

Chapitre 2

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MP2I

Exercice 10

Énoncé

1. Exprimer $\tan(a + b + c)$ en fonction de $\tan(a)$, $\tan(b)$ et $\tan(c)$.
2. Généraliser à $\tan\left(\sum_{k=1}^n a_k\right)$.

Solution proposée

1. Soit $(a, b, c) \in \mathbb{R}^3$ tel que $\tan(a)$, $\tan(b)$ et $\tan(c)$ soient définis.

$$\begin{aligned}\tan(a + b + c) &= \frac{\tan(a + b) + \tan(c)}{1 - \tan(a + b) \tan(c)} \\ &= \frac{\frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)} + \tan(c)}{1 - \frac{\tan(a) + \tan(b)}{1 - \tan(a) \tan(b)} \tan(c)} \\ &= \frac{\tan(a) + \tan(b) + \tan(c)(1 - \tan(a) \tan(b))}{1 - \tan(a) \tan(b) - \tan(c)(\tan(a) + \tan(b))} \\ &= \frac{\tan(a) + \tan(b) + \tan(c) - \tan(a) \tan(b) \tan(c)}{1 - \tan(a) \tan(b) - \tan(a) \tan(c) - \tan(c) \tan(b)}\end{aligned}$$

2. Soit $(a_1, a_2, a_3, a_4) \in \mathbb{R}^4$ tel que $\forall i \in \{1, 2, 3, 4\}$, $\tan(a_i)$ est défini.

$$\begin{aligned}\tan\left(\sum_{k=1}^4 a_k\right) &= \frac{\tan\left(\sum_{k=1}^3 a_k\right) + \tan(a_4)}{1 - \tan\left(\sum_{k=1}^3 a_k\right) \tan(a_4)} \\ &= \frac{\frac{\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k)}{1 - \sum_{k=1}^3 \prod_{i=1; i \neq k}^3 a_k} + \tan(a_4)}{1 - \left(\frac{\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k)}{1 - \sum_{k=1}^3 \prod_{i=1; i \neq k}^3 a_k}\right) \tan(a_4)} \\ &= \frac{\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k) + \tan(a_4) \left(1 - \sum_{k=1}^3 \prod_{i=1; i \neq k}^3 a_k\right)}{1 - \sum_{k=1}^3 \prod_{i=1; i \neq k}^3 a_k - \left(\sum_{k=1}^3 \tan(a_k) - \prod_{k=1}^3 \tan(a_k)\right) \tan(a_4)} \\ &= \frac{\sum_{k=1}^4 \tan(a_k) - \prod_{k=1}^3 \tan(a_k) - \sum_{k=1}^3 \prod_{i=1; i \neq k}^4 a_k}{1 - \sum_{k=1}^3 \prod_{i=1; i \neq k}^3 a_k - \tan(a_4) \sum_{k=1}^3 \tan(a_k) + \prod_{k=1}^4 \tan(a_k)}\end{aligned}$$