Intro to Probability

QL 1.1

Learning Outcomes

By the end of this class, you will be able to...

1. Write down functions to compute probability and conditional probability
2. Use Bayes formula to compute conditional probability

Probability Basic Concepts

- **Probability** as a general concept can be defined as the *chance of an event occurring.*
- Processes such as flipping a coin, rolling a die, or drawing a card from a deck are called **probability experiments.**
- A **probability experiment** is a chance process that leads to well-defined results called outcomes.
- An **outcome** is the result of a single trial of a probability experiment.
- **Sample space**: It is the set of all possible outcomes of a probability experiment.

Probability Basic Concepts

Some sample spaces for various probability experiments are shown here.

Experiment	Sample space
Toss one coin	Head, tail
Roll a die	1, 2, 3, 4, 5, 6
Answer a true/false question	True, false
Toss two coins	Head-head, tail-tail, head-tail, tail-head

Rolling Dice

3 min

Find the sample space for rolling two dice.

Activity Solution

Sample Space for Rolling Two Dice

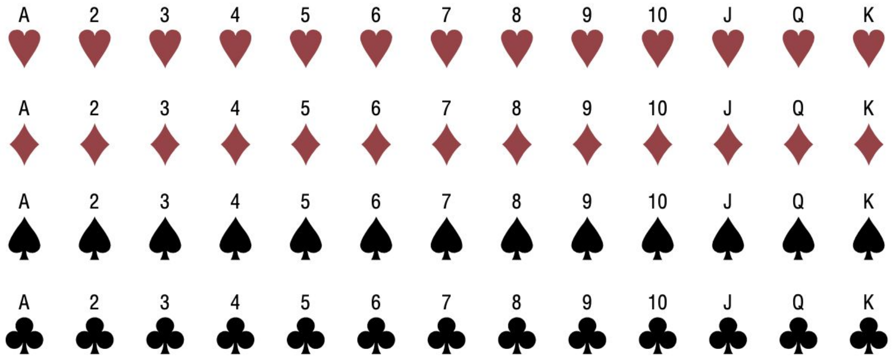
Die 1	Die 2					
	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Example

Find the sample space for drawing one card from an ordinary deck of cards.

Solution

Since there are 4 suits (hearts, clubs, diamonds, and spades) and 13 cards for each suit (ace through king), there are 52 outcomes in the sample space.



Gender of Children

5 mins

Find the sample space for the gender of the children if a family has three children. Use B for boy and G for girl and X for non-binary.

Hint: You can use a python program to print them all.

Tree diagram

A **tree diagram** is a device consisting of line segments emanating from a starting point and also from the outcome point. It is used to determine all possible outcomes of a probability experiment.

Event

- An outcome was defined previously as the result of a single trial of a probability experiment. In many problems, one must find the probability of two or more outcomes. For this reason, it is necessary to distinguish between an outcome and an event.
- An **event** consists of a set of outcomes of a probability experiment.
    - An event can be one outcome or more than one outcome.

Probability Formula

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Formula for Classical Probability

The probability of any event E is

$$\frac{\text{Number of outcomes in } E}{\text{Total number of outcomes in the sample space}}$$

This probability is denoted by

$$P(E) = \frac{n(E)}{n(S)}$$

This probability is called *classical probability,* and it uses the sample space S.

Drawing Cards

10 min

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1. Find the probability of getting a red ace when a card is drawn at random from an ordinary deck of cards.

Info: there are 52 cards in an ordinary deck of cards and two red aces.

2. If a family has three children, find the probability that two of the three children are girls.

Info: Assume there are 3 genders: Boy, Girl and X.

Break 10 mins

Probability Rules

- Probability Rule 1:
  - The probability of any event E is a number (either a fraction or decimal) between and including 0 and 1. This is denoted by 0 <= P(E) <= 1.
- Probability Rule 2:
  - If an event E cannot occur (i.e., the event contains no members in the sample space), its probability is 0.
- Probability Rule 3:
  - If an event E is certain, then the probability of E is 1.

Probability Rules - Examples

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Rolling a Die

When a single die is rolled,

A. find the probability of getting a 9.
B. what is the probability of getting a number less than 7?

Probability Rules

- Probability Rule 4:
  - The sum of the probabilities of all the outcomes in the sample space is 1.

Example:

in the roll of a fair die, each outcome in the sample space has a probability of 1/6. Hence, the sum of the probabilities of the outcomes is

Outcome	1		2		3		4		5		6
Probability	$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$		$\frac{1}{6}$
Sum	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6}$	$+$	$\frac{1}{6} = \frac{6}{6} = 1$

Complementary Events

- The **complement of an event E** is the set of outcomes in the sample space that are not included in the outcomes of event E.
- The complement of E is denoted by E (read “E bar”).

Finding Complements

2 mins

Find the complement of each event.

a. Rolling a die and getting a 4
b. Selecting a letter of the alphabet and getting a vowel
c. Selecting a day of the week and getting a weekday

Rule for Complementary Events

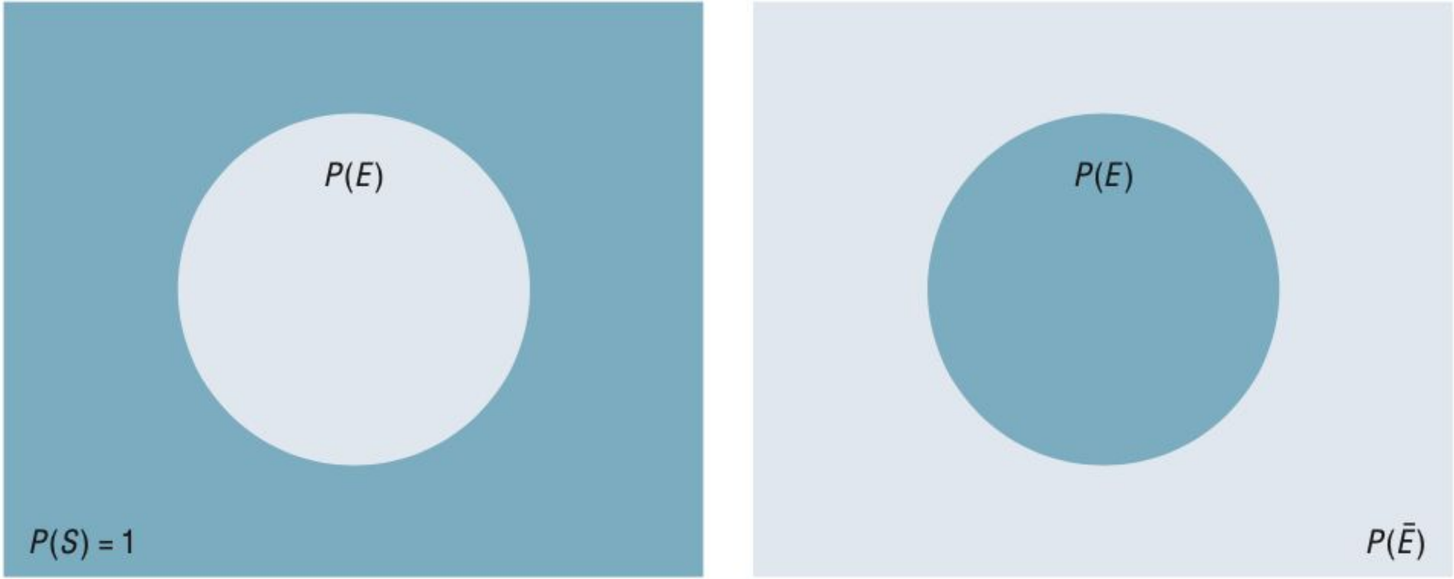
$P(\bar{E}) = 1 - P(E)$ or

$P(E) = 1 - P(\bar{E})$ or

$P(E) + P(\bar{E}) = 1$

- If the probability of an event or the probability of its complement is known, then the other can be found by subtracting the probability from 1
- Example; If the probability that a person lives in an industrialized country of the world is 1/5 , find the probability that a person does not live in an industrialized country.

Venn Diagram



(a) Simple probability

(b) $P(\bar{E}) = 1 - P(E)$

Empirical Probability

- Classical vs empirical probability
- A researcher for the AAA asked 50 people who plan to travel over the Thanksgiving holiday how they will get to their destination :

Method	Frequency
Drive	41
Fly	6
Train or bus	3
	50

What's the probability of selecting a person who is driving (the person was randomly selected from a pool of people who plan to travel over the thanksgiving holiday)?

Empirical Probability

Formula for Empirical Probability

Given a frequency distribution, the probability of an event being in a given class is

$$P(E) = \frac{\text{frequency for the class}}{\text{total frequencies in the distribution}} = \frac{f}{n}$$

This probability is called *empirical probability* and is based on observation.

Distribution of Blood Types

5 mins

Distribution of Blood Types Activity

The Addition Rules for Probability

1. At a large political gathering, you might wish to know, for a person selected at random, the probability that the person is a female or is a Republican. Possibilities to consider:
   a. The person is a female.
   b. The person is a Republican.
   c. The person is both a female and a Republican.
2. At the same gathering there are Republicans, Democrats, and Independents. If a person is selected at random, what is the probability that the person is a Democrat or an Independent?
   a. The person is a Democrat.
   b. The person is an Independent.

Mutually Exclusive Events

Two events are ***mutually exclusive*** events if they cannot occur at the same time

Determine which events are mutually exclusive and which are not, when a single die is rolled:

1. Getting an odd number and getting an even number
2. Getting a 3 and getting an odd number
3. Getting an odd number and getting a number less than 4
4. Getting a number greater than 4 and getting a number less than 4

Addition Rules - Mutually Exclusive

Addition Rule 1

When two events A and B are mutually exclusive, the probability that A or B will occur is

$$P(A \text{ or } B) = P(A) + P(B)$$

A day of the week is selected at random. Find the probability that it is a weekend day.

Addition Rule - Not Mutually Exclusive

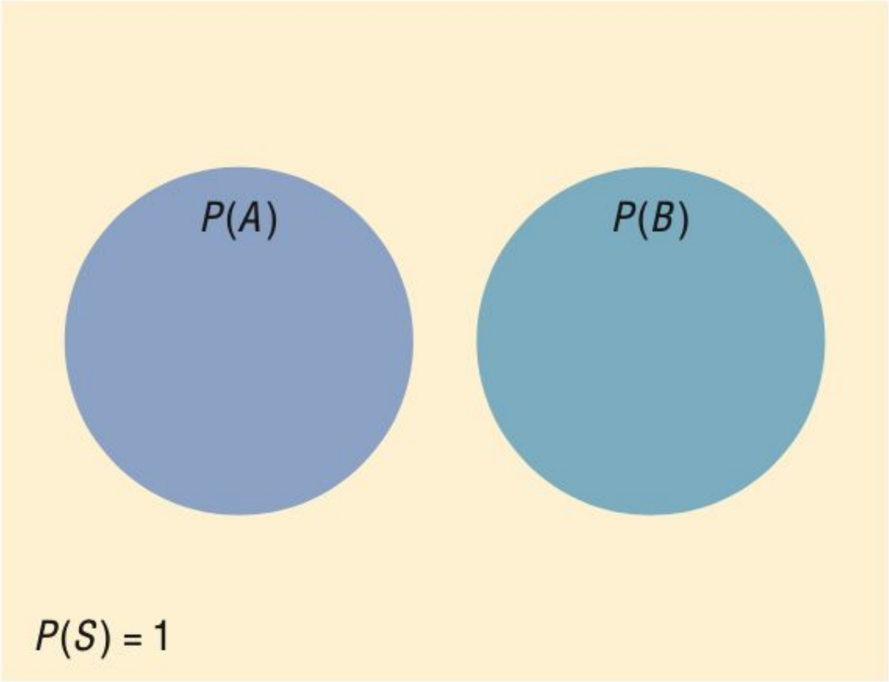
Addition Rule 2

If A and B are *not* mutually exclusive, then

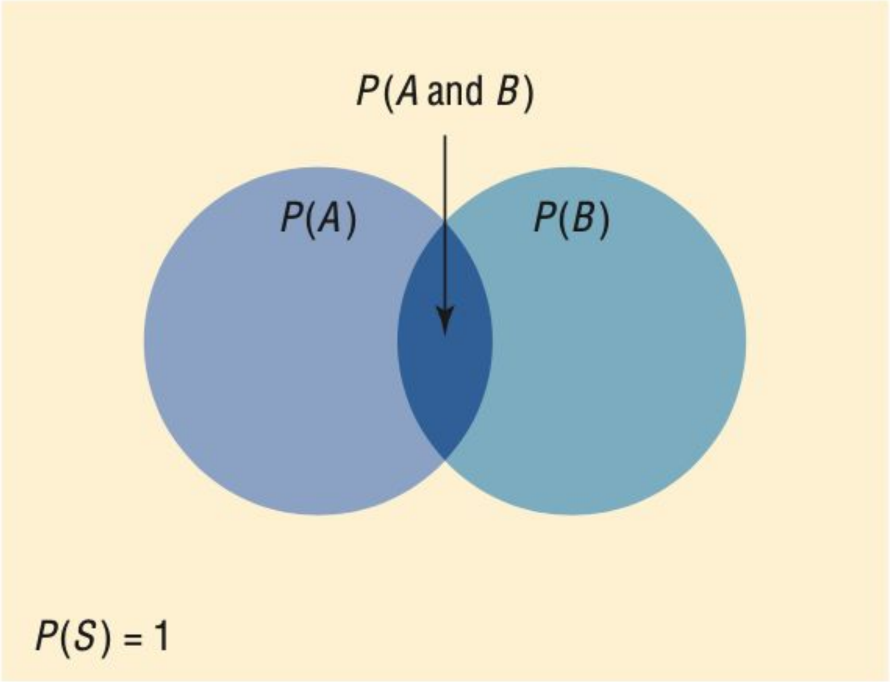
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

In a hospital unit there are 8 nurses and 5 physicians; 7 nurses and 3 physicians are females. If a staff person is selected, find the probability that the subject is a nurse or a male.

Addition Rules - Venn Diagram



(a) Mutually exclusive events
$P(A \text{ or } B) = P(A) + P(B)$



(b) Nonmutually exclusive events
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Independent events

Two events A and B are ***independent*** events if the fact that A occurs does not affect the probability of B occurring.

Independent Events

3 mins

Give some examples for

independent and dependent events.

Independent Events

Examples for **independent** events:

- Tossing a coin and then roll a die
- Rolling a die and getting a 6, and then rolling a second die and getting a 3.
- Drawing a card from a deck and getting a queen, replacing it, and drawing a second card and getting a queen.

Examples for dependent events:

- Taking out a blue ball from a bag of mixed red/blue balls, then taking out another blue ball (the possibility of getting a blue ball for the second time will change)

Multiplication Rule

Multiplication Rule 1

When two events are independent, the probability of both occurring is

$$P(A \text{ and } B) = P(A) \cdot P(B)$$

A coin is flipped and a die is rolled. Find the probability of getting a head on the coin and a 4 on the die.

Multiplication Rule Extended

Multiplication rule 1 can be extended to three or more independent events by using the formula

$$P(A \text{ and } B \text{ and } C \text{ and } \ldots \text{ and } K) = P(A) \cdot P(B) \cdot P(C) \cdots P(K)$$

A Harris poll found that 46% of Americans say they suffer great stress at least once a week. If three people are selected at random, find the probability that all three will say that they suffer great stress at least once a week.(Source: 100% American.)

Dependent Events

When the outcome or occurrence of the first event affects the outcome or occurrence of the second event in such a way that the probability is changed, the events are said to be ***dependent*** events.

Here are some examples of dependent events:

- Drawing a card from a deck, not replacing it, and then drawing a second card.
  Selecting a ball from an urn, not replacing it, and then selecting a second ball.
  Being a lifeguard and getting a suntan.
- Having high grades and getting a scholarship.
- Parking in a no-parking zone and getting a parking ticket.

Conditional Probability

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Formula for Conditional Probability

The probability that the second event B occurs given that the first event A has occurred can be found by dividing the probability that both events occurred by the probability that the first event has occurred. The formula is

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

P(B|A) = The probability that event B occurs given that event A has already occurred.

References

1. McGraw-Hill. ISBN 0070428646.