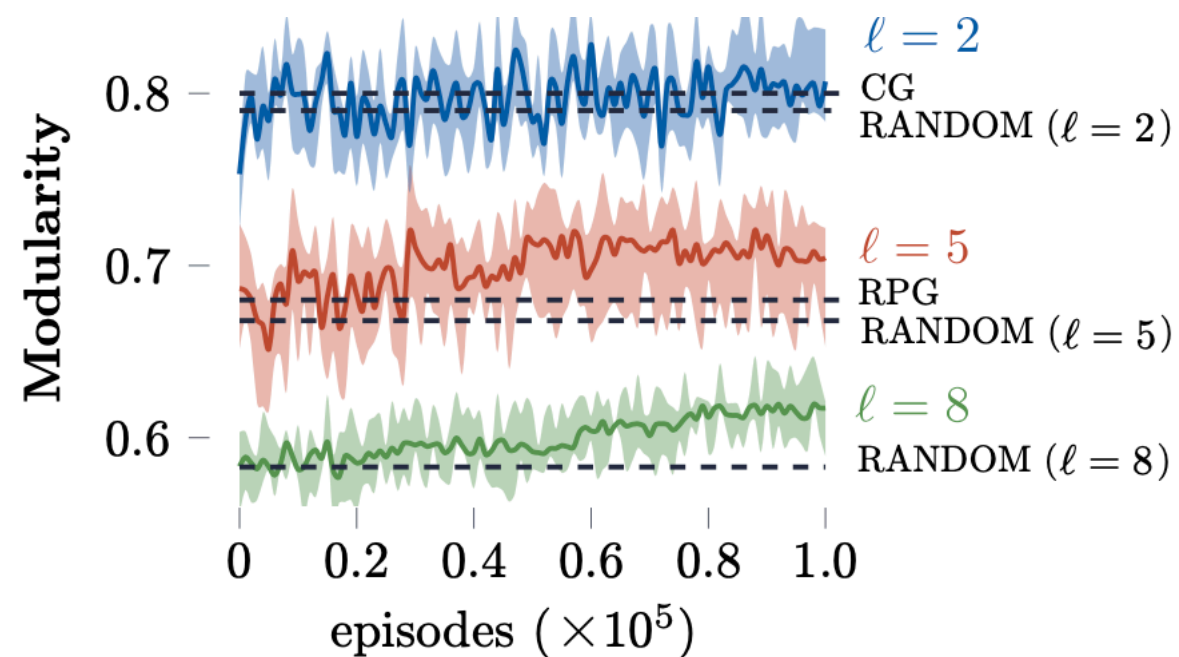
Emergence of Community Structure

Intuition: Community structure emerges when **all** agents are **only** in pursuit of **bonding** capital.

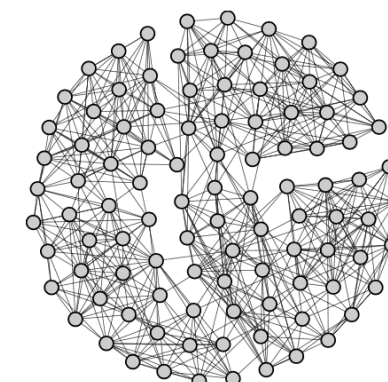
Preference weights setting: $w_i = 1$ for all $i \in N$.

Indicator of community structure: **modularity**.

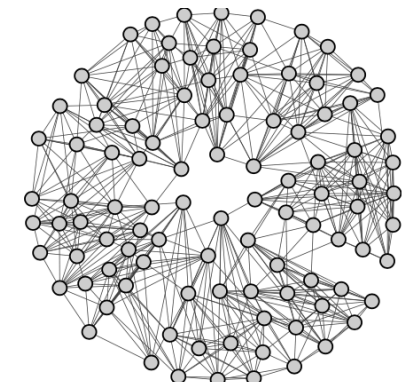
Baselines: **Caveman graph** (CG) model and **random partition graph** (RPG)



configuration



episode 10^4



episode 10^5

Results

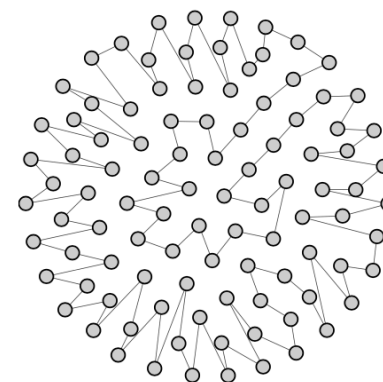
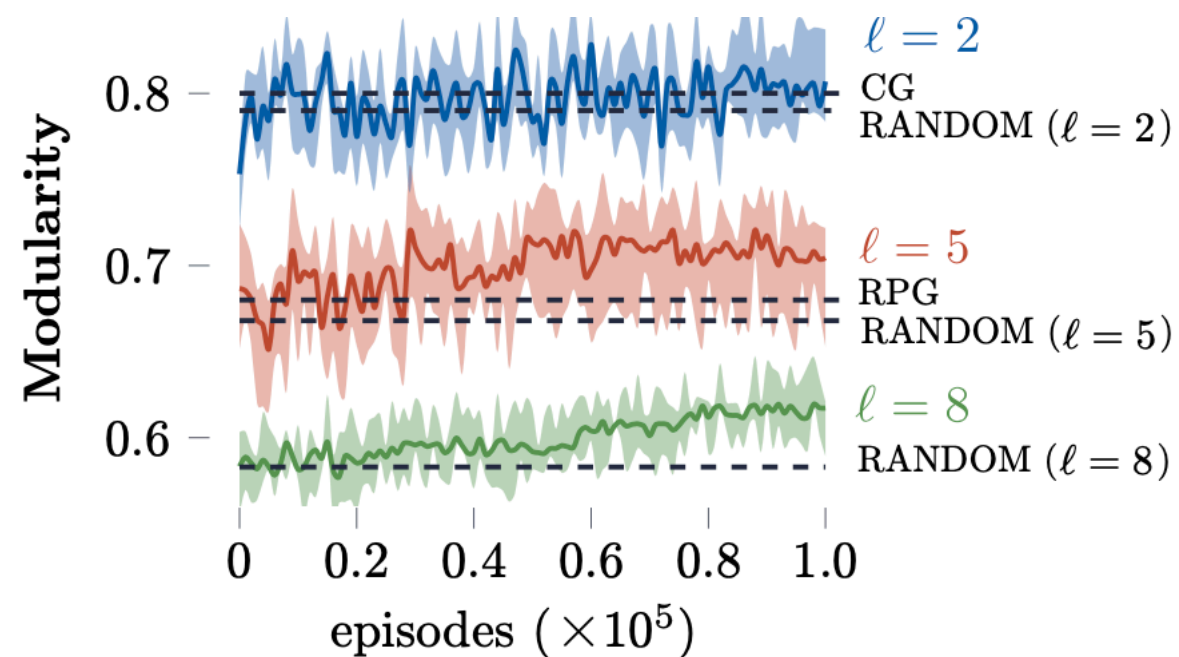
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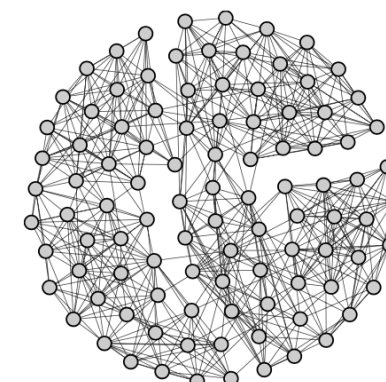
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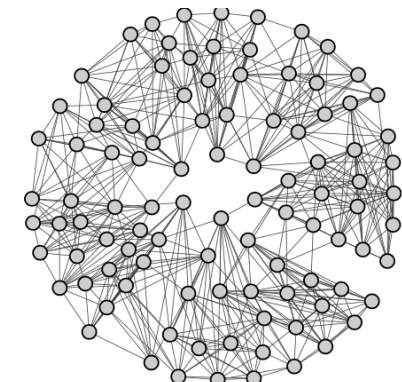
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configuration



episode 10^4



episode 10^5

Result: We observe the emergence of community structure with high modularity!

Results

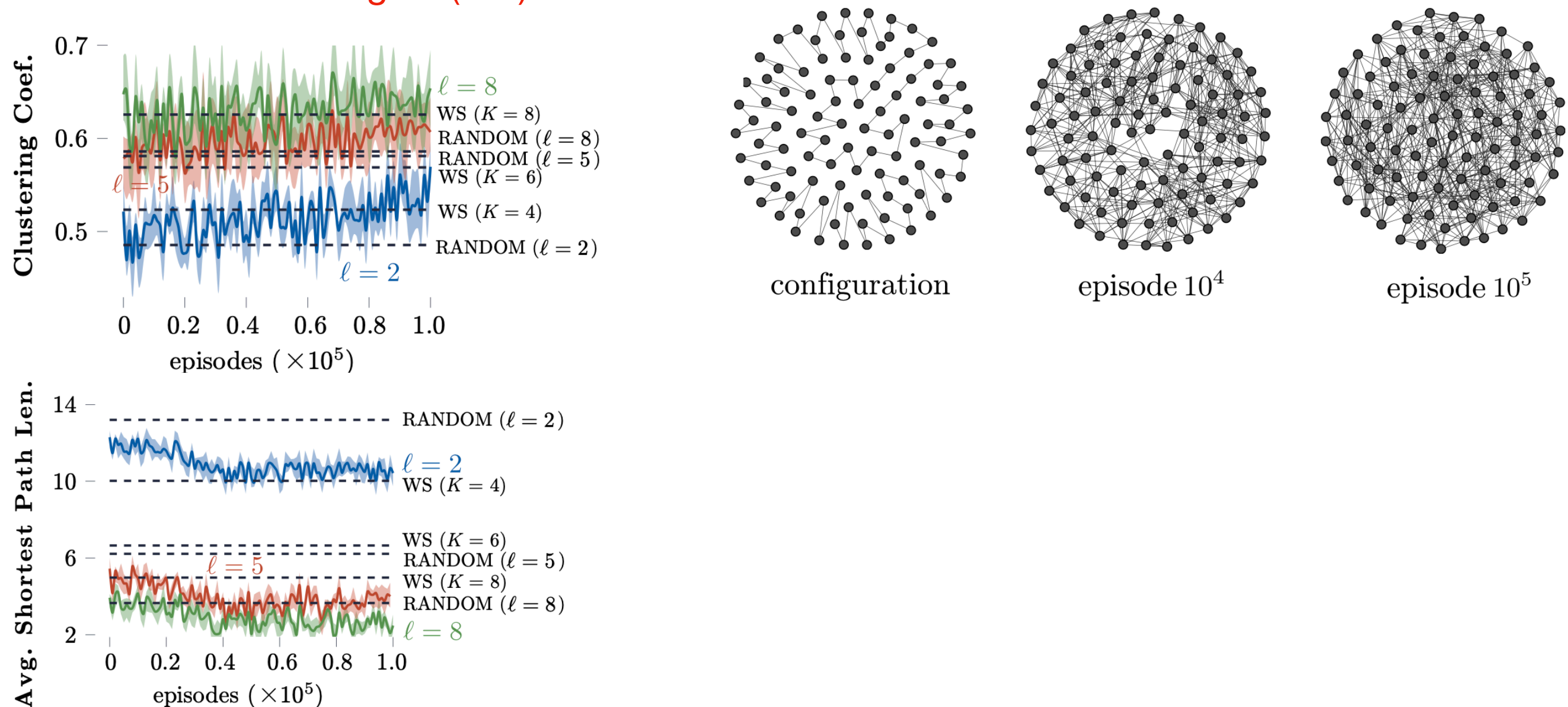
Emergence of Small World Structure

Intuition: Small-World structure emerges when **all** agents are **only** in pursuit of **bridging** capital.

Preference weights setting: $w_i = 0$ for all $i \in N$.

Indicators of small-world structure: **clustering coefficient** and **avg. shortest path length**.

Baselines: **Watts-Strogatz (WS) model**.



Results

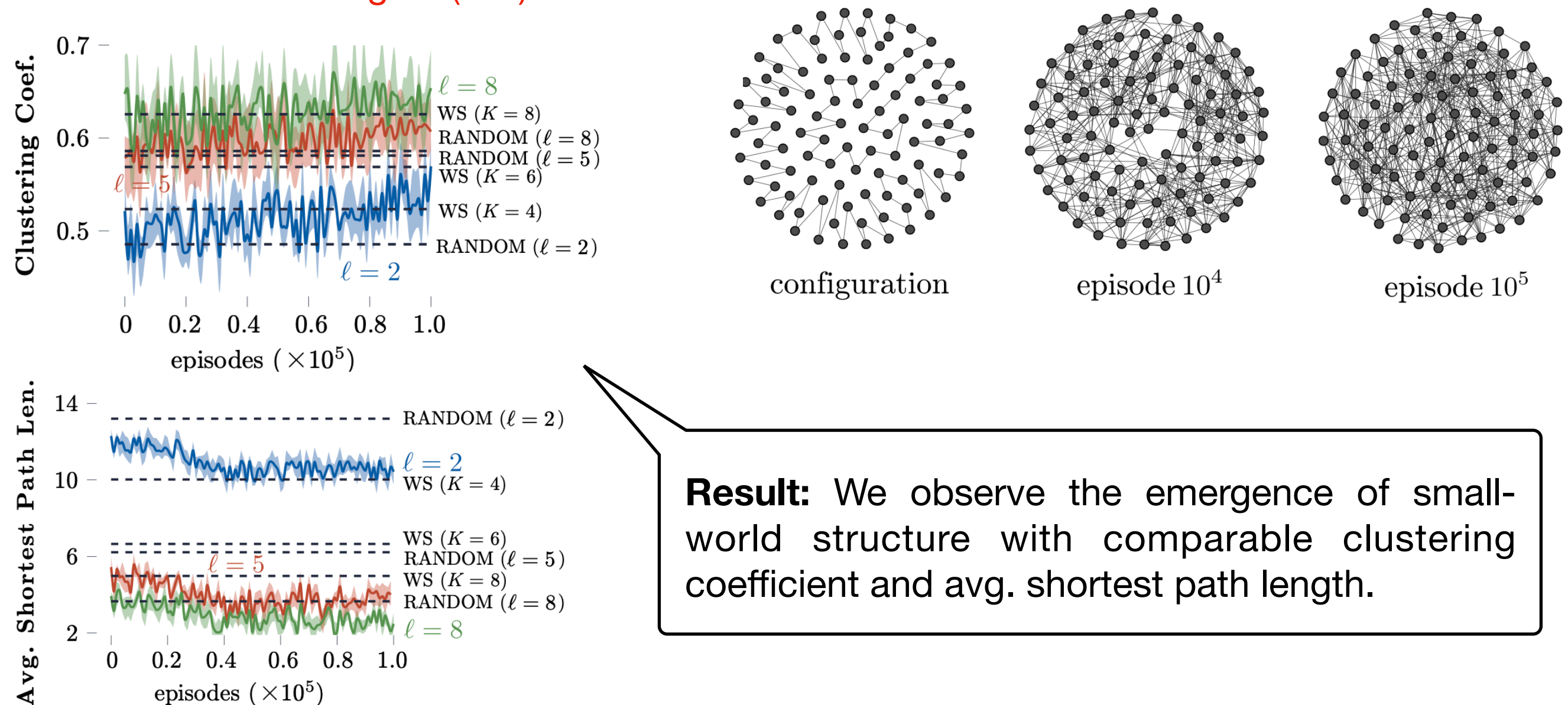
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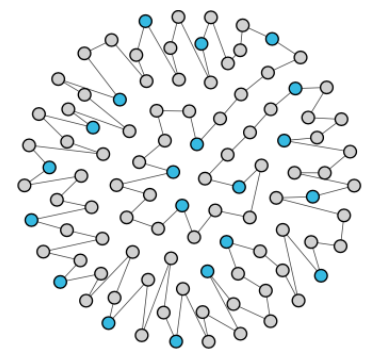
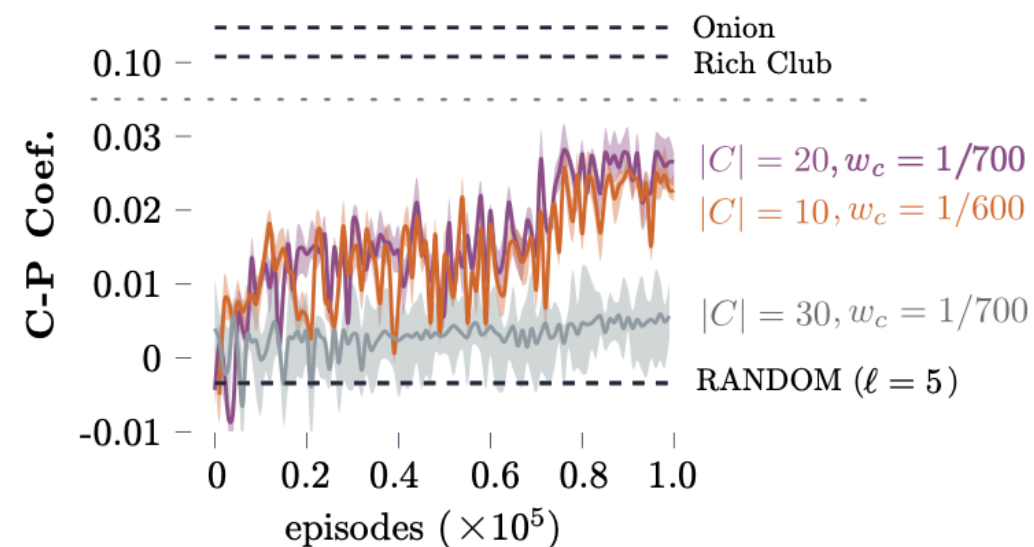
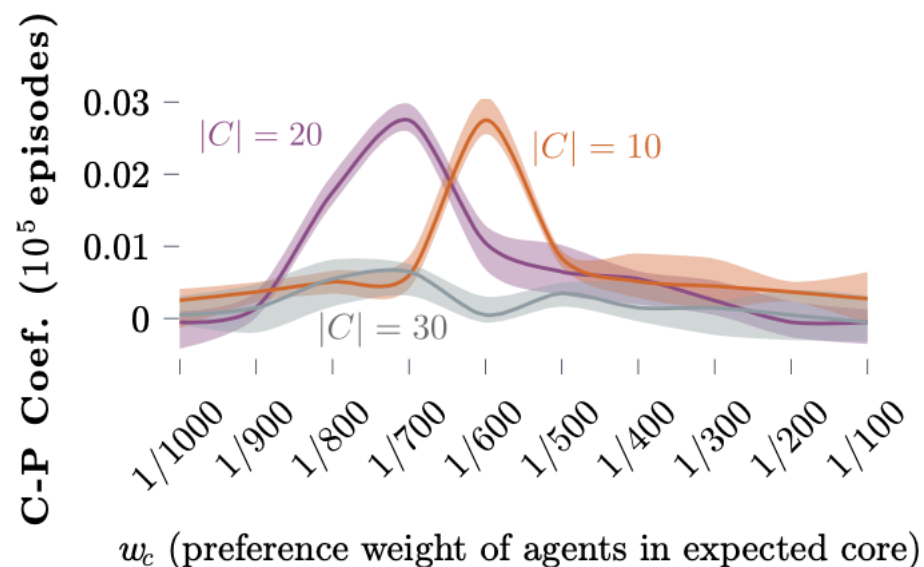
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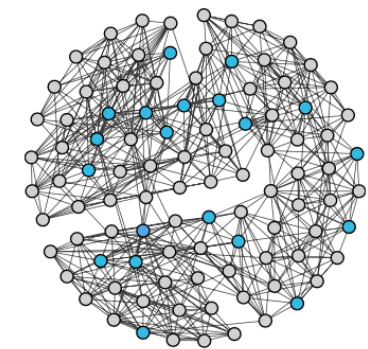
Results

Emergence of Core-Periphery Structure

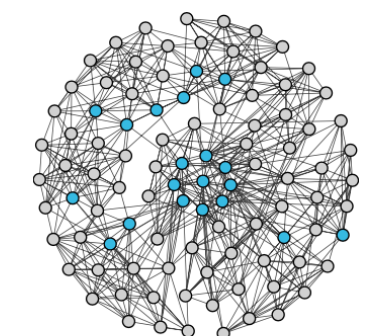
- **Intuition:** Core-periphery structure emerges when a **group** of agents (periphery) are only in pursuit of bonding capital, while the other **group** of agents (core) show **large** preferences to bridging and **small** preference to bonding capital.
- **Preference weights:** Randomly select a subset $C \subset N$ (expected core), for all agents $c \in C$, set w_c to **small** values. For all remaining agents $p \in N \setminus C$, set $w_p = 1$.
- **Indicators of C-P structure:** **core-periphery coefficient**.
- **Baselines:** **rich club model** and **onion model**.



configuration



episode 10^4

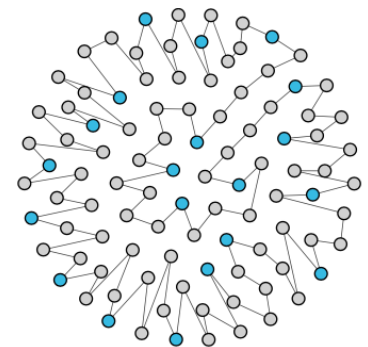
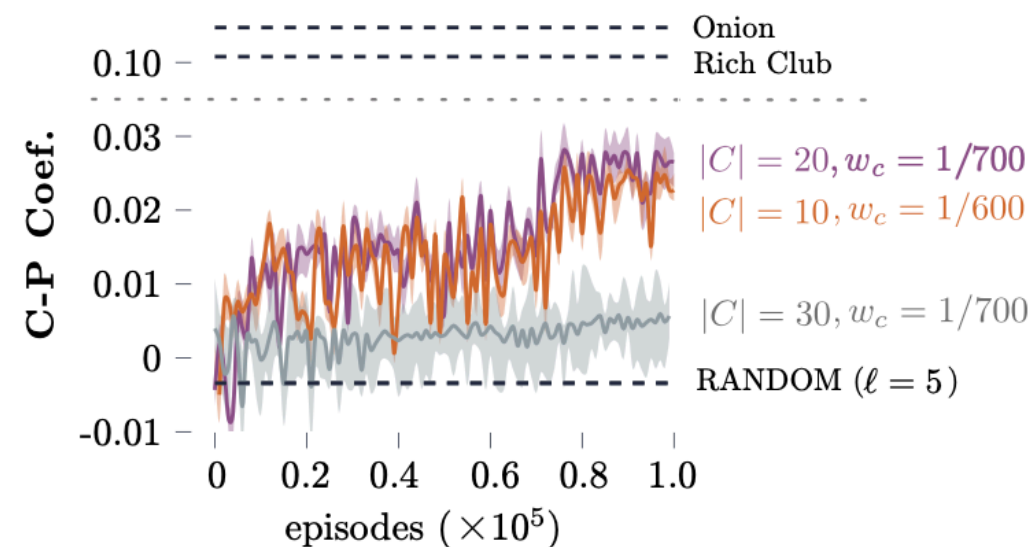
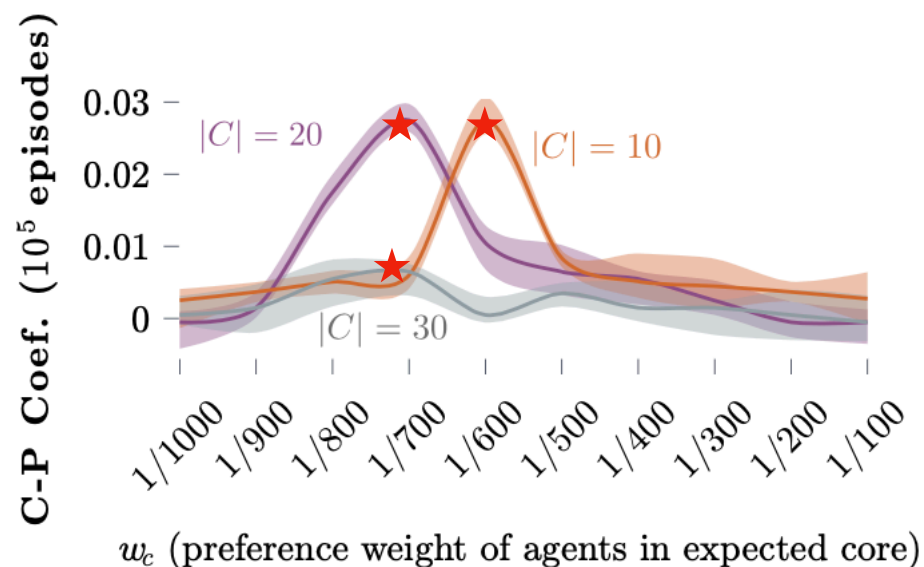


episode 10^5

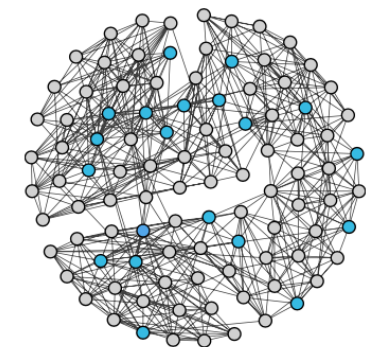
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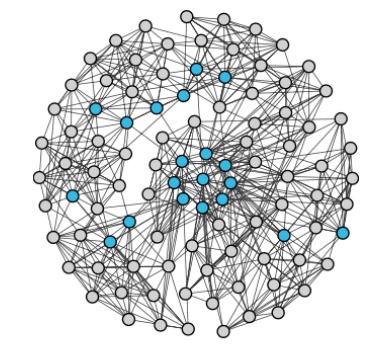
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configuration



episode 10^4

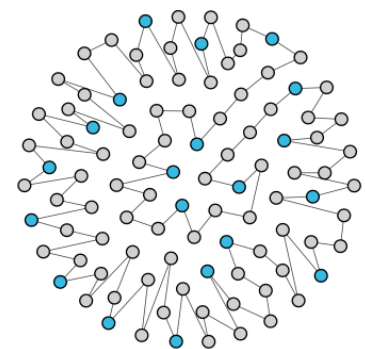
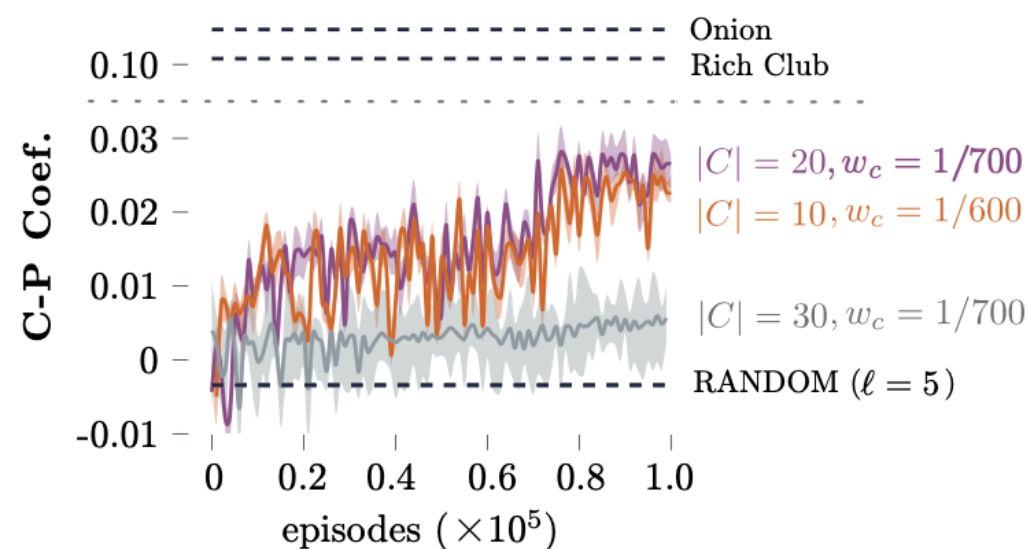
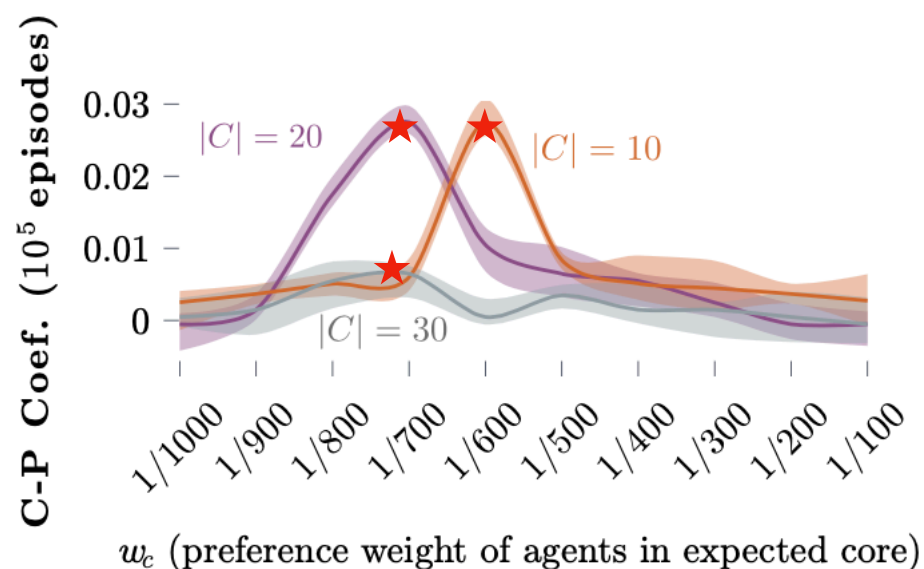


episode 10^5

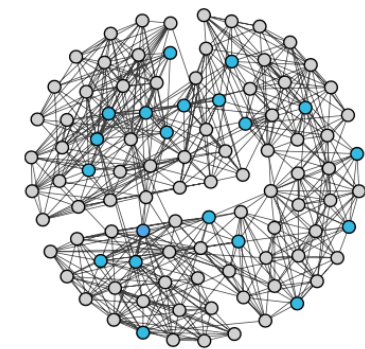
Results

Emergence of Core-Periphery Structure

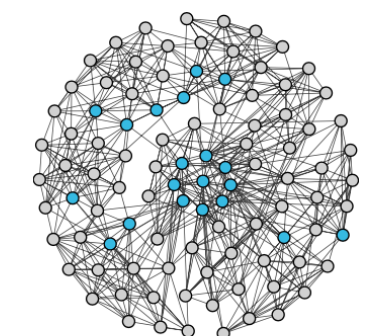
- **Intuition:** Core-periphery structure emerges when a **group** of agents (periphery) are only in pursuit of bonding capital, while the other **group** of agents (core) show **large** preferences to bridging and **small** preference to bonding capital.
- **Preference weights:** Randomly select a subset $C \subset N$ (expected core), for all agents $c \in C$, set w_c to **small** values. For all remaining agents $p \in N \setminus C$, set $w_p = 1$.
- **Indicators of C-P structure:** **core-periphery coefficient**.
- **Baselines:** **rich club model** and **onion model**.



configuration



episode 10^4



episode 10^5

Result: We observe the emergence of core-periphery structure under proper settings.

Conclusion

1. **A novel model for network formation:** social capital games.
2. **A new utility concept:** two types of social capital.
3. **A learning method:** introduce MARL to network formation.
4. **A unified explanation for the emergence of social structures:** our framework unifies the explanation for the natural emergence of classical social network structures.

Thank you!

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