1 Competition, Profitability, and Discount Rate

1.1 Some Concepts

Hodrick-Prescott:

The Hodrick–Prescott filter (also known as Hodrick–Prescott decomposition) is a mathematical tool used in macroeconomics, especially in real business cycle theory, to remove the cyclical component of a time series from raw data.

Earning-price ratio: The price-earnings ratio, also known as P/E ratio, P/E, or PER, is the ratio of a company's share (stock) price to the company's earnings per share. The ratio is used for valuing companies and to find out whether they are overvalued or undervalued.

$$P/E = \frac{\text{Share Price}}{\text{Earnings per Share}}$$

Profit margin:

$$\text{net profit margin} = \frac{\text{net profit}}{\text{revenue}} = \frac{\text{revenue} - \text{cost}}{\text{revenue}}$$

2 baseline model

2.1 customer base and demand

we denote the industry-level consumption index by C_t, consisting of a basket of firm-level composites $C_{i,t}$. More precisely, the industry-level consumption index C_t is determined by a Dixit-Stiglitz constant-elasticity-of-substitution aggregation:

$$C_t = \left[\sum_{i=1}^{2} \left(\frac{M_{i,t}}{M_t} \right)^{\frac{1}{\eta}} C_{i,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \text{ with } M_t = \sum_{i=1}^{2} M_{i,t} \tag{1}$$

- where $C_{i,t}$ is the amount of firm i's products purchased by consumers, and the parameter $\eta > 1$ captures the elasticity of substitution among goods produced by different firms in the same industry.
- The greater the industry-level consumption index C_t is, the more utility consumers derive from it.
- Intuitively, the weight $M_{i,t}/M_t$ captures consumers' relative "taste" for firm $i's$ products; In other words, the greater the weight $M_{i,t}/M_t$ is, the higher the utility consumers derive from consuming a unit of firm $i's$ products, all other things being equal.

Let $P_{i,t}$ denote the price of firm $i's$ goods. Given the price system $P_{i,t}$ for $i = 1, 2$ and the industry-level consumption index C_t, the demand for firm $i's$ goods $C_{i,t}$ can be obtained by solving a standard expenditure minimization problem:

$$C_{i,t} = \frac{M_{i,t}}{M_t} \left(\frac{P_{i,t}}{P_t} \right)^{-\eta} C_t, \quad \text{with industry price index } P_t = \left[\sum_{j=1}^{2} \left(\frac{M_{i,t}}{M_t} \right) P_{i,t}^{1-\eta} \right]^{\frac{1}{1-\eta}} \tag{2}$$

The demand for firm $i's$ goods increases with $M_{i,t}$, all else unchanged. From a firm's perspective, it is natural to think of consumers' taste $M_{i,t}$ as firm $i's$ customer base (or customer capital) and

M_t as the industry's total customer customer base. The share $M_{i,t}/M_t$ can be interpreted as the customer base share of firm i. Moreover, Eq (2) implies that firm i has a greater influence on the price index P_t when it possesses a larger share of customer base, $M_{i,t}/M_t$. Thus, firm i has the incentive to accumulate $M_{i,t}$ to increase demand and gain market power.

Consumers would naturally like to purchase more products C_t to gain higher utility, but doing so incurs a cost that is dependent on the price index P_t for the basket of goods.

we postulate an industry-level demand curve

$$C_t = \mathscr{D}(P_t) \tag{3}$$

where M_t is an endogenous stochastic process that captures the total customer base of the industry. The coefficient $\epsilon > 1$ captures the industry's price elasticity of demand. A common microfoundation for such an isoelastic industry demand curve is that a continuum of industries exist in the economy producing differentiated industry-level baskets of goods, with the elasticty of substitution across industries being ϵ and the preference weight for an industry's goods equal to its customer base M_t.

$$C_{i,t} = M_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\eta} P_t^{-\epsilon}, \quad \text{with } P_t = \left[\sum_{j=1}^{2}\left(\frac{M_{i,t}}{M_t}\right) P_{i,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \tag{4}$$

Endogenous price elasticity of duopolists

$$\begin{aligned} -\frac{\partial \ln C_{i,t}}{\partial \ln P_{i,t}} &= \underbrace{\mu_{i,t}\left[-\frac{\partial \ln C_t}{\partial \ln P_t}\right]}_{\text{cross-industry}} + \underbrace{(1-\mu_{i,t})\left[-\frac{\partial \ln\left(C_{i,t}/C_t\right)}{\partial \ln\left(P_{i,t}/P_t\right)}\right]}_{\text{within-industry}} \\ &= \mu_{i,t}\epsilon + (1-\mu_{i,t})\,\eta, \end{aligned} \tag{5}$$

where $\mu_{i,t}$ is the (revenue) market share of firm i, defined as follows:

$$\mu_{i,t} = \frac{P_{i,t}C_{i,t}}{P_t C_t} = \left(\frac{P_{i,t}}{P_t}\right)^{1-\eta}\frac{M_{i,t}}{M_t}$$

2.2 Evolution of customer base

We assume that a change in market leaders in an industry (i.e., leadership turnover) occurs exogenously with intensity λ. We use the Poisson process N_t to characterize the exogenous occurrence of leadership turnover in the industry.

Each of the new leaders has the same initial customer base $\overline{M} > 0$. This assumption technically ensures that the industry-level customer base M_t is mean-reverting and stationary in the model.

Deep habits and demand shocks:

We model the evolution of firm $i's$ customer base, conditioned on $dN_t =$ (i.e. leadership turnover does not take place over [t,t+dt]), as follows:

$$\mathrm{d}M_{i,t}/M_{i,t} = \left[\alpha\left(C_{i,t}/M_{i,t}\right)^h - \rho\right]\mathrm{d}t + \varsigma \mathrm{d}Z_t + \sigma_M\ \mathrm{d}W_{i,t} \tag{6}$$

- The term $\alpha(C_{i,t}/M_{i,t})^h dt$ captures the endogenous accumulation of customer base.
- Intuitively, by setting a lower price $P_{i,t}$, firm i increase the contemporaneous demand flow rate $C_{i,t}$ according to Eq(4), thereby allowing it to accmulate a larger customer base over [t,t+dt].

- The parameter $\alpha > 0$ captures the speed of accumulation. A greater α indicates that customer base accumulation is more sensitive to contemporaneous demand $C_{i,t}$.
- The parameter $h \in [0,1]$ captures the relative importance of contemporaneous demand in accumulating customer base.

Next, we summarize the role of the endogenous drift term $\alpha(C_{i,t}/M_{i,t})^h dt$ in the evolution equation.

- First, it generates consumption inertia.
- Second, setting a lower profit margin leads to not only higher contemporaneous demand $C_{i,t}$, but also a larger future customer base and demand on average.
- Third, according to Eq(5), the firm with a smaller customer base $M_{i,t}$ in the industry faces a higher price elasticity of demand, and thus sets a lower price.
- Last, the endogenous drift term can generate different growth rates for different industries.

The constant term ρ in Eq(6) captures customer base depreciation due to industry-level reasons such as mortality. The standard Brownian motion Z_t captures economy-wide aggregate shock, and $W_{i,t}$ idiosyncratic shocks, to firm $i's$ customer base.

2.3 Production and profit margins

Firms produce differentiated goods using capital, rented at a competitive rental rate $w = r + \delta$. The risk-free rate is r and the capital depreciation rate is δ. Each firm uses an AK production technology. Over $[t, t+dt]$, firm i produces a flow of goods with intensity

$$Y_{i,t} = AK_{i,t} \tag{7}$$

where $K_{i,t}$ is the amount of capital rented by firm i at t. Given productivity level A, the marginal cost of producing one unit of goods is ω/A. Without loss of generality, we normalize $A = 1$. Thus, the firm incurs cost with intensity $\omega Y_{i,t}$ in producing a flow of goods with intensity $Y_{i,t}$ over $[t, t+dt]$. Given the demand $C_{i,t}$ and price $P_{i,t}$, firm $i's$ optimal profits over $[t, t+dt]$ are

$$\begin{aligned} \text{Earnings } s_{i,t} &= \max_{Y_{i,t} \geq 0} (P_{i,t} - \omega) Y_{i,t} \\ \text{subject to the demand constraint} Y_{i,t} &\leq C_{i,t} \end{aligned} \tag{8}$$

Therefore, the firm finds it optimal to choose $P_{i,t} > \omega$ and produce up to $Y_{i,t} = C_{i,t}$ in equilibrium. The optimal net profits can be written as

$$\text{Earnings }_{i,t} = (P_{i,t} - \omega) C_{i,t}, \text{ with } P_{i,t} > \omega. \tag{9}$$

All net profits are paid out as dividends because the model has no financial friction.

$$GP_{i,t} \equiv \underbrace{\frac{(P_{i,t} - \omega) C_{i,t}}{K_{i,t}}}_{\text{gross profitability}} = \underbrace{\frac{P_{i,t} - \omega}{P_{i,t}}}_{\text{profit margin}} \times \underbrace{\frac{P_{i,t} C_{i,t}}{K_{i,t}}}_{\text{asset turnover}} \tag{10}$$

The former reflects the price-cost relation shaped by the degree of competition. The firm-level and industry-level profit margins are denoted by

$$\theta_{i,t} \equiv \frac{P_{i,t} - \omega}{P_{i,t}} \text{ and } \theta_t \equiv \frac{P_t - \omega}{P_t}, \text{ respectively.} \tag{11}$$

The relation between θ_t and $\theta_{i,t}$ directly follows from Eq(2) and is

$$1-\theta_t=\left[\sum_{j=1}^{2}\left(\frac{M_{i,t}}{M_t}\right)\left(1-\theta_{i,t}\right)^{\eta-1}\right]^{\frac{1}{\eta-1}} \tag{12}$$

The profit margin, rather than the marginal price, is amined in this paper for the following reasons.

- First, we are concerned with asset pricing, and thus it is the profit margin, rather than the nominal price tag, that matters here.
- Second, the purpose of competition and even price wars is not to reduce compeitiors' prices, but to destory their profit margins.
- Third, accurate and detailed data of retail prices and firms' marginal costs for a broad set of industries are not available.
- Fourth, even if they were available, the implicit discounts, coupons, rebates, and gifts are not easily observable to economists.
- Last, price levels cannot be meaningfully compared across industries, but profit margins can.

Strategic complementary

$$\text{Earnings }_{i,t}=\Pi_i\left(\theta_{i,t},\theta_{j,t}\right)M_{i,t} \tag{13}$$

where $\Pi_i\left(\theta_{i,t},\theta_{j,t}\right)$ describes the profits per unit of customer base, given by

$$\Pi_i\left(\theta_{i,t},\theta_{j,t}\right)=\omega^{1-\epsilon}\theta_{i,t}\left(1-\theta_{i,t}\right)^{\eta-1}\left(1-\theta_t\right)^{\epsilon-\eta} \tag{14}$$

2.4 Stochastic discount factor

We directly specify the stochastic discount factor Λ_t (SDF) , which evolves as follows:

$$\frac{\mathrm{d}\Lambda_t}{\Lambda_t}=-r_f\ \mathrm{d}t-\gamma_t\ \mathrm{d}Z_t-\zeta\mathrm{d}Z_{\gamma,t} \tag{15}$$

where Z_t and $Z_{\gamma,t}$ are independent standard Brownian motions, and r_f is the equilibrium risk-free rate. we directly specify the evolution of the time-varing discount rate γ_t :

$$\mathrm{d}\gamma_t=-\varphi\left(\gamma_t-\bar{\gamma}\right)\mathrm{d}t-\pi\mathrm{d}Z_{\gamma,t}\text{ with }\varphi,\bar{\gamma},\pi>0 \tag{16}$$

We assume $\zeta>0$ to capture the well-documented countercyclical market price of risk whose primitative economic mechanism can be, for example, time-varying risk aversion.

2.5 Solutions of the Nash equilibrium

Noncollusive equilibria. The noncollusive equilibrium is characterized by profit-margin scheme $\Theta^N(\cdot)=\left(\theta_1^N(\cdot),\theta_2^N(\cdot)\right)$, which is a pair of functions defined in state space $\mathscr{X}$ such that each firm i chooses profit margin $\theta_{i,t}\equiv\theta_i\left(x_t\right)$ to maximize shareholder value $V_{i,t}^N\equiv V_i^N\left(x_t\right)$ under the assumption that its competitor j will set the one-shot Nash-equilibrium profit margin $\theta_{j,t}^N\equiv\theta_j^N\left(x_t\right)$. The superscript N stands for the noncollusive equilibrium. Following the recursive formulation in dynamic games for characterizing the Nash equilibrium.

$$\begin{aligned}0=\max_{\theta_{i,t}}&\underbrace{\Lambda_t\left[\Pi_i\left(\theta_{i,t},\theta_{j,t}^N\right)M_{i,t}-\lambda V_{i,t}^N\right]\mathrm{d}t}_{\text{earnings and displacement loss}}\\&+\underbrace{\mathbb{E}_t\left[\mathbf{d}\left(\Lambda_tV_{i,t}^N\right)\mid\theta_{i,t},\theta_{j,t}^N\right]}_{\text{if not displaced}}\end{aligned} \tag{17}$$

The term $\Pi_i\left(\theta_{i,t}, \theta_{j,t}^N\right) M_{i,t}$ describes the earnings of firm i if it chooses profit margin $\theta_{i,t}$. The term $\lambda V_{i,t}^N$ is the expected loss due to possible displacement of firm i's market leadership. The term $\mathbb{E}_t\left[\ \mathrm{d}\left(\Lambda_t V_{i,t}^N\right) \mid \theta_{i,t}, \theta_{j,t}^N\right]$ is the expected valuation change if the market leadership does not change over $[t, t+\mathrm{d}t]$.

Collusive equilibrium:

In particular, if one firm deviates from the collusive profit margin scheme, then with probability $\xi \mathrm{d}t$ over $[t, t+dt]$ the other firm will implement a punishment strategy in which it will forever set the noncollusive profit margin.

We use the idiosyncratic Poisson process $N_{i,t}$ to characterize whether a firm can successfully implement a punishment strategy.

Formally, the set of incentive-compatible collusion agreements, denoted by $\mathscr{C}$, consists of all continuous profit-margin schemes $\Theta^C(\cdot) \equiv \left(\theta_1^C(\cdot), \theta_2^C(\cdot)\right)$, such that the following incentive compatibility (IC) constraints are satisfied:

$$V_i^D(x) \leq V_i^C(x), \text{ for all } x \in \mathscr{X} \text{ and } i = 1, 2 \tag{18}$$

Here, $V_{i,t}^C \equiv V_i^C(x_t)$ is firm i's value in the collusive equilibrium, pinned down recursively according to

$$0 = \underbrace{\Lambda_t\left[\Pi_i\left(\theta_{i,t}^C, \theta_{j,t}^C\right) M_{i,t} - \lambda V_{i,t}^C\right] \mathrm{d}t}_{\text{earnings and displacement loss}} + \underbrace{\mathbb{E}_t\left[\ \mathrm{d}\left(\Lambda_t V_{i,t}^C\right) \mid \theta_{i,t}^C, \theta_{j,t}^C\right]}_{\text{if not displaced}} \tag{19}$$

where $\theta_{i,t}^C = \theta_i^C(x_t)$ with $i = 1, 2$ are the collusive profit margins.

Further, $V_{i,t}^D = V_i^D(x_t)$ is firm $i's$ highest shareholder value if it deviates from the the implicit collusion:

$$\begin{aligned} 0 = \max_{\theta_{i,t}} & \underbrace{\Lambda_t\left[\Pi_i\left(\theta_{i,t}, \theta_{j,t}^C\right) M_{i,t} - \xi\left(V_{i,t}^D - V_{i,t}^N\right) - \lambda V_{i,t}^D\right] \mathrm{d}t}_{\text{earnings, displacement loss, and punishment loss}} \\ & + \underbrace{\mathbb{E}_t\left[\ \mathrm{d}\left(\Lambda_t V_{i,t}^D\right) \mid \theta_{i,t}, \theta_{j,t}^C\right]}_{\text{if not displaced or punished}} \end{aligned} \tag{20}$$

- The term $\Pi_i(\theta_{i,t}, \theta_{j,t}^C) M_{i,t}$ describes the earnings that firm i gains by setting profit margin $\theta_{i,t}$.
- The term $\Lambda V_{i,t}^D$ is the expected loss due to possible displacement of firm $i's$ market leadership.
- The term $\zeta(V_{i,t}^D - V_{i,t}^N)$ is the expected loss due to possible punishment for firm i's deviation behavior.
- The term $\mathbb{E}_t[d(\Lambda_t V_{i,t}^D)|\theta_{i,t}, \theta_{j,t}^N]$ is the expected valuation change if the market leadership does not change or the deviator is not punished over $[t, t+dt]$.

In fact, there exist infinitely many elements in $\mathscr{C}$ and hence infinitely many collusive equilibirum. We focus on a subset of $\mathscr{C}$, denoted by $\overline{\mathscr{C}}$, consisting of all profit-margin schemes $\Theta^C(\cdot)$ such that the IC constraints are binding state by state, i.e., $V_i^D(x) = V_i^C(x)$ for all $x \in \mathscr{X}$ and $i = 1, 2$.

We further narrow our focus to the "Pareto-efficient frontier" of $\overline{\mathscr{C}}$, denoted by $\overline{\mathscr{C}}_p$, consisting of all pairs of $\Theta^C(\cdot)$ such that there does not exist another pair $\widetilde{\Theta}^c(\cdot) \in \overline{\mathscr{C}}$ with $\tilde{\theta}_i(x) \geq \theta_i(x)$ for all $x \in \mathscr{X}$ and $i = 1, 2$ and with strict inequality holding fr some x and i.

Discussions on model ingredients: State variables and shocks:

By exploiting the model's homogeneity in M_t, we can reduce the model to two state variables, $M_{1,t}/M_{i,t}$ and γ_t, when characterizing the industry's equilibrium. In particular, the value function of firm i can be represented by $V_i^C(M_{1,t}, M_{2,t}, \gamma_t) = v_i^C(M_{1,t}/M_t, \gamma_t)M_t$.

The state variables $(M_{1,t}/M_t, \gamma_t)$ are driven by two aggregate shocks Z_t and $Z_{\gamma,t}$, and three idiosyncratic shocks $W_{i,t}, W_{2,t}$, and N_t.

The aggregate demand shock Z_t and aggregate discount rate shock $Z_{\gamma,t}$ are crucial to our core endogenous competition mechanism for the following reasons:

- the aggregate demand shock Z_t shows up in both the customer-base and the SDF to ensure that the variation in discount rates γ_t
- the aggregate discount-rate shock $Z_{\gamma,t}$ carries a negative market price of risk ζ and generates cross-sectional asset pricing implications.

3 Empirical analyses

In this section, we empirically test the main predictions of our model. Section 4.1 describe the data and the discount rate measure. We also construct a leadership turnover measure by estimating a logistic regression model.

we then conduct our empirical tests in three steps

- In section 4.2, we first provide empirical evidence to support our model's main implications for risk premium and profitability. we show that profitability comoves negatively with discount rates and that comovement is more pronounced in more profitable industries.
- In section 4.3, we push one step further in terms of testing the theoretical implications for the turnover rate of the industry's market leaders and the joint cross-sectional asset pricing implication. We show that the leadership turnover measure is correlated negatively with profitability as predicted by model.
- In section 4.4, **we push another step further to directly test the unique predictions of our core competition mechanism** We explore the impact of the variation in the industry market structure on firms' endogenous competition behavior by examing the changes in the sensitivity of profitability to discount rates.

3.1 Data and empirical measures

Measures of discount rate

- In this paper, we use the smoothed earnings-price ratio.

Leadership turnover measure

- we define market leaders as the top two firms ranked by sales in a given industry.
- we estimate the leadership turnover rate using a logistic regression model. Specifically, we assume that the marginal probability of market leadership turnover follows a logistic regression model given by

$$\mathbb{P}\left(\mathbb{1}^{t\to t+2}_{\text{turnover},i} = 1\right) = \frac{1}{1+\exp\left(-b_0 - b_1 x_{i,t}\right)}, \tag{21}$$

- where $\mathbb{1}^{t\to t+2}_{\text{turnover},i}$ is an indicator that equals one if the market leaders of industry$i$ in year $t+2$ are different from those in year t, and $x_{i,t}$ is a column vector of explanatory variables whose values are known at the end of year t.

3.2 Profitability and stock returns

In this section, we test the model's predictions on the relation between discount rates, profitability, and stock returns.

3.2.1 Exposure of proditability to discount rates

Time-series comovement
Test: profitability comoves negatively with discount rates.

- To test this prediction, we regress the year-on-year changes in the average profitability on the changes in the smoothed earning-price ratio.
- when discount rates rise, both the average gross profitability and net profitability drop significantly. By decomprosing profitability into asset turnover and profit margins, columns 3-5 further show that both asset turnover and profit margins comove negatively with discount rates.

Cross-industry heterogeneity
Our model also implies that the negative comovement between profitability and discount rates is mroe pronounced in more profitable industries

- To test this implication, we split industries into quintiles based on their gross profitability and examine the sensitivity of net profitability to discount rates.
- We focus on the net profitability because it reflects firms' net cash flows, which are directly related to asset prices (i.e., firm values) in our model.

3.2.2 Exposure of gross profitability spreads to discount rates

In this subsection, we show that gross profitability spreads (high minus low) are significantly positive at the cross-industry, within-industry and cross-firm levels.
Gross profitability premia

- **Because our model emphasizes the heterogenous variations in competition intensity across industries, we primarily focus on the cross-industry GP spread in the data.**
- **Test:** our model predicts that industries with higher profitability are more exposed to fluctuations in discount rates and thus have higher expected sotck returns.
- we sort all SIC4 industries into quintiles and examine their returns.

Heterogeneous exposure of GP spreads to discount-rate shocks

- **Test:** our model predicts more profitable industries are exposed to discount-rate shocks to a greater degree and thus the cross-industry load negatively on discount-rate shocks.

3.3 Empirical tests on the implications of market leadership turnover

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3.1 Customer base and demand

$$C_t = \left[\sum_{i=1}^{2} \left(\frac{M_{i,t}}{M_t}\right)^{\frac{1}{\eta}} C_{i,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \text{ with } M_t = \sum_{i=1}^{2} M_{i,t} \quad (1)$$

the industry-level consumption index
(consisting of a basket of firm-level composites $C_{i,t}$)

- $C_{i,t}$ is the amount of firm i's product purchased by consumers
- parameter $\eta > 1$ captures the elasticity of substitution among goods produced by different firms in the same industry.
- $\frac{M_{i,t}}{M_t}$ captures cosumers' relative "taste" for i's product

$$C_{i,t} = \frac{M_{i,t}}{M_t} \left(\frac{P_{i,t}}{P_t}\right)^{-\eta} C_t \quad \text{with industry price index } P_t = \left[\sum_{j=1}^{2} \left(\frac{M_{i,t}}{M_t}\right) P_{i,t}^{1-\eta}\right]^{\frac{1}{1-\eta}} \quad (2)$$

In Eq(2), the demand for firm i's goods increase with $M_{i,t}$.

- consumers' taste $M_{i,t}$ as firm i's custom base (Customer capital)
- $\frac{M_{i,t}}{M_t}$ can be interpreted as the customer base share of firm i.

Consumer
- → purchase more product to gain higher utility
- trade out
- → incur a cost that is dependent on Price index P_t

To capture this

industry demand curve: $C_t = M_t P_t^{-\epsilon}$ (3)

- M_t is an endogenous stochastic process that captures the total customer base of the industry
- $\epsilon > 1$ captures the industry's price elasticity of demand

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combining Eq(2) and Eq(3)

$$C_{i,t} = M_{i,t}\left(\frac{P_{i,t}}{P_t}\right)^{-\eta} P_t^{-\epsilon}, \quad \text{with } P_t = \left[\sum_{j=1}^{2}\left(\frac{M_{i,t}}{M_t}\right)\cdot P_{i,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}$$

which is a standard CES demand system.

Endogenous price elasticity of duopolists

$$-\frac{\partial \ln C_{i,t}}{\partial \ln P_{i,t}} = \underbrace{\mu_{i,t}\left[-\frac{\partial \ln C_t}{\partial \ln P_t}\right]}_{\text{cross-industry}} + \underbrace{(1-\mu_{i,t})\left[-\frac{\partial \ln(C_{i,t}/C_t)}{\partial \ln(P_{i,t}/P_t)}\right]}_{\text{within-industry}}$$

$$= \mu_{i,t}\,\epsilon + (1-\mu_{i,t})\,\eta$$

$\mu_{i,t}$ is the (revenue) market share of firm i, defined as

$$\mu_{i,t} = \frac{P_{i,t}C_{i,t}}{P_t C_t} = \left(\frac{P_{i,t}}{P_t}\right)^{1-\eta}\frac{M_{i,t}}{M_t} \quad (5)$$

3.2 Evolution of customer base

Market follower $\xrightarrow{\text{Challenge}}$ Market leader (are replaced rapidly)

$\lambda \Rightarrow$ change in market leader in an industry exogenously intensity (Poisson Process N_t)

Deep habits and demand shocks

Firm can attracting consumers by
- undercutting profit margins
- using strategies such as lowering prices
- offering discounts
- increasing marketing expense

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$C_{i,t}$ can boost the firm's demand
① consumption inertia
② information frictions
③ switching costs

Modeling the evolution of firm i's customer base over [t, t+dt])
(Conditioned on $dN_t = 0$, i.e. leadership turnover doesn't take place

- ρ capture customer base depreciation due to industry-level reasons
- standard Brownian motion Z_t capture economy-wide aggregate shock

$$\frac{dM_{i,t}}{M_{i,t}} = \left[\alpha\left(\frac{C_{i,t}}{M_{i,t}}\right)^h - \rho\right]dt + \varsigma dZ_t + \sigma_M d\cancel{M_{i,t}}W_{i,t} \quad (6)$$

- the term $\alpha(C_{i,t}/M_{i,t})^h dt$ captures the endogenous accumulation of customer base.
- $W_{1,t}$ and $W_{2,t}$ can be interpreted as idiosyncratic demand ("or taste") shock

- $\alpha > 0$, the speed of accumulation

$h \in [0,1]$ relative importance

3.3 Production and Profit margins

Firms produce differentiated goods using capital, rented at a competitive rate $w \equiv r + \delta$ ⇒ capital depreciation

r: risk-free rate

Over [t, t+dt], firm i produces a flow of goods with intensity

$$Y_{i,t} = A \; K_{i,t}$$

- productivity level
- marginal cost is $\frac{w}{A}$
- the amount of capital rented by firm i at t.
- The rental cost is $wK_{i,t}dt$ over [t, t+dt]

The firm incurs cost with intensity $wY_{i,t}$ in producing goods with intensity $Y_{i,t}$ over $[t, t+dt]$

Given the demand $C_{i,t}$ and price $P_{i,t}$, firm i's optimal profits over $[t, t+dt]$ are:

$$\text{Earnings}_{i,t} = \max_{Y_{i,t} \geq 0} (P_{i,t} - w) Y_{i,t} \quad (8)$$

subject to the demand constraint $Y_{i,t} \leq C_{i,t}$

optimal $\Downarrow$ choose ① $P_{i,t} > w$

② produce up to $Y_{i,t} = C_{i,t}$

$$\text{Earning}_{i,t} = (P_{i,t} - w) C_{i,t} \quad \text{with } P_{i,t} > w \quad (9)$$

model has no financial friction ⇒ All net profits are paid out as dividends

$$\underbrace{GP_{i,t} = \frac{(P_{i,t} - w) C_{i,t}}{K_{i,t}}}_{\text{gross profitability}} = \underbrace{\frac{P_{i,t} - w}{P_{i,t}}}_{\text{profit margin}} \times \underbrace{\frac{P_{i,t} C_{i,t}}{K_{i,t}}}_{\text{asset turnover}} \quad (10)$$

gross profitability $\Downarrow$ capture the economic gain, which concern investors.

$$\theta_{i,t} = \frac{P_{i,t} - w}{P_{i,t}} \qquad \theta_t = \frac{P_t - w}{P_t} \quad (11)$$

firm level profit margin — industry level profit margin

The relationship between θ_t and $\theta_{i,t}$ follow from (2) and is

$$1 - \theta_t = \left[\sum_{j=1}^{2} \left(\frac{M_{i,t}}{M_t} \right) (1 - \theta_{i,t})^{\eta - 1} \right]^{\frac{1}{\eta - 1}}$$

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Strategic complementary

Substituting Eq.(2) into Eq.(9) gives

$$\text{Earning}_{i,t} = \Pi_i(\theta_{i,t}, \theta_{j,t}) M_{i,t} \quad (13)$$

describes the profits per unit of custom base

$$\Pi_i(\theta_{i,t}, \theta_{j,t}) = w^{1-\epsilon}\, \theta_{i,t}(1-\theta_{i,t})^{\eta-1}(1-\theta_t)^{\epsilon-\eta} \quad (14)$$

3.4 Stochastic discount factor

SDF (Λ_t) evolves as

$$\frac{d\Lambda_t}{\Lambda_t} = -r_f\,dt - r_t\,dZ_t - \xi\,dZ_{r,t} \quad (16)$$

equilibrium risk-~~free~~free

$\xi > 0$ capture the countercyclical market price of risk

the evolution of the time-varying discount rate r_t:

$$dr_t = -\varphi(r_t - \bar{r})dt - \pi\,dZ_{r,t}, \text{ with } \varphi, \bar{r}, \pi > 0 \quad (17)$$

3.5 Solutions of Nash equilibria

"payoff-relevant" physical states $x_t = \{M_{1,t}, M_{2,t}, r_t\}$

Noncollusive equilibria

profit-margin scheme $\Theta^N(\cdot) = (\theta_1^N(\cdot), \theta_2^N(\cdot))$, which is a pair of functions defined in ~~sa~~ state space $\mathcal{X}$ such that each firm i chooses profit margin $\theta_{i,t} = \theta_i(x_t)$ to maximize share holder value $V_{i,t}^N = V_i^N(x_t)$

(the superscript N stands for noncollusive equilibrium) optimization problems can be formulated recursively using Hamilton - Jacobi - Bellman (HJB) equations: (18)

$$0 = \max_{\theta_{i,t}} \underbrace{\Lambda_t[\Pi_i(\theta_{i,t}, \theta_{j,t}^N) M_{i,t} - \lambda V_{i,t}^N]dt}_{\text{earnings and displacement loss}} + \underbrace{E_t[d(\Lambda_t V_{i,t}^N) | \theta_{i,t}, \theta_{j,t}^N]}_{\text{if not displaced}}$$

- the earning of firm i if it chooses profit margin $\theta_{i,t}$

- expected loss due to possibe displacement of firm i's market leadership

Collusive equilibria

incentive compatibility (IC) constraints are satisfied:

$$V_i^D(x) \leq V_i^C(x), \text{ for all } x \in \mathcal{X} \text{ and } i = 1,2 \quad (19)$$

Here, $V_{i,t}^C \equiv V_i^C(x_t)$ is firm value in the collusive equilibrium, pinned down recursively according to

$$0 = \underbrace{\Lambda_t[\Pi_i(\theta_{i,t}^C, \theta_{j,t}^C) M_{i,t} - \lambda V_{i,t}^C]dt}_{\text{earnings and displacement loss}} + \underbrace{E_t[d(\Lambda_t V_{i,t}^C) | \theta_{i,t}^C, \theta_{j,t}^C]}_{\text{if not displaced}} \quad (20)$$