

Trigonométrie

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MP2I

Exercice 1

Énoncé Calculer $\sin\left(\frac{45\pi}{12}\right)$ et $\cos\left(\frac{45\pi}{12}\right)$

Solution proposée On a

$$\begin{aligned}\sin\left(\frac{45\pi}{12}\right) &= \sin\left(\frac{3^2 \times 5 \times \pi}{2^2 \times 3}\right) \\ &= \sin\left(\frac{15\pi}{4}\right) \\ &= \sin\left(4\pi - \frac{\pi}{4}\right) \\ &= -\sin\left(\frac{\pi}{4}\right) \\ &= -\frac{\sqrt{2}}{2}\end{aligned}$$

et

$$\begin{aligned}\cos\left(\frac{45\pi}{12}\right) &= \cos\left(4\pi - \frac{\pi}{4}\right) \\ &= \cos\left(\frac{\pi}{4}\right) \\ &= \frac{\sqrt{2}}{2}\end{aligned}$$

Exercice 2

Énoncé Résoudre dans \mathbb{R} l'équation $\sin\left(x + \frac{3\pi}{4}\right) = \cos\left(2x - \frac{\pi}{3}\right)$.

Solution proposée Soit $x \in \mathbb{R}$,

$$\sin\left(x + \frac{3\pi}{4}\right) = \cos\left(2x - \frac{\pi}{3}\right) \Leftrightarrow \sin\left(x + \frac{3\pi}{4}\right) = \sin\left(2x - \frac{\pi}{3} + \frac{\pi}{2}\right)$$

Donc

$$\begin{aligned}&\begin{cases} x + \frac{3\pi}{4} \equiv 2x + \frac{\pi}{6} [2\pi] \\ \text{ou} \\ x + \frac{3\pi}{4} \equiv \pi - 2x - \frac{\pi}{6} [2\pi] \end{cases} \\ \Leftrightarrow &\begin{cases} -x \equiv \frac{2\pi}{12} - \frac{9\pi}{12} [2\pi] \\ \text{ou} \\ 3x \equiv \frac{12\pi}{12} - \frac{9\pi}{12} - \frac{2\pi}{12} [2\pi] \end{cases}\end{aligned}$$

$$\iff \begin{cases} x \equiv \frac{7\pi}{12} [2\pi] \\ \text{ou} \\ x \equiv \frac{\pi}{36} \left[\frac{2\pi}{3} \right] \end{cases}$$

Exercice 3

Énoncé Résoudre dans \mathbb{R} l'équation $\sin\left(5x + \frac{\pi}{2}\right) = \sin(2x)$.

Solution proposée Soit $x \in \mathbb{R}$, Donc

$$\begin{aligned} \sin\left(5x + \frac{\pi}{2}\right) &= \sin(2x) \\ \iff \begin{cases} 5x + \frac{\pi}{2} &\equiv 2x [2\pi] \\ \text{ou} \\ 5x + \frac{\pi}{2} &\equiv \pi - 2x [2\pi] \end{cases} \\ \iff \begin{cases} 3x &\equiv -\frac{\pi}{2} [2\pi] \\ \text{ou} \\ 7x &\equiv \pi - \frac{\pi}{2} [2\pi] \end{cases} \\ \iff \begin{cases} x &\equiv -\frac{\pi}{6} \left[\frac{2\pi}{3} \right] \\ \text{ou} \\ x &\equiv \frac{\pi}{14} \left[\frac{2\pi}{7} \right] \end{cases} \end{aligned}$$

Exercice 10

Énoncé

1. Exprimer $\tan(a + b + c)$ en fonction de $\tan(a)$, $\tan(b)$ et $\tan(c)$.
2. Généraliser à $\tan\left(\sum_{k=1}^n a_k\right)$.

Solution proposée

1. Soit $(a, b, c) \in \mathbb{R}^3$ tel que $\tan(a)$, $\tan(b)$ et $\tan(c)$ soient définis.

$$\begin{aligned} \tan(a + b + c) &= \frac{\tan(a + b) + \tan(c)}{1 - \tan(a + b) \tan(c)} \\ &= \frac{\frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)} + \tan(c)}{1 - \frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)} \tan(c)} \\ &= \frac{\tan(a) + \tan(b) + \tan(c)(1 - \tan(a) \tan(b))}{1 - \tan(a) \tan(b) - \tan(c)(\tan(a) + \tan(b))} \\ &= \frac{\tan(a) + \tan(b) + \tan(c) - \tan(a) \tan(b) \tan(c)}{1 - \tan(a) \tan(b) - \tan(a) \tan(c) - \tan(c) \tan(b)} \end{aligned}$$

2. Soit $(a_1, a_2, a_3, a_4) \in \mathbb{R}^4$ tel que $\forall i \in \{1, 2, 3, 4\}$, $\tan(a_i)$ est défini.

$$\begin{aligned}
\tan\left(\sum_{k=1}^4 a_k\right) &= \frac{\tan\left(\sum_{k=1}^3 a_k\right) + \tan(a_4)}{1 - \tan\left(\sum_{k=1}^3 a_k\right) \tan(a_4)} \\
&= \frac{\frac{\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k)}{1 - \sum_{k=1}^3 \prod_{i=1, i \neq k}^3 a_k} + \tan(a_4)}{1 - \left(\frac{\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k)}{1 - \sum_{k=1}^3 \prod_{i=1, i \neq k}^3 a_k}\right) \tan(a_4)} \\
&= \frac{\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k) + \tan(a_4) \left(1 - \sum_{k=1}^3 \prod_{i=1, i \neq k}^3 \tan(a_k)\right)}{1 - \sum_{k=1}^3 \prod_{i=1, i \neq k}^3 \tan(a_k) - \left(\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k)\right) \tan(a_4)} \\
&= \frac{\sum_{k=1}^4 \tan(a_k) - \prod_{k=1}^3 \tan(a_k) - \sum_{k=1}^3 \prod_{i=1, i \neq k}^4 \tan(a_k)}{1 - \sum_{k=1}^3 \prod_{i=1, i \neq k}^3 \tan(a_k) - \tan(a_4) \sum_{k=1}^3 \tan(a_k) + \prod_{k=1}^4 \tan(a_k)}
\end{aligned}$$