ANALYSIS OF KALMAN FILTERING TECHNIQUE FOR GNSS POSITION ACCURACY IMPROVEMENT

A Project Report Submitted By

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in partial fulfilment of the requirements for the award of the Degree of

Bachelor of Engineering
in
Electronics and Communication Engineering

from
Visvesvaraya Technological University, Belagavi

NITTE EDUCATION TRUST | NMAM INSTITUTE OF TECHNOLOGY

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May-June 2019

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Abstract

GPS satellites are used to calculate the geo-location information of a GPS receiver on or near the surface of the earth. These receivers work by triangulating the signal by having an unobstructed line of sight with a minimum of 4 GPS satellites orbiting the earth at any given time. The positioning accuracy of these GPS receivers vary widely with respect to the cost, with low-cost receivers having accuracy around few hundred meters to high-cost receivers having accuracy around a few centimetres. The proposed project deals with improving the accuracy of low-cost GPS receivers by a significant amount by incorporating sensor data from other parameters through the process of Kalman Filtering. Kalman Filter uses a predictive approach to estimate the position of the object by combining speed and heading information. The result of the implementation has been analysed to prove that there is an improvement in accuracy of low-cost GPS receivers.

Acknowledgement

This project would not have been successful without the encouragement, guidance and support by various personalities. First and foremost, I would like to express my sincere gratitude towards my project guide **Dr. Rekha Bhandarkar**, Professor and Head, Department of Electronics and Communication Engineering, N. M. A. M. Institute of Technology, Nitte, for her guidance and inspiration throughout the project work.

I wish to acknowledge the support of **Dr. Niranjan N. Chiplunkar**, Principal, N. M. A. M. Institute of Technology, Nitte, for providing motivational academic environment.

My sincere gratitudes are due to **Dr. Subramanya Bhat**, Associate Professor, Department of Electronics and Communication Engineering, N. M. A. M. Institute of Technology, Nitte, and the Project Co-ordinator who has supported me throughout the course of the project.

I would like to express my immense gratitude to **Accord Software and Systems Pvt. Ltd., Bangalore**, for its all-round support.

I express my sincere thanks to **Mr. Balakrishna S.**, Principal Technical Leader - Systems, Accord Software and Systems, Pvt. Ltd., Bangalore, for support and assistance during the course of this project.

I would like to express my sincere thanks to all the teaching and non-teaching staff of the department of Electronics and Communication Engineering, N. M. A. M. Institute of Technology, Nitte, for their patience and help during my project work.

Finally, I would like to thank all my friends and well-wishers who have helped me whenever needed and supported me.

Mahesh Devadiga

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Chapter 1

Introduction

1.1 General Introduction

GPS which stands for Global Positioning Service is a satellite-based radio-navigation system that is used to provide geo-location service to any GPS receiver on or near the surface of the Earth. At present, GPS has been used in a lot of fields including military, navigation, aviation, transportation and surveying. Over the recent years the accuracy of the GPS receivers have improved significantly from having positioning errors from meters to centimetres. But with the improvement in accuracy, the cost of these GPS receivers have also increased. In the field of commercial marine navigation, low-cost low-precision GPS receivers are used because accuracy is not a priority. The cost of GPS receivers vary based on their positioning accuracy. The project aims at improving the accuracy of low-cost GPS receivers in ships by fusing data from other parameters on-board the ship through the process of Kalman Filtering to provide a position estimate which is more precise than the measurement recorded by the GPS receiver alone.
Kalman Filter uses a set of measurements over time, and produces estimates of unknown variables that tend to be precise than those based on single measurements. It is a two-step process. The first step called the prediction step makes an estimate based on the dynamics of the system. The next step is called the correct step, where based on the uncertainty of the system and the difference between the predicted and sensor measurements, the estimate is corrected. The speed and heading measurements are extracted from the sensors which are more accurate. By using these measurements and incorporating them in the filter, a more accurate estimate of the position can be calculated.

1.2 Objective

To improve the positioning accuracy in ships which use low-cost low-precision GPS receivers.

1.3 Problem Statement

In the field of marine navigation, location data of the ship is of utmost importance. GPS provides the fastest method for mariners to navigate and determine location. High-precision GPS receivers are used in most ships which tend to be very costly. Sometimes in small ships and cruisers, low-precision GPS receivers are preferred, which have an accuracy of about a few hundred meters. While at sea, accurate position is needed to ensure the vessel reaches its destination in the safest, most economical way possible. The goal of this project is to find an economical method of improving the accuracy of the low-cost GPS receivers.

1.4 Proposed Method/Technique

The proposed method involves improving the positioning accuracy of low-cost receivers by using data from speed and heading sensors. The data is used to make a optimal estimate through Kalman Filtering.

1.5 Methodology

The proposed method involves using two low-cost GPS receivers placed on the ship. The speed and heading data are available from sensors installed on board the ship. The data from speed, heading and GPS sensors are fed into the Kalman Filter which is designed to output an optimal estimate based on the dynamics of the system. The stepwise approach is given below

- Acquisition of GPS data in terms of lat/lon from the GPS receivers, and speed, heading data from sensors onboard the ship.

- Converting the data received to a common frame of reference - ECEF frame of reference.

- Get the position estimate using Kalman Filter with the converted data as inputs.

1.6 Literature Survey

Minghui Zhao, Jianhua Wang, Shanjia Zhang and Chen Zhang published a paper ”Method for improving positioning accuracy by using double low-precision GPS” [1]. It made an attempt to improve positioning accuracy using two low-cost GPS receivers. These two receivers were placed at the extreme ends of an unmanned boat. In the proposed system, the centre position was calculated by averaging the data from the two sensors and the result was then passed through a Kalman Filter. The filter was designed in such a way to provide better estimate using two low accuracy GPS receivers. The algorithm designed was tested on an unmanned boat and the results implied a improvement in positioning accuracy.

Simon D. Levy posted an on-line article ”The Extended Kalman Filter: An Interactive Tutorial for Non-Experts” which explains how Kalman Filter can be designed to combine direct and indirect data from multiple sensors to provide a better estimate. It showed insights on how a Kalman Filter can be designed to implement sensor fusion, wherein a better estimate is calculated based on various other indirect data. [2]

K. Rajaduraimanickam and J. Shanmugam and G. Anitha published a paper ”ADDR-GPS data fusion using Kalman filter algorithm”. The paper discusses methods on combining data from Heading of an UAV vehicle with GPS to improve its positioning accuracy. The Kalman Filter was designed to take into consideration multiple other sensor measurements like the gyroscopic, acceleration, pose, speed, heading and altitude to provide a better Kalman estimate. Compared to other indirect measurements, the speed and heading data were given a better preference. The results showed a significant improvement in positioning accuracy. [3]

1.7 Organization of the Report

Chapter 2 briefs about the design of the Kalman Filter
Chapter 3 discusses about the software tools used
Chapter 4 gives an overview of the system
Chapter 5 explains the Algorithms and Flowcharts
Chapter 6 explains the Results
Chapter 7 gives Conclusion of the Project

Chapter 2

Design Of The Kalman Filter

2.1 Kalman Filter

The Kalman Filter uses series of measurements over time, and produces estimates of unknown variables that tend to be more precise than those based on single measurements alone. It operates on streams of noisy data to produce an optimal estimate of the underlying system state. The Kalman Filter uses a prediction followed by correction in order to determine the states of the filter. The main concept of the Kalman Filter is that by knowing the dynamics of the state, the filter will predict the next state. The correction part involves adjusting the predicted state, by scaling the difference between the measured and the predicted value. The recursive loop of the predict-correct process is shown in Figure 2.1

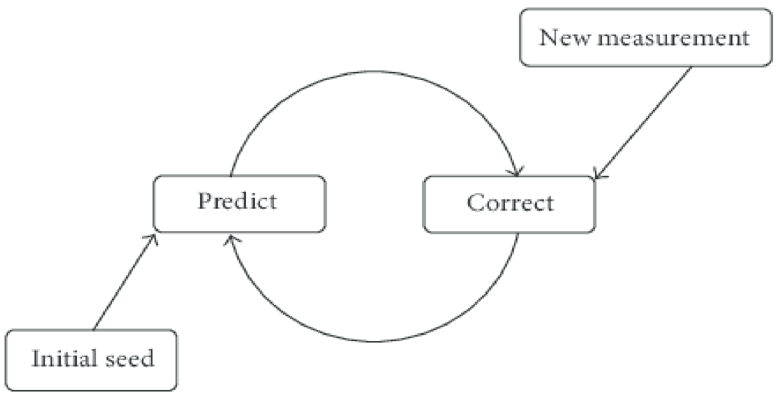


Figure 2.1: Cycle of Predict and Correct Process [4]

The Kalman Filter is explained in terms of equations below. The predict-correct format is applied recursively at each time step. First, the state vector is predicted from the state dynamics in eq 2.1

$$\hat{x}_k = A\hat{x}_{k-1} + Bu_{k-1} \tag{2.1}$$

where $\hat{x}_k$ is the predicted state vector, $\hat{x}_{k-1}$ is the previous estimated state vector, u is the input vector, and A and B are the system matrices.
The state error covariance matrix is also predicted by using eq. 2.2

$$P_k = A_{k-1}P_{k-1}A_{k-1}^T + Q \tag{2.2}$$

where P_k is the predicted state covariance matrix, P_{k-1} is the previous estimated state covariance matrix, and Q is process noise covariance matrix. Once the predicted equations are obtained, the Kalman Gain Matrix is calculated using eq. 2.3.

$$K_k = P_k H_k^T (H_k P_k H_k^T + R_k)^{-1} \tag{2.3}$$

where H is called the observation matrix which is responsible for defining the output equation and R is the measurement noise covariance.
The state vector is then updated by scaling the difference between the measurement of output, z_k, and the predicted output, $H_k x_k$ as defined in eq. 2.4

$$\hat{x}_k = \hat{x}_k + K_k(z_k - H_k\hat{x}_k) \tag{2.4}$$

The state covariance is then updated by using eq. 2.5

$$P_k = (I - K_k H_k)P_k \tag{2.5}$$

where I is identity matrix.

2.1.1 Effect Of Noise Covariance Assumption

The covariance matrices R, Q and P_0 have a significant effect on the performance of the Kalman Filter. Considering a large Q implies considering a high uncertainity in the state equations, which means that the predicted states are not reliable and should be corrected more with the sensor measurements. Similarly, considering a large R implies a large uncertainity in the measurement, which means that the sensor measurements are not reliable, and the predicted states should be corrected less. A diagram detailing the phenomenon is shown in Figure 2.2

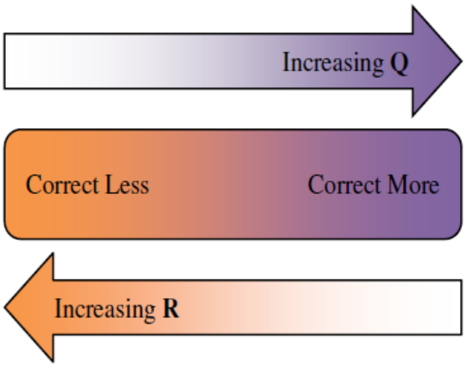


Figure 2.2: Noise Covariance Assumptions Effect on Filter Operation [5]

2.2 Design and Calculation of Parameters necessary for the Filter

The goal of the Kalman Filter for the project is to provide a better position estimate by using data from GPS, speed and heading sensors. The Kalman Filter is a linear estimator and works only for linear systems. The GPS receivers output data in terms of latitude/longitude which is a non-linear system. Also the speed and heading data are in m/s and $degrees$ also. These parameters are converted to a common frame of reference, which is the Earth-Centered-Earth-Fixed(ECEF) frame of reference, which is explained in detail in Appendix A. Using kinematics, the equations to predict the next position based on the velocity of the system is shown in eqs. (2.6a) to (2.6c). The equations eqs. (2.6d) to (2.6f) represent the calculations of predicted state velocity.

$$pos(x)_k = pos(x)_{k-1} + vel(x)_{k-1} * \Delta t \tag{2.6a}$$
$$pos(y)_k = pos(y)_{k-1} + vel(y)_{k-1} * \Delta t \tag{2.6b}$$
$$pos(z)_k = pos(z)_{k-1} + vel(z)_{k-1} * \Delta t \tag{2.6c}$$
$$vel(x)_k = vel(x)_{k-1} + \Delta V_x \tag{2.6d}$$
$$vel(y)_k = vel(y)_{k-1} + \Delta V_y \tag{2.6e}$$
$$vel(z)_k = vel(z)_{k-1} + \Delta V_z \tag{2.6f}$$

where $pos(x)$, $pos(y)$ and $pos(z)$ are the positions in the ECEF coordinate system. $vel(x)$, $vel(y)$ and $vel(z)$ are the velocity components in the ECEF frame. ΔV_x, ΔV_y and ΔV_z are the change in velocity.

The state vector x, contains values which will be estimated by the filter. Therefore we have the matrix eq. (2.7)

$$x_k = \begin{bmatrix} pos(x)_k \\ pos(y)_k \\ pos(z)_k \\ vel(x)_k \\ vel(y)_k \\ vel(z)_k \end{bmatrix} \tag{2.7}$$

Now writing eqs. (2.6a) to (2.6f) in matrix form, we have matrix eq. (2.8)

$$\begin{bmatrix} pos(x)_k \\ pos(y)_k \\ pos(z)_k \\ vel(x)_k \\ vel(y)_k \\ vel(z)_k \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} pos(x)_{k-1} \\ pos(y)_{k-1} \\ pos(z)_{k-1} \\ vel(x)_{k-1} \\ vel(y)_{k-1} \\ vel(z)_{k-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{bmatrix} \tag{2.8}$$

Comparing eq. (2.7) with eq. (2.1) we get,

$$A = \begin{bmatrix} 1 & 0 & 0 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta t & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta t \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{2.9}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \Delta V_x \\ \Delta V_y \\ \Delta V_z \end{bmatrix} \tag{2.10}$$

Since low-cost GPS recievers are used, their accuracy range around 100 meters. So the position error covariances($\sigma_x^2, \sigma_y^2, \sigma_z^2$) are assumed to be $10000m^2$. Also the speed and heading sensors are far more accurate and their variances($\sigma_v el(x)^2, \sigma_v el(y)^2, \sigma_v el(z)^2$)

are assumed to be around $25m^2$. So we have,

$$R = \begin{bmatrix} 1000000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 10000 & 0 & 0 & 0 & 0 \\ 0 & 0 & 10000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 25 & 0 \\ 0 & 0 & 0 & 0 & 0 & 25 \end{bmatrix} \tag{2.11}$$

The value of Q is set based on the performace of the filter which is determined by studying the filter results.

Chapter 3

Software Description

3.1 MATLAB

MATLAB or Matrix Laboratory is a programming language developed by MathWorks. MATLAB is primarily intended for numerical computations and is used to solve many technical computing problems, especially those that involve matrices and vector formulations, in a fraction of the time it would take to write a program in a scalar non-interactive language such as C or C++. It allows matrix manipulations, plotting of functions and data, implementation of algorithms and interactive toolboxes. Since the project involves Kalman filter which has recursive matrix calculations, MATLAB is very suitable in this environment. It also has a vast variety of inbuilt functions which can be used to format data and functions. It also allows easy plotting of variables which allows easy understanding of the process.

Chapter 4

System Overview

The proposed method involves collecting raw data from the two low-precision GPS receivers placed at the extreme ends on the hull of the ship. The data from these two receivers are averaged, which helps improve the signal-noise-ratio(SNR) of the signal. The speed and heading data taken from the on-board sensors, are split into speed in the north and east components. The GPS data received is in the geodetic coordinate system(Appendix A) and the velocity is in the ENU coordinate system (Appendix A). For defining the state dynamics of the filter, the GPS and velocity data are converted to a common frame of reference, which is the ECEF coordinate system (Appendix A).

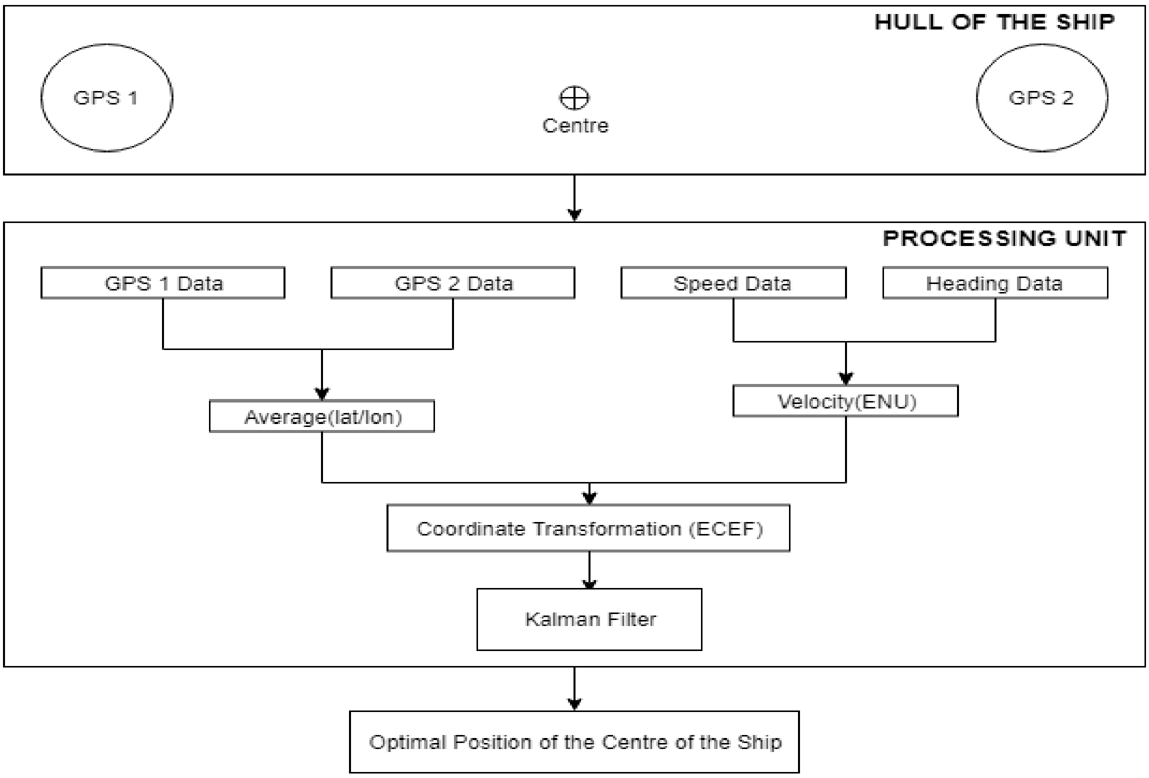


Figure 4.1: Block Diagram of The Proposed System

All the necessary data now being in the ECEF coordinate system are passed to the Kalman Filter, where the filter calculates the optimal estimate of the position. The parameters necessary for the filter are designed based on the dynamics of the system. The resulting position estimate is then converted back to geodetic coordinate system, and the filtered position is displayed. The block diagram of the proposed system is shown in Figure 4.1

Chapter 5

Algorithm and Flowchart

5.1 Coordinate Conversion

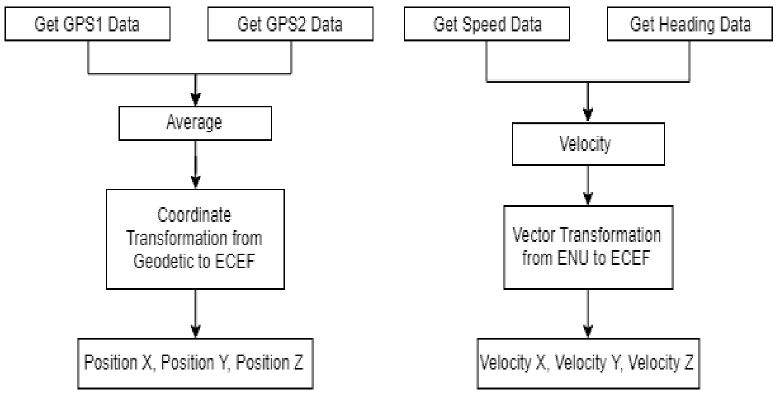


Figure 5.1: Flowchart of the Coordinate Conversion Process

The proposed method has two coordinate conversion processes. Since the Kalman Filter is a linear estimator, the equations used to define the dynamics of the system must also be linear. But the positional data we receive from the GPS sensors are in the format of latitude, longitude and altitude, which is a non-linear system. Hence necessary coordinate transformations are made to convert from geodetic to ECEF system. This applies even for the speed and heading data, which is in the ENU coordinate system. All data from the sensors are converted to a common ECEF frame of reference. The flowchart of the coordinate conversion process is shown in Figure 5.1

5.2 Kalman Filter

The Kalman Filter needs to calculate three main parameters, which are the Kalman gain, current estimate and error in estimate. These calculations are recursive.

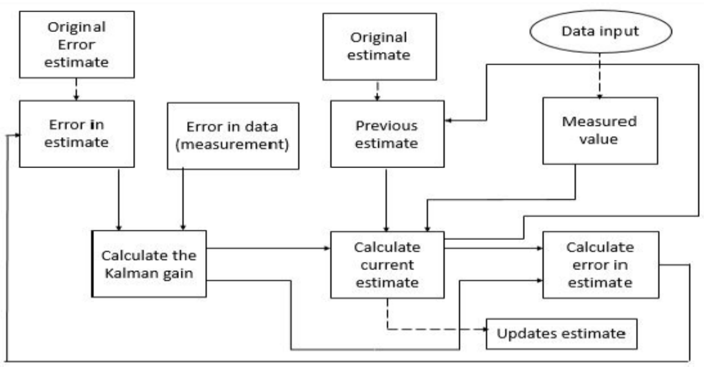


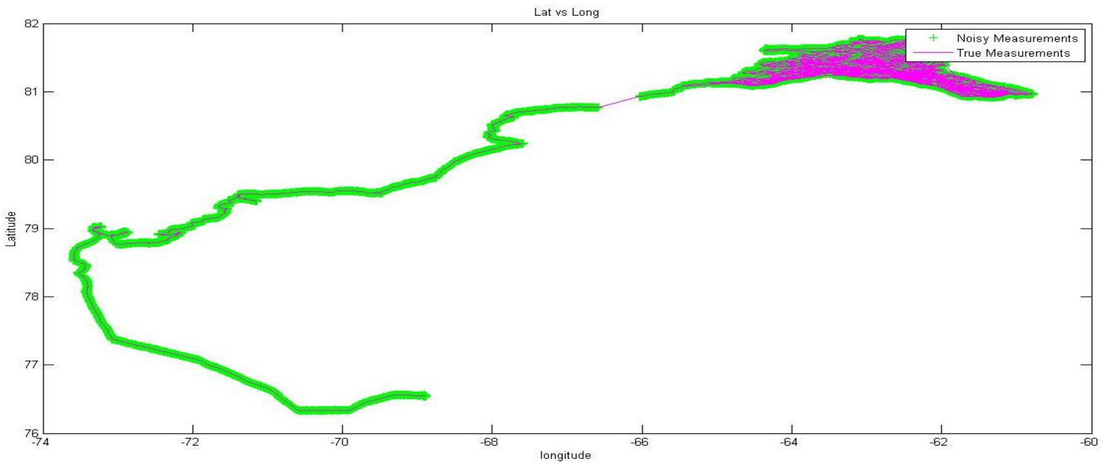
Figure 5.2: Flowchart of the Kalman Filter Process

The Kalman gain is calculated based on the error in the measurements. After the Kalman gain is calculated the current estimate is calculated based on the previous estimate. At the end, the result from the current estimate is used to update the previous estimates to the system. The flowchart of the Kalman Filter process is shown in Figure 5.2

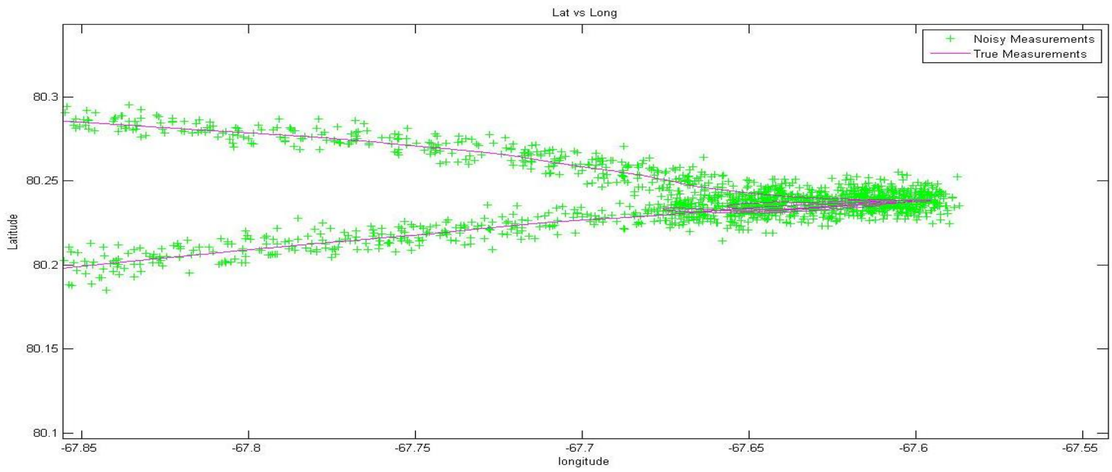
Chapter 6

Results

The project was aimed at proposing a method to improve accuracy in low-cost GPS receivers by combining the data from speed and heading sensors. The algorithm developed was tested on dataset generated by a high-precision GPS receiver. It contains 6,00,000 samples of data which provide the position of the ship in time intervals of $5\ ms$. This dataset also includes measurements of other parameters such as the speed, heading, pressure, wind-direction, current speed, etc. recorded by the control panel of the ship at the same instants. Gaussian errors are added to the positional data in the dataset to simulate a low-precision GPS receiver. The gaussian errors added are of 0 mean and having variance of about 100 meters. The original data with no noise referred to as the true measurement is used to compare the results of the Kalman Filtered data.
The data logged by the high-precision GPS receiver is now converted to low-precision GPS receiver data as a result of adding gaussian errors which are shown in Figure 6.1a to Figure 6.1c.
The low-precision GPS data is then passed through the Kalman Filter algorithm and the results are compared with the high-precision GPS data as shown in Figure 6.2a to Figure 6.2c.
A more detailed plot of latitude and longitude data with respect to time is shown in Figure 6.3a to Figure 6.3c.
The error in the filtered positional data and the data from individual low-precision GPS receivers with respect to the true measurement is compared in Figure 6.4a and Figure 6.4b.
A significant improvement in accuracy of the positional data is observed.



(a)



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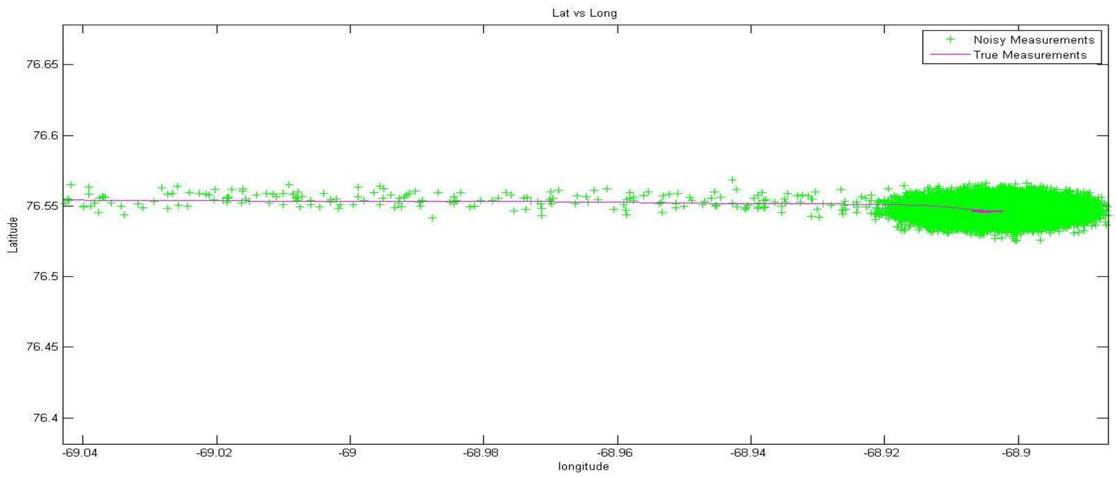
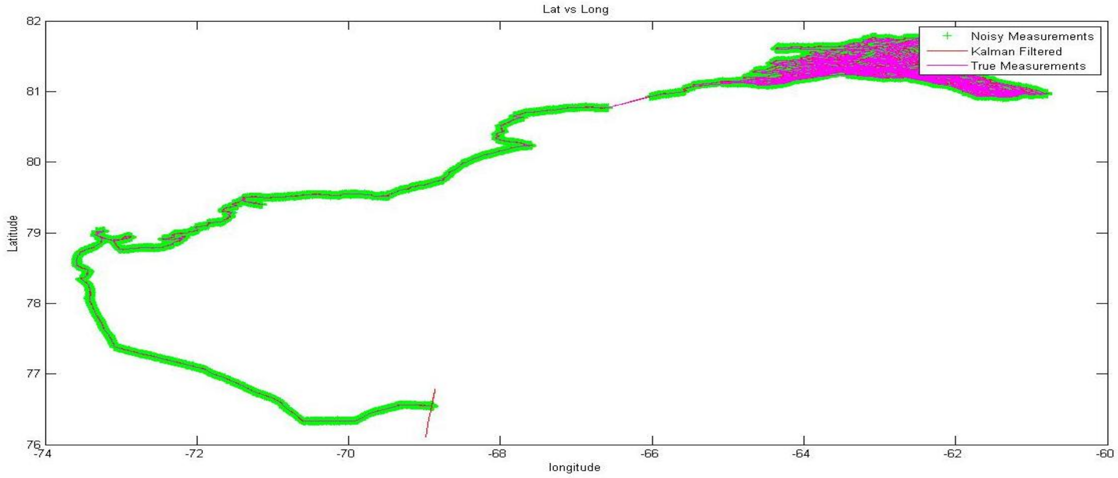


(c)

(d) Plot of data logged by the GPS receiver and the low-precision version simulated by adding gaussian errors



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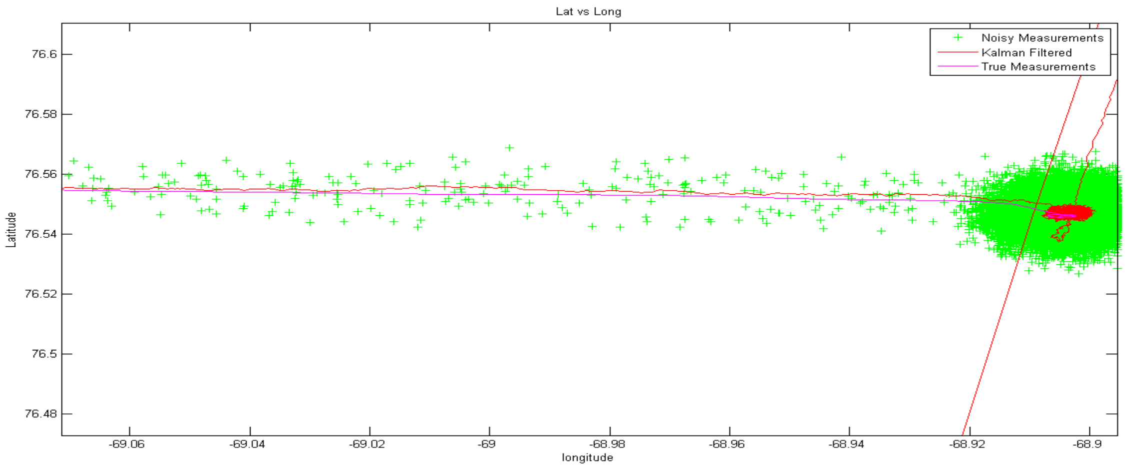

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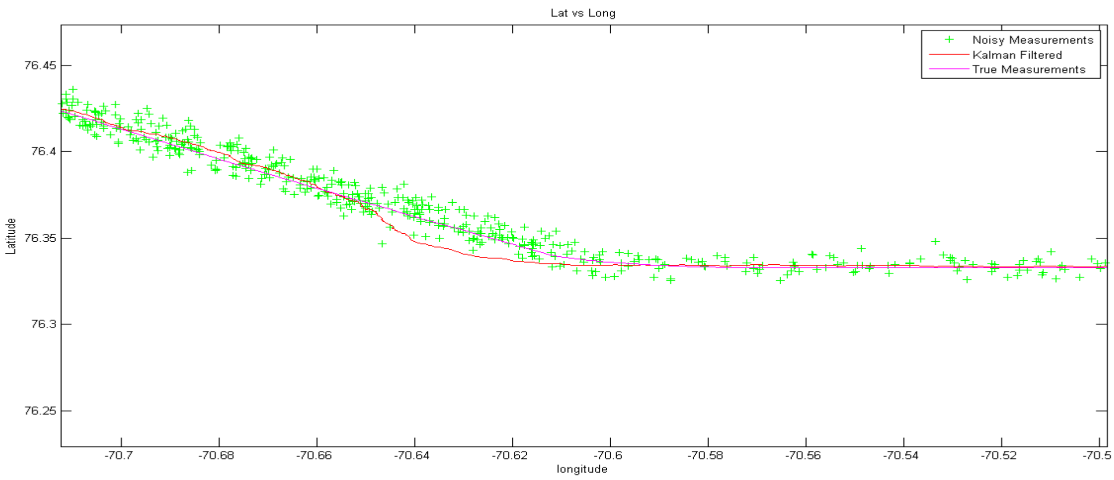
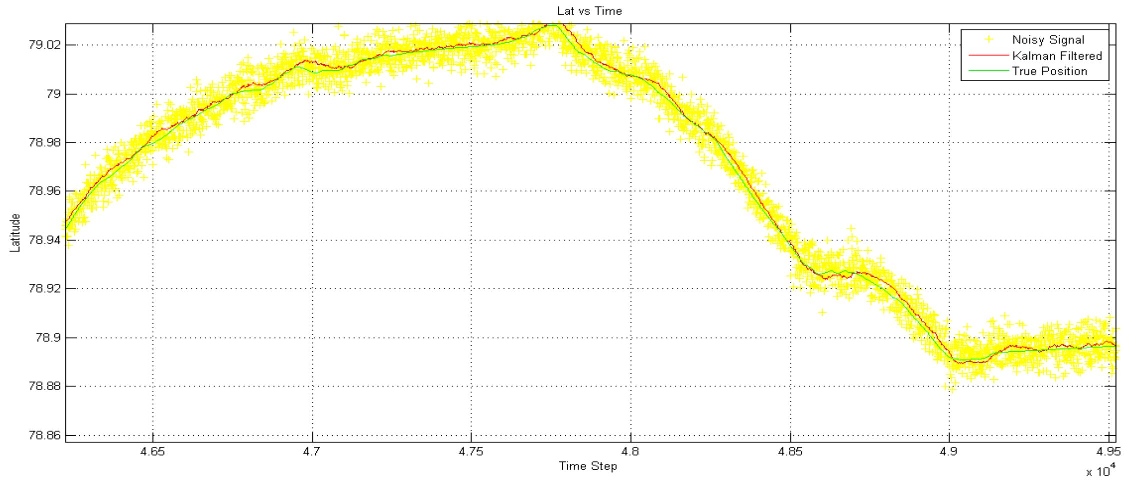


(c)

Figure 6.2: Plot of optimal Kalman estimate with respect to the low-precison GPS measurements



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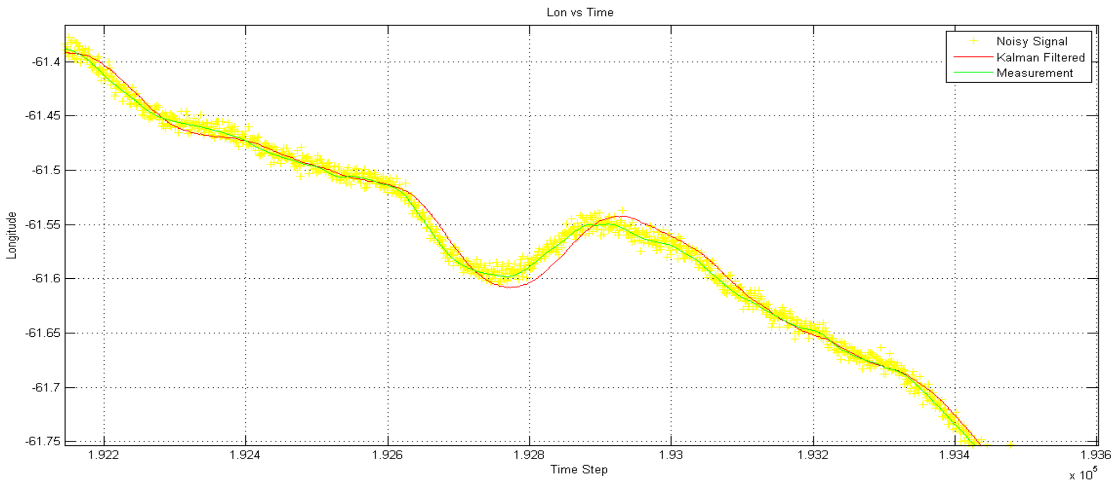

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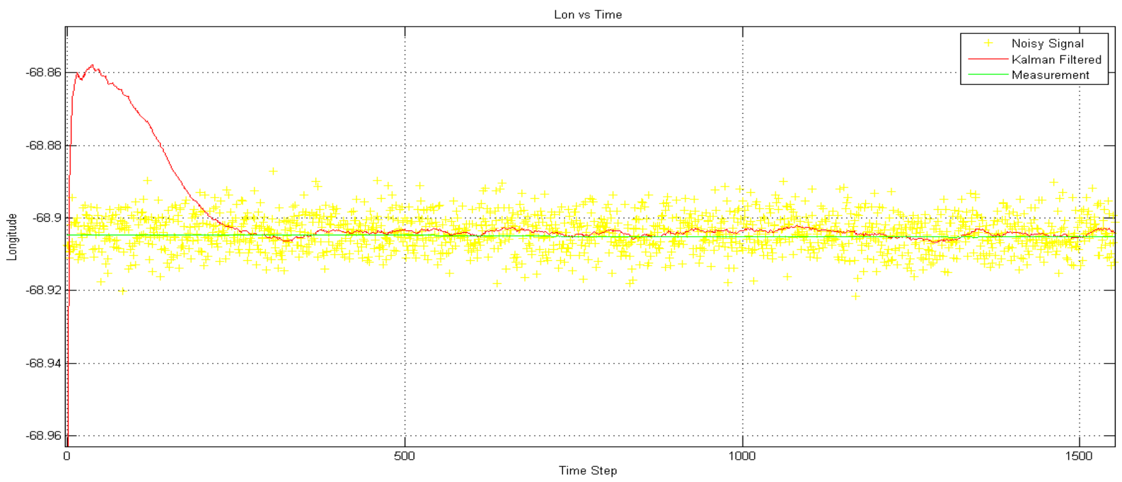
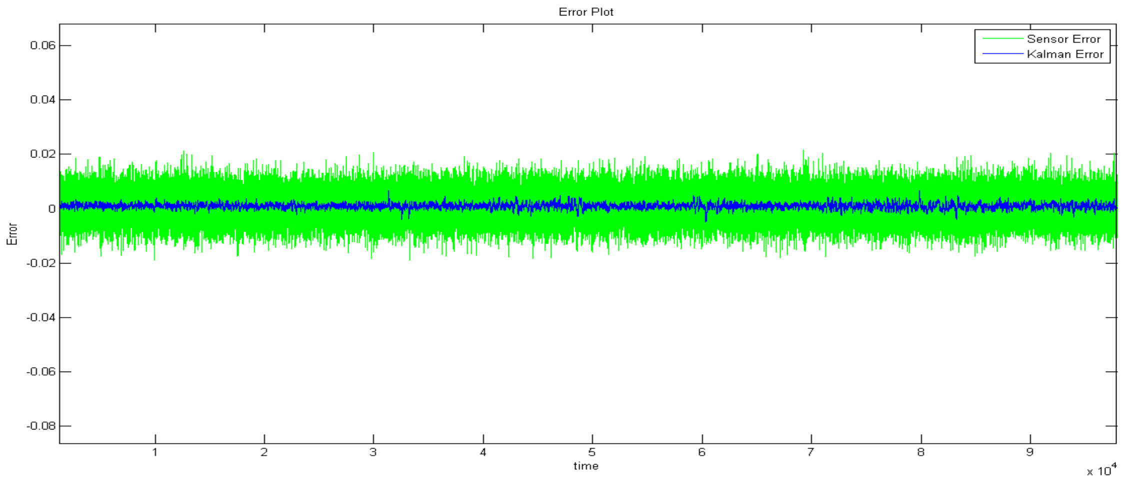


(c)

Figure 6.3: (a) Plot of low-precison sensor latitude data and the Kalman Filtered Estimate. (b) Plot of low-precison sensor longitude data and the Kalman Filtered Estimate. (c) Kalman Estimate (longitude) converging to the true measurement. (d) Kalman Estimate (latitude) converging to the true measurement.



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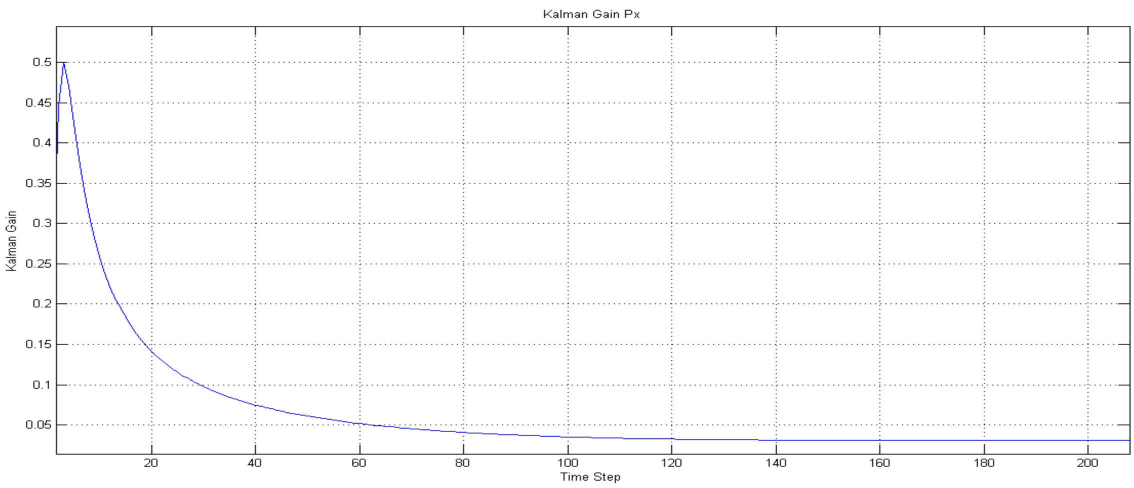

(b)

Figure 6.4: (a) Plot of error in position with respect to the true measured data. (b) Convergence of Kalman Gain.

Chapter 7

Conclusion and Future Scope

7.1 Conclusion

GPS technology is used widely in fields ranging from military to commercial applications. Accuracy is an important factor in some applications. High-precision GPS receivers provide accurate data within few meters but they are very expensive and also take up much power. By making use of data from multiple other sensors, a better estimate in position can be calculated. Based on the results observed the proposed system improves the positioning accuracy of low-precision GPS receivers significantly. The approach is economical and also inexpensive compared to the alternative.

7.2 Future Scope

The positioning accuracy of the proposed system could be improved and made more reliable by considering various other sensor readings, like the wind speed, wind direction, water current, etc. An Extended Kalman Filter can be designed which is more complex and also more sophisticated. The Kalman Filter designed can be optimised to decrease the processing time.

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Appendix A

ENU, ECEF and Geodetic Coordinate System

A.1 Introduction

A geographic coordinate system is a coordinate system that allows every location on earth to be determined by a set of numbers, letters or symbols. There are different ways of representing the coordinates. The coordinates are sometimes chosen such that one parameter denotes the vertical position and two or more other parameters denote the horizontal position, or alternatively they can be represented as a 3D-cartesian vector. The most commonly used coordinate system is the geodetic coordinate system in which coordinates are represented in terms of latitude, longitude and elevation. The following sections describe the different geographic coordinate systems in detail.

A.1.1 Geodetic coordinate System

In a geodetic system, the Earth's surface is approximated by an ellipsoid, and locations near the surface are described in terms of latitude(Φ), longitude(λ) and height(h).

The latitude(Φ) of a point on the Earth's surface is the angle between the equitorial plane and the straight line which passes through that point from the center of the Earth. The North Pole is 90° N, and the South pole is 90° S. The 0° paralles is designated as the Equator and it divides the globe into two hemispheres, the Northen and Southern Hemisphere.

The longitude(λ) of a point on the Earth's Surface is the angle East or West of a reference meridian to another meridian that passes through the point. The combination of these two components specify the position at any point on the surface of the Earth. The ENU system is represented in Fig. A.1

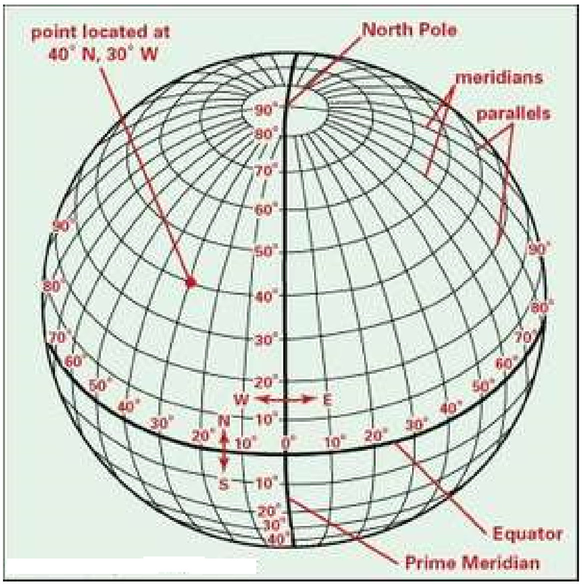

Figure A.1: Geodetic System [6]

A.1.2 ENU coordinate System

East-North-Up or ENU coordinate system is based on the local vertical direction and the Earth's axis of rotation. It consists of three'coordinates, one representing the position along the northen axis, one along the local eastern axis and one representing the vertical position. These coordinate systems are generally used for representing state vectors in avionics and marine fields. These frames are location dependent. The ENU system is represented in Fig A.2

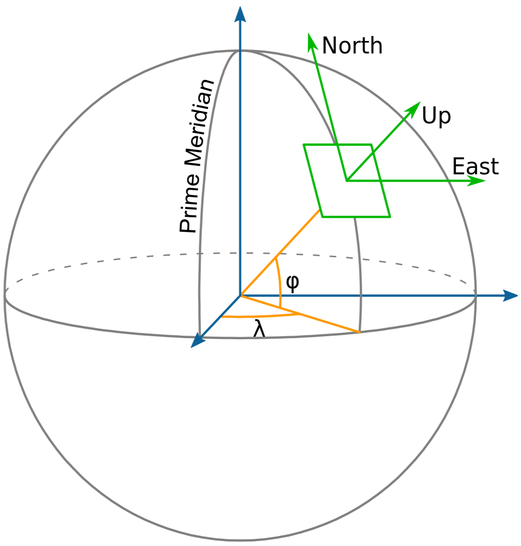

Figure A.2: ENU Coordinate System [7]

A.1.3 ECEF Coordinate System

Earth-Centered-Earth-Fixed or ECEF, is a geographic coordinate system which represents the psotions in terms of cartesian X, Y and Z coordinates. The origin is defined at the center of mass of the Earth. The z-axis extends towards the True North. The x-axis intersects the sphere of the earth at 0° latitude and 0° longitude. The ECEF rotates with the Earth, and therefore the coordinates of a point on the surface of the Earth remain fixed.The ECEF system is shown in Fig. A.3

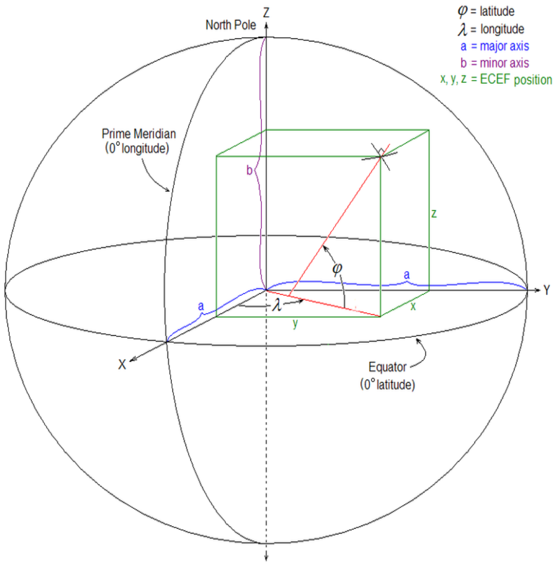

Figure A.3: ECEF Coordinate System [8]

A.2 Conversion

A.2.1 Geodetic System to ECEF System

Geodetic coordinates can be converted to ECEF using the following equation.

$$X = (N(\phi) + h)\cos\phi\cos\lambda \tag{A.1a}$$

$$Y = (N(\phi) + h)\cos\phi\sin\lambda \tag{A.1b}$$

$$Z = \left(\frac{b^2}{a^2}N(\phi) + h\right)\sin\phi \tag{A.1c}$$

where

$$N(\phi) = \frac{a}{\sqrt{1 - e^2\sin^2\phi}} \tag{A.2a}$$

a and b are the equitorial radius and the polar radius. The prime vertical radius of curvature $N(\phi)$ is the distance from the surface to the Z-axis along the ellipsoid normal. ϕ, λ and h are the latitude, longitude and height of the point.

A.2.2 ENU System to ECEF System

By using a local reference point in ECEF system, we can convert any point to ENU using the following transformation matrix.

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} -\sin\lambda & -\sin\lambda\cos\lambda & \cos\phi\cos\lambda \\ \cos\lambda & -\sin\phi\sin\lambda & \cos\phi\sin\lambda \\ 0 & \cos\phi & \sin\phi \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} + \begin{bmatrix} X_r \\ Y_r \\ Z_r \end{bmatrix} \tag{A.3}$$

where x, y and z are the local ENU coordinates or vectors. X_r, Y_r and Z_r are the reference points in the ECEF system.