The General 2^k Factorial Design

The General 2^k Factorial Design

k factors being tested at two-level each:

- k main effects
- $\binom{k}{2}$ two-factor interaction
- $\binom{k}{3}$ three-factor interaction

$\vdots$

- one k-factor interaction

2^k-1 effects of factors/interactions

Complete replicate has 2^k runs

Building a 2^k Design

1) Build a 2^2 design

Treat	x_A	x_B
(1)	-1	-1
a	+1	-1
b	-1	+1
ab	+1	+1

Building a 2^k Design

1) Build a 2^2 design

2) Double it for the 3rd factor

$2^3 = 8$ treatments

$x_C = -1$ for upper half

$x_C = +1$ for lower half

Treat	x_A	x_B	x_C
(1)	-1	-1	-1
a	+1	-1	-1
b	-1	+1	-1
ab	+1	+1	-1
c	-1	-1	+1
ac	+1	-1	+1
bc	-1	+1	+1
abc	+1	+1	+1

Building a 2^k Design

1) Build a 2^2 design

2) Double it for the 3rd factor

$2^3 = 8$ treatments

$x_C = -1$ for upper half

$x_C = +1$ for lower half

3) Double it for the 4th factor

$2^4 = 16$ treatments

$x_D = -1$ for upper half

$x_D = +1$ for lower half

Treat	x_A	x_B	x_C	x_D
(1)	-1	-1	-1	-1
a	+1	-1	-1	-1
b	-1	+1	-1	-1
ab	+1	+1	-1	-1
c	-1	-1	+1	-1
ac	+1	-1	+1	-1
bc	-1	+1	+1	-1
abc	+1	+1	+1	-1
d	-1	-1	-1	+1
ad	+1	-1	-1	+1
bd	-1	+1	-1	+1
abd	+1	+1	-1	+1
cd	-1	-1	+1	+1
acd	+1	-1	+1	+1
bcd	-1	+1	+1	+1
$abcd$	+1	+1	+1	+1

Building a 2^k Design

1) Build a 2^2 design

2) Double it for the 3rd factor

$2^3 = 8$ treatments

$x_C = -1$ for upper half

$x_C = +1$ for lower half

3) Double it for the 4th factor

$2^4 = 16$ treatments

$x_D = -1$ for upper half

$x_D = +1$ for lower half

4) Double it for the 5th factor

$2^5 = 32$ treatments

$x_E = -1$ for upper half

$x_E = +1$ for lower half

Treat	x_A	x_B	x_C	x_D	x_E
(1)	-1	-1	-1	-1	-1
a	+1	-1	-1	-1	-1
b	-1	+1	-1	-1	-1
ab	+1	+1	-1	-1	-1
c	-1	-1	+1	-1	-1
ac	+1	-1	+1	-1	-1
bc	-1	+1	+1	-1	-1
abc	+1	+1	+1	-1	-1
d	-1	-1	-1	+1	-1
ad	+1	-1	-1	+1	-1
bd	-1	+1	-1	+1	-1
abd	+1	+1	-1	+1	-1
cd	-1	-1	+1	+1	-1
acd	+1	-1	+1	+1	-1
bcd	-1	+1	+1	+1	-1
$abcd$	+1	+1	+1	+1	-1

Treat	x_A	x_B	x_C	x_D	x_E
e	-1	-1	-1	-1	+1
ae	+1	-1	-1	-1	+1
be	-1	+1	-1	-1	+1
abe	+1	+1	-1	-1	+1
ce	-1	-1	+1	-1	+1
ace	+1	-1	+1	-1	+1
bce	-1	+1	+1	-1	+1
$abce$	+1	+1	+1	-1	+1
de	-1	-1	-1	+1	+1
ade	+1	-1	-1	+1	+1
bde	-1	+1	-1	+1	+1
$abde$	+1	+1	-1	+1	+1
cde	-1	-1	+1	+1	+1
$acde$	+1	-1	+1	+1	+1
$bcde$	-1	+1	+1	+1	+1
$abcde$	+1	+1	+1	+1	+1

The General 2^k Design: Analysis of Variance

main factors

two-factor interactions

three-factor interactions

k-factor interaction

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_0	p-value
A	SS_A	1			
B	SS_B	1			
$\vdots$	$\vdots$	$\vdots$			
K	SS_K	1			
AB	SS_{AB}	1			
AC	SS_{AC}	1			
$\vdots$	$\vdots$	$\vdots$			
JK	SS_{JK}	1			
ABC	SS_{ABC}	1			
$\vdots$	$\vdots$	$\vdots$			
IJK	SS_{IJK}	1			
$\vdots$	$\vdots$	$\vdots$			
$ABC \cdots K$	$SS_{ABC \cdots K}$	1			
Error	SS_E	$2^k(n-1)$			
Total	SS_T	$2^k \times n - 1$			

2^k-1 effects of factors/interactions

1 degree of freedom

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Analysis Procedure for a 2^k Design

Analysis of variance (ANOVA table)

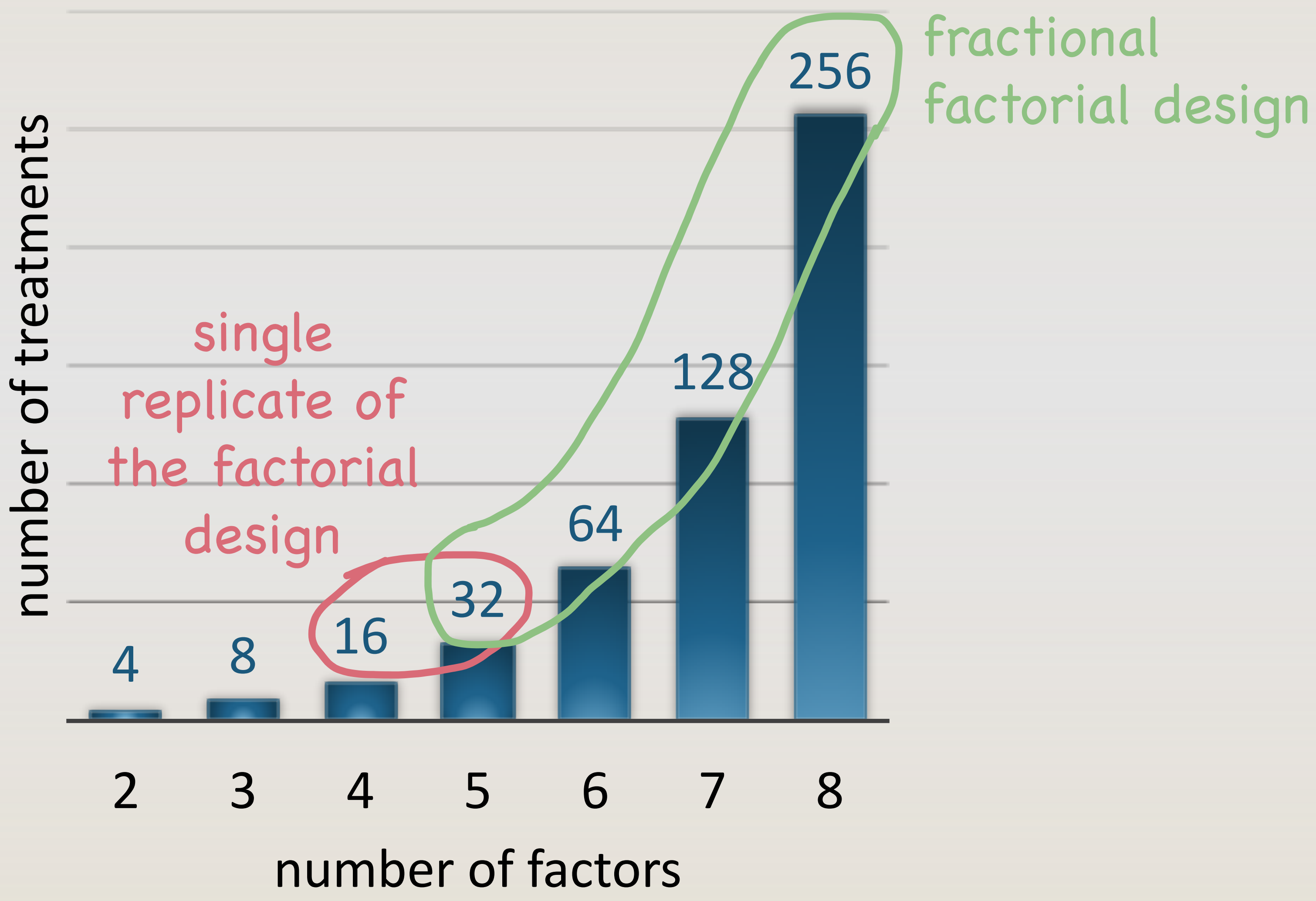
Regression model for coded and natural variables

Check of model adequacy: R^2 and residuals plot

Final presentation: contour plots & effects table

Single Replicate 2^k Design

A Single Replicate of the 2^k Design



A Single Replicate of the 2^k Design

ANOVA Table:

How to estimate the error?

Source	Sum of Squares	Freedom	Mean Square	F_0	p-value
A	SS_A	1			
$\vdots$	$\vdots$	$\vdots$			
$ABC\cdots K$	$SS_{ABC\cdots K}$	1			
Error	SS_E	$2^k(n-1)$			
Total	SS_T	$2^k \times n - 1$			

$$F_0 = \frac{MS}{MS_E}$$

$$MS_E = \frac{SS_E}{df_E}$$

single replicate:
n=1

df error = 0

There is no evaluation of the internal error

Sparsity of Effects Principle

Most systems are dominated by main effects and low-order interactions.

A study* on 113 response variables from 43 published experiments showed:

40 % of main effects are significant

11 % of two-factor interactions are significant

Three-factor interactions are rare

Main effects are about 4-times higher than two-factor effects

If a factor or interaction i is not significant:
$$E[MS_i] \cong MS_E$$

*Li, Sudarsanan and Frei (2006). https://doi.org/10.1002/cplx.20123

A Single Replicate of the 2^k Design

ANOVA Table:

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_0	p-v
A	SS_A	1			
$\vdots$	$\vdots$	$\vdots$			
$ABC\cdots K$	$SS_{ABC\cdots K}$	1			
Error	SS_E	$2^k(n-1)$			
Total	SS_T	$2^k \times n - 1$			

single replicate:
n=1

df error = 0

Sparsity of effects principle:
High-order interactions are negligible and can be used to estimate error.

There is no evaluation of the internal error

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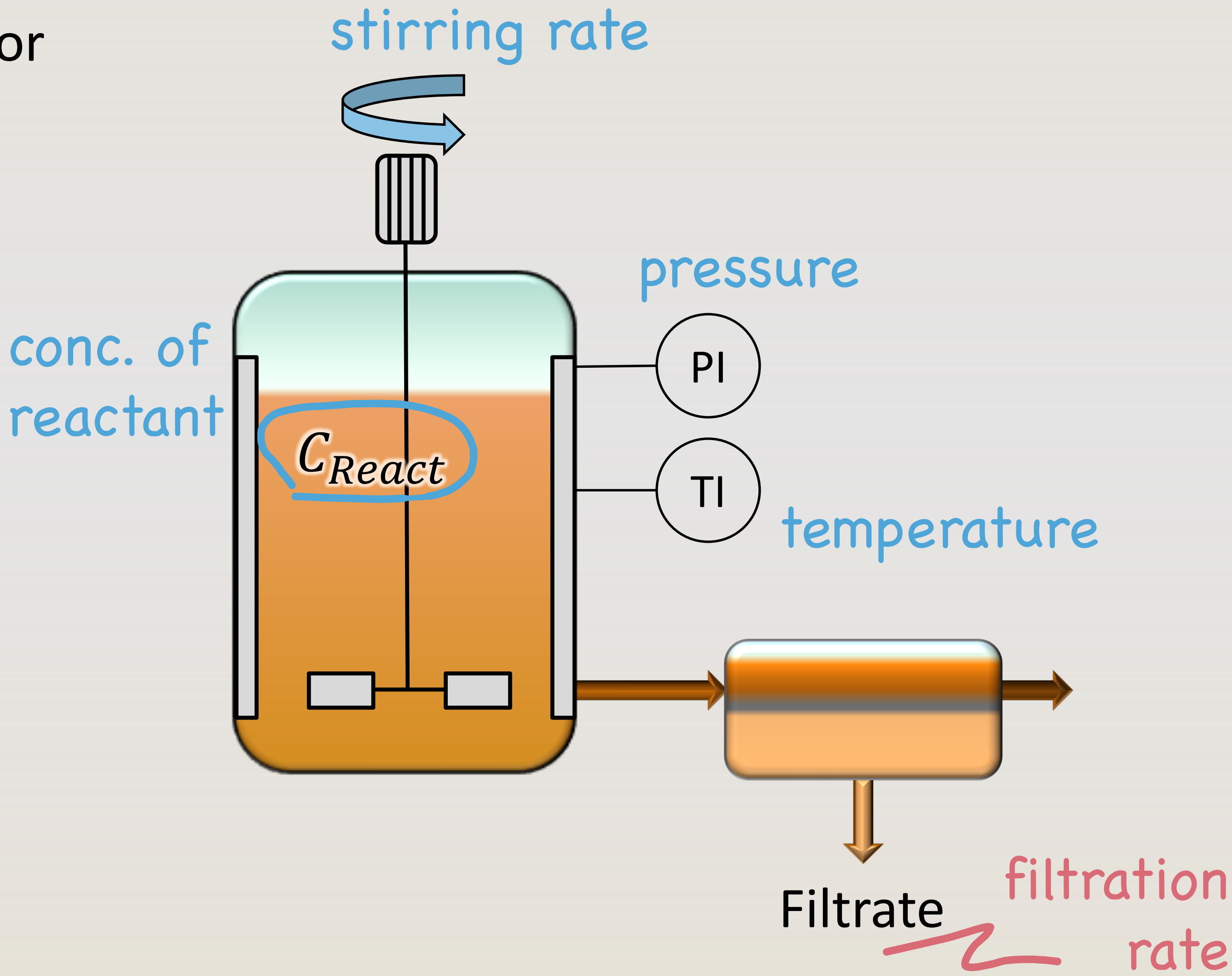
A Single Replicate of the 2^4 Design

The drawing shows a chemical reactor followed by a filtration system.

The experimenter wants to evaluate how the variables:

- Temperature
- Pressure
- Concentration of reactant
- Stirring rate

affect filtration rate?



A Single Replicate of the 2^4 Design

The drawing shows a chemical reactor followed by a filtration system.

The experimenter wants to evaluate how the variables:

- **Temperature**
- **Pressure**
- **Concentration of reactant**
- **Stirring rate**

affect filtration rate?

Table with factor levels:

Factor	**low level -1**	**high level +1**
A: Temperature (T, ˚C)	40	80
B: Pressure (P, atm)	1.5	2.5
C: Concentration (C, g/L)	50	100
D: Stirring rate (ω, rpm)	200	400

Table with the experimental matrix of the single-replicate 2^4 Factorial Design:

Treatment	x_A	x_B	x_C	x_D	A: T (˚C)	B: P (atm)	C: C (g/L)	D: ω (rpm)	Filt. rate (L/min)
(1)	-1	-1	-1	-1	40	1.5	50	200	45
a	+1	-1	-1	-1	80	1.5	50	200	71
b	-1	+1	-1	-1	40	2.5	50	200	48
ab	+1	+1	-1	-1	80	2.5	50	200	65
c	-1	-1	+1	-1	40	1.5	100	200	68
ac	+1	-1	+1	-1	80	1.5	100	200	60
bc	-1	+1	+1	-1	40	2.5	100	200	80
abc	+1	+1	+1	-1	80	2.5	100	200	65
d	-1	-1	-1	+1	40	1.5	50	400	43
ad	+1	-1	-1	+1	80	1.5	50	400	100
bd	-1	+1	-1	+1	40	2.5	50	400	45
abd	+1	+1	-1	+1	80	2.5	50	400	104
cd	-1	-1	+1	+1	40	1.5	100	400	75
acd	+1	-1	+1	+1	80	1.5	100	400	86
bcd	-1	+1	+1	+1	40	2.5	100	400	70
$abcd$	+1	+1	+1	+1	80	2.5	100	400	96

Table with the experimental matrix of the single-replicate 2^4 Factorial Design:

Treatment	x_A	x_B	x_C	x_D	A: T (˚C)	B: P (atm)	C: C (g/L)	D: ω (rpm)	Filt. rate (L/min)
(1)	-1	-1	-1	-1	40	1.5	50	200	45
a	[illegible]	[illegible]	[illegible]	-1	80	1.5	50	200	71
b	[illegible]	[illegible]	[illegible]	-1	40	2.5	50	200	48
ab	+1	+1	-1	-1	80	[illegible]	[illegible]	[illegible]	65
c	-1	-1	+1	-1	40	[illegible]	[illegible]	[illegible]	68
ac	+1	-1	+1	-1	80	1.5	100	200	[illegible]
bc	-1	+1	+1	-1	40	2.5	100	200	[illegible]
abc	+1	+1	+1	-1	80	2.5	100	200	[illegible]
d	-1	-1	-1	+1	40	1.5	50	400	[illegible]
ad	+1	-1	-1	+1	80	1.5	50	400	100
bd	-1	+1	-1	+1	40	2.5	50	400	45
abd	+1	+1	-1	+1	80	2.5	50	400	104
cd	-1	-1	+1	+1	40	1.5	100	400	75
acd	+1	-1	+1	+1	80	1.5	100	400	86
bcd	-1	+1	+1	+1	40	2.5	100	400	70
$abcd$	+1	+1	+1	+1	80	2.5	100	400	96

Standard 2^4 Design

Natural Variables

Experimental Results

Table with the experimental matrix of the single-replicate 2^4 Factorial Design:

Treatment	x_A	x_B	x_C	x_D	A: T (˚C)	B: P (atm)	C: C (g/L)	D: ω (rpm)	Filt. rate (L/min)
(1)	-1	-1	-1	-1	40	1.5	50	200	45
a	+1	[illegible]	[illegible]	[illegible]	[illegible]	1.5	50	200	71
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.5	50	200	48
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2.5	50	200	65
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.5	100	200	68
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	1.5	100	200	60
bc	-1	+1	+1	-1	40	2.5	100	200	80
abc	+1	+1	+1	-1	80	2.5	100	200	65
d	-1	-1	-1	+1	40	1.5	50	400	43
ad	+1	-1	-1	+1	80	1.5	50	400	100
bd	-1	+1	-1	+1	40	2.5	50	400	45
abd	+1	+1	-1	+1	80	2.5	50	400	104
cd	-1	-1	+1	+1	40	1.5	100	400	75
acd	+1	-1	+1	+1	80	1.5	100	400	86
bcd	-1	+1	+1	+1	40	2.5	100	400	70
$abcd$	+1	+1	+1	+1	80	2.5	100	400	96

The 16 runs were performed in a random order:
completely randomized experiment

Single Replicate 2^4 Design: Analysis of Variance of the Complete Model

```
> anova1 <- aov(filt ~ Factor_T*Factor_P*Factor_C*Factor_W, data = filtration)
> summary(anova1)
                                  Df Sum Sq Mean Sq
Factor_T                           1 1870.6  1870.6
Factor_P                           1   39.1    39.1
Factor_C                           1  390.1   390.1
Factor_W                           1  855.6   855.6
Factor_T:Factor_P                  1    0.1     0.1
Factor_T:Factor_C                  1 1314.1  1314.1
Factor_P:Factor_C                  1   22.6    22.6
Factor_T:Factor_W                  1 1105.6  1105.6
Factor_P:Factor_W                  1    0.6     0.6
Factor_C:Factor_W                  1    5.1     5.1
Factor_T:Factor_P:Factor_C         1   14.1    14.1
Factor_T:Factor_P:Factor_W         1   68.1    68.1
Factor_T:Factor_C:Factor_W         1   10.6    10.6
Factor_P:Factor_C:Factor_W         1   27.6    27.6
Factor_T:Factor_P:Factor_C:Factor_W 1    7.6     7.6
>
```

main factors

two-factor interactions

three-factor interactions

four-factor interaction

There is no:
Residuals
F_0
p-value

single replicate:
n=1

df error = 0

There is no estimate of the internal error

Single Replicate 2^4 Design: Analysis of Variance of the Complete Model

```
> anova1 <- aov(filt ~ Factor_T*Factor_P                  filtration)
> summary(anova1)
                                  Df
Factor_T                           1
Factor_P                           1
Factor_C                           1
Factor_W                           1
Factor_T:Factor_P                  1
Factor_T:Factor_C                  1 1314.1  1314.1
Factor_P:Factor_C                  1   22.6    22.6
Factor_T:Factor_W                  1 1105.6  1105.6
Factor_P:Factor_W                  1    0.6     0.6
Factor_C:Factor_W                  1    5.1     5.1
Factor_T:Factor_P:Factor_C         1   14.1    14.1
Factor_T:Factor_P:Factor_W         1   68.1    68.1
Factor_T:Factor_C:Factor_W         1   10.6    10.6
Factor_P:Factor_C:Factor_W         1   27.6    27.6
Factor_T:Factor_P:Factor_C:Factor_W 1    7.6     7.6
>
```

Sparsity of effects principle:

High-order interactions are negligible and can be used to estimate error.

Three and four-factor interactions will be used to estimate the residuals

Single Replicate 2^4 Design: Analysis of Variance of the Complete Model

```
> anova1 <- aov(filt ~ Factor_T*Factor_P                  filtration)
> summary(anova1)
                                   Df
Factor_T                            1
Factor_P                            1
Factor_C                            1
Factor_W                            1
Factor_T:Factor_P                   1    0.1
Factor_T:Factor_C                   1 1314.1  1314.1
Factor_P:Factor_C                   1   22.6    22.6
Factor_T:Factor_W                   1 1105.6  1105.6
Factor_P:Factor_W                   1    0.6     0.6
Factor_C:Factor_W                   1    5.1     5.1
Factor_T:Factor_P:Factor_C          1   14.1    14.1
Factor_T:Factor_P:Factor_W          1   68.1    68.1
Factor_T:Factor_C:Factor_W          1   10.6    10.6
Factor_P:Factor_C:Factor_W          1   27.6    27.6
Factor_T:Factor_P:Factor_C:Factor_W 1    7.6     7.6
>
```

df_E 5

SS_E 127.8

Sparsity of effects principle:

High-order interactions are negligible and can be used to estimate error.

Three and four-factor interactions will be used to estimate the residuals

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Single Replicate 2^4 Design: Analysis of Variance

```
> anova2 <- aov(filt ~ (Factor_T+Factor_P+Factor_C+Factor_W)^2, data = filtration)
> summary(anova2)
                  Df Sum Sq Mean Sq F value   Pr(>F)    
Factor_T           1 1870.6  1870.6  73.176 0.000360 ***
Factor_P           1   39.1    39.1   1.528 0.271297    
Factor_C           1  390.1   390.1  15.259 0.011337 *  
Factor_W           1  855.6   855.6  33.469 0.002172 ** 
Factor_T:Factor_P  1    0.1     0.1   0.002 0.962478    
Factor_T:Factor_C  1 1314.1  1314.1  51.406 0.000821 ***
Factor_T:Factor_W  1 1105.6  1105.6  43.249 0.001220 ** 
Factor_P:Factor_C  1   22.6    22.6   0.883 0.390613    
Factor_P:Factor_W  1    0.6     0.6   0.022 0.887871    
Factor_C:Factor_W  1    5.1     5.1   0.198 0.674909    
Residuals          5  127.8    25.6                     
---
Signif. codes:  0 '**
```

main factors

two-factor interactions

$$SS_E = SS_{TPC} + SS_{TP\omega} + SS_{TC\omega} + SS_{PC\omega} + SS_{TPC\omega}$$

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Single Replicate 2^4 Design: Analysis of Variance

```
> anova2 <- aov(filt ~ (Factor_T+Factor_P+Factor_C+Factor_W)^2, data = filtration)
> summary(anova2)
                  Df Sum Sq Mean Sq F value   Pr(>F)
Factor_T           1 1870.6  1870.6  73.176 0.000360 ***
Factor_P           1   39.1    39.1   1.528 0.271297
Factor_C           1  390.1   390.1  15.259 0.011337 *
Factor_W           1  855.6   855.6  33.469 0.002172 **
Factor_T:Factor_P  1    0.1     0.1   0.002 0.962478
Factor_T:Factor_C  1 1314.1  1314.1  51.406 0.000821 ***
Factor_T:Factor_W  1 1105.6  1105.6  43.249 0.001220 **
Factor_P:Factor_C  1   22.6    22.6   0.883 0.390613
Factor                                  2 0.887871
Facto                                   3 0.674909
Resid
---
Signif                                  0.05 '.' 0.1 ' ' 1
```

significant

significant

significant

Main factors: T, C, ω

Two-factor interactions:
$T \times C, T \times \omega$

Single Replicate 2^4 Design: Analysis of Variance

```
> anova2 <- aov(filt ~ (Factor_T+Factor_P+Factor_C+Factor_W)^2, data = filtration)
> summary(anova2)
                  Df Sum Sq Mean Sq F value   Pr(>F)    
Factor_T           1 1870.6  1870.6  73.176 0.000360 ***
Factor_P           1   39.1    39.1   1.528 0.271297    
Factor_C           1  390.1   390.1  15.259 0.011337 *  
Factor_W           1  855.6   855.6  33.469 0.002172 ** 
Factor_T:Factor_P  1    0.1     0.1   0.002 0.962478    
Factor_T:Factor_C  1 1314.1  1314.1  51.406 0.000821 ***
Factor_T:Factor_W  1 1105.6  1105.6  43.249 0.001220 ** 
Factor_P:Factor_C  1   22.6    22.6   0.883 0.390613    
Factor_P:Factor_W  1    0.6     0.6   0.022 0.887871    
Factor_C:Factor_W  1    5.1     5.1   0.198 0.674909    
Residuals          5  127.8    25.6                     
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The pressure (Factor_P) was not significant, nor any of its interactions

Run a 3-factor ANOVA (T, C, ω).

The 2 pressure levels will be considered as replicates

Single Replicate 2^4 Design: Analysis of Variance

```
> anova3 <- aov(filt ~ (Factor_T+Factor_C+Factor_W)^2, data = filtration)
> summary(anova3)
                  Df Sum Sq Mean Sq F value   Pr(>F)    
Factor_T           1 1870.6  1870.6   88.58 5.91e-06 ***
Factor_C           1  390.1   390.1   18.47 0.001997 ** 
Factor_W           1  855.6   855.6   40.51 0.000131 ***
Factor_T:Factor_C  1 1314.1  1314.1   62.23 2.48e-05 ***
Factor_T:Factor_W  1 1105.6  1105.6   52.35 4.89e-05 ***
Factor_C:Factor_W  1    5.1     5.1    0.24 0.636118    
Residuals          9  190.1    21.1                     
---
Signif                                  0.05 '.' 0.1 ' ' 1
```

Main factors: T, C, ω

Two-factor interactions:
$T \times C, T \times \omega$

Single Replicate 2^4 Design: Analysis of Variance

```
> anova3 <- aov(filt ~ (Factor_T+Factor_C+Factor_W)^2,
> summary(anova3)
                  Df Sum Sq Mean Sq F value   Pr(>F)
Factor_T           1 1870.6  1870.6   88.58 5.91e-06 ***
Factor_C           1  390.1   390.1   18.47 0.001997 **
Factor_W           1  855.6   855.6   40.51 0.000131 ***
Factor_T:Factor_C  1 1314.1  1314.1   62.23 2.48e-05 ***
Factor_T:Factor_W  1 1105.6  1105.6   52.35 4.89e-05 ***
Factor_C:Factor_W  1    5.1     5.1    0.24 0.636118
```

T, C and ω

the p-values are lower

the factors are more significant

```
> anova2 <- aov(filt ~ (Factor_T+Factor_P+Factor_C+Facto        ration)
> summary(anova2)
                  Df Sum Sq Mean Sq F value   Pr(>F)
Factor_T           1 1870.6  1870.6  73.176 0.000360 ***
Factor_P           1   39.1    39.1   1.528 0.271297
Factor_C           1  390.1   390.1  15.259 0.011337 *
Factor_W           1  855.6   855.6  33.469 0.002172 **
Factor_T:Factor_P  1    0.1     0.1   0.002 0.962478
Factor_T:Factor_C  1 1314.1  1314.1  51.406 0.000821 ***
Factor_T:Factor_W  1 1105.6  1105.6  43.249 0.001220 **
Factor_P:Factor_C  1   22.6    22.6   0.883 0.390613
```

T, C, P and ω

Single Replicate 2^4 Design: Analysis of Variance

```
> anova3 <- aov(filt ~ (Factor_T+Factor_C+Factor_W)^2,
> summary(anova3)
                  Df Sum Sq Mean Sq F value   Pr(>F)
Factor_T           1 1870.6  1870.6   88.58 5.91e-06 ***
Factor_C           1  390.1   390.1   18.47 0.001997 **
Factor_W           1  855.6   855.6   40.51 0.000131 ***
Factor_T:Factor_C  1 1314.1  1314.1   62.23 2.48e-05 ***
Factor_T:
Factor_C:
```

```
Residuals          9  190.1    21.1
```

T, C and ω

the p-values are lower

the factors are more significant

degrees of freedom of the residuals term

df_E F_{crit}

```
> anova2 <- aov(filt ~ (Factor_T+Factor_P+Factor_C+Facto                ration)
> summary(anova2)
                  Df Sum Sq Mean Sq F value   Pr(>F)
Factor_T           1 1870.6  1870.6  73.176 0.000360 ***
Factor_P           1   39.1    39.1   1.528 0.271297
Factor_C           1  390.1   390.1  15.259 0.011337 *
Factor_W           1  855.6   855.6  33.469 0.002172 **
Factor_T:Factor_P  1    0.1     0.1   0.002 0.962478
Factor_T:Factor_C  1 1314.1  1314.1  51.406 0.000821 ***
Factor_T:Factor_W  1 1
Factor_P:Factor_C  1
```

```
Residuals          5  127.8    25.6
```

T, C, P and ω

Single Replicate 2^4 Design: Analysis of Variance

```
> anova3 <- aov(filt ~ (Factor_T+Factor_C+Factor_W)^2, data = filtration)
> summary(anova3)
                  Df Sum Sq Mean Sq F value   Pr(>F)    
Factor_T           1 1870.6  1870.6   88.58 5.91e-06 ***
Factor_C           1  390.1   390.1   18.47 0.001997 ** 
Factor_W           1  855.6   855.6   40.51 0.000131 ***
Factor_T:Factor_C  1 1314.1  1314.1   62.23 2.48e-05 ***
Factor_T:Factor_W  1 1105.6  1105.6   52.35 4.89e-05 ***
Factor_C:Factor_W  1    5.1     5.1    0.24 0.636118    
Residuals          9  190.1    21.1                     
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

ANOVA

residuals

Single Replicate 2^4 Design: Analysis of Variance

T C ω

```
> anova4 <- aov(filt ~ Factor_T+Factor_C+Factor_W + Factor_T:Factor_C + Factor_T:Factor_W,
> summary(anova4)
                  Df Sum Sq Mean Sq F value   Pr(>F)
Factor_T           1 1870.6  1870.6   95.86 1.93e-06 ***
Factor_C           1  390.1   390.1   19.99   0.0012 **
Factor_W           1  855.6   855.6   43.85 5.92e-05 ***
Factor_T:Factor_C  1 1314.1  1314.1   67.34 9.41e-06 ***
Factor_T:Factor_W  1 1105.6  1105.6   56.66 2.00e-05 ***
Residuals         10  195.1    19.5
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

$T\times C$ $T\times\omega$

all factors and interactions are now significant

Backwards Elimination:

The technique of the stepwise elimination of non-significant factors and interactions.

It is a widely used procedure in the analysis of experimental results.

Single Replicate 2^4 Design: Analysis of Variance

T $\quad C \quad \omega$

```
> anova4 <- aov(filt ~ Factor_T+Factor_C+Factor_W + Factor_T:Factor_C + Factor_T:Factor_W,
```

$T\times C \qquad T\times\omega$

```
 Sq F value    Pr(>F)
0.6    95.86 1.93e-06 ***
0.1    19.99   0.0012 **
5.6    43.85 5.92e-05 ***
4.1    67.34 9.41e-06 ***
5.6    56.66 2.00e-05 ***
9.5

Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Regression Model:

Main factors: T, C, ω

Two-factor interactions: $T\times C, T\times\omega$

all factors and interactions are now significant

Backwards Elimination:

The technique of the stepwise elimination of non-significant factors and interactions.

It is a widely used procedure in the analysis of experimental results.

The 2^4 Factorial Design: Regression Model and Table of Effects

```
> coded.model <- lm(filt ~ xT + xC+ xW + xT*xC + xT*xW, data=filtration)
> summary(coded.model)

Call:
lm(formula = filt ~ xT + xC + xW + xT * xC + xT * xW, data = filtration)

Residuals:
    Min      1Q  Median      3Q     Max 
-6.3750 -1.5000  0.0625  2.9062  5.7500 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   70.062      1.104  63.444 2.30e-14 ***
xT            10.812      1.104   9.791 1.93e-06 ***
xC             4.938      1.104   4.471   0.0012 ** 
xW             7.313      1.104   6.622 5.92e-05 ***
xT:xC         -9.063      1.104  -8.206 9.41e-06 ***
xT:xW          8.312      1.104   7.527 2.00e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.417 on 10 degrees of freedom
Multiple R-squared:  0.966,	Adjusted R-squared:  0.9489 
F-statistic: 56.74 on 5 and 10 DF,  p-value: 5.14e-07
```

The 2^4 Factorial Design: Regression Model and Table of Effects

```
> coded.model <- lm(filt ~ xT + xC+ xW + xT*xC + xT*xW, data=filtration)
> summary(coded.model)

Call:
lm(formula = filt ~ xT + xC + xW + xT * xC + xT * xW, data = filtration)

Residuals:
    Min      1Q  Median
-6.3750 -1.5000  0.0625  2

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)   70.062      1.104  63.444 2.30e-14 ***
xT            10.812      1.104   9.791 1.93e-06 ***
xC             4.938      1.104   4.471   0.0012 **
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xT:xC         -9.063      1.104  -8.206 9.41e-06 ***
xT:xW          8.312      1.104   7.527 2.00e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

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F-statistic: 56.74 on 5 and 10 DF,  p-value: 5.14e-07
```

b_i

$$b_i = \frac{Effect_i}{2}$$

$2 \times b_i$

Table of Effects:

	Effect	p-value
Temperature (T)	21.624	< 0.001
Concentration (C)	9.876	0.0012
Stirring Rate (ω)	14.626	< 0.001
$T \times C$	-18.126	< 0.001
$T \times \omega$	16.624	< 0.001

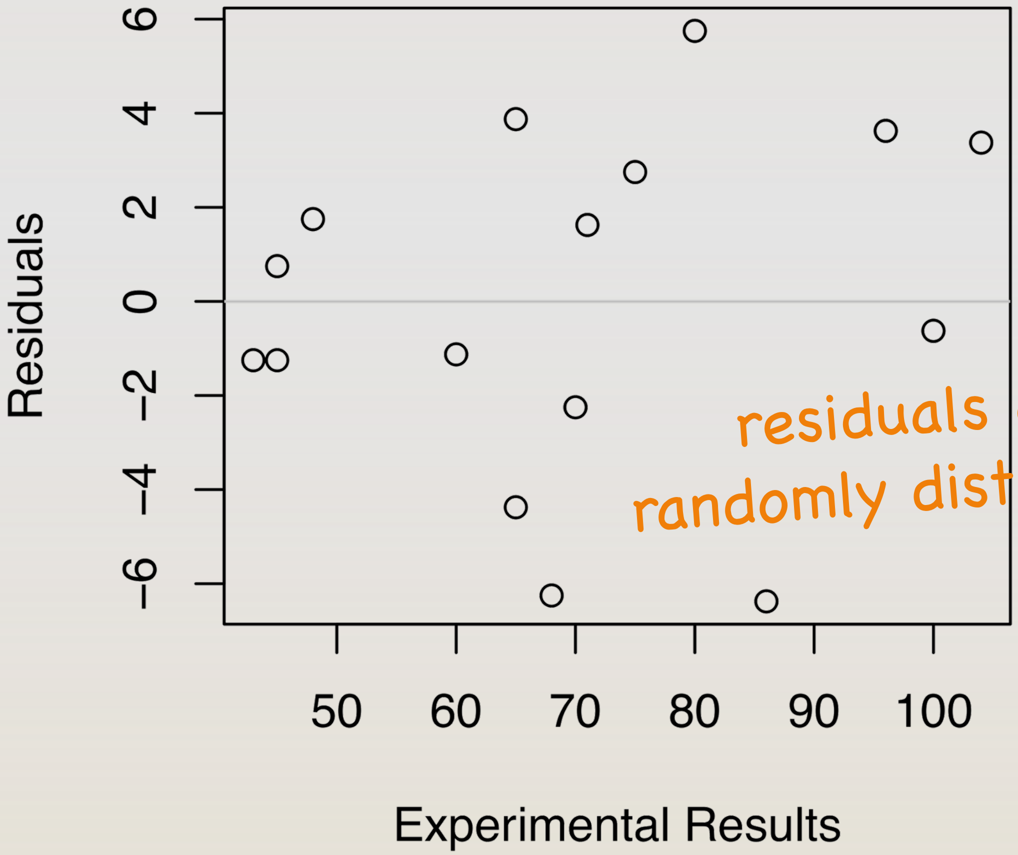
The 2^4 Factorial Design: Regression Model and Model Adequacy

$$FiltRate = -5.875 + 0.653T + 1.285C - 0.17652\omega - 0.18125T\times C + 0.0041562T\times\omega$$

regression model for the natural variables

$R^2 = 0.966$

96.6% of the data variability is explained by the model



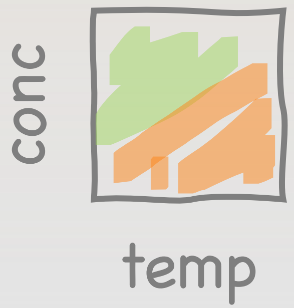
residuals are randomly distributed

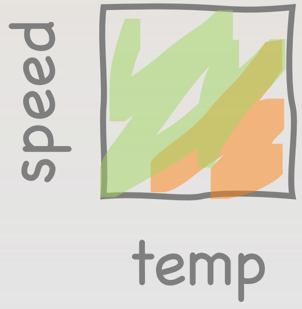
The 2^4 Factorial Design: Contour Plots

$$FiltRate = -5.875 + 0.653T + 1.285C - 0.17652\omega - 0.181125T\times C + 0.0041562T\times\omega$$

$R^2 = 0.966$

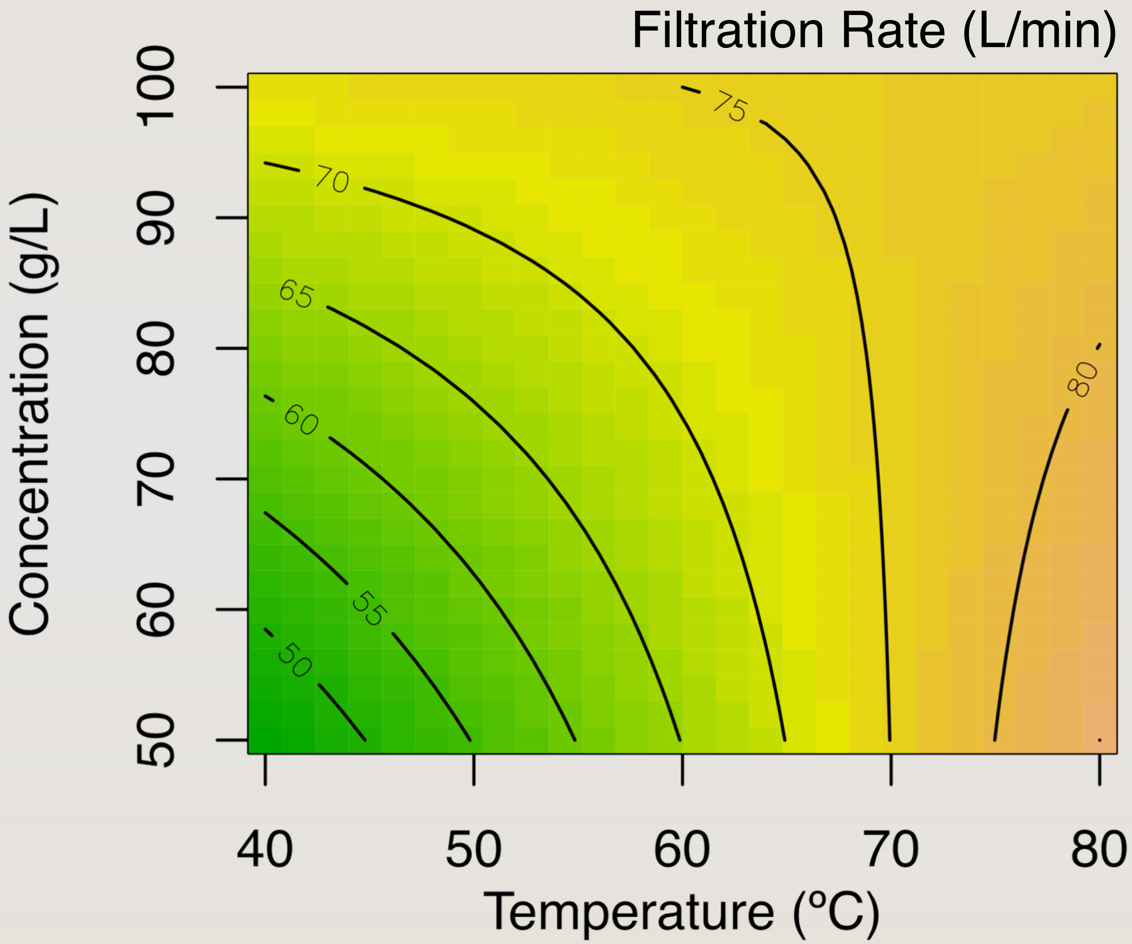
Which contour plots are the best to show the behaviour of the system?

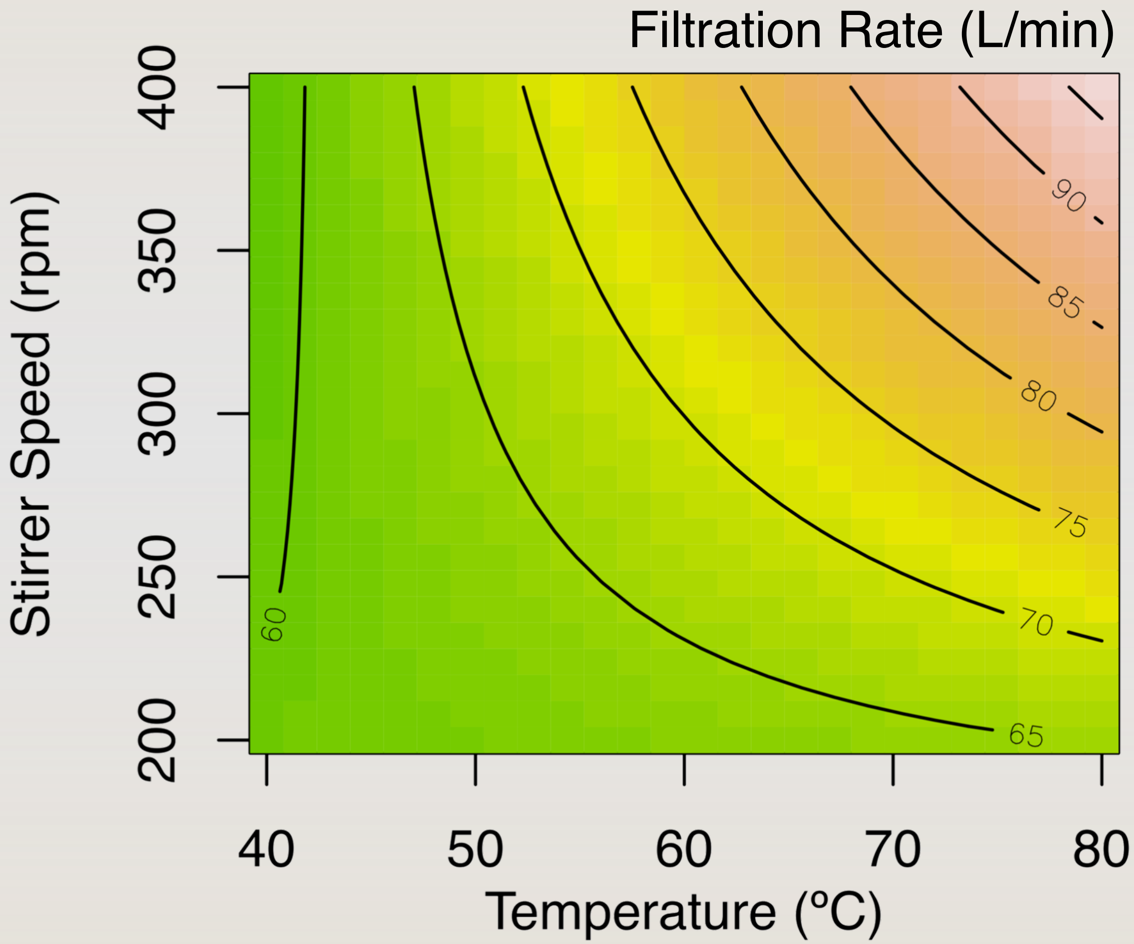




As the interactions $T\times C$ and $T\times\omega$ are significant,
the behaviour of the filtration rate with temperature depends on the concentration and on the stirrer speed,
thus, these factors must be shown as axis in a contour plot.

The 2^4 Factorial Design: Contour Plots





The 2^4 Factorial Design: Contour Plots

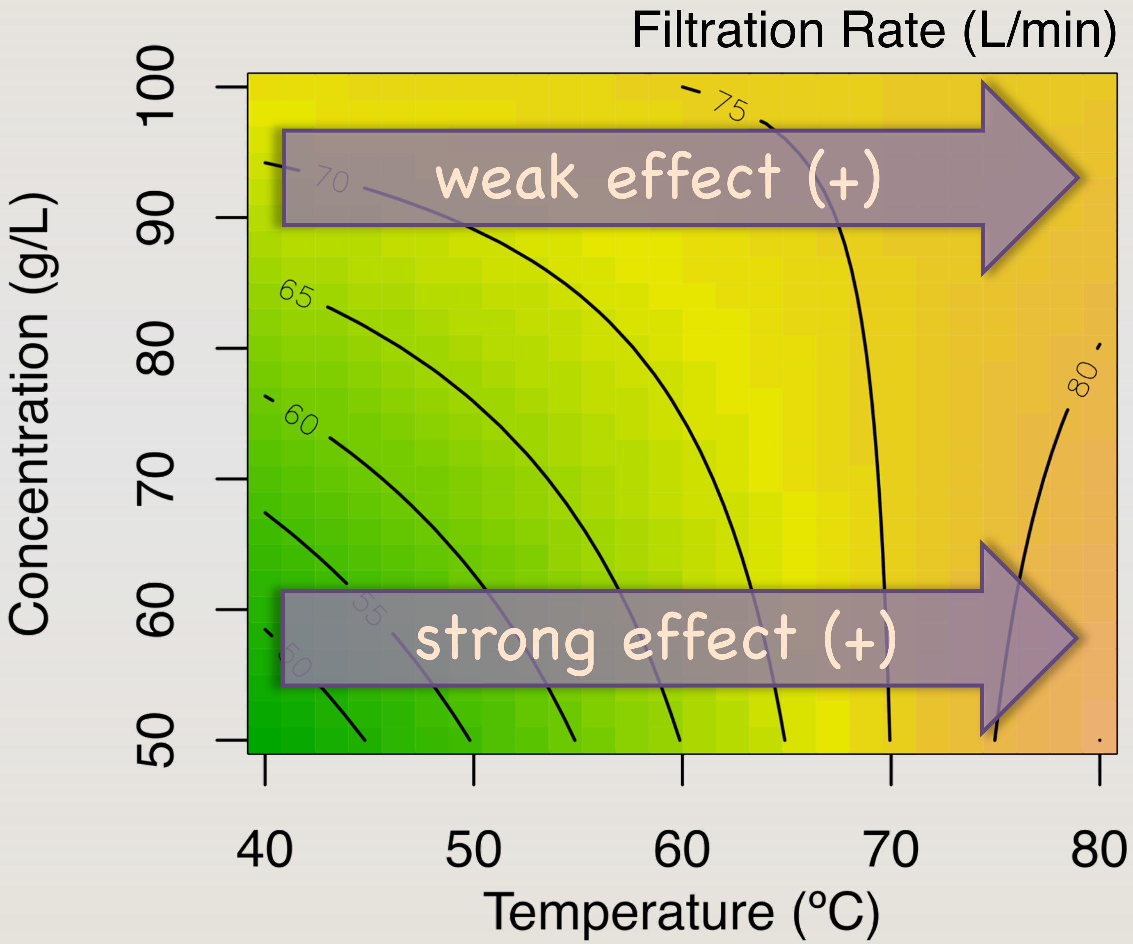


Table of Effects:

	Effect	p-value
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Concentration (C)	9.876	0.0012
Stirring Rate (ω)	14.626	< 0.001
$T\times C$	-18.126	< 0.001
$T\times\omega$	16.624	< 0.001

The 2^4 Factorial Design: Contour Plots

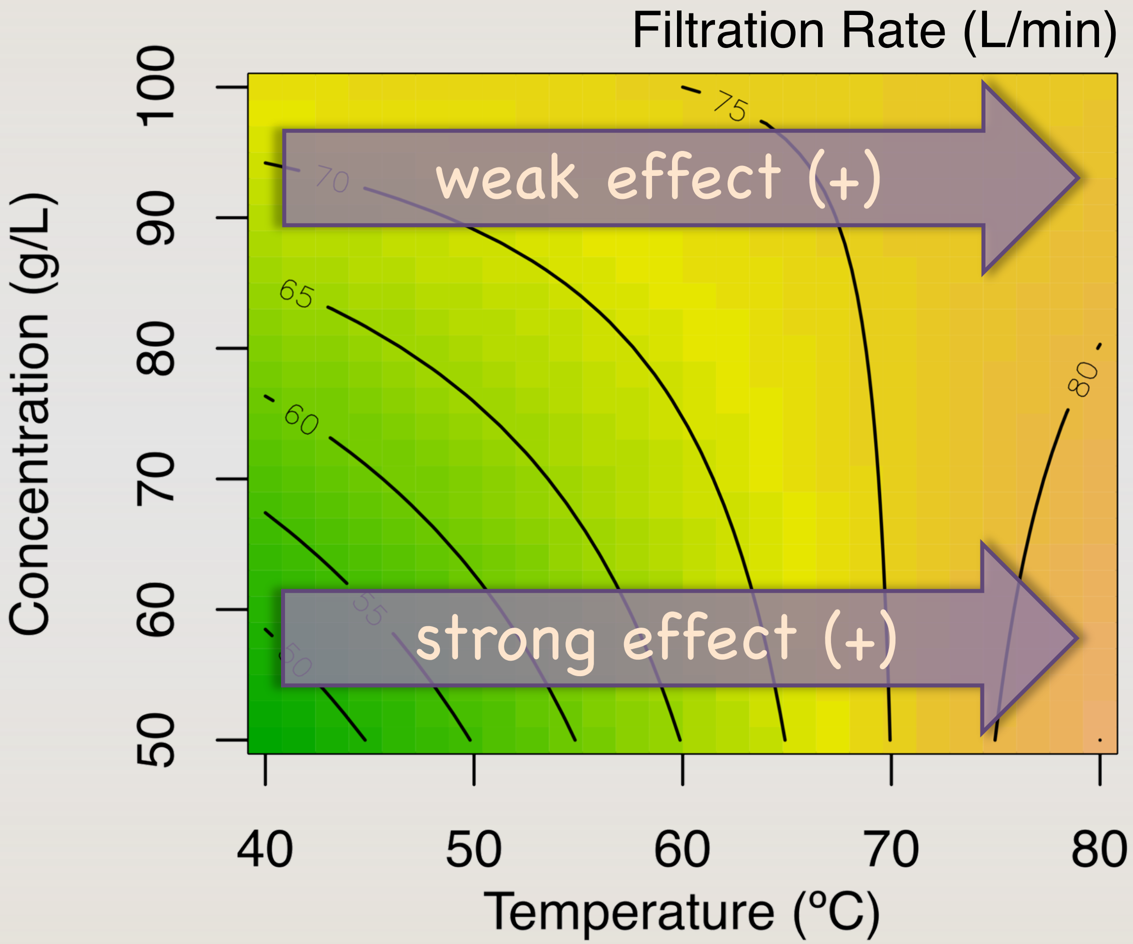


Table of Effects:

	Effect	p-value
Temperature (T)	21.624	< 0.001
Concentration (C)	9.876	0.0012
Stirring Rate (ω)	14.626	< 0.001
$T\times C$ ~~$-T$~~ T	$+18.126$	< 0.001
$T\times\omega$	16.624	< 0.001

When concentration is at its low level:

$$x_C = -1$$

$$Effect\ T = 21.624 + 18.126 = 39.75$$

The 2^4 Factorial Design: Contour Plots

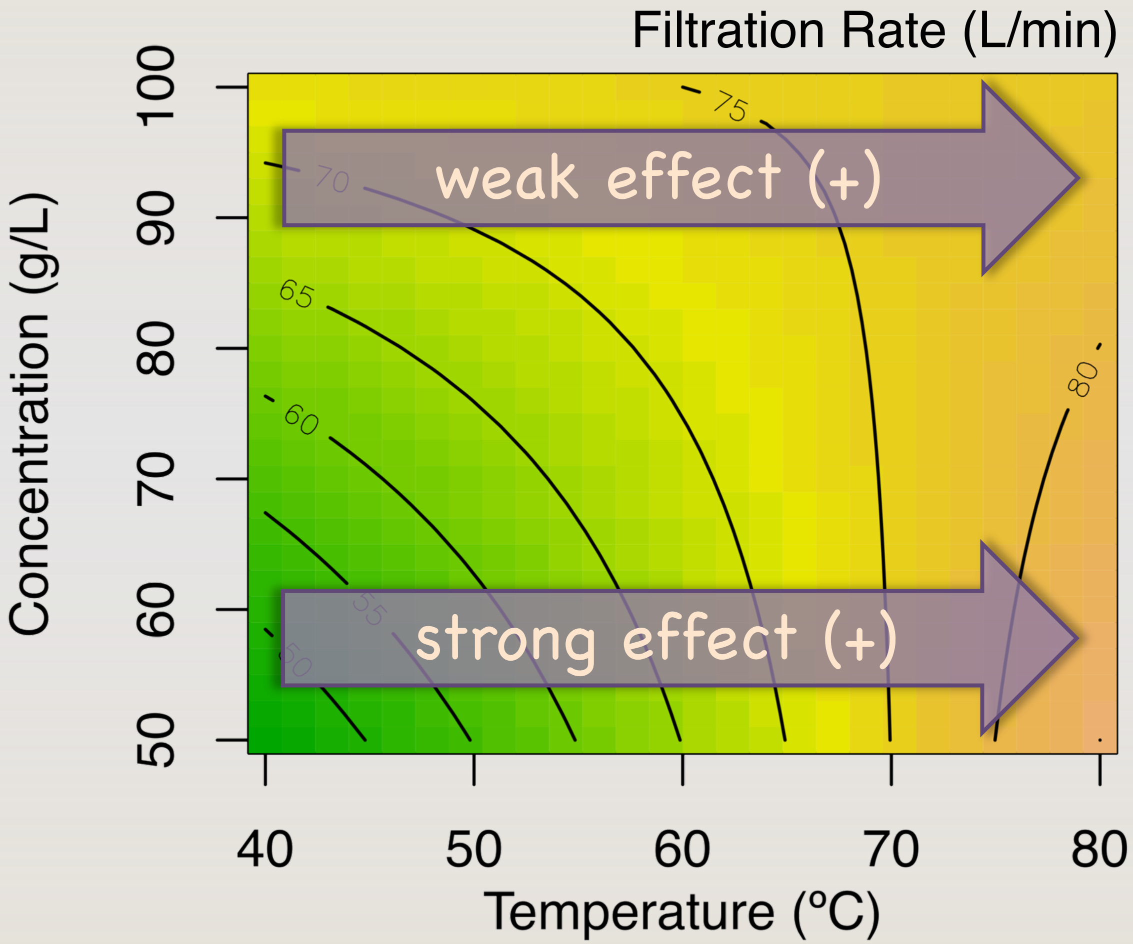


Table of Effects:

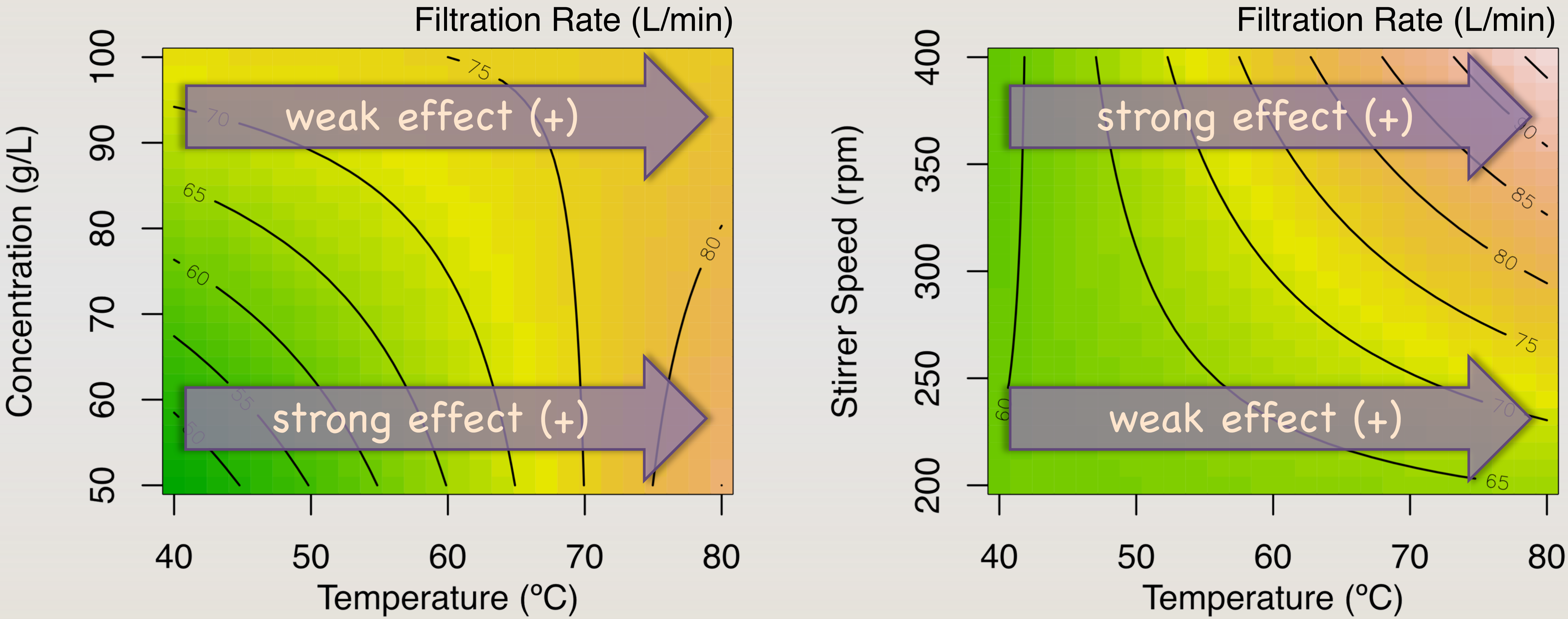
	Effect	p-value
Temperature (T)	21.624	< 0.001
Concentration (C)	9.876	0.0012
Stirring Rate (ω)	14.626	< 0.001
$T \times C$ T	-18.126	< 0.001
$T \times \omega$	16.624	< 0.001

When concentration is at its high level:

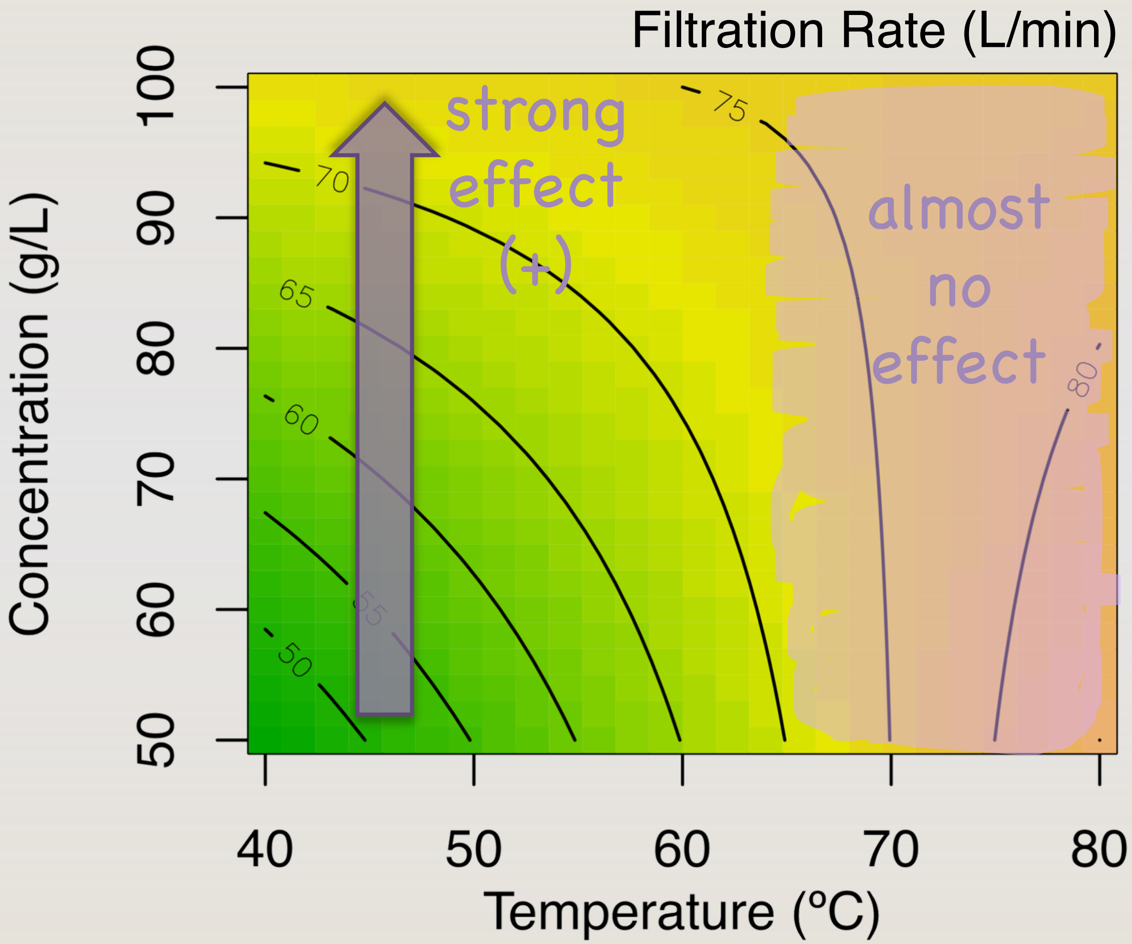
$$x_C = +1$$

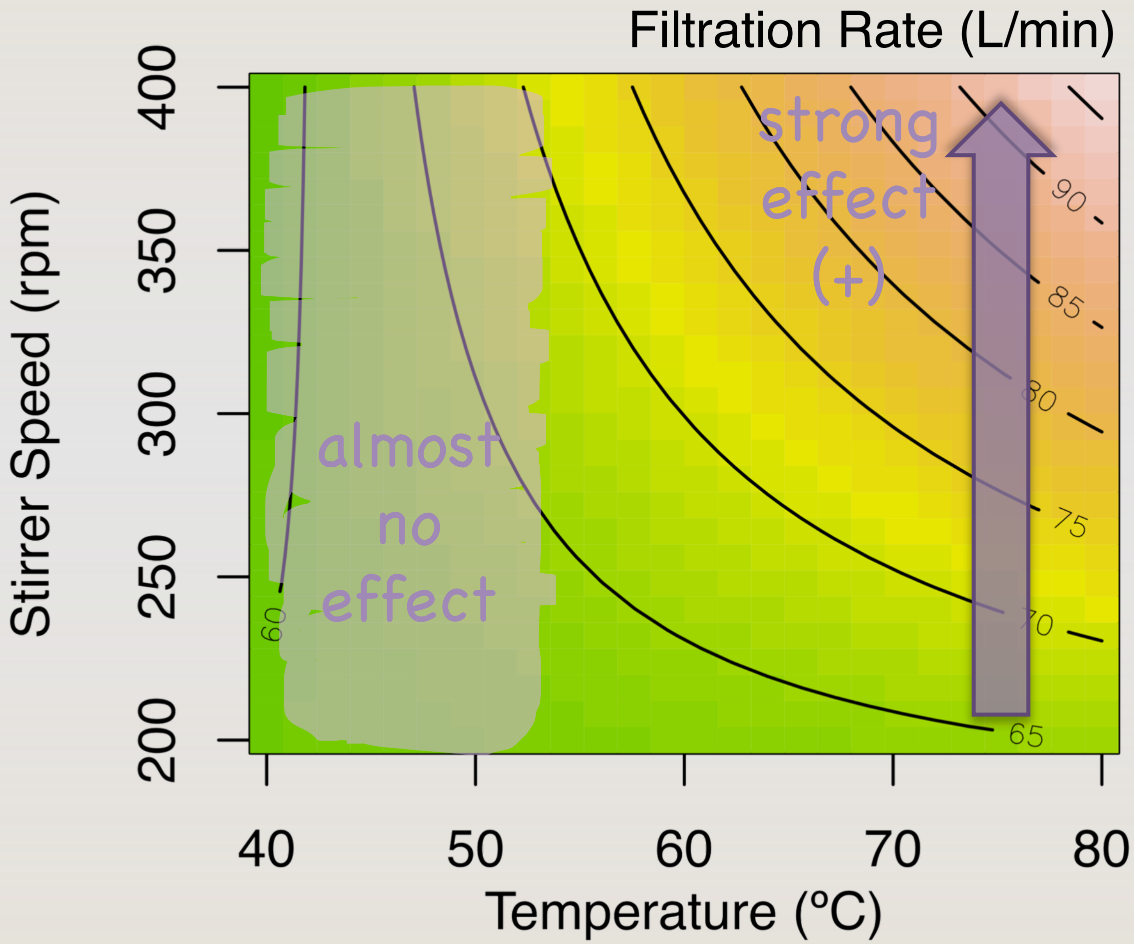
$$Effect\ T = 21.624 - 18.124 = 3.50$$

The 2^4 Factorial Design: Contour Plots

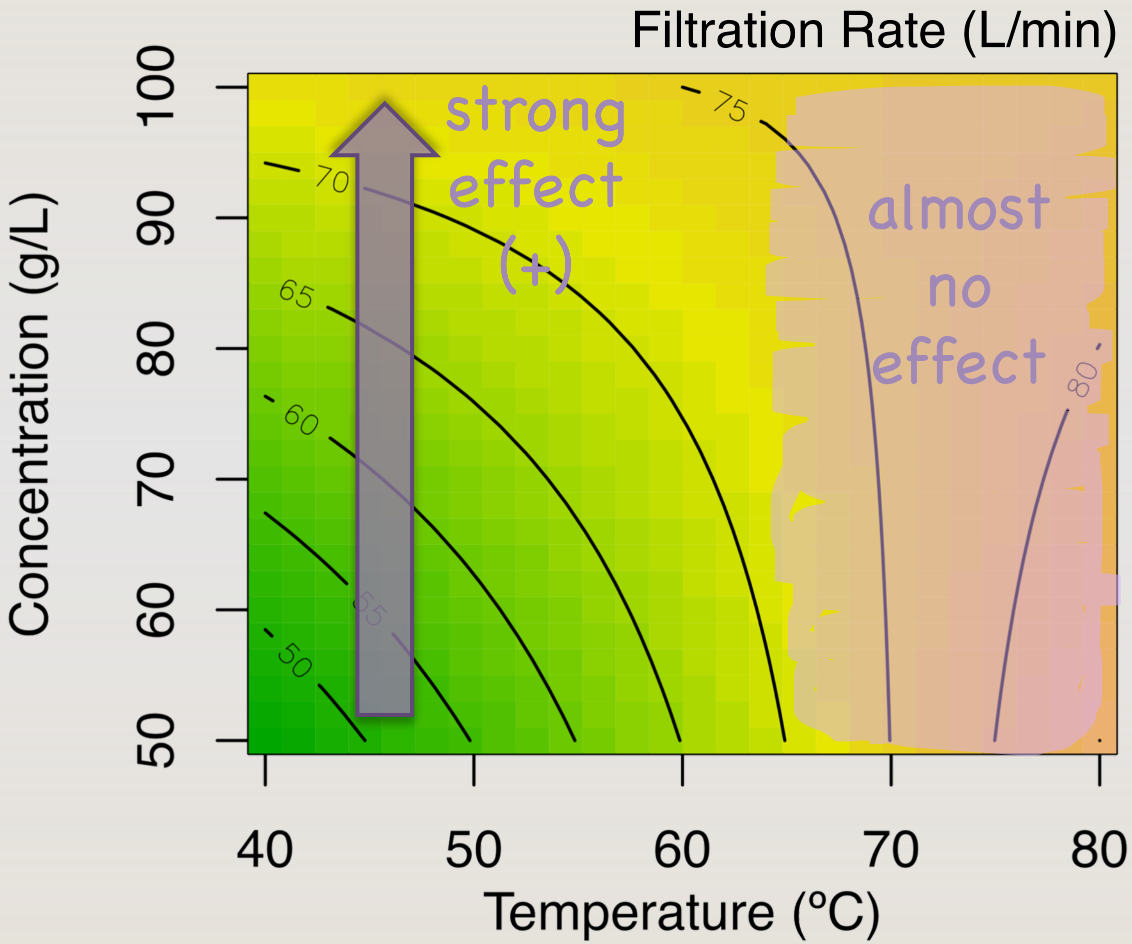


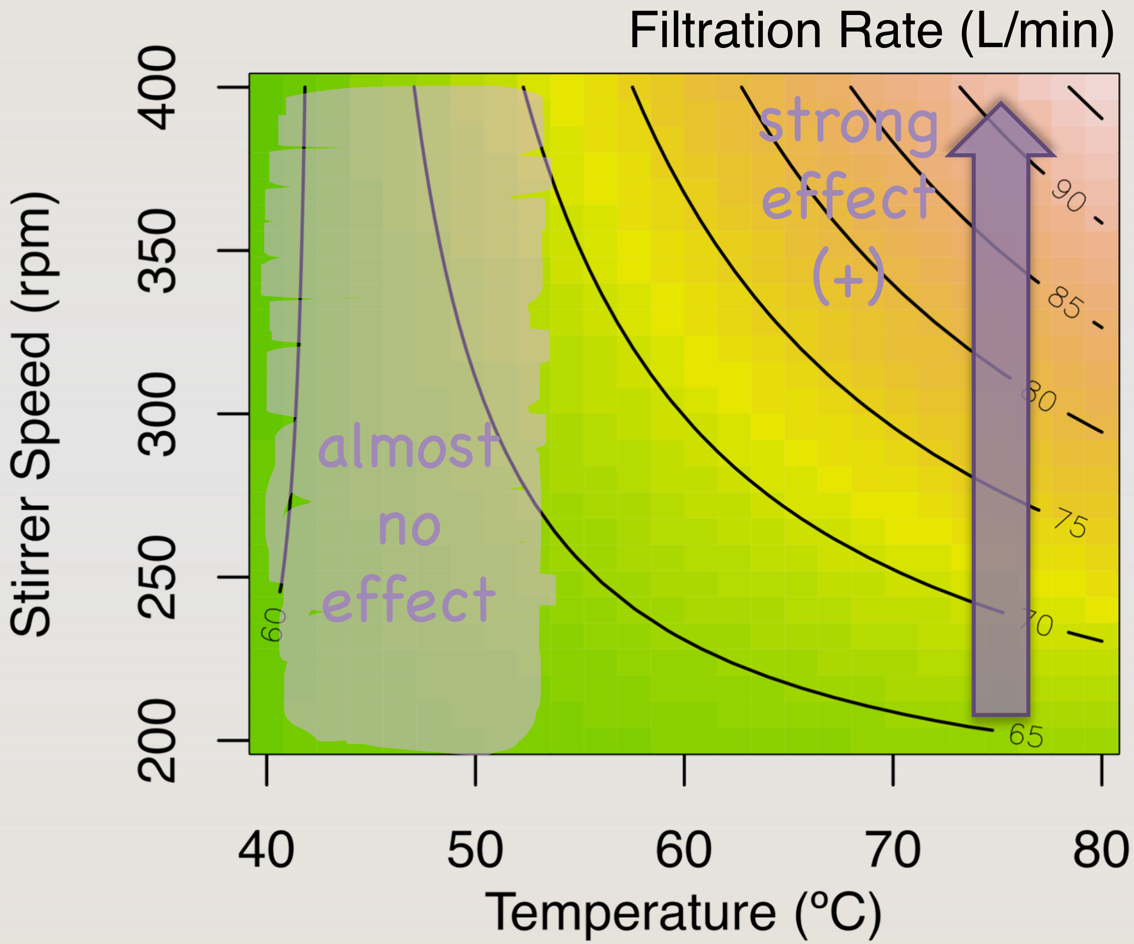
The 2^4 Factorial Design: Contour Plots



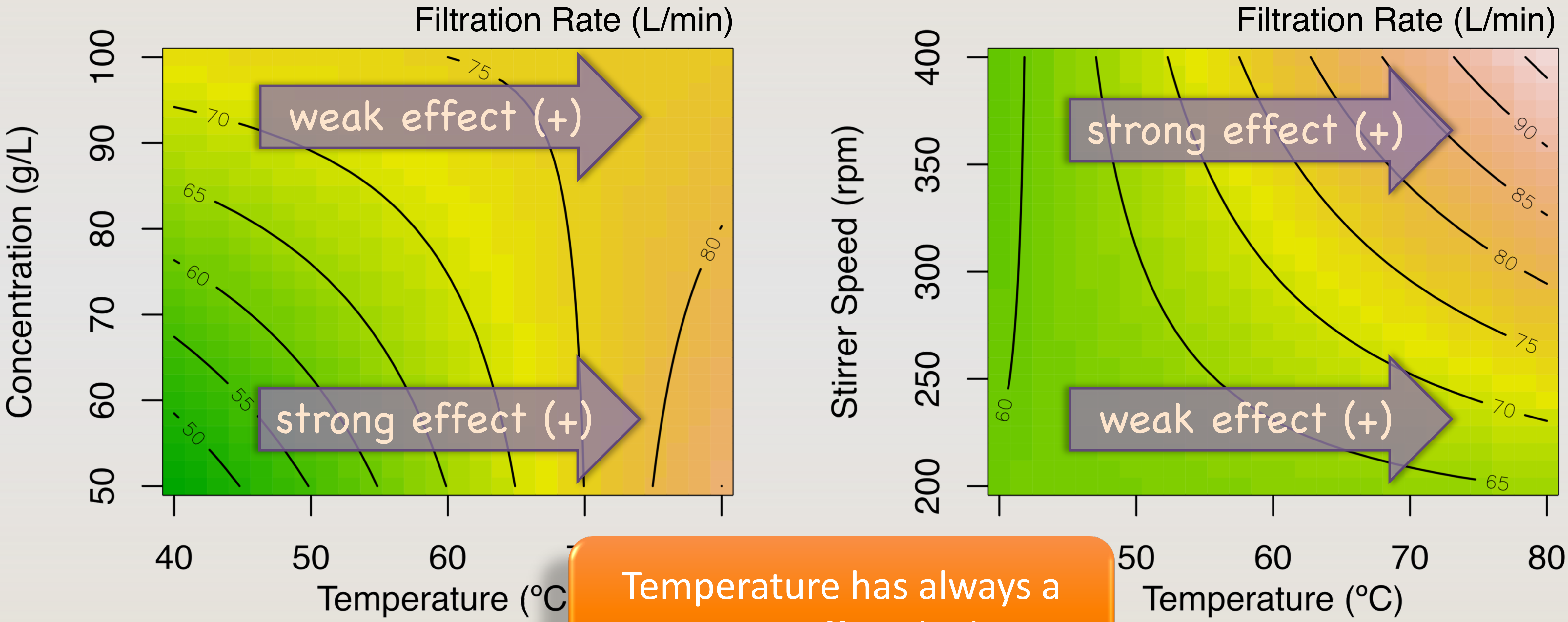


The 2^4 Factorial Design: Choosing the Best Operational Parameters



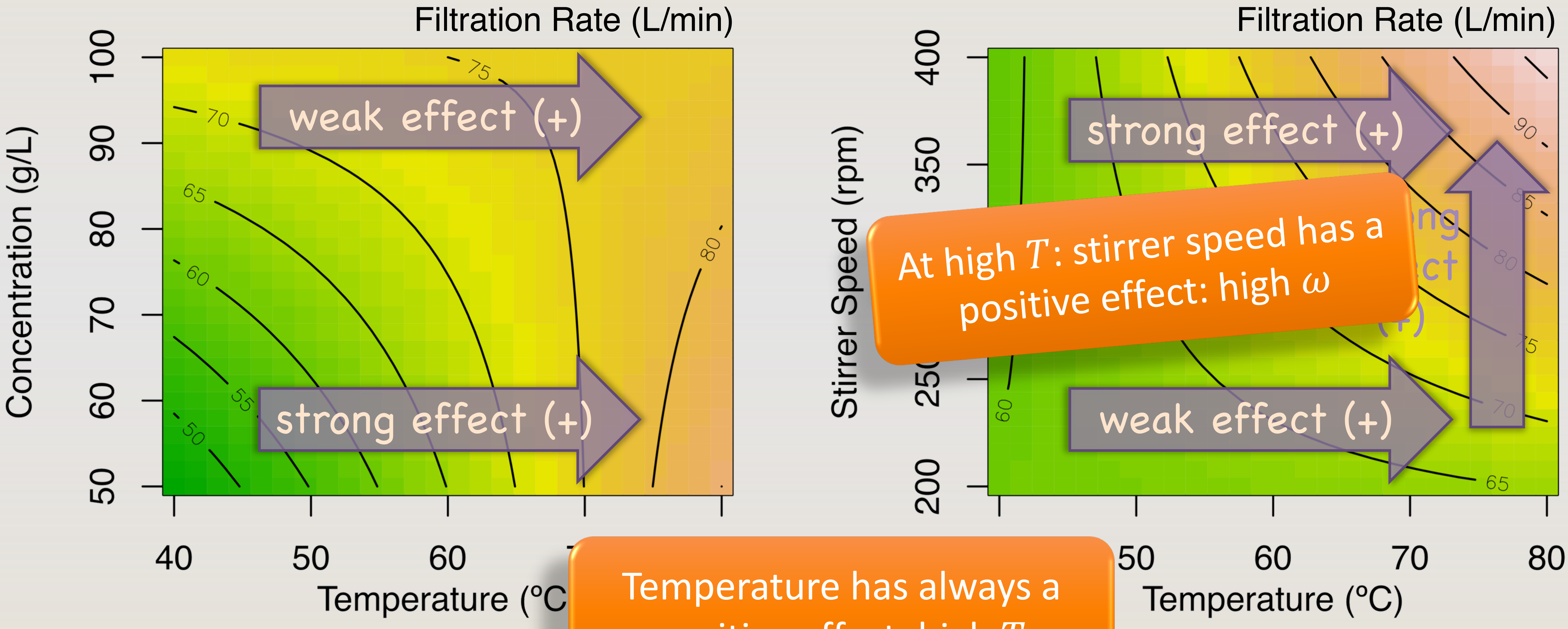


The 2^4 Factorial Design: Choosing the Best Operational Parameters



Temperature has always a positive effect: high T

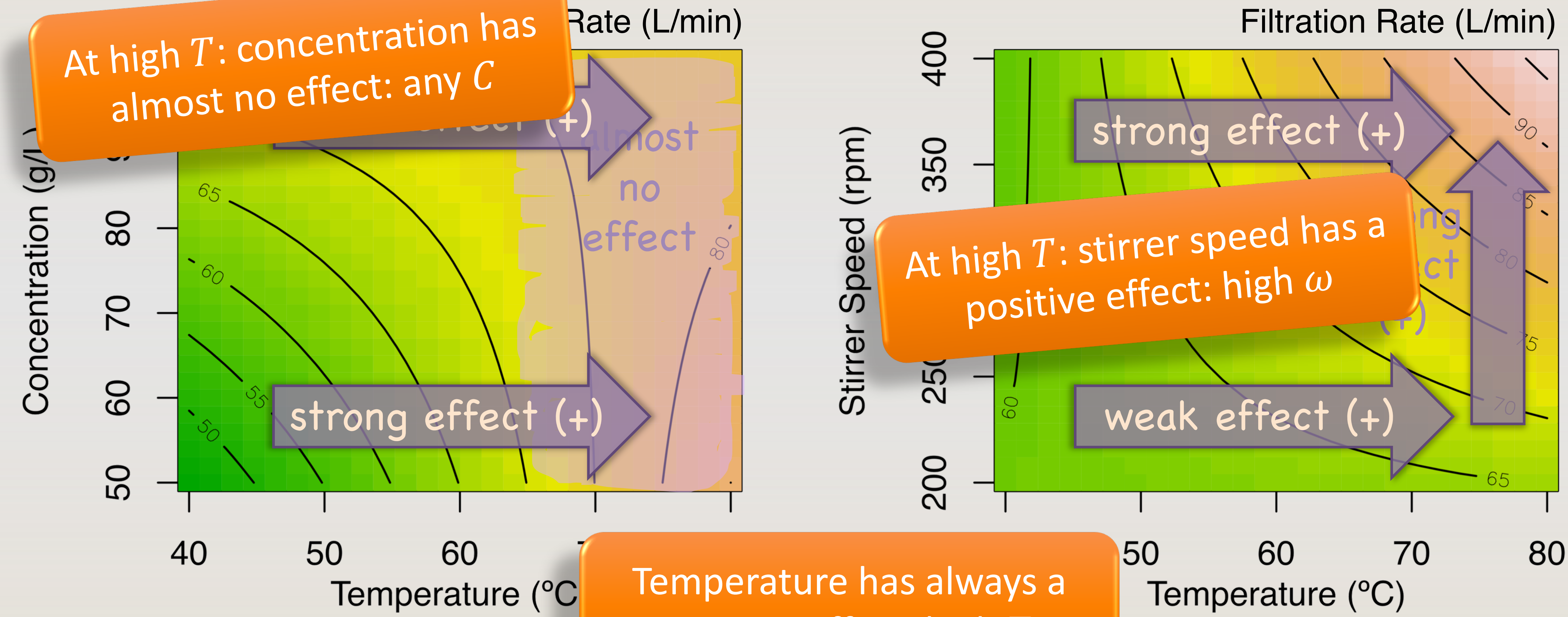
The 2^4 Factorial Design: Choosing the Best Operational Parameters



At high T: stirrer speed has a positive effect: high ω

Temperature has always a positive effect: high T

The 2^4 Factorial Design: Choosing the Best Operational Parameters



At high T: concentration has almost no effect: any C

At high T: stirrer speed has a positive effect: high ω

Temperature has always a positive effect: high T

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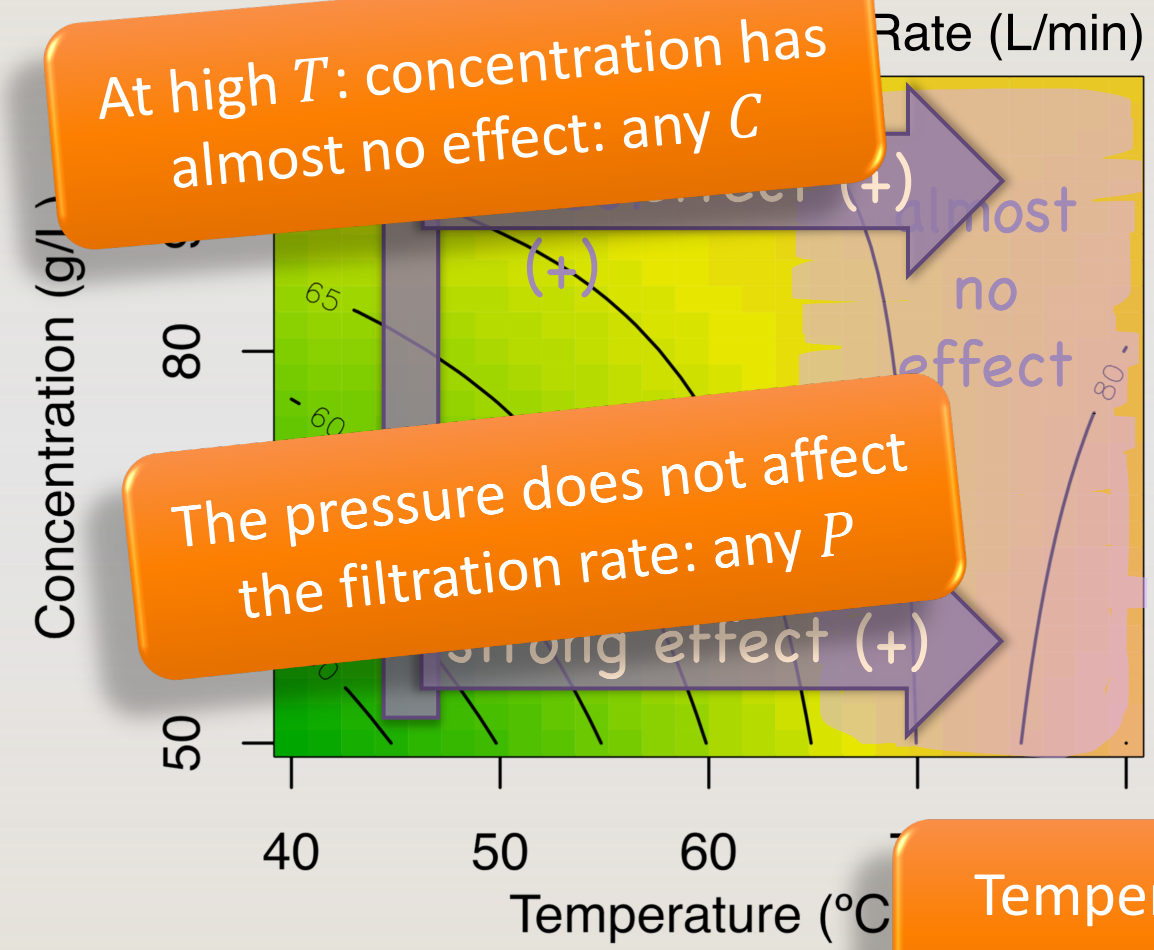
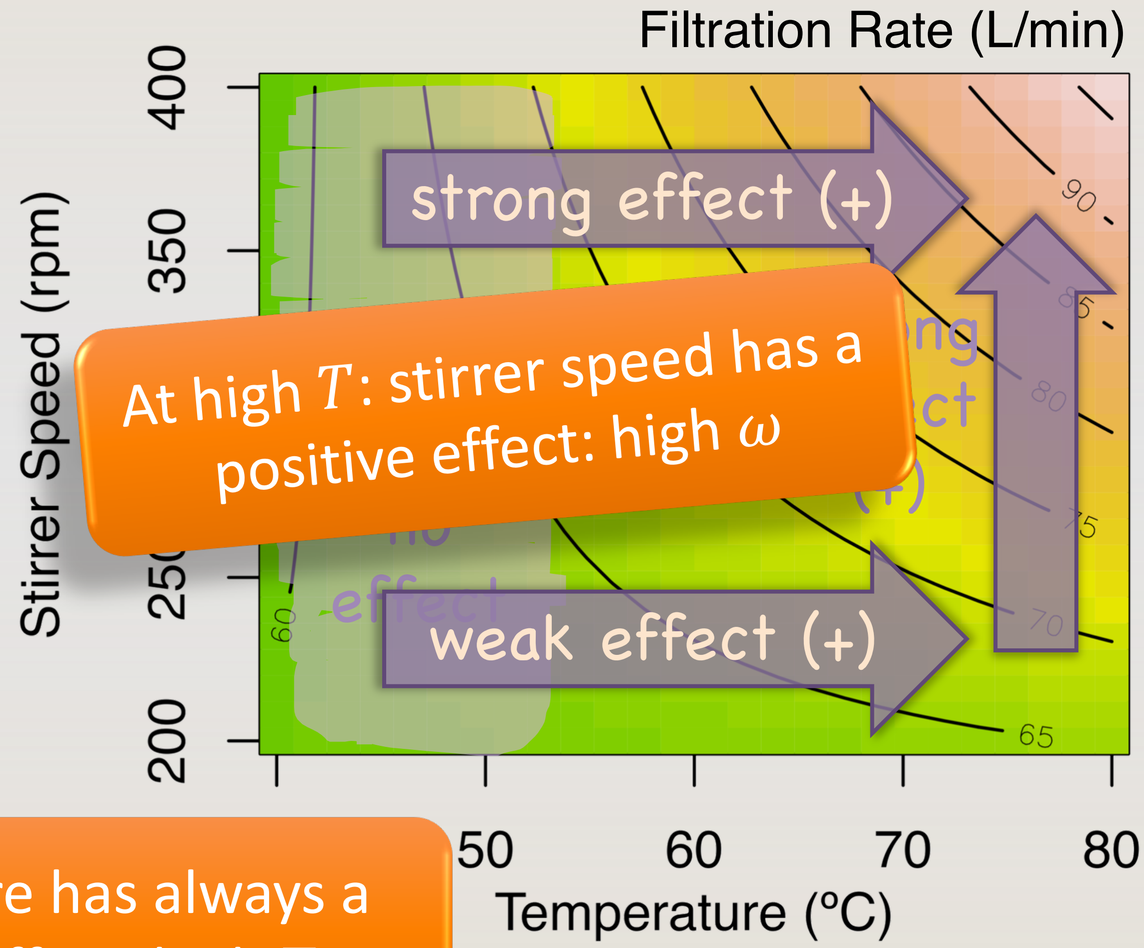
The 2^4 Factorial Design: Choosing the Best Operational Parameters



At high T: concentration has almost no effect: any C

The pressure does not affect the filtration rate: any P



At high T: stirrer speed has a positive effect: high ω

Temperature has always a positive effect: high T

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