TD 5 : Calculs de complexité et récursivité

Amar AHMANE
MP2I

Algorithme de Karatsuba

5

$$U_0 = \sum_{k=0}^{n-1} u_k 2^k$$
$$U_1 = \sum_{k=n}^{2n-1} u_k 2^{k-n}$$
$$V_0 = \sum_{k=0}^{n-1} v_k 2^k$$
$$U_1 = \sum_{k=n}^{2n-1} v_k 2^{k-n}$$

6 On a

$$\begin{aligned}
uv &= (2^n U_1 + U_0)(2^n V_1 + V_0) \\
&= 2^{2n} U_1 V_1 + 2^n U_1 V_0 + 2^n V_1 U_0 + U_0 V_0 \\
&= 2^{2n} U_1 V_1 + 2^n U_1 V_0 + 2^n V_1 U_0 + 2^n U_0 V_0 - 2^n U_0 V_0 + 2^n U_1 V_1 - 2^n U_1 V_1 + U_0 V_0 \\
&= (2^{2n} + 2^n) U_1 V_1 + 2^n (U_1 V_0 + U_0 V_0 - U_0 V_0 - U_1 V_1) + (2^n + 1) U_0 V_0 \\
&= (2^{2n} + 2^n) U_1 V_1 + 2^n (U_1 - U_0)(V_1 - V_0) + (2^n + 1) U_0 V_0
\end{aligned}$$