

Number Bases



CS 1.3 - Core Data Structures

Welcome to CS 1.3!

Self Paced Course Structure

- 1 hour: Optional lecture/activities
- 15 min: Break
- 1.5 hours: Required Lab Time (working on modules)

Modules

- Self assessments
- Only Mastery portion required
- Base and Stretch are for your own learning if you choose to complete them
- Videos, links, reading, exercises

Check In Activity

- Break into groups and write/discuss
- One thing you are most excited to learn
- One thing you are most nervous about
- One question you have for me



Students, write your response!

- Be able to explain the concept of representing numbers in different formats
- Be able to understand the differences between base 10, base 2, and base 16 number systems
- Be able to explain how to convert between various number bases at a high level

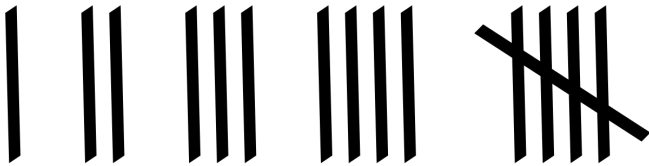
Number Bases: There are many ways to represent numbers

Back in the day: Unary Numbers

Basically there's one symbol which represents 1 and it's repeated

5: 11111

3: 

15: 

Eventually number systems evolved to use more than one symbol

Roman: I, II, III, IV, V, X, L, M etc.

Only gave unique symbols to 5, 10, 50, 100, 1000, etc.

Egyptian Numerals



You can even make your own!

Troll counting system from Terry Pratchett's Discworld series

one, two, three, many, many-one, many-two, many-three, many many,
many-many-one, many-many-two, many-many-three, many many many,
many-many-many-one, many-many-many-two, many-many-three, LOTS.

Activity: 10 mins, make your own number system

Think about how would design your own number system

If you want to use emojis: ctrl + cmd + spacebar brings up the emoji palette



Students, draw anywhere on this slide!

What if we gave each number their own symbol?

The Hindu-Arabic Numeral System

Use the position of the numeral as well

“We always add and never subtract. And each position is 10 more than the one before it.

So, 35 means “add 3×10 to 5×1 ” and 456 means $4 \times 100 + 5 \times 10 + 6 \times 1$. This “positional decimal” setup is the Hindu-Arabic number system we use today.

Source: <https://betterexplained.com/articles/numbers-and-bases/>

Base 10 (decimal)

Most modern day cultures use base-10, a.k.a. “decimal.”

We have 10 digits in base-10. (0 through 9)

9 rolls over to 10.

99 rolls over to 100.



<https://www.youtube.com/watch?v=DKavhec9fGE>

Base 2 (binary)

Computers use base-2, a.k.a. “binary.”

There are 2 digits in base-2. (0 and 1)

1 rolls over to 10.

11 rolls over to 100.

“There are 10 kinds of people in the world. Those who understand binary, and those who don’t.”

–Old Programming Proverb

Counting in Binary

0, 1, 10, 11, 100, 101, 110, 111,
1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111,
10000, 10001, 10010, 10011, 10100, 10101, 10110, 10111,
11000, 11001, 11010, 11011, 11100, 11101, 11110, 11111,
100000, 100001, 100010, 100011, 100100, 100101, 100110, 100111,
101000, 101001, 101010, 101011, 101100, 101101, 101110, 101111,
110000, 110001, 110010, 110011, 110100, 110101, 110110, 110111,
111000, 111001, 111010, 111011, 111100, 111101, 111110, 111111,
...

Binary to Decimal

$$\begin{array}{r} 101001010011 \\ * \\ \hline 2^{11} \ 2^{10} \ 2^9 \ 2^8 \ 2^7 \ 2^6 \ 2^5 \ 2^4 \ 2^3 \ 2^2 \ 2^1 \ 2^0 \\ \hline 2048 \ 0 \ 512 \ 0 \ 0 \ 64 \ 0 \ 16 \ 0 \ 0 \ 2 \ 1 \\ \hline 2643_{10} \end{array}$$

Let's Practice Binary to Decimal!

1 0 0 1 0



Students, draw anywhere on this slide!

- Paper: <https://curriculum.code.org/csp-18/unit1/5/>
- Digital: https://docs.google.com/spreadsheets/d/1v4_WCGzdvi6_X5fj_vGu3xt3D3TWGTET5q9yTRJlir0/copy

Decimal to Binary

Remainder Method, keep dividing by 2:

<https://owlcation.com/stem/How-to-Convert-Decimal-to-Binary-and-Binary-to-Decimal>

Successive Division by 2

$$\begin{array}{r} 2 \overline{) 29} \\ 2 \overline{) 14} \\ 2 \overline{) 7} \\ 2 \overline{) 3} \\ 2 \overline{) 1} \\ 0 \end{array}$$

Remainders

1 LSB
0
1
1
1 MSB

Read the remainders
from the bottom up

29 decimal = 11101 binary



Let's Practice Decimal to Binary!

25 / 2 ---->



Students, draw anywhere on this slide!

Computers sometimes also use base-16, a.k.a. “hexadecimal” or simply “hex.”

There are 16 digits in base-16. (0-9 and A-F)

9 continues to A. F rolls over to 10.

99 continues to 9A. FF rolls over to 100.

There are 16 digits in base-16. (**0–9** and **A–F**)

$$A_{16} = 10_{10}$$

$$D_{16} = 13_{10}$$

$$B_{16} = 11_{10}$$

$$E_{16} = 14_{10}$$

$$C_{16} = 12_{10}$$

$$F_{16} = 15_{10}$$

Hex is often prefixed with 0x

`0xAF32010016 == AF32010016`

Counting in Hex

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F,

10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1A, 1B, 1C, 1D, 1E, 1F,

20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2A, 2B, 2C, 2D, 2E, 2F,

...

90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 9A, 9B, 9C, 9D, 9E, 9F,

A0, A1, A2, A3, A4, A5, A6, A7, A8, A9, AA, AB, AC, AD, AE, AF,

B0, B1, B2, B3, B4, B5, B6, B7, B8, B9, BA, BB, BC, BD, BE, ...

Hex to Decimal

Check out this article: <https://owlcation.com/stem/Convert-Hex-to-Decimal>

Post more resources you find on piazza!

Let's Practice Hex to Decimal!

0x D 3 F 2

$$(13 \times 16^3) + (3 \times 16^2) + (15 \times 16^1) + (2 \times 16^0)$$

0



Students, draw anywhere on this slide!

How would we go from Decimal to Hex?



Students, write your response!

Let's Practice Decimal to Hex!

$84 / 16 \rightarrow 5$ remainder of 4

$5 / 16$ r5

54



Students, draw anywhere on this slide!

Every hex digit is 4 binary digits (bits)

$5_{16} \leftrightarrow 0101_2$

$10_{16} \leftrightarrow 0001\ 0000_2$

$8_{16} \leftrightarrow 1000_2$

$72_{16} \leftrightarrow 0111\ 0010_2$

$B_{16} \leftrightarrow 1011_2$

$A6_{16} \leftrightarrow 1010\ 0110_2$

$F_{16} \leftrightarrow 1111_2$

$FF_{16} \leftrightarrow 1111\ 1111_2$

Let's Practice Hex to Binary!

0x D 3 F 2



Students, draw anywhere on this slide!

Think about this...

Based on what we've learned how would we convert from decimal to base 4?

Base 4 to decimal?

What about any arbitrary base n ?



Students, write your response!

Module 1: Number Bases